NATURAL SCIENCES TRIPOS Part III

Tuesday 13 January 2004 2.00 to 3.30

EXPERIMENTAL AND THEORETICAL PHYSICS (4)
Particle Physics

Answer **two** *questions only. The approximate number of marks allotted to each part of a question is indicated in the right margin where appropriate. The paper contains THREE sides and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 A high energy particle of energy E_1 is deflected through an angle θ in an elastic collision with a stationary particle of mass M. Show that the particle loses energy $\nu = E_1 - E_3 = -q^2/2M$ in the collision, where $q^2 = -2E_1E_3(1-\cos\theta)$. [6]

The differential cross section for the elastic scattering of a high energy electron on a stationary proton is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\theta/2}\frac{E_3}{E_1}\cos^2\frac{\theta}{2}\left[\frac{G_E^2+\tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2}\right]$$

where $\tau = -q^2/4M^2$. Outline a general strategy which would allow the functions $G_E(q^2)$ and $G_M(q^2)$ to be measured using a monoenergetic incident electron beam whose energy can be varied and a particle spectrometer which can be positioned over a range of angles to the beam direction. What value of the beam energy should be chosen to obtain $|q^2| = 2.5\,\mathrm{GeV}^2$ if the spectrometer is positioned at an angle $\theta = 60°$? Sketch the measured forms of $G_E(q^2)$ and $G_M(q^2)$ for the proton and neutron and explain what information can be extracted from these measurements. [9]

[Proton mass = 0.938 GeV.]

In deep-inelastic lepton-nucleon scattering, the variables x and y are defined as $x = -q^2/2M\nu$ and $y = p_2.q/p_2.p_1$ where p_1 and p_2 are the four-momenta of the incident lepton and nucleon, respectively. Show that x can be interpreted as the fractional momentum carried by the struck quark, and that $y = \frac{1}{2}(1-\cos\theta^*)$ where θ^* is the lepton scattering angle in the lepton-quark centre of mass frame. Determine the shape of the θ^* angular distributions corresponding to differential cross sections of the form $d\sigma/dy$ = constant and $d\sigma/dy \propto (1-y)^2$. [9]

The differential cross section for neutrino-proton scattering is

$$\frac{d^2\sigma^{\nu p}}{dx\,dy} = \frac{G_F^2 s}{\pi} x\left[d(x) + (1-y)^2\overline{u}(x)\right]$$

where $s = (p_1+p_2)^2$. Draw Feynman diagrams corresponding to each of the terms on the right-hand side of this expression, and explain how the y-dependence of each term can be understood from helicity considerations. Sketch the form of the functions $d(x)$ and $\overline{u}(x)$. [6]

2 Write brief accounts of **two** of the following:

(a) CP violation; [15]

(b) QCD and colour forces; [15]

(c) electroweak unification and experimental tests thereof; [15]

(d) open questions in the Standard Model and their possible resolution. [15]

3 Answer **both** parts:

(a) Give an example of a Feynman diagram corresponding to the transition of a $\mathrm{B_d^0}$ meson (quark content $\overline{\mathrm{b}}\mathrm{d}$) into a $\overline{\mathrm{B}}{}_\mathrm{d}^0$ meson (quark content $\mathrm{b}\overline{\mathrm{d}}$). Give an example of a Feynman diagram for the decay of a $\mathrm{B_d^0}$ meson into a semileptonic final state, and draw the diagram for the corresponding $\overline{\mathrm{B}}{}_\mathrm{d}^0$ decay. [4]

Neglecting CP violation, the mass eigenstates $\mathrm{B_H}$ and $\mathrm{B_L}$ are given by $\mathrm{B_H} = (\mathrm{B_d^0} + \overline{\mathrm{B}}{}_\mathrm{d}^0)/\sqrt{2}$ and $\mathrm{B_L} = (\mathrm{B_d^0} - \overline{\mathrm{B}}{}_\mathrm{d}^0)/\sqrt{2}$. Show that the transition rate for an initial $\mathrm{B_d^0}$ to oscillate into a $\overline{\mathrm{B}}{}_\mathrm{d}^0$ after a proper time t is proportional to $\mathrm{e}^{-\varGamma t}(1 - \cos \varDelta m t)$ where $\varGamma = \varGamma_\mathrm{H} = \varGamma_\mathrm{L}$ is the common total width of the $\mathrm{B_H}$ and $\mathrm{B_L}$ states and $\varDelta m = m_\mathrm{H} - m_\mathrm{L}$ is their mass difference. Show that the probability that an initial $\mathrm{B_d^0}$ oscillates into a $\overline{\mathrm{B}}{}_\mathrm{d}^0$, integrated over all proper times, is $P(\mathrm{B_d^0} \to \overline{\mathrm{B}}{}_\mathrm{d}^0) = \frac{1}{2}x^2/(1 + x^2)$ where $x = \varDelta m/\varGamma$. [8]

[$\int_0^\infty \mathrm{e}^{-\alpha t} \cos \beta t \, \mathrm{d}t = \alpha/(\alpha^2 + \beta^2)$ where α and β are constants.]

An experiment produces pairs of uncorrelated $\mathrm{B_d^0}$ and $\overline{\mathrm{B}}{}_\mathrm{d}^0$ mesons, which are detected through their decays into semileptonic final states. 29.7% of charged lepton pairs from the decay of $\mathrm{B_d^0}$ and $\overline{\mathrm{B}}{}_\mathrm{d}^0$ mesons are found to have leptons of the same electric charge ($\mathrm{e^\pm e^\pm}$, $\mathrm{e}^\pm\mu^\pm$ or $\mu^\pm\mu^\pm$). Determine the mass difference $\varDelta m$. [5]

[$\mathrm{B_d^0}$ meson lifetime $= 1.5 \times 10^{-12}$ s; $\hbar = 6.582 \times 10^{-25}$ GeV s.]

(b) Summarise the current experimental evidence that neutrinos can oscillate from one flavour into another. Explain the rôle of the PMNS matrix and outline the information on neutrino masses and mixing angles that can be extracted from experiments on neutrino oscillations. [8]

In the K2K experiment, a neutrino beam of average energy 1.2 GeV and containing predominantly ν_μ neutrinos is directed towards a detector at a distance of 250 km. A total of 56 events are observed in which a neutrino interacts to produce a μ^- in the final state, compared to an expectation of 80 such events if neutrinos are assumed not to oscillate. Give the most likely explanation for this observation and show that it is approximately consistent with other neutrino oscillation measurements. [5]

[The oscillation probability for two neutrino flavours ν_α and ν_β is given by $P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \sin^2(\varDelta m^2 L/4E)$. $\hbar c = 0.197$ GeV fm.]

END OF PAPER