NATURAL SCIENCES TRIPOS Part III

Tuesday 14 January 2003 2.00 to 3.30

EXPERIMENTAL AND THEORETICAL PHYSICS (4)
Particle Physics

Answer **two** *questions only. The approximate number of marks allotted to each part of a question is indicated in the right margin where appropriate. The paper contains FIVE sides and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 Describe the various symmetries and conservation laws based on SU(2) and SU(3) which are relevant to elementary particles and their interactions. [8]

(a) The D^0 and D^+ mesons have quark content $c\bar{u}$ and $c\bar{d}$, respectively, and have approximately equal mass. The D^{*0} and D^{*+} form a similar pair of mesons at higher mass, the dominant decays of the D^{*+} being $D^{*+} \to D^0\pi^+$ and $D^{*+} \to D^+\pi^0$. Draw a possible Feynman diagram for each of these decays. Write down the isospin quantum numbers of the mesons involved in each decay, and obtain a prediction for their relative branching ratio assuming isospin to be an exact symmetry of the strong interactions. [8]

[The isospin ladder operators T_+ and T_- have the properties

$$T_+ \left|I, I_3\right\rangle = \sqrt{I(I+1) - I_3(I_3+1)}\, \left|I, I_3+1\right\rangle ,$$
$$T_- \left|I, I_3\right\rangle = \sqrt{I(I+1) - I_3(I_3-1)}\, \left|I, I_3-1\right\rangle .]$$

(b) The colour ladder operators $T_\pm$, $U_\pm$ and $V_\pm$ have the properties

$$T_+ \left|g\right\rangle = \left|r\right\rangle , \qquad U_+ \left|b\right\rangle = \left|g\right\rangle , \qquad V_+ \left|b\right\rangle = \left|r\right\rangle ,$$
$$T_- \left|r\right\rangle = \left|g\right\rangle , \qquad U_- \left|g\right\rangle = \left|b\right\rangle , \qquad V_- \left|r\right\rangle = \left|b\right\rangle ,$$

and the eight gluons of QCD have colour wavefunctions

$$r\bar{g},\ r\bar{b},\ b\bar{g},\ b\bar{r},\ g\bar{r},\ g\bar{b},\ \frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g}),\ \frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b}).$$

By considering the effect of the colour ladder operators on the state $\left|rr\right\rangle$, or otherwise, show that the possible colour states of a two-quark system consist of a sextet of states which are symmetric under particle interchange, and a triplet of states which are antisymmetric under particle interchange. [5]

Show that the force arising from single-gluon exchange between a pair of quarks is repulsive if the two quarks are in any of the colour states belonging to the sextet but attractive if the two quarks are in one of the colour triplet states. Explain briefly why the latter result does not necessarily imply that hadrons containing only two valence quarks can exist as free particles. [9]

2 Write brief accounts of **two** of the following:

(a) electromagnetic showers and electromagnetic calorimeters; [15]

(b) deep-inelastic electromagnetic scattering; [15]

(c) the CKM matrix; [15]

(d) the Higgs boson. [15]

3 Explain what is meant by the statement that charged current weak interactions have a $V - A$ structure, and indicate some of the experimental observations which led to this conclusion. What constraints does this form of interaction impose on the chirality and helicity of particles and antiparticles interacting with W bosons? [7]

The matrix element for the decay $\mathrm{W}^- \to \mathrm{e}^-\overline{\nu}_e$ is

$$M_{\mathrm{fi}} = \frac{g_{\mathrm{W}}}{\sqrt{2}}\epsilon_\mu(p_1)\bar{u}(p_3)\gamma^\mu\frac{1}{2}(1-\gamma^5)v(p_4)$$

where p_1 is the 4-momentum of the W^- and p_3 and p_4 are the 4-momenta of the electron and the antineutrino, respectively. Working in the W^- rest frame, neglecting the electron mass, and taking the 4-momentum of the electron to be $p_3 = (E, E\sin\theta, 0, E\cos\theta)$, show that the lepton current is equal to $2E(0, -\cos\theta, -i, \sin\theta)$. Given that the decay rate is

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{p^*}{32\pi^2 m_{\mathrm{W}}^2}\,|M_{\mathrm{fi}}|^2,$$

where p^* is the centre of mass momentum, show that the angular distribution $d\Gamma/d\cos\theta$ of the electron is proportional to $(1\pm\cos\theta)^2$ or to $\sin^2\theta$, depending on the initial polarisation of the W^-. For an unpolarised sample of W^- bosons, show that the decay rate is

$$\Gamma(\mathrm{W}^- \to \mathrm{e}^-\overline{\nu}_e) = \frac{g_{\mathrm{W}}^2 m_{\mathrm{W}}}{48\pi}\;.$$

Write down the other allowed two-body decays of the W^-, and determine the branching ratio for the $\mathrm{W}^- \to \mathrm{e}^-\overline{\nu}_e$ decay. [16]

The figure on page 5 shows the $\cos\theta_{\mathrm{e}}^*$ distribution of electrons and positrons from $\mathrm{W}^\pm$ decays observed by the UA1 experiment in high energy proton-antiproton collisions, where θ_{e}^* is the angle of the electron (positron) with respect to the proton (antiproton) direction in the $\mathrm{W}^\pm$ rest frame. Given that the $\mathrm{W}^\pm$ bosons are produced dominantly by the annihilation of a valence quark from the proton with a valence antiquark from the antiproton, draw the relevant leading-order Feynman diagrams for the $\mathrm{W}^\pm$ production and decay. Explain how the UA1 data can be understood by considering the allowed helicity states of the particles involved, drawing diagrams to illustrate your arguments. [7]

You *may* require the following information:

The Dirac spinors are

$$u_\uparrow = \sqrt{E}\begin{pmatrix} c \\ s \\ c \\ s \end{pmatrix}, \quad u_\downarrow = \sqrt{E}\begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix}, \quad v_\uparrow = \sqrt{E}\begin{pmatrix} s \\ -c \\ -s \\ c \end{pmatrix}, \quad v_\downarrow = \sqrt{E}\begin{pmatrix} c \\ s \\ c \\ s \end{pmatrix}$$

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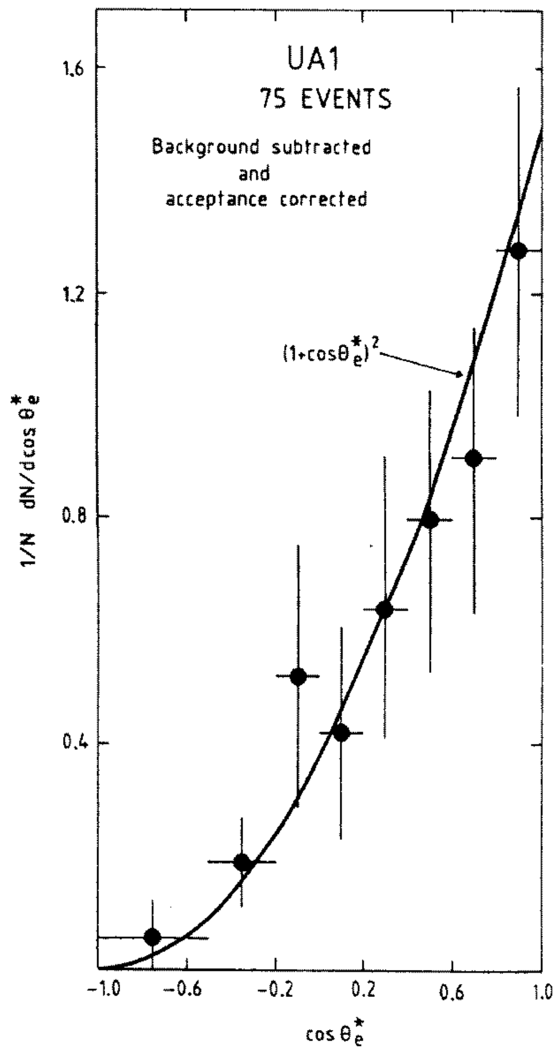
where $c \equiv \cos\theta/2$, $s \equiv \sin\theta/2$. The Dirac gamma-matrices are

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \qquad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix},$$

$$\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

The polarisation vectors for a massive spin 1 particle at rest are

$$\epsilon_+^\mu = -\frac{1}{\sqrt{2}}(0,1,i,0), \quad \epsilon_-^\mu = \frac{1}{\sqrt{2}}(0,1,-i,0), \quad \epsilon_L^\mu = (0,0,0,1)\ .$$



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