NATURAL SCIENCES TRIPOS   Part III

Tuesday 15 January 2002   2.00 to 3.30

EXPERIMENTAL AND THEORETICAL PHYSICS (4)
Particle Physics

*Answer two questions only. The approximate number of marks allotted
to each part of a question is indicated in the right margin where
appropriate. The paper contains FOUR sides and is accompanied
by a book giving values of constants and containing mathematical
formulae which you may quote without proof.*

1   The matrix element for the decay H → f̄f of the Higgs boson H to a fermion (quark or lepton) f and its antiparticle f̄ is

$$M_{fi} = \frac{g_W m_f}{2 m_W} \bar{u}(p_2) v(p_3)$$

where g_W is the weak decay constant, m_W is the mass of the $W^{\pm}$ boson, m_f is the mass of the fermion f, and p_2 and p_3 are the 4-momenta of f and f̄, respectively.

In the rest frame of the Higgs boson, with the fermion travelling in the $+z$ direction, show that $\bar{u}_i(p_2) v_j(p_3)$ $(i, j = 1, 2)$ is non-zero for only two of the four possible combinations and interpret this result.   [6]

Given that the decay rate is

$$\Gamma = \frac{p^*}{8 \pi m_H^2} \langle |M_{fi}|^2 \rangle$$

where p^* is the centre of mass momentum of either final state particle, show that

$$\Gamma = N_c \frac{G_F}{\sqrt{2}} \frac{m_f^2 m_H}{4\pi} \left(1 - \frac{4 m_f^2}{m_H^2} \right)^{3/2}$$

where N_c is the number of colour degrees of freedom of the fermion f.   [6]

List all possible decays of the type H → f̄f for a Higgs mass $m_H = 100\,\text{GeV}$. Neglecting the (small) contributions from other types of decay, and taking suitable approximate values for fermion masses where needed, estimate the total width and lifetime of a Higgs boson of this mass, and the three largest branching ratios to f̄f final states.   [6]

Draw a leading order Feynman diagram for the Higgs boson production process $e^+ e^- \rightarrow H Z^0$ and describe the various types of final state which result once the H and Z^0 bosons have decayed. Explain how events of this type can be identified and distinguished from events due to non-Higgs background processes, indicating the detection techniques which are important in this respect.   [5]

Summarise the current experimental knowledge of the Higgs boson mass, both from direct searches for Higgs boson production and from indirect approaches.   [4]

State the Higgs boson decay modes which are expected to be of importance for the experimental detection of Higgs bosons at future hadron colliders, and draw a Feynman diagram illustrating a possible Higgs boson production mechanism at such a machine.   [3]

[You may require the following information:

$G_F = 1.166 \times 10^{-5}\,\text{GeV}^{-2}$, $G_F/\sqrt{2} = g_W^2/8 m_W^2$.
$\hbar = 6.582 \times 10^{-25}\,\text{GeV s}$.

The Dirac spinors are:

$$u_1 = N \begin{pmatrix} 1 \\ 0 \\ \dfrac{p_z}{E+m} \\ \dfrac{p_x + ip_y}{E+m} \end{pmatrix}, \quad u_2 = N \begin{pmatrix} 0 \\ 1 \\ \dfrac{p_x - ip_y}{E+m} \\ \dfrac{-p_z}{E+m} \end{pmatrix},$$

$$v_1 = N \begin{pmatrix} \dfrac{p_x - ip_y}{E+m} \\ \dfrac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix}, \quad v_2 = N \begin{pmatrix} \dfrac{p_z}{E+m} \\ \dfrac{p_x + ip_y}{E+m} \\ 1 \\ 0 \end{pmatrix},$$

where $N = \sqrt{E+m}$.]

2   Write brief accounts of **two** of the following:

(a) the roles of SU(2) and SU(3) symmetry in particle physics;   [15]

(b) deep-inelastic scattering of neutrinos and antineutrinos;   [15]

(c) experimental techniques for the separation of charged particle types (e, μ, π, K, p) based on the measurement of velocity;   [15]

(d) elastic $e^-\mu^-$, e^-p and e^-n scattering.   [15]

3   The K^0 and $\overline{K}^0$ mesons have quark content $d\bar{s}$ and $s\bar{d}$, respectively, and belong to a meson nonet with quantum numbers $J^{PC} = 0^{-+}$. Explain how CP eigenstates K_1 and K_2 can be constructed in the $K^0 - \overline{K}^0$ system, and explain how, neglecting CP violation, these can be related to the states K_S and K_L.   [5]

Draw Feynman diagrams for the allowed decays of K^0 and $\overline{K}^0$ mesons into the final states $\pi^+ e^- \bar{\nu}_e$ and $\pi^- e^+ \nu_e$. A beam of neutral kaons of momentum $100\,\text{GeV}$ is produced in an initial state which is known to consist purely of K^0 mesons. At a point a distance $17.8\,\text{m}$ downstream of the production point, the decay rate into $\pi^+ e^- \bar{\nu}_e$ is observed to be equal to the decay rate into $\pi^- e^+ \nu_e$. Neglecting the effects of CP violation, derive expressions for the dependence of the $\pi^+ e^- \bar{\nu}_e$ and $\pi^- e^+ \nu_e$ decay rates on the distance along the beam direction and estimate the difference Δm between the K_S and K_L masses. Show also that, neglecting CP violation, the partial widths $\Gamma(K_L \to \pi^- e^+ \nu_e)$ and $\Gamma(K_L \to \pi^+ e^- \bar{\nu}_e)$ would be expected to be equal.   [8]

Describe briefly the first experimental observation showing that CP is not a symmetry of the weak interactions, explaining why this conclusion is required and how the data can be understood if the K_L meson is a state of the form   [4]

$$K_L = \frac{1}{\sqrt{1 + |\epsilon|^2}} \left(K_2 + \epsilon K_1\right) .$$

Show that a non-zero asymmetry

$$\delta = \frac{\Gamma(K_L \to \pi^- e^+ \nu_e) - \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e)}{\Gamma(K_L \to \pi^- e^+ \nu_e) + \Gamma(K_L \to \pi^+ e^- \bar{\nu}_e)}$$

is now expected, and derive an expression for δ in terms of ϵ.   [8]

Outline the role of the CKM matrix in charged current weak interactions and summarise the current experimental knowledge of the elements of this matrix. Explain briefly how CP violation can be accommodated within the Standard Model, and indicate how and to what extent the parameters determining the level of CP violation are constrained by current data.   [5]

[The K^0 mass is $0.498\,\text{GeV}$. $\hbar c = 0.197\,\text{GeV fm}$.]

END OF PAPER