UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Physics	NST3PHY
MASTER OF ADVANCED STUDY IN PHYSICS	MAPY
NATURAL SCIENCES TRIPOS: Part III Astrophysics	NST3AS

Tuesday 13 January 2015 14.00 to 16.00

MAJOR TOPICS

Paper 1/PP (Particle Physics)

Answer **two** *questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper contains 15 sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

You should use a **separate Answer Book** *for each question.*

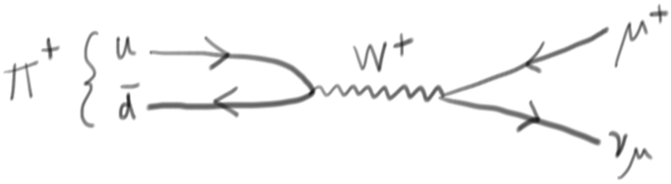
STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2 × 20-page answer books	Mathematical formulae handbook
Rough workpad	Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

Draft: 22nd December 2014

1 **a**

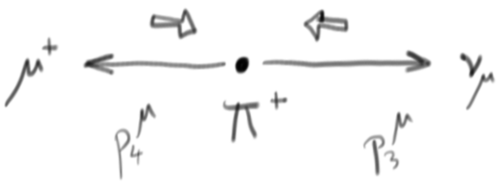
Bookwork



b

Bookwork

The π^+ is a spin-0 meson, si in its rest frame we have a equal and opposite muon and neutrino momenta, and they must have equal and opposite helicities. The neutrino is a a particle, and so is produced in a left handed (LH) chiral state by the W-boson. As an effectively massless particle, the neutrino's LH chiral state is co-incident with a LH helicity state and therefore to conserve total spin the anti-muon must also be in a LH helicity state:



c

Unseen in this form, though similar to lectures

$$J^\mu = \bar{u}_\downarrow(p_3)\gamma^\mu \frac{1}{2}(1-\gamma^5)v_\downarrow(p_4) \tag{1}$$

By aligning the z-axis with the neutrino direction, we can take $\theta = 0$ for the neutrino, $\theta = \pi$ for the anti-muon, and $\phi = 0$ for both. Accordingly:

$$v_\downarrow(p_4) = \sqrt{E+m}\begin{pmatrix} 0 \\ \alpha \\ 0 \\ 1 \end{pmatrix}$$

and

$$u_\downarrow(p_3) = \sqrt{p}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

in which p is the magnitude of the three momentum of either of the daughters of the pion in its rest frame, m is the muon mass, $E = \sqrt{m^2 + p^2}$, and $\alpha = \frac{p}{E+m}$.

Using the supplied gamma matrices, the students can compute that

$$1 - \gamma^5 = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

and hence

$$(1 - \gamma^5) v_\downarrow(p_4) = \sqrt{(}E + m) \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \alpha \\ 0 \\ 1 \end{pmatrix} \quad (2)$$

$$= \sqrt{E + m} \begin{pmatrix} 0 \\ \alpha - 1 \\ 0 \\ -\alpha + 1 \end{pmatrix} \quad (3)$$

$$\propto \sqrt{E + m}(1 - \alpha). \quad (4)$$

Therefore

$$J^\mu \propto \sqrt{p}\sqrt{E + m}(1 - \alpha). \quad (5)$$

We therefore see that

$$J^\mu \propto \sqrt{p}\sqrt{E + m}(1 - \alpha) \quad (6)$$

$$= \sqrt{p}\sqrt{E + m}(1 - \frac{p}{E + m}) \quad (7)$$

$$= \sqrt{p}\sqrt{E + m}\frac{E + m - p}{E + m} \quad (8)$$

$$= \frac{\sqrt{p}(E + m - p)}{\sqrt{E + m}} \quad (9)$$

as required.

d

Unseen in this form, though similar to lectures

From the previous result,

$$
\begin{aligned}
(J^{\mu}\pi_{\mu})^2 &\propto \frac{p(E+m-p)^2}{E+m} && (10)\\
&= \frac{p(E^2+m^2+p^2+2Em-2Ep-2mp)}{E+m} && (11)\\
&= \frac{p(2E^2+2Em-2Ep-2mp)}{E+m} && (12)\\
&= \frac{2p(E-p)(E+m)}{E+m} && (13)\\
&\propto p(E-p). && (14)
\end{aligned}
$$

Using the supplied two-body decay-rate together with the result just proved,

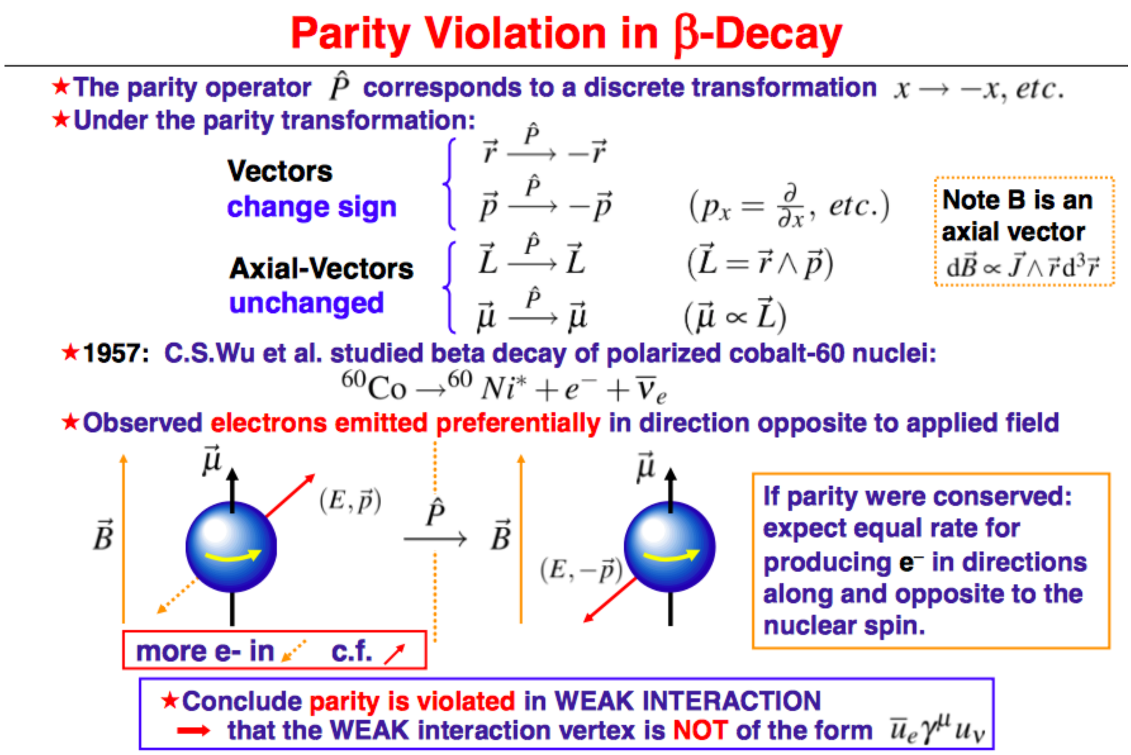
$$
\frac{\Gamma(\pi^+ \to e^+\nu_e)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = \left(\frac{p_e}{p_\mu}\right)\frac{p_e(E_e-p_e)}{p_\mu(E_\mu-p_\mu)} \qquad (15)
$$

which works out to be about 1.273×10^{-4} using the data supplied in the question.

e

Bookwork

Here it is expected that an answer will include a description of Madame Wu's experiment:



and will note that this experiment unambiguously determined that this process did not respect parity as a symmetry of nature, since the experimental data observed (electrons departing preferentially antiparallel to the spin direction) would not have been invariant under a parity transformation on a virtual representation of the experiment..

An answer will go on to describe the forward backward asymmetry of the Z-boson at LEP

★ And the Matrix elements become

$$|M_{RR}|^2 = \frac{g_Z^4 s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2 \qquad \text{etc.}$$

★ In the limit where initial and final state particle mass can be neglected:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|M_{fi}|^2$$

(page 31)

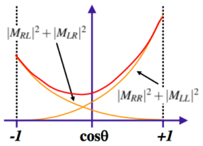
★ Giving:

$$\frac{d\sigma_{RR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$

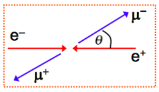
$$\frac{d\sigma_{LL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_L^\mu)^2(1+\cos\theta)^2$$

$$\frac{d\sigma_{LR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$

$$\frac{d\sigma_{RL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$



★ Because $|M_{LL}|^2+|M_{RR}|^2 \neq |M_{LR}|^2+|M_{RL}|^2$, the differential cross section is asymmetric, i.e. parity violation (although not maximal as was the case for the W boson).



Forward-Backward Asymmetry

★ On page 495 we obtained the expression for the differential cross section:

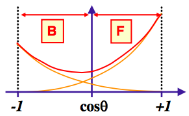
$$\langle|M_{fi}|\rangle^2 \propto [(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2](1+\cos^2\theta) + [(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2]\cos\theta$$

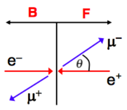
★ The differential cross sections is therefore of the form:

$$\frac{d\sigma}{d\Omega} = \kappa\times[A(1+\cos^2\theta)+B\cos\theta] \qquad \begin{cases} A = [(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2] \\ B = [(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2] \end{cases}$$

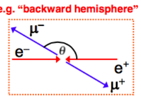
★ Define the FORWARD and BACKWARD cross sections in terms of angle incoming electron and out-going particle

$$\sigma_F \equiv \int_0^1 \frac{d\sigma}{d\cos\theta}d\cos\theta \qquad \sigma_B \equiv \int_{-1}^0 \frac{d\sigma}{d\cos\theta}d\cos\theta$$



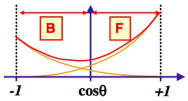


e.g. "backward hemisphere"



★ The level of asymmetry about cosθ=0 is expressed in terms of the Forward-Backward Asymmetry

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$



• Integrating equation (1):

$$\sigma_F = \kappa\int_0^1 [A(1+\cos^2\theta)+B\cos\theta]d\cos\theta = \kappa\int_0^1 [A(1+x^2)+Bx]dx = \kappa\left(\frac{4}{3}A+\frac{1}{2}B\right)$$

$$\sigma_B = \kappa\int_{-1}^0 [A(1+\cos^2\theta)+B\cos\theta]d\cos\theta = \kappa\int_{-1}^0 [A(1+x^2)+Bx]dx = \kappa\left(\frac{4}{3}A-\frac{1}{2}B\right)$$

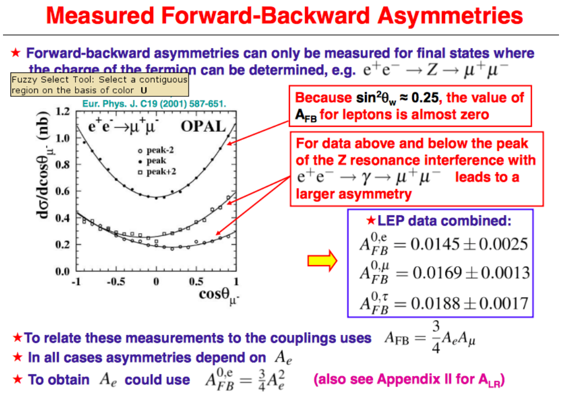
★ Which gives:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{B}{(8/3)A} = \frac{3}{4}\left[\frac{(c_L^e)^2-(c_R^e)^2}{(c_L^e)^2+(c_R^e)^2}\right]\cdot\left[\frac{(c_L^\mu)^2-(c_R^\mu)^2}{(c_L^\mu)^2+(c_R^\mu)^2}\right]$$

★ This can be written as

$$A_{FB} = \frac{3}{4}A_e A_\mu \qquad \text{with} \qquad A_f \equiv \frac{(c_L^e)^2-(c_R^e)^2}{(c_L^e)^2+(c_R^e)^2} = \frac{2c_V^f c_A^f}{(c_V^f)^2+(c_A^f)^2} \qquad (4)$$

★ Observe a non-zero asymmetry because the couplings of the Z to LH and RH particles are different. Contrast with QED where the couplings to LH and RH particles are the same (parity is conserved) and the interaction is FB symmetric



A first class answer should conclude by noting that, though the forward-backward asymmetry of the Z is a consequence of the parity volation in the weak interation, the forward backward asymmetry of the Z-boson does not, in itself, show that the Standard Model violates parity. This is because a parity inversion on LEP would result in the forward direction being mapped to the forward direction, and the backward direction being mapped to the backward direction (since forward means μ^- goes in same direction that e^- was going, and both of these directions invert themselves under parity). The FB asmmetry, therefore, would be invariant under a parity transofmration, and so provides no direct evidence for parity violation.

2 CP violation in the neutral Kaon system

- Describe parity.
- Describe charge conjugation.
- Assuming CPT, CP violation implies violation of T.
- In SM two place where CP arises: PMNS matrix and CKM matrix.
- CKM and PMNS matrices are unitary.
- For three generations can have a complex phase which gives CP violation
- CP violation not possible for two generations.
- CP violation observed in kaon system
- Describe main features of CP violation in kaons CP eigenstates
- CP even decays to $\pi\pi$ and CP odd decays to $\pi\pi\pi$ CP states roughly correspond to KS and KL
- At long distance have pure KL beam
- But KL observed to decay to $\pi\pi$ at level of 0.1
- CP violation enters in box diagrams because $V_{ij} \neq V_{ij}^*$
- CP violation in SM not sufficient to explain baryon dominated universe

Experimental and theoretical aspects of neutrino oscillations

- Theoretical
  - difference between mass $\nu_{1,2,3}$ and flavour $\nu_{e,\mu,\tau}$ eigenstates of neutrinos
  - neutrinos produced in states of definite flavour
  - neutrinos propagate as states of definite mass
  - unitary matrix to relate the two type of basis
  - matrix has single parameter if there are only two flavours
  - matrix ('PMNS' matrix) has four free parameters (three angles, one CP violating phase) if there are three flavours
  - Time evolution of mass states $\nu_1(x,t) = e^{-ip_\mu x^\mu}\nu_1(0,0)$ is trivial, and can be used to evolve flavour states in time, but first expressing the flavour states (via the PMNS matrix) in a mass basis.
  - Probability for seeing neutrino in given flavour at time of later observation is then obtained by looking for the component of that flavour at the observation time, by re-expressing back in terms of flavour basis.
  - For Neutrino oscillations to occur, needs two things to be present:
    * Non-zero mass difference between different neutrino mass eigenstates sets **wavelength** of oscillation $\lambda = \frac{4\pi E}{\Delta m^2}$

∗degree of mixing between flavours (i.e. PMNS matrix controlled numbers) set **amplitude** of oscillation

•Experimental

–Task of experiments is to constrain (or over constrain) the PMNS (mixing) matrices and the mass differences.

–In order to look at as many types of nutrino flavour as possible need many different productioin processes (solar furnace, neutrino beam, cosmic ray, nuclear reactor, etc).

–In order to see different parts of neutrino oscillations need different length scales (e.g. from Chooz at 500m to the diameter of the earth at Super-K) or different energies (beam-line vs reactor) to stretch or shink oscillation wavelength.

–To see different flavours at point of detection, need spectrum of energies to overcome fixed-target production thresholds (e.g. to see muons in charge-current interation) and variety of detection technologies.

–Tau leptons are too heavy to allow tau-neutrino detection in most circumstances, but can count neutral-current elastic scattering rates.

–Draw Feynman diagrams for the two main detection mechanisms (inverse beta for charge-current and elastic scattering via Z-boson for neutral current).

–Current data favours one large mass difference and one small mass difference

–Mention some experiments and what distinguishes them:

CHOOZ and KamlandFormer at short $\sim$ 1 km lengthscales, other at longer $\sim$ 200 km lengthscales, see reactor (anti)neutrinos via $\bar{\nu}_e + p \to e^+ + n$ followed by e^+e^- annihilation to two photons and delayed photon signal from netron capture (on Gadolinium in CHOOZ, on deuteron in Kamland). signal double coincidence detect annihilation photons + neutron CHOOZ negative result sets limit on θ_{13}. Example sheet question in course covered recent Daya-Bay result on non-zero θ_{13}, but not covered in lectures. KamLand positive results compatible with solar neutrino. KamLand + SNO gives precise measurement of $\Delta m^2_{12} \sim 8^{-5} eV^2$ and θ_{12}.

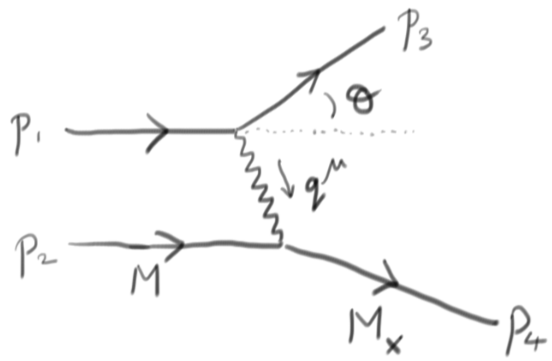
SuperKwater Cherenkov detection, can see e and μ rings (fuzzy or sharp for PID) at appropriate energies. Detectors solar ν_e disappearance relative to SSM, detects atmospheric ν_e disappearance relative to atmosphetic ν_μ (near maximal mixing).

SNOIn many ways like SuperK, but with the added benefit of ability to see neutral current interactions and thereby count rates for all three neutrino flavoursat Solar Neutrino energies: CC rate proportional to ν_e rate only, NC rate (Z-boson splitting a deuteron) proportional to all three flavours, elastic scattering rate prop to $\nu_e + 0.15(\nu_\mu + \nu_\tau)$ due to special role of electrons in matter. Sees total flux consistent with SSM.

MINOSBeam line experiment, high enough energy to make and observe muon neutrinos via CC interaction. See $\Delta m^2 \sim 2.5 \times 10^{-3} eV^2$.
–scintillator detectors + brief description

3 **a**

Bookwork



$$p_2^\mu + q^\mu = p_4^\mu \implies (p_2 + q)^2 = p_4^2 \tag{16}$$
$$\implies M^2 + q^2 + 2p_2 \cdot q = M_X^2 \tag{17}$$
$$\implies M^2 + q^2 + 2M(E_1 - E_3) = M_X^2 \tag{18}$$
$$\implies q^2 + 2M\nu = M_X^2 - M^2 \tag{19}$$
$$\implies \frac{q^2}{2M} + \nu = \frac{M_X^2 - M^2}{2M}. \tag{20}$$

But in elastic collisions $M_X = M$ and so we have

$$q^2 + 2M\nu = 0 \implies \frac{q^2}{2M} + \nu = 0. \tag{21}$$

b

Unseen; similar calculations in lectures

To perform the integral:

$$\int \delta\left(\nu + \frac{q^2}{2M}\right) dE_3 \tag{22}$$

one must first get the integrand to depend entirely on E_1, E_3 and θ, in which E_1 is considered fixed.

$$\int \delta\left(\nu + \frac{q^2}{2M}\right) dE_3 = \int \delta\left(E_1 - E_3 + \frac{(p_1^\mu - p_3^\mu)^2}{2M}\right) dE_3 \tag{23}$$
$$= \int \delta\left(E_1 - E_3 + \frac{0 + 0 - 2p_1 \cdot p_3}{2M}\right) dE_3 \tag{24}$$
$$= \int \delta\left(E_1 - E_3 - \frac{E_1 E_3 (1 - \cos\theta)}{M}\right) dE_3 \tag{25}$$
$$= \left|-1 - \frac{E_1(1 - \cos\theta)}{M}\right|^{-1}_{\left(\frac{q^2}{2M} + \nu = 0\right)} \tag{26}$$
$$= \left(1 + \frac{2E_1 E_3 (1 - \cos\theta)}{2E_3 M}\right)^{-1}_{\left(\frac{q^2}{2M} + \nu = 0\right)} \tag{27}$$
$$= \left(1 + \frac{-q^2}{2E_3 M}\right)^{-1}_{\left(\frac{q^2}{2M} + \nu = 0\right)} \tag{28}$$
$$= \left(1 + \frac{\nu}{E_3}\right)^{-1} \tag{29}$$
$$= \left(\frac{E_3 + (E_1 - E_3)}{E_3}\right)^{-1} \tag{30}$$
$$= \frac{E_3}{E_1} \tag{31}$$

wherein E_3 must be the value of E_3 that solves $\frac{q^2}{2M} + \nu = 0$ given E_1, M and θ.

The δ-function makes $\frac{d\sigma}{dE_3 d\Omega}$ zero at all values of E_3 except that for which the scattering is elastic.

c

Unseen, but simplification of what's seen in lectures

In the supplied model, if p_2^μ is the initial proton momentum, then $p_u^\mu = \frac{1}{3} p_2^\mu$ is the momentum of one u-quark, and similarly for the other quarks. Evidently $p_u^2 = \frac{1}{9} p_2^2 = \frac{1}{9} M^2$ and therefore $m = \frac{M}{3}$.

In the elastic scattering seen in lectures, only one 'variable' θ (or, without loss of generality, q^2) was needed to parametrise events. This was because the scattering could always be assumed to take place in the $\phi = 0$ plane, and since E_1 and M could be regarded as fixed, all other quantities could be derived from knowledge of θ (or q^2). In the inelastic scattering seen in lectures, a second variable (e.g. x) was needed, in addition to *theta* (or q^2), to parametrise the additional degree of freedom generated by the uncertain momentum fraction of the struck quark. In the inelastic scattering present in this question, however, the struck quark has a 100% fixed momentum (of one third of the proton) so unlike lectures we will only need a single degree of freedom to characterise these inelastic events. Without loss of generality we take that parameter to be θ.

We may therefore use the differential cross section supplied at the beginning of the question (with M replaced by $m = M/3$) to represent the inner eleastic parton-parton cross section contained within the inelastic proton scattering process. We must multiply this cross section by a factor $2\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2$ to account for the fact that the inelastic scattering involves the projectile interacting with either of two charge $\frac{2}{3}$ u-quarks or one charge $-\frac{1}{3}$ d-quark. This makes the differential cross section $\frac{d\sigma}{d\Omega}$ for the inelastic scattering:

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{a^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \left(\frac{E_3}{E_1}\right) \left[\cos^2 \frac{\theta}{2} - \frac{q^2}{2m^2} \sin^2 \frac{\theta}{2}\right] \left(2\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right) \\ &= \frac{a^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \left(\frac{E_3}{E_1}\right) \left[\cos^2 \frac{\theta}{2} - \frac{q^2}{2m^2} \sin^2 \frac{\theta}{2}\right] \qquad (32)\end{aligned}$$

in which E_3 must now be the value of E_3 solving $\frac{q^2}{2m} + \nu = 0$ for given θ, not solving $\frac{q^2}{2M} + \nu = 0$ as previously. Since $\nu = E_1 - E_3$, this latter constraint can be removed, or rather replaced, by a integral and a δ-function:

$$\frac{d\sigma}{d\Omega} = \frac{a^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \int_{E_3} \left(\frac{E_3}{E_1}\right) \left[\cos^2 \frac{\theta}{2} - \frac{q^2}{2m^2} \sin^2 \frac{\theta}{2}\right] \delta\left(\frac{q^2}{2m} + \nu\right) dE_3 \qquad (33)$$

or equivalently:

$$\begin{aligned}\frac{d\sigma}{d\Omega dE_3} &= \frac{a^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \left(\frac{E_3}{E_1}\right) \left[\cos^2 \frac{\theta}{2} - \frac{q^2}{2m^2} \sin^2 \frac{\theta}{2}\right] \delta\left(\frac{q^2}{2m} + \nu\right) \qquad (34) \\ &= \frac{a^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \left(\frac{E_3}{E_1}\right) \left[\cos^2 \frac{\theta}{2} + \frac{\nu}{m} \sin^2 \frac{\theta}{2}\right] \delta\left(\frac{q^2}{2m} + \nu\right), \qquad (35)\end{aligned}$$

the presence of the δ-function allowing the final replacement. By comparing the form of the differential cross section given above to that supplied in the question, we see that:

$$\frac{F_2(\nu, q^2)}{\nu} = \delta\left(\frac{q^2}{2m} + \nu\right) \quad \text{and} \qquad (36)$$

$$\frac{2F_1(\nu, q^2)}{M} = \frac{\nu}{m}\delta\left(\frac{q^2}{2m} + \nu\right) \qquad (37)$$

or equivalently

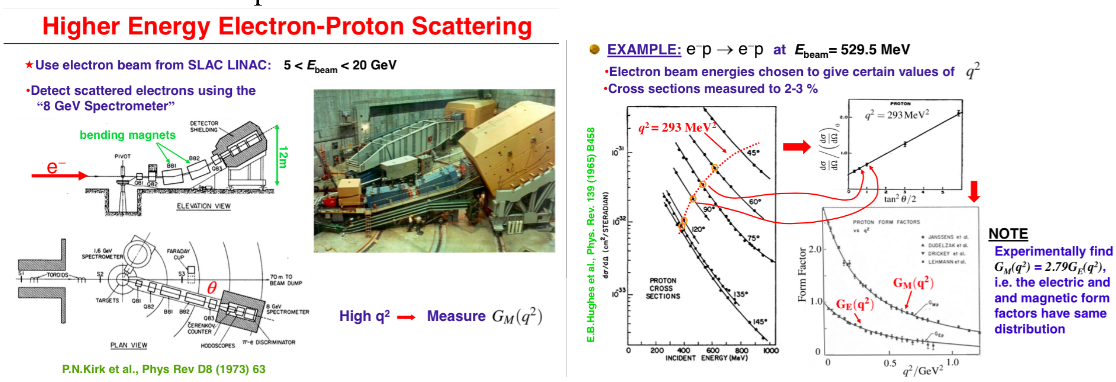
$$F_2(\nu, q^2) = \frac{2}{3}F_1(\nu, q^2) = \nu\delta\left(\frac{q^2}{2m} + \nu\right). \qquad (38)$$

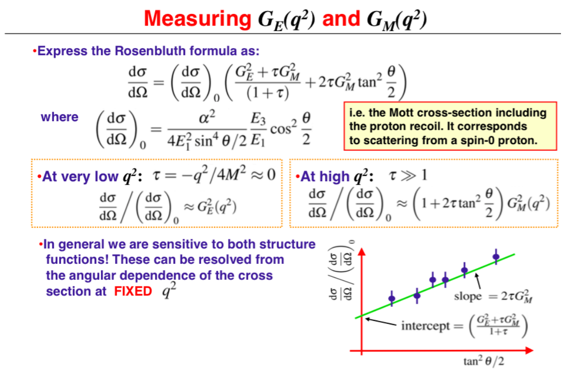
[Aside: The first equality in (23) is recognisable as the Callen-Gross relation $F_2 = 2xF_1$ for a fixed value of x, namely $\frac{1}{3}$. The second equality in (23) may be shown (though the question does not require or expect this!) to be equal to $F_2(\nu, q^2) = \frac{1}{3}\delta\left(x - \frac{1}{3}\right)$, where $x = -\frac{q^2}{2M\nu}$, which is consistent with the form expected for the usual structure functions, namely: $F_2(x) = x\Sigma_i e_i^2 u_i(x)$, where u_i is the parton distribution function for the i-th parton, and e_i is the charge of the i-th parton (in units of e).]

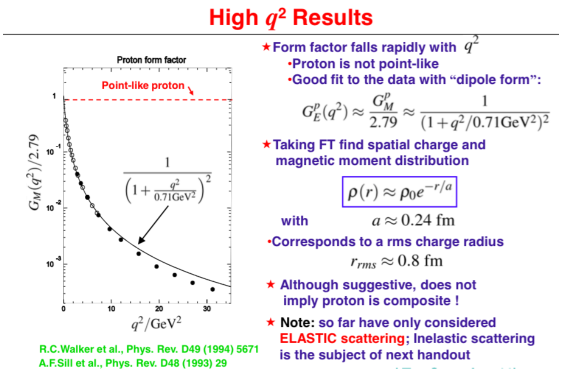
d

Bookwork

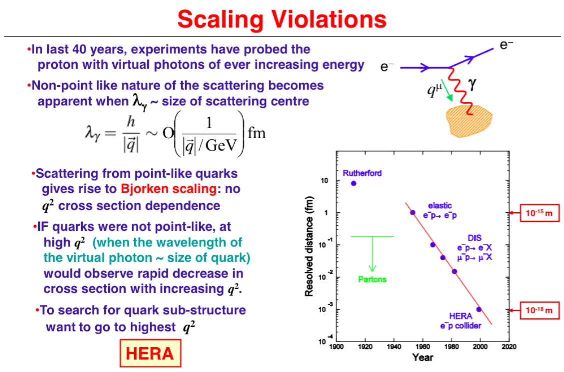
Here it is anticipated that the students will reproduce some of the description of the SLAC Linac Experiments described in the lectures in slides 169-172:

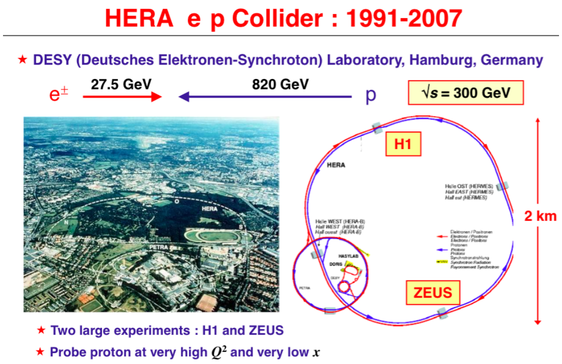


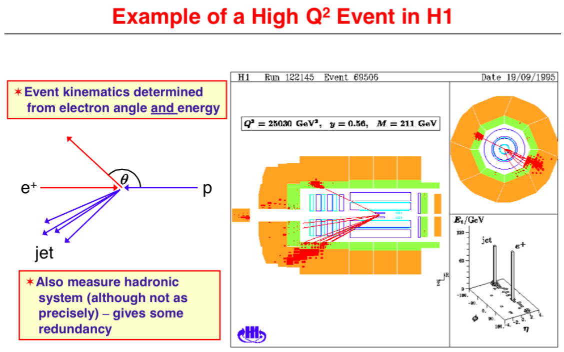


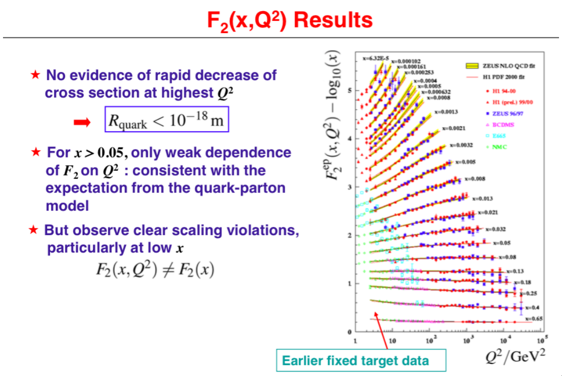


and the HERA experiments described in slides 199-202:









The comment on ‘the extent to which the results agreed with the theoretical predictions’ part of the question should probably include some description of the most naive parton model being endored by SLAC (Callen-Gross relations and Bjorjken Scaling), but should go on to note that scaling violations ($F(x) \to F(x, q^{\prime})$) were clearly seen at low x at HERA, but are ultimately still believed to be consitent with less-simplistic version of the parton-model (momentum sharing with gluons, etc). The answer will probably include descirption of the quark content of the proton, sketches of the PDFs for the valence quarks, sea quarks and gluons, a description of how the quark PDFs were determined from the experimental data. A decription should be given of the method by which the total momentum carried by quarks vs gluons has been determined, and a note that it has been found to be roughly fifty-fifty shared between gluons and quarks.

END OF PAPER