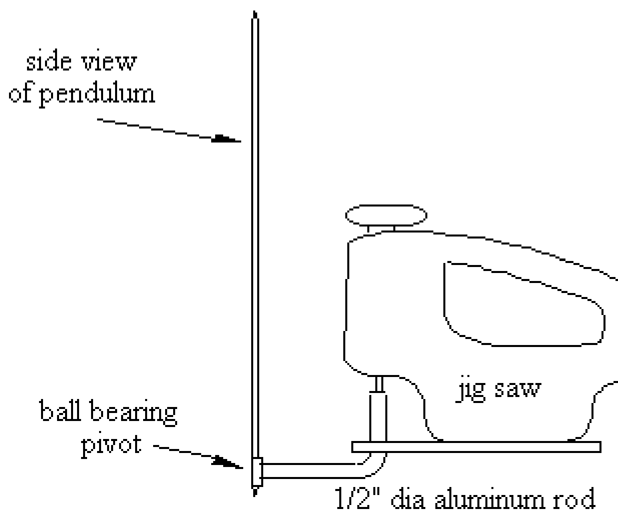
Inverted Pendulum

simple harmonic motion - driven oscillator - phase stability - Mathieu equation

What it shows:
An inverted pendulum in unstable equilibrium becomes stable by the application of an oscillating force. Indeed, the driven upside-down pendulum is so stable that even a push to "knock it down" does not perturb it enough to become unstable - after a few lateral excursions it returns to its vertical position as if gravity was pulling it up.

How it works:
The physical pendulum is a 45 cm long x 3 cm wide by 3 mm (1/8") thick aluminum strip mounted on a ball-bearing pivot and can rotate 360°. Its pivot is driven by a 3/4" stroke, variable speed, jig saw. [1] The basic idea was adapted from Michaelis (reference 3). While holding the pendulum in the inverted position, the saw is switched on and brought up to speed until the pendulum becomes stable. Because the length of the stroke is well above the critical amplitude of vibration, the range of driving frequencies for the pendulum to remain in stable equilibrium is quite broad.



If the driving point executes sinusoidal vibrations, the equation of motion of the pendulum is Mathieu's equation. A complete analysis of the motion can entail considerable complexity and the reader is referred to the references below. To understand what is physically going on, consider the following scenario. Suppose the pendulum is "falling down" in the sense that gravity is pulling on the center-of-gravity and causing it to rotate about the pivot. If the pivot is pulled down (by the jig saw) then a torque about the center-of-gravity will produce a counter rotation of the pendulum towards the vertical. The pivot point is then lifted up while the pendulum is near the vertical. The pendulum overshoots the vertical and begins to fall down on the opposite side. The pivot is pulled down..., etc. Thus the amplitude, frequency, and phase (relative to the pendulum's free motion) of the pivot's vibration all conspire to stabilize the pendulum and keep it near the vertical position.

Setting it up:
This is trivial as the demo is hand-held and quite portable! Since the pendulum moves with speeds of a few m/sec, it should be directed away from the audience and the operator ought to wear safety goggles as a precaution (the pendulum is press-fit onto the bearing and you can never tell when disaster will strike).

Comments:
This demo is so elegantly simple and its behavior goes against all expectation. In addition to its obvious demonstration purpose, Michaelis has pointed out that it can be used as "an analog in understanding such diverse phenomena as the theory of accelerators (strong focusing), inertial confinement, magneto-plasma confinement, rf plasma confinement, liquid confinement in inverted vessels, long-distance laser beam transmission (periodic focusing and defocusing with gas lenses analogous to accelerator strong focusing), containment of charged particles, and quadrupole mass filters." Is it possible that all problems in quantum electrodynamics can be solved by various Sears power tools? Rating***

References:

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Related Stability Demonstrations:

H. Winter and H.W. Ortjohann, Am J Phys **59**, 807 (1991). Simple demonstration of storing macroscopic particles in a "Paul trap"
L.W. Alvarez, R. Smits, and G. Senecal, Am J Phys **43**, 293 (1975). Mechanical analog of the synchrotron, illustrating phase stability and two-dimensional focusing

1 Sears Craftsman Auto Scroller Saw (model 315.172090)

main categories oscillation/waves demos