Peterhouse / Cambridge

Sagitta biases.

Perspective from the emu analysis.

https://twiki.cern.ch/twiki/bin/viewauth/AtlasProtected/EmuChargeFlavourAsymmetry

Christopher Lester

16th September 2020

https://arxiv.org/abs/2007.07624

Bias correction

$$p' = p\,(1 + q\;p_{\mathrm{T}}\;\delta_{\mathrm{sagitta}})^{-1}$$

I forgot the T here.

[Can be fixed easily if rest of idea is worth pursuing.]

https://arxiv.org/abs/2007.07624

Above paper contains TWO ways of estimating sagitta bias corrections

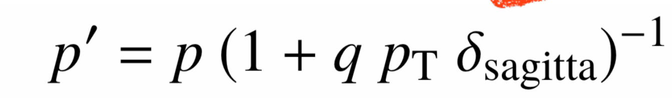
↙ sec 6.1.1

↘ sec 6.1.2

[see next slides]

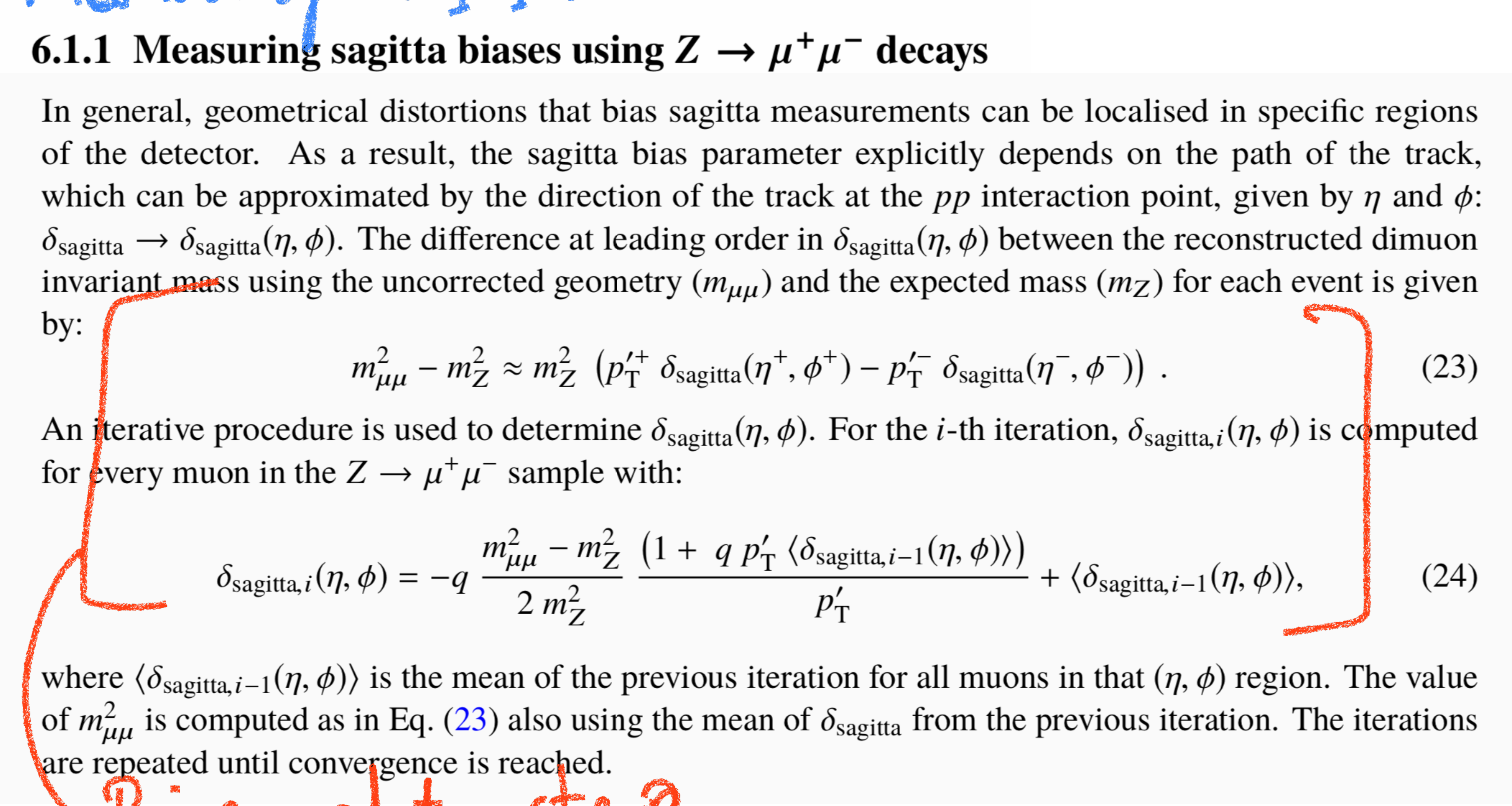
https://arxiv.org/abs/2007.07624

Bias correction



$$p' = p\,(1 + q\,p_{\mathrm{T}}\,\delta_{\mathrm{sagitta}})^{-1}$$

Method of 6.1.1 :



6.1.1 Measuring sagitta biases using $Z \to \mu^+\mu^-$ decays

In general, geometrical distortions that bias sagitta measurements can be localised in specific regions of the detector. As a result, the sagitta bias parameter explicitly depends on the path of the track, which can be approximated by the direction of the track at the pp interaction point, given by η and ϕ: $\delta_{\mathrm{sagitta}} \to \delta_{\mathrm{sagitta}}(\eta, \phi)$. The difference at leading order in $\delta_{\mathrm{sagitta}}(\eta, \phi)$ between the reconstructed dimuon invariant mass using the uncorrected geometry ($m_{\mu\mu}$) and the expected mass (m_Z) for each event is given by:

$$m^2_{\mu\mu} - m^2_Z \approx m^2_Z \left(p'^{+}_{\mathrm{T}}\,\delta_{\mathrm{sagitta}}(\eta^+, \phi^+) - p'^{-}_{\mathrm{T}}\,\delta_{\mathrm{sagitta}}(\eta^-, \phi^-)\right) . \tag{23}$$

An iterative procedure is used to determine $\delta_{\mathrm{sagitta}}(\eta, \phi)$. For the i-th iteration, $\delta_{\mathrm{sagitta},i}(\eta, \phi)$ is computed for every muon in the $Z \to \mu^+\mu^-$ sample with:

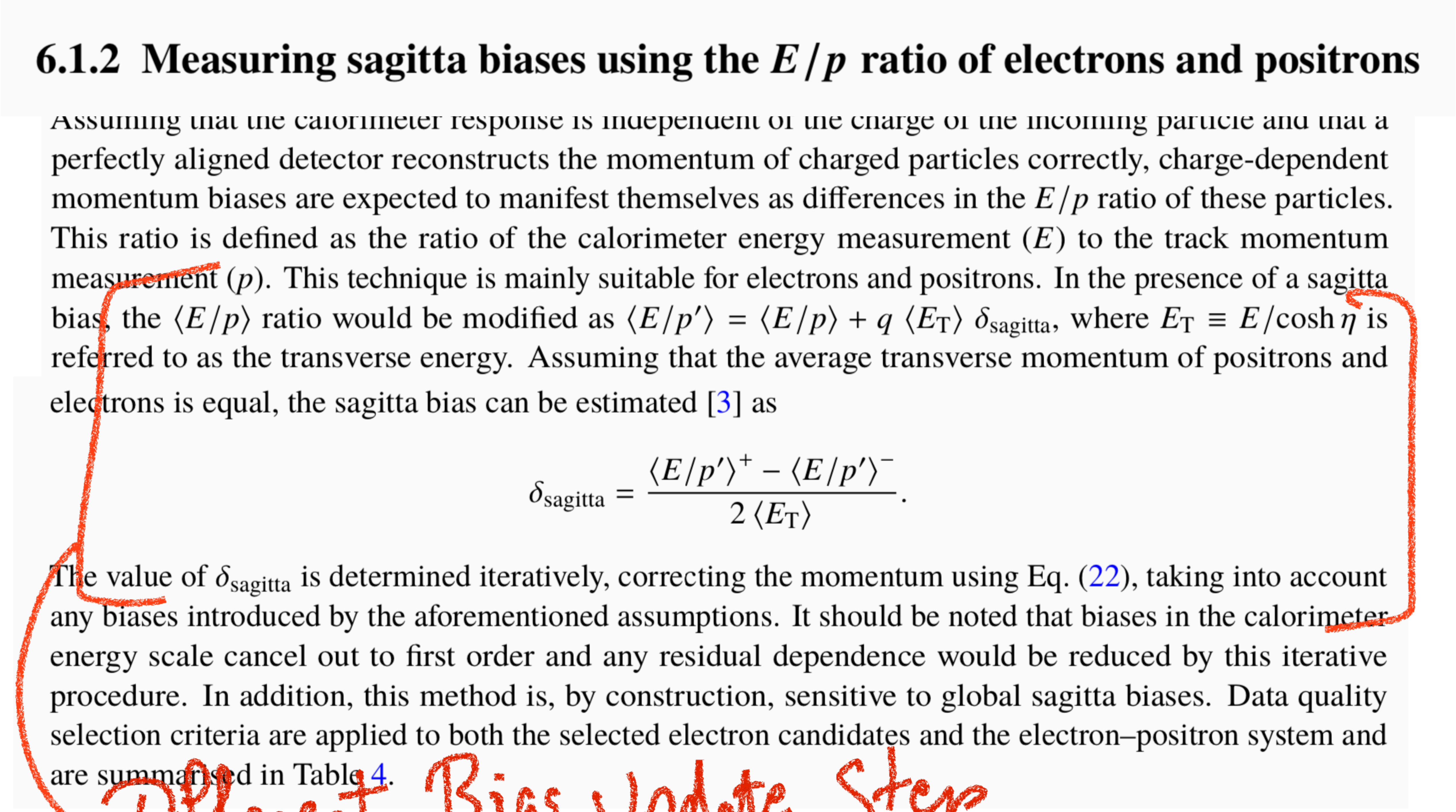
$$\delta_{\mathrm{sagitta},i}(\eta, \phi) = -q\,\frac{m^2_{\mu\mu} - m^2_Z}{2\,m^2_Z}\,\frac{\left(1 +\ q\,p'_{\mathrm{T}}\,\langle\delta_{\mathrm{sagitta},i-1}(\eta, \phi)\rangle\right)}{p'_{\mathrm{T}}} + \langle\delta_{\mathrm{sagitta},i-1}(\eta, \phi)\rangle, \tag{24}$$

where $\langle\delta_{\mathrm{sagitta},i-1}(\eta, \phi)\rangle$ is the mean of the previous iteration for all muons in that (η, ϕ) region. The value of $m^2_{\mu\mu}$ is computed as in Eq. (23) also using the mean of $\delta_{\mathrm{sagitta}}$ from the previous iteration. The iterations are repeated until convergence is reached.

Bias update step

https://arxiv.org/abs/2007.07624

Method of 6.1.2:



6.1.2 Measuring sagitta biases using the E/p ratio of electrons and positrons

Assuming that the calorimeter response is independent of the charge of the incoming particle and that a perfectly aligned detector reconstructs the momentum of charged particles correctly, charge-dependent momentum biases are expected to manifest themselves as differences in the E/p ratio of these particles. This ratio is defined as the ratio of the calorimeter energy measurement (E) to the track momentum measurement (p). This technique is mainly suitable for electrons and positrons. In the presence of a sagitta bias, the $\langle E/p \rangle$ ratio would be modified as $\langle E/p' \rangle = \langle E/p \rangle + q \, \langle E_{\mathrm{T}} \rangle \, \delta_{\text{sagitta}}$, where $E_{\mathrm{T}} \equiv E/\cosh\eta$ is referred to as the transverse energy. Assuming that the average transverse momentum of positrons and electrons is equal, the sagitta bias can be estimated [3] as

$$\delta_{\text{sagitta}} = \frac{\langle E/p' \rangle^{+} - \langle E/p' \rangle^{-}}{2 \, \langle E_{\mathrm{T}} \rangle}.$$

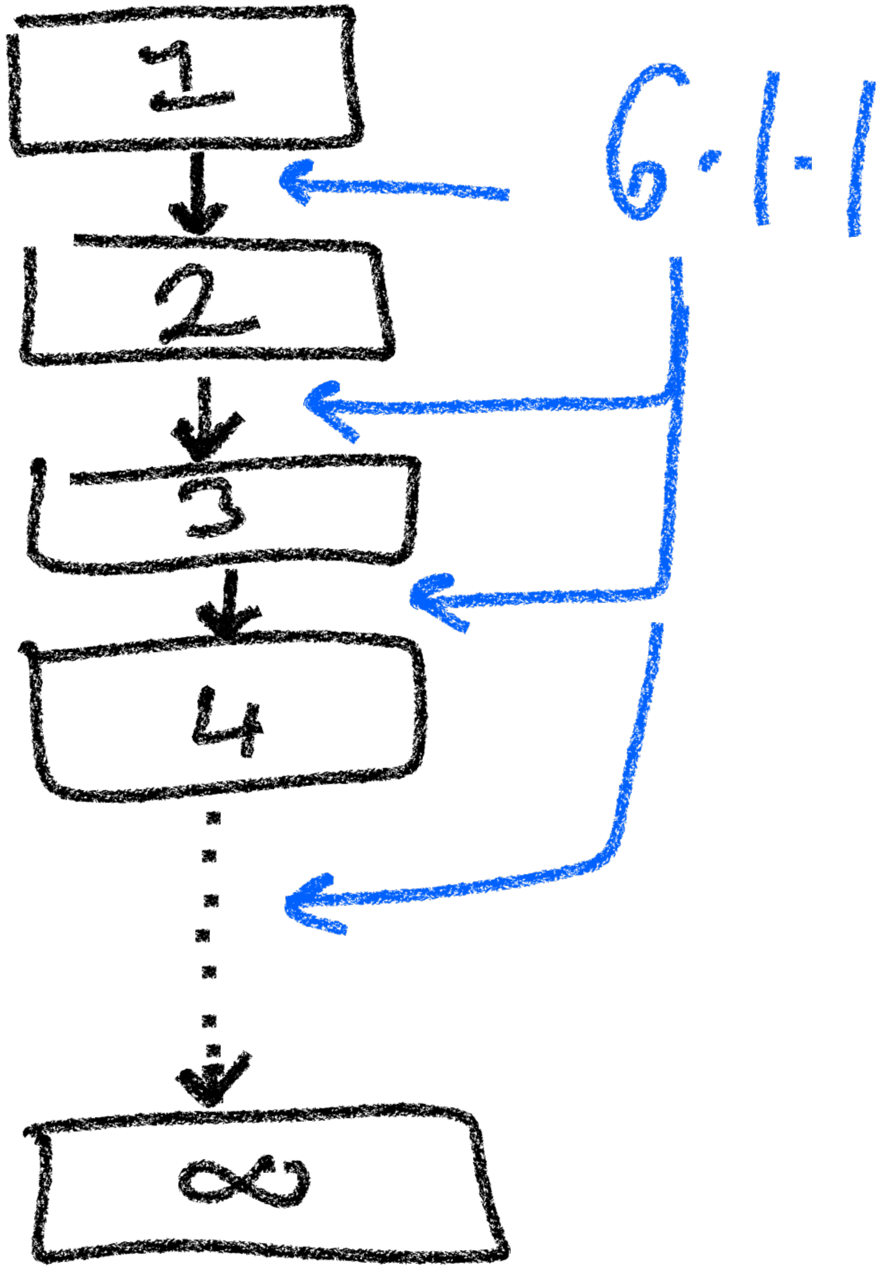
The value of δ_{sagitta} is determined iteratively, correcting the momentum using Eq. (22), taking into account any biases introduced by the aforementioned assumptions. It should be noted that biases in the calorimeter energy scale cancel out to first order and any residual dependence would be reduced by this iterative procedure. In addition, this method is, by construction, sensitive to global sagitta biases. Data quality selection criteria are applied to both the selected electron candidates and the electron–positron system and are summarised in Table 4.

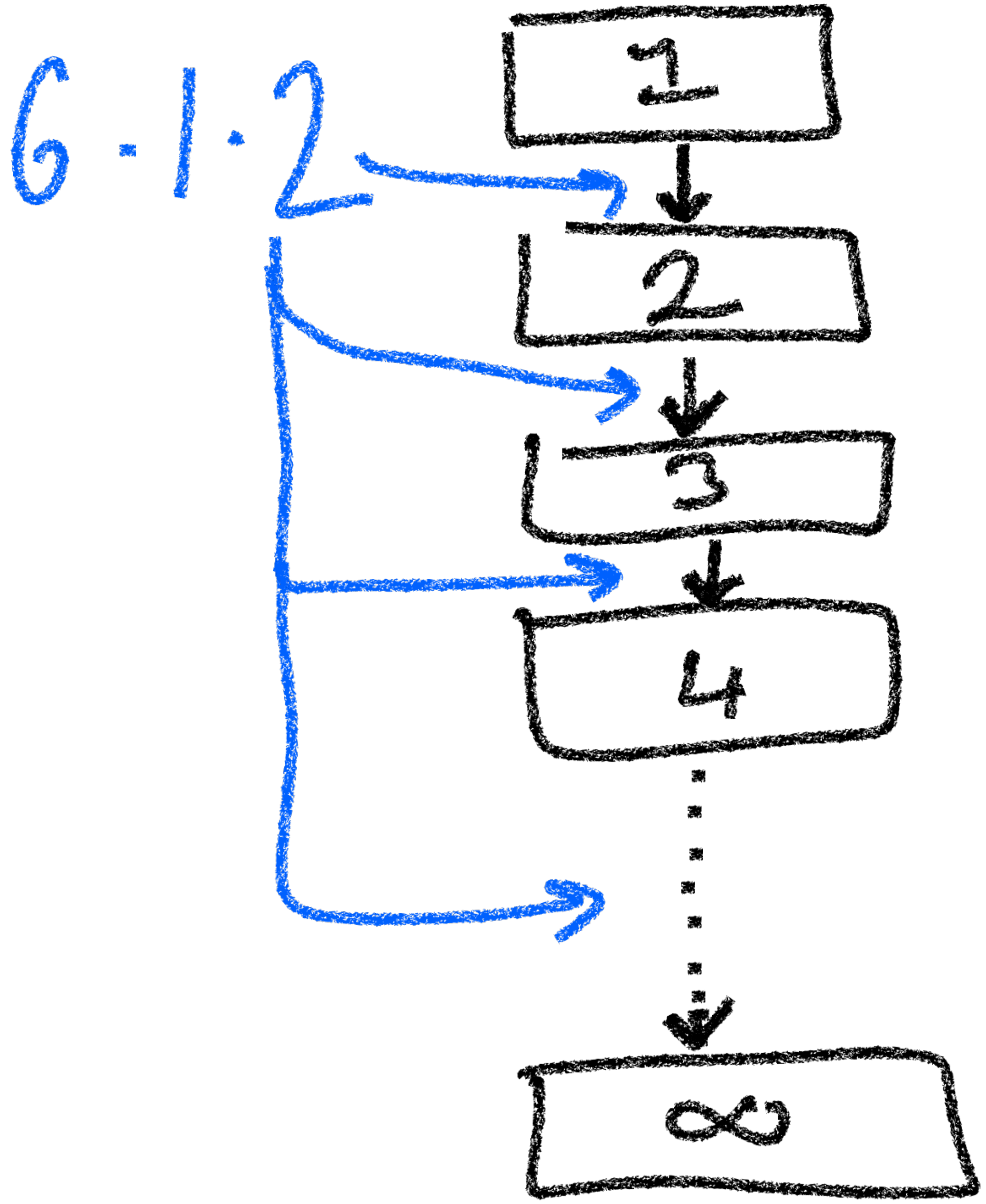
Different Bias Update Step

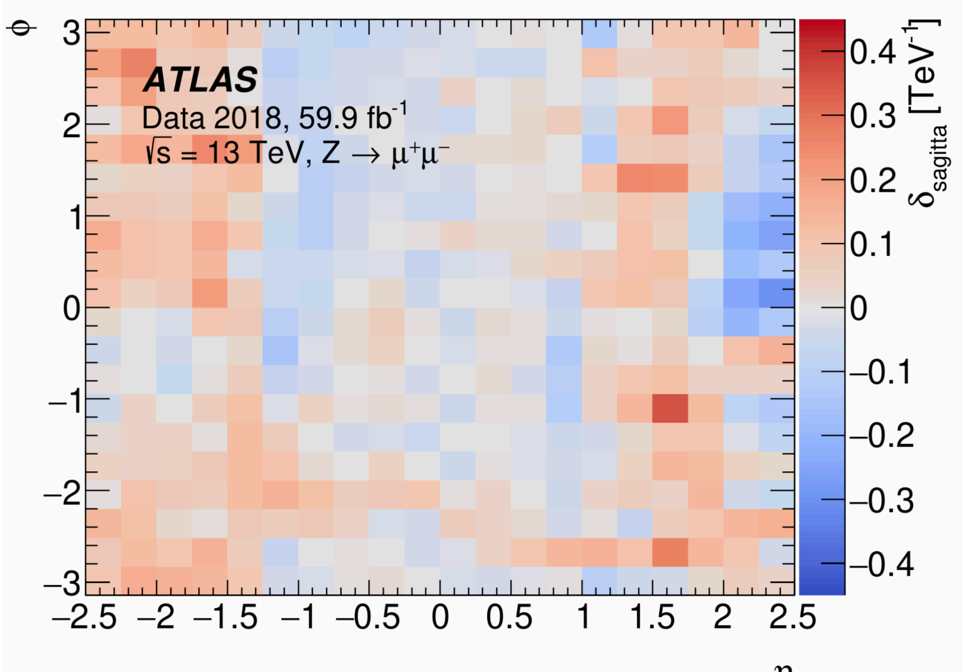
It is the nature of the

UPDATE
STEPS

which determine the biases.

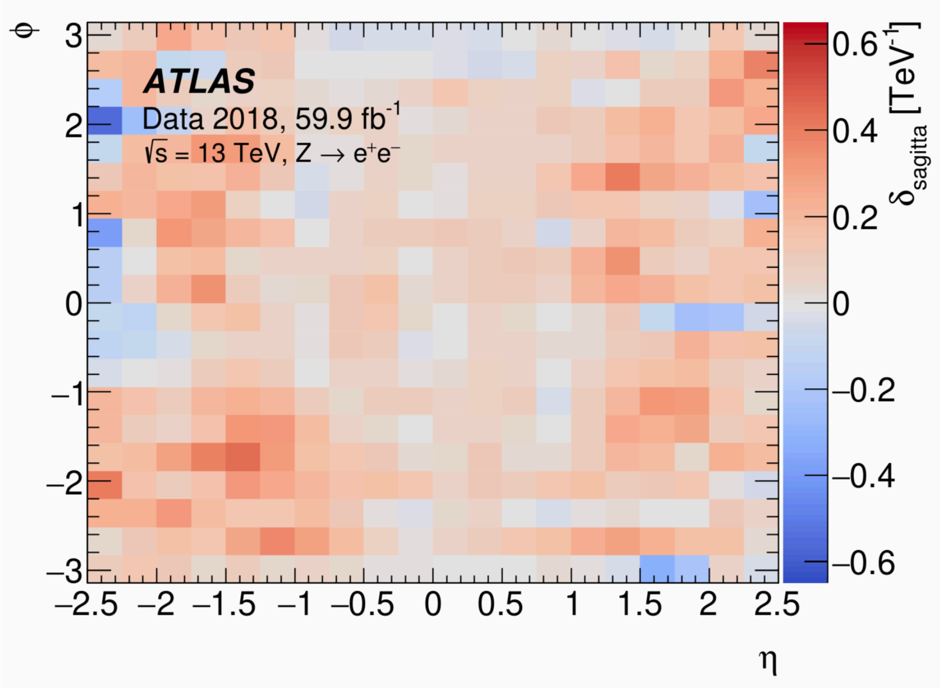






Bluer

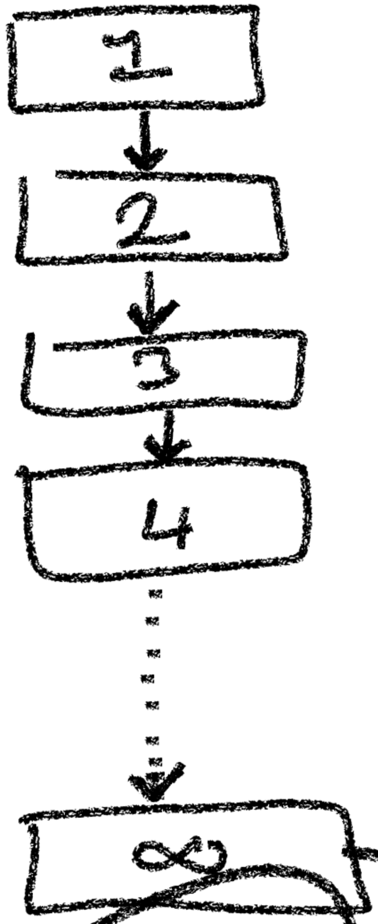
Redder

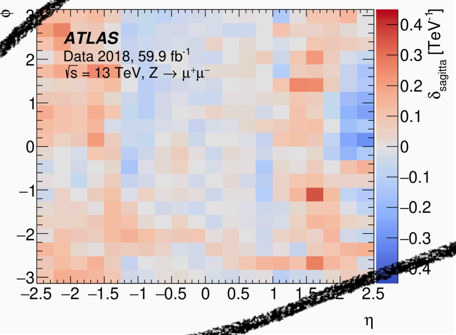


QUESTION:

What are the DESIRED PROPERTIES of the final biases?

6-1-1





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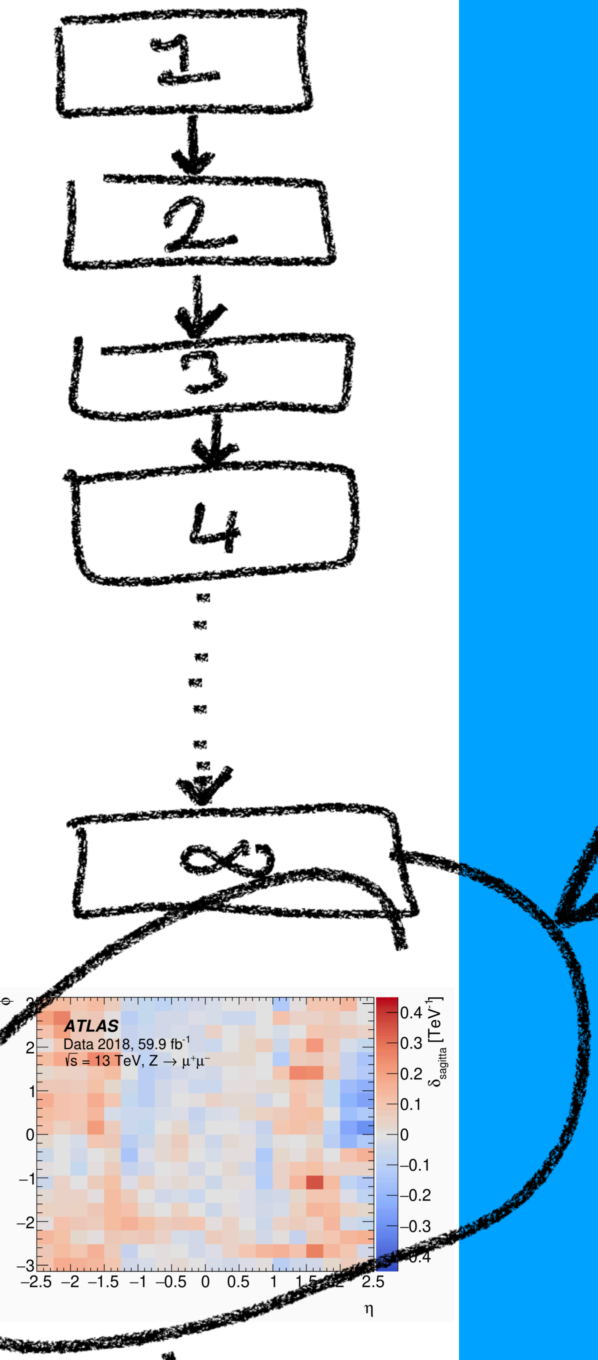
$Z \to \mu\mu$

What single DEFINING PROPERTY do these biases have?

6-1-1



Bluer

$Z \to \mu\mu$

What single DEFINING PROPERTY do these biases have?

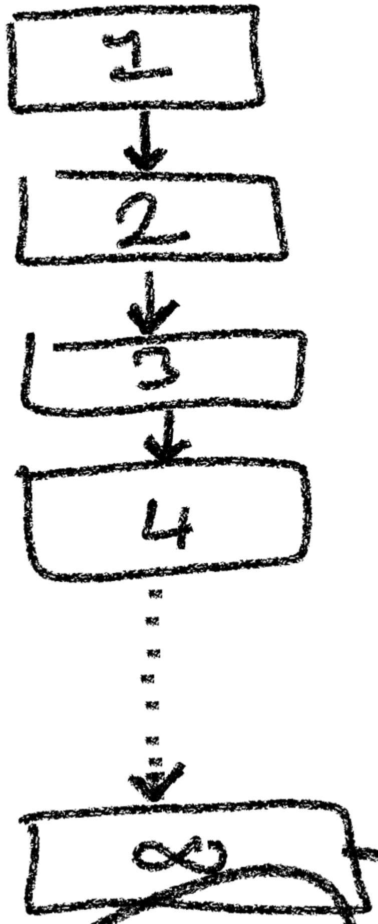
Do they minimise $\sum_i |m_{\mu\mu}^2 - m_Z^2|$?

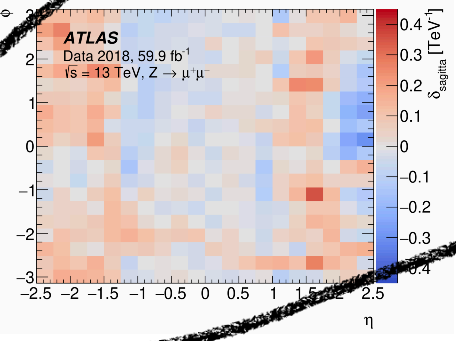
" $\sum_i (m_{\mu\mu}^2 - m_Z^2)^2$?

" $\text{Var}(m_{\mu\mu}^2)$?

" Something else ?

6-1-1





Bluer

$Z \to \mu\mu$

What single DEFINING PROPERTY do these biases have?

Do they minimise $\sum_i |m_{\mu\mu}^2 - m_Z^2|$?

$\sum_i (m_{\mu\mu}^2 - m_Z^2)^2$?

$\mathrm{Var}(m_{\mu\mu}^2)$?

Something else ?

I don't know!

6·1·1 6·1·2

Bluer Redder

Z→μμ E/p

What about these biases?

What is their DEFINING PROPERTY?

What about these biases?

What is their DEFINING PROPERTY?

I DON'T KNOW!

The reason I don't know is because the biases are defined by an

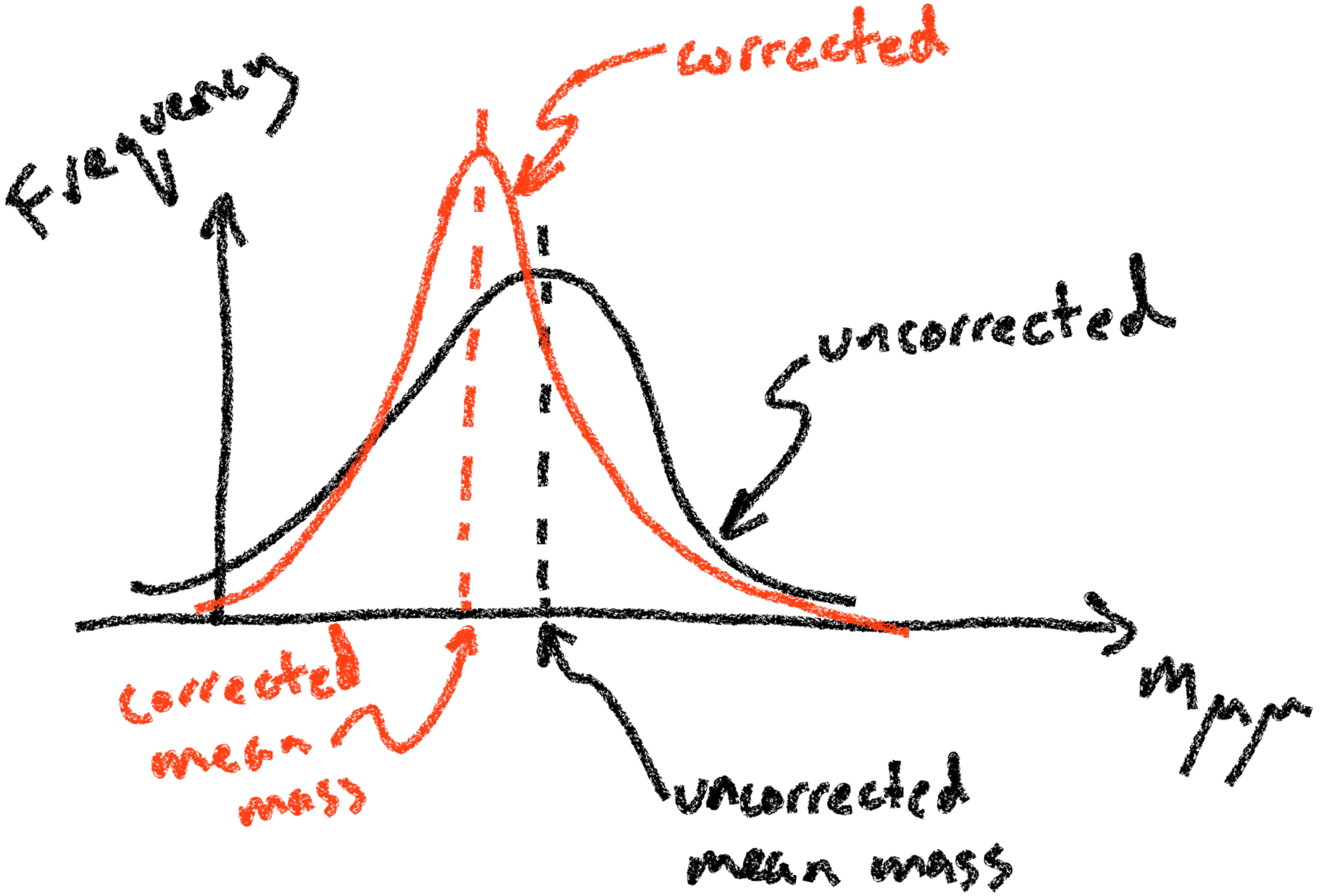
UPDATE STEP

not by an

EXPLICIT GOAL

Example of an Explicit Goal: (CGL 1)

$Z \to \mu\mu$

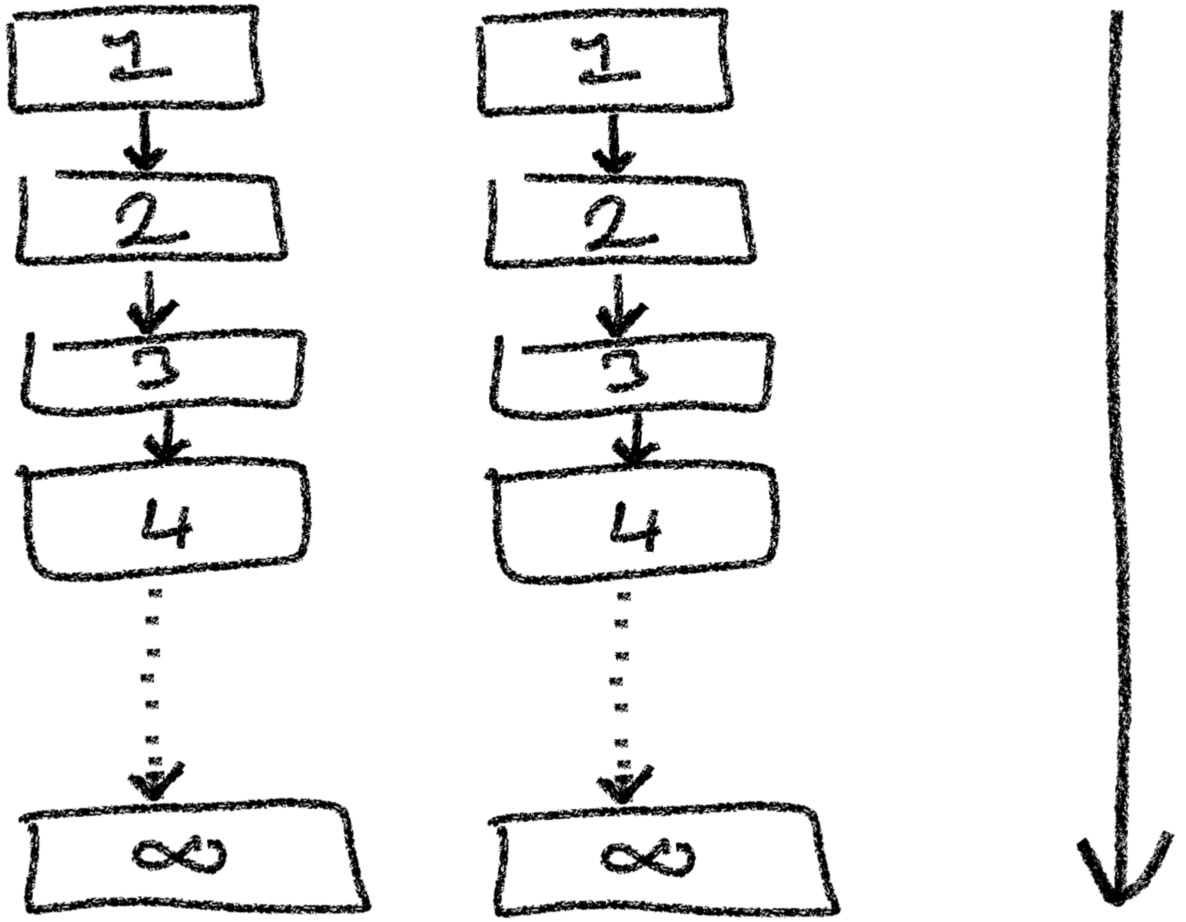


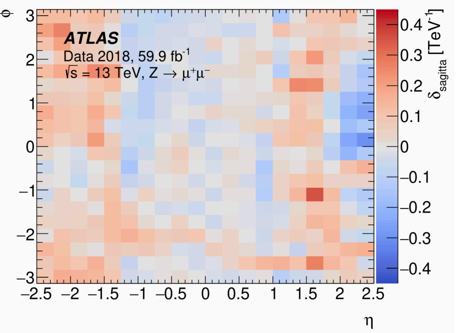
Define bins corrections to be those which $\left\{ \text{Minimise } \mathrm{Var}\left[m^2_{\mu\mu}\right] \right\}$?

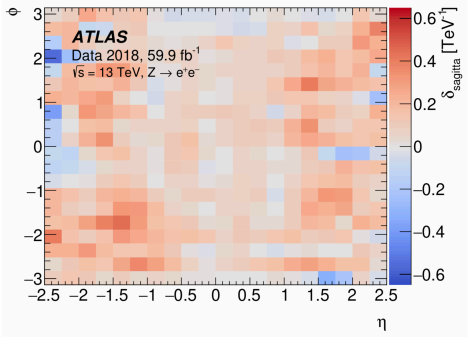
6·1·1

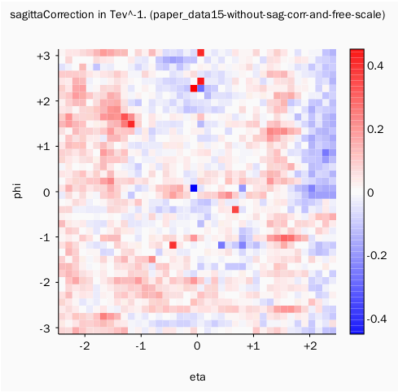
6·1·2

CGL 1









Bluer

Redder

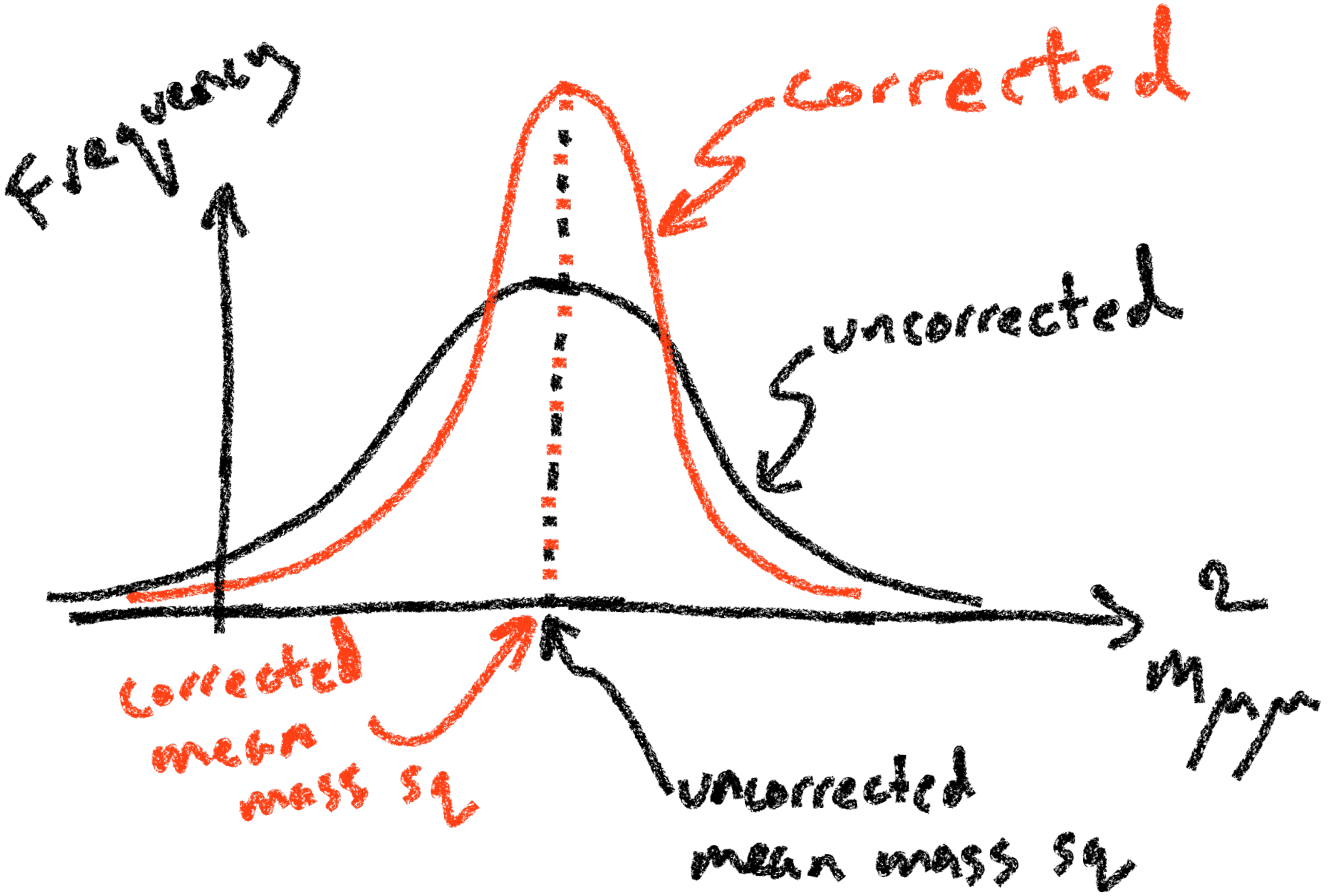
Minimal $\mathrm{Var}[m^2_{ll}]$

$Z \to \mu\mu$

E/p

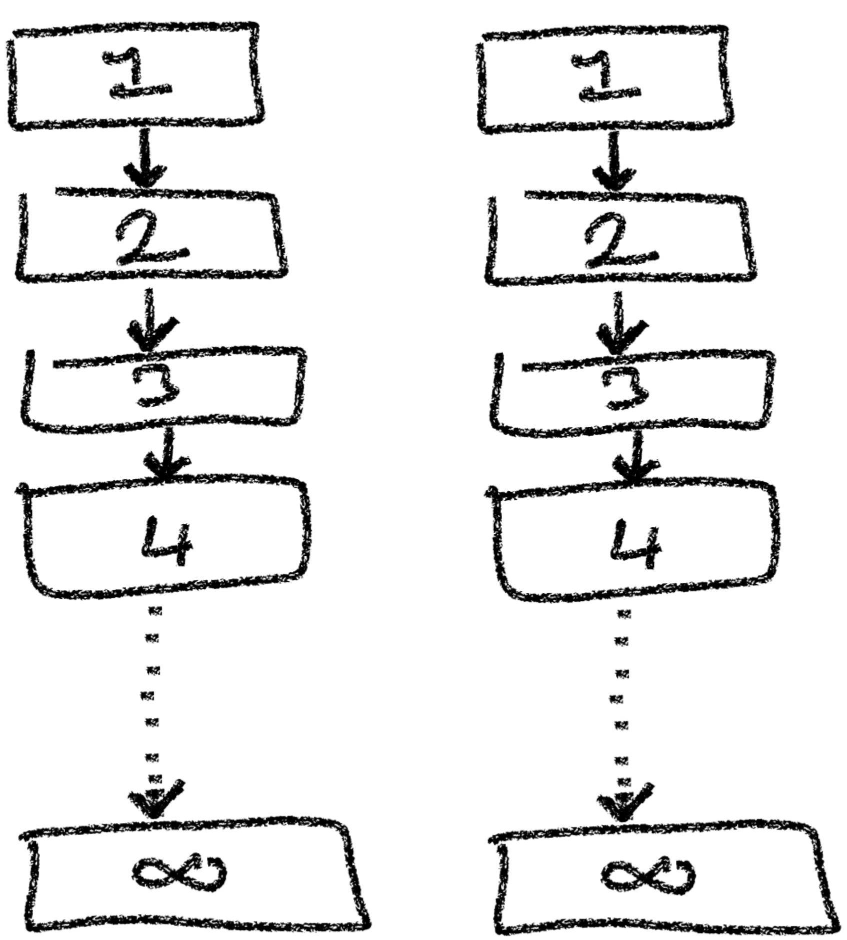
Example of an Explicit Goal : (CGL 2)

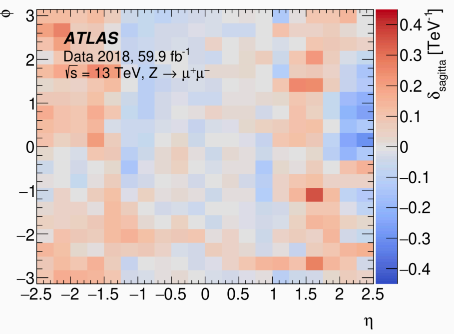
$Z \to \mu\mu$

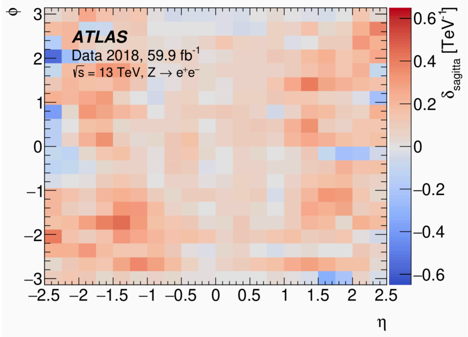


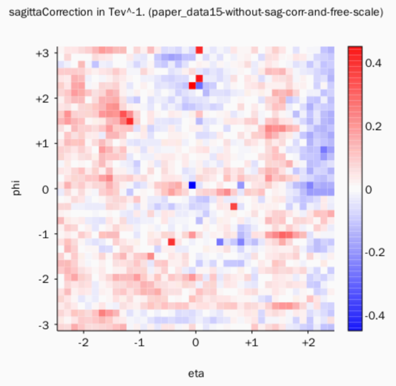
Define bins corrections to be those which $\left\{ \begin{array}{l} \text{Minimise } \mathrm{Var}\left[m^2_{\mu\mu}\right] \\ \text{without changing the mean mass squared} \end{array} \right\}$?

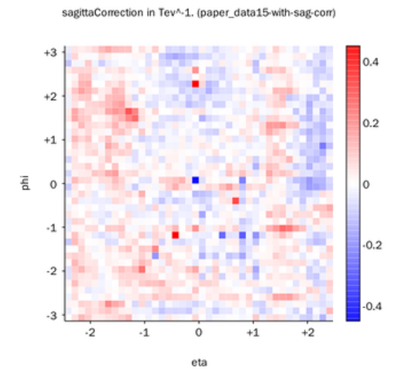
6.1.1 6.1.2 CGL 1 CGL 2











Bluer

Redder

Minimal $\mathrm{Var}[m_{ll}^2]$

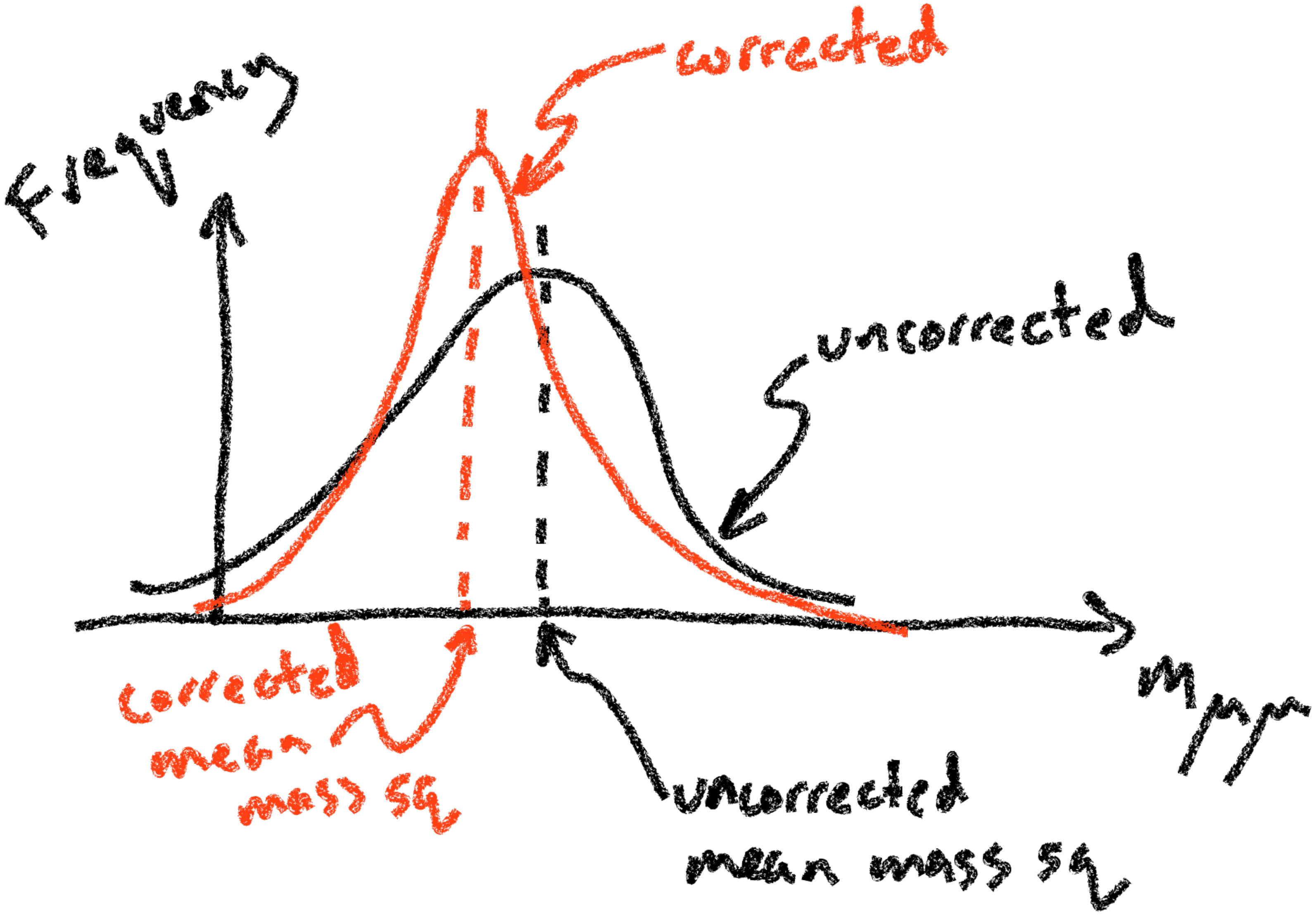
Minimal $\mathrm{Var}[m_{ll}^2]$ subject to $\langle m_{ll}^2 \rangle = \langle m_{ll}^2 \rangle_0$

$Z \to \mu\mu$

E/p

Example of an Explicit Goal: (CGL3)

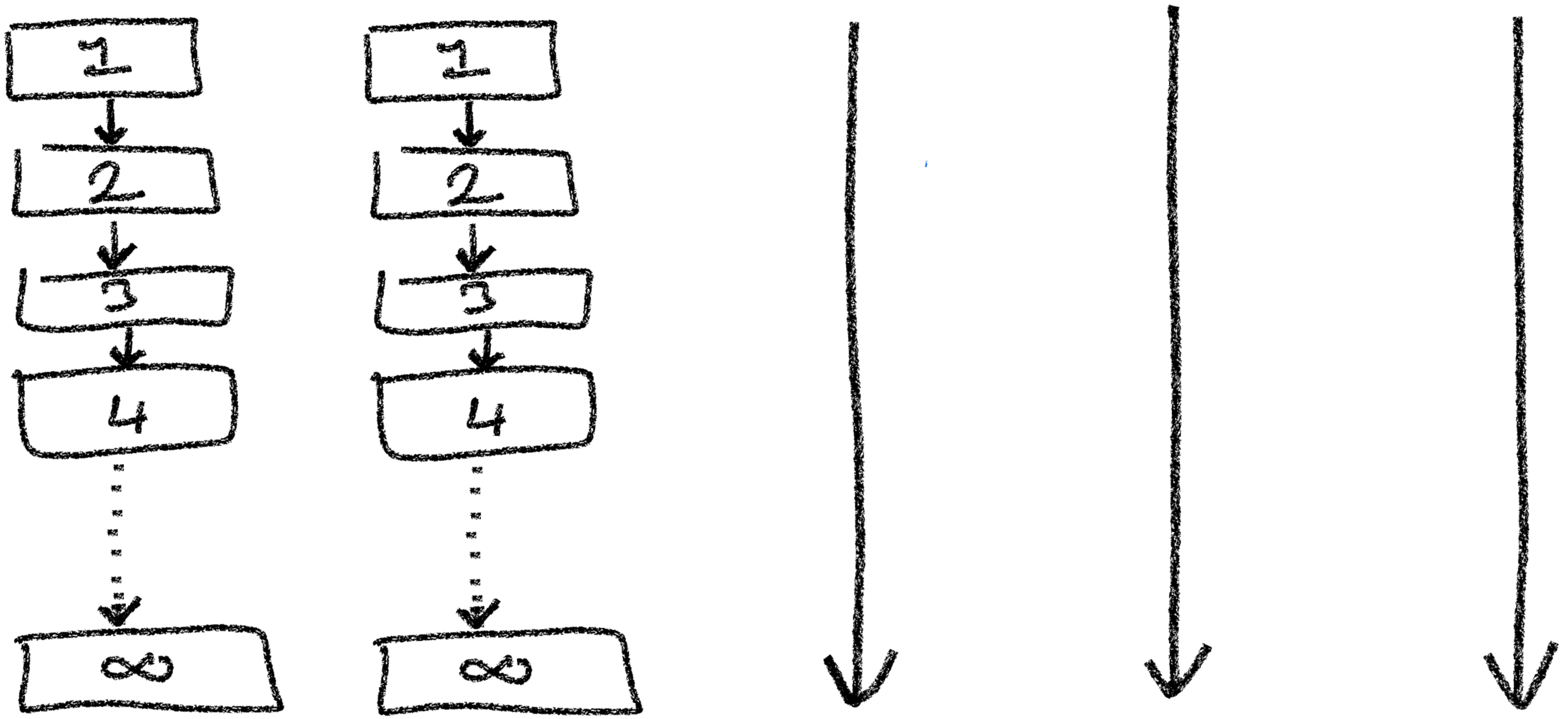
$Z \to \mu\mu$

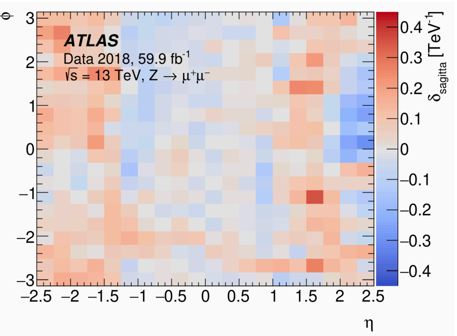


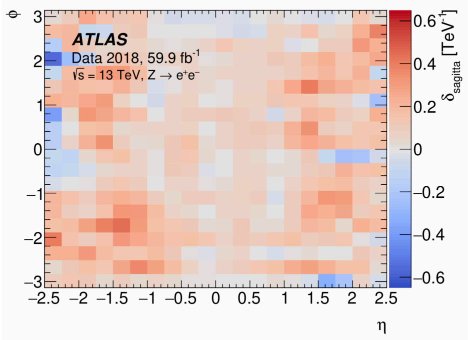
Define bins corrections to be those which

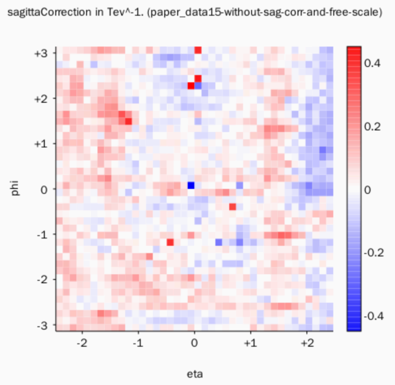
$$\left\{ \begin{array}{l} \text{Minimise } \mathrm{Var}\left[m^2_{\mu\mu}\right] \\ \text{subject to requiring} \\ \langle m^2_{\mu\mu} \rangle = (91.2\ \mathrm{GeV})^2 \end{array} \right\} ?$$

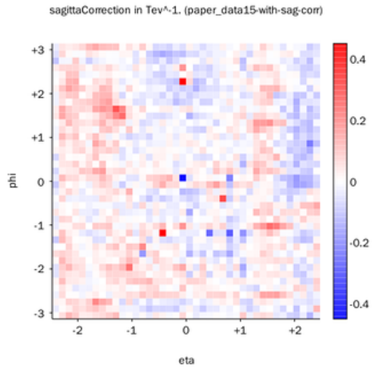
6.1.1 | 6.1.2 | CGL 1 | CGL 2 | CGL 3











?

Bluer

$Z \to \mu\mu$

Redder

E/p

Minimal $\mathrm{Var}[m_{ll}^2]$

Minimal $\mathrm{Var}[m_{ll}^2]$ subject to $\langle m_{ll}^2 \rangle = \langle m_{ll}^2 \rangle_0$

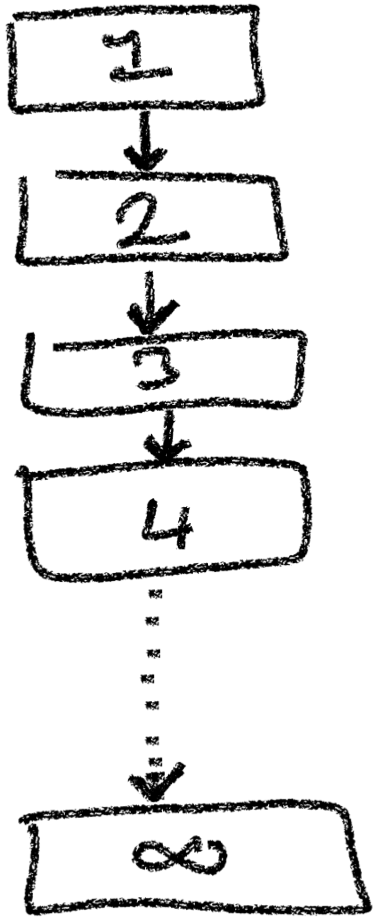
Minimal $\mathrm{Var}[m_{ll}^2]$ subject to $\langle m_{ll}^2 \rangle = 91.2$ GeV

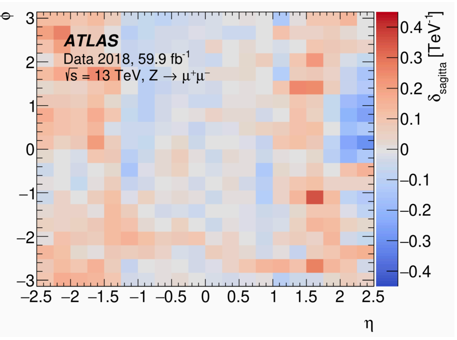
All of my examples may be inappropriate.

— But: —

You can define Your Own explicit objectives!

6.1.1





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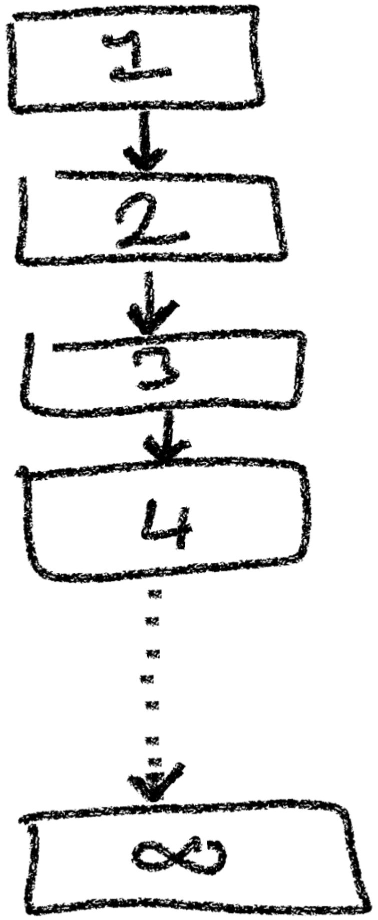
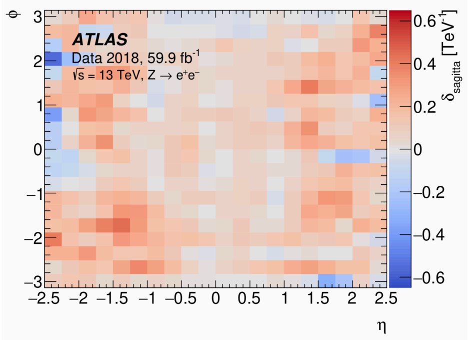
$Z \to \mu\mu$

6.1.2

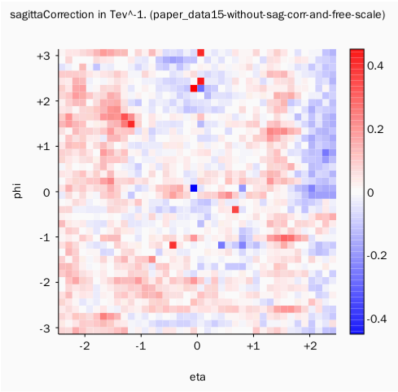




Redder

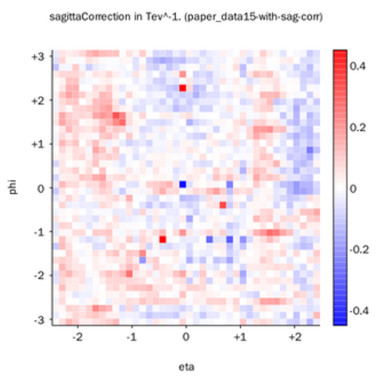
E/p

CGL 1



Minimal $\mathrm{Var}[m^2_{ll}]$

CGL 2



Minimal $\mathrm{Var}[m^2_{ll}]$ subject to $\langle m^2_{ll}\rangle = \langle m^2_{ll}\rangle_0$

CGL 3

?

Minimal $\mathrm{Var}[m^2_{ll}]$ subject to $\langle m^2_{ll}\rangle = 91.2$ GeV

Other

?

Whatever you want! ☺

How do we find bias corrections given an explicit objective?

https://gitlab.cern.ch/emus/OSDFChargeFlavourAsymmCode/-/blob/master/sagitta/lester/DOCS/SagittaBias.pdf ← write up here

Details not helpful for this forum....

....but main principles can be highlighted.

① Represent an arbitrary bias corrections in a vector $\underline{\delta} = (\delta_1, \delta_2, \ldots, \delta_B)$

one element for each bin of correction.

---

② Use method of Lagrange Multipliers to write down function to be minimised:

$$\mathcal{L} = f(\underline{\delta}, \underline{\lambda}, \text{data})$$

unknown bias corrections

Lagrange multipliers

③ Note that solution (desired bias corrections $\underline{\delta}$) will be solution to:

$$\frac{\partial L}{\partial \delta_1} = \frac{\partial L}{\partial \delta_2} = \dots = \frac{\partial L}{\partial \delta_B} = \frac{\partial L}{\partial \lambda_1} = \frac{\partial L}{\partial \lambda_2} = \dots = 0 . \tag{$\circledast$}$$

---

④ [OPTIONAL] Since the δ's should be small, linearise ⊛ in the δ's.

---

⑤ Solve ⊛ using standard linear (or non-linear) solver as appropriate.

⑥ REMARK

The explicit goals of CGL 1, CGL 2 and CGL3 do not need linearising.
They naturally end up in the form:

$$M \underline{\delta} = \underline{k}$$

a B vector which takes negligible time to fill.

a B×B matrix whose fill time is proportional to # of calibration events: (1,000,000 events/second)

Solving time depends on B. For B=40×40 solution takes ~ 10 secs.

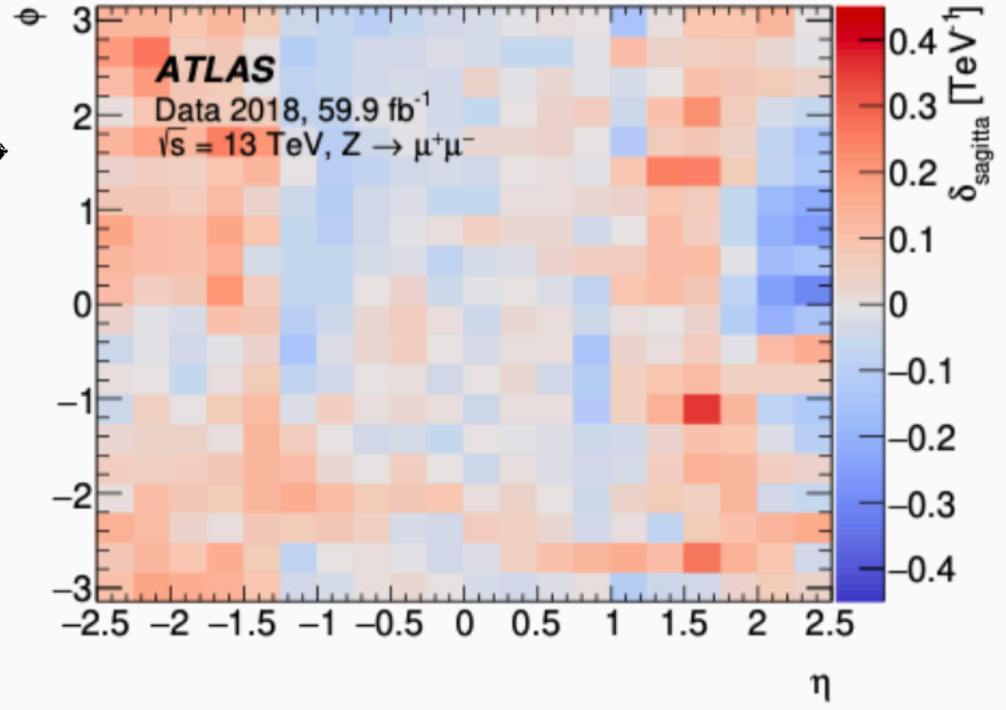
Using "EIGEN"

http://eigen.tuxfamily.org/

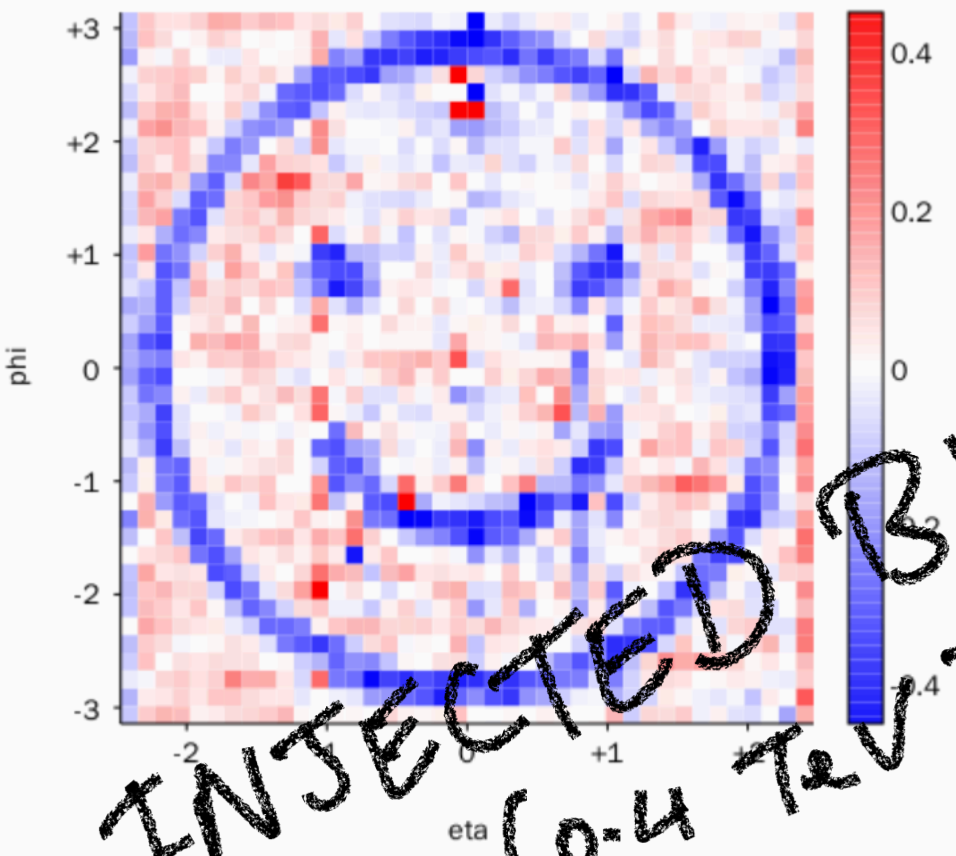
1,000,000 events Data 15

https://arxiv.org/abs/2007.07624

Z→μμ

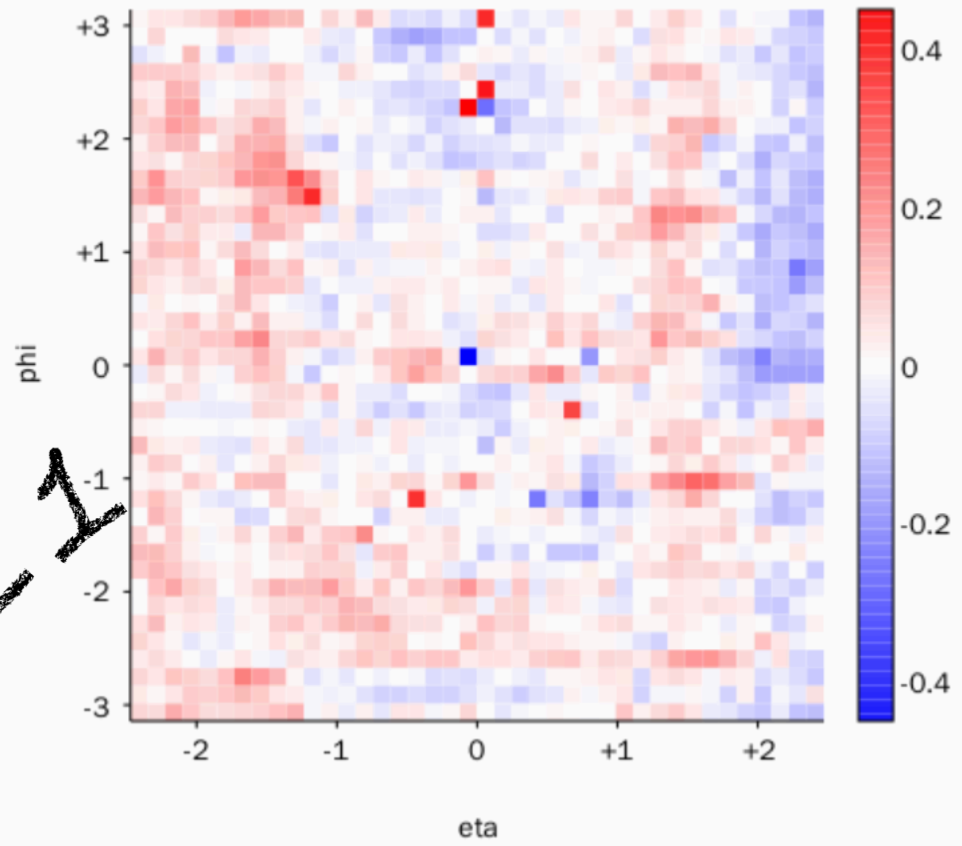


sagittaCorrection in Tev^-1. (paper_data15-without-sag-corr-and-blue-smiley)



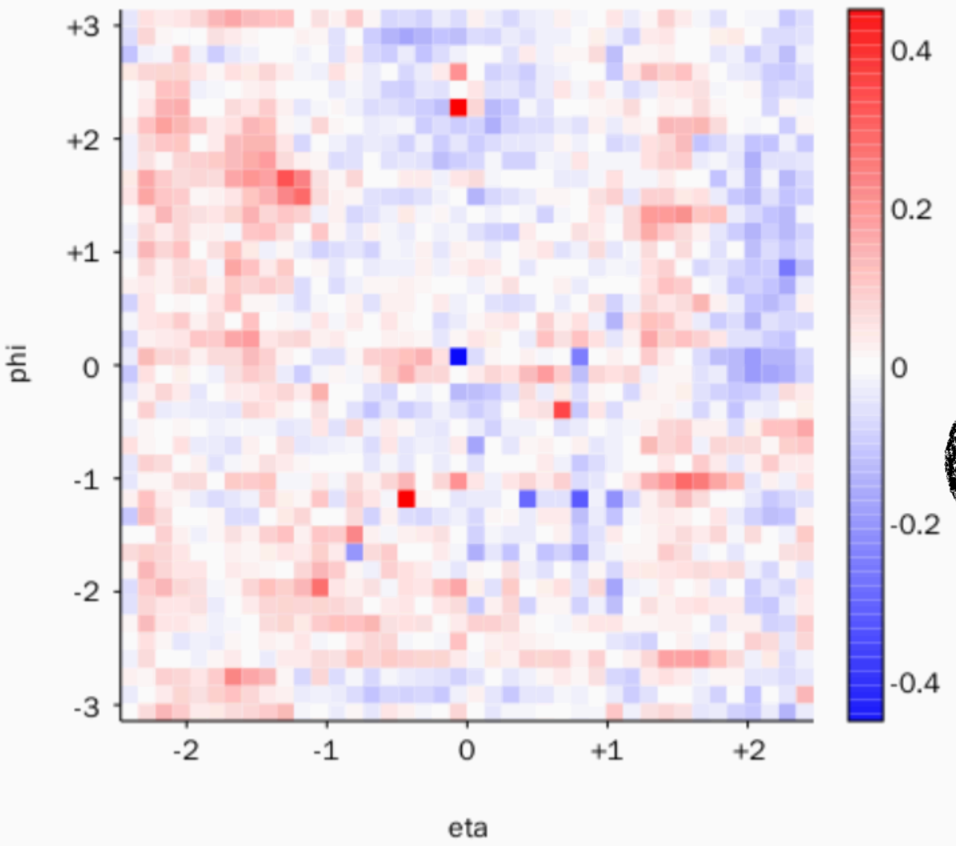
INJECTED BIAS
(0.4 TeV^{-1})

sagittaCorrection in Tev^-1. (paper_data15-without-sag-corr-and-free-scale)



CGL 1

sagittaCorrection in Tev^-1. (paper_data15-without-sag-corr)



CGL 2

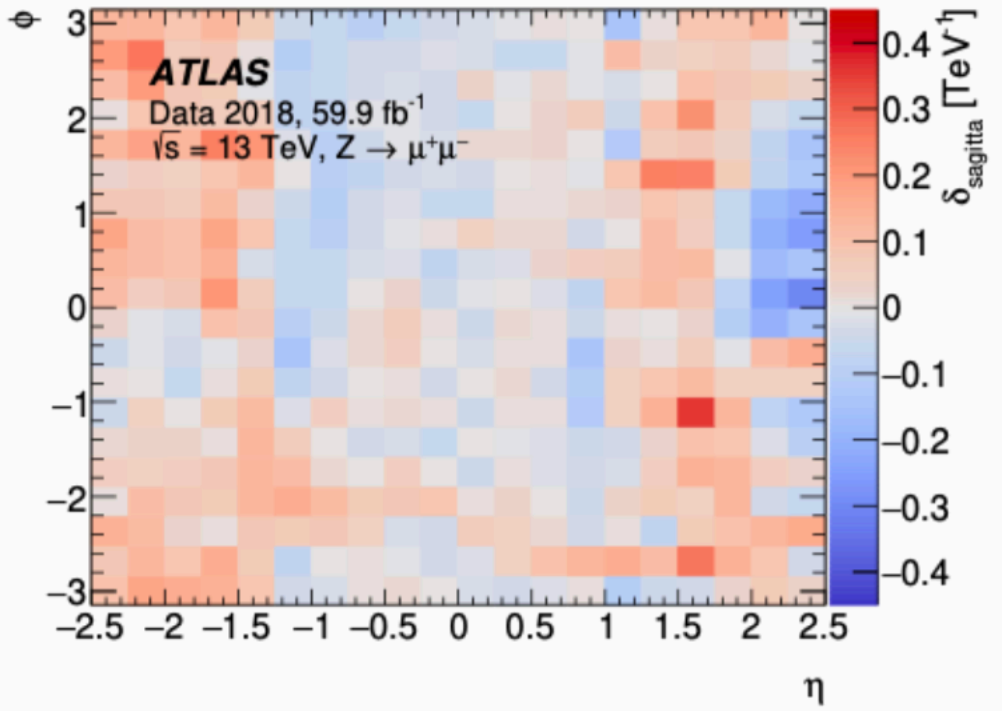
Summary

(p vs p_T)

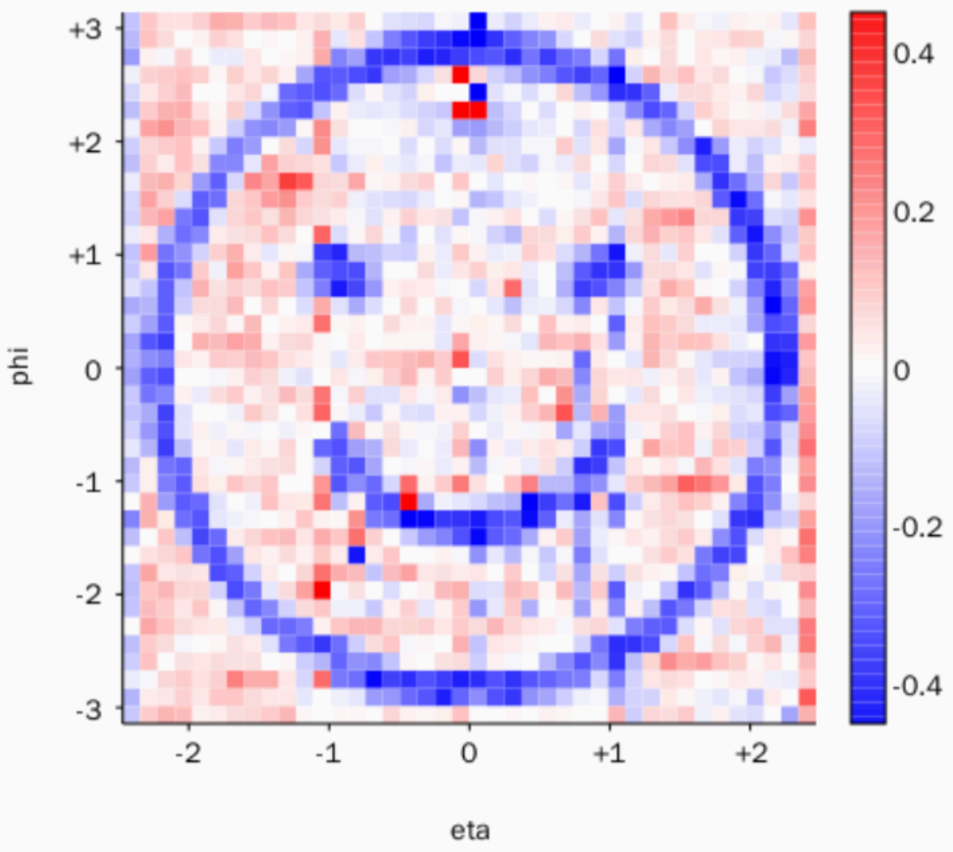
- My method has at least one (fixable) bug.
- Proof-of-Principle implementation here → https://gitlab.cern.ch/emus/OSDFChargeFlavourAsymmCode/-/blob/master/sagitta/lester/root_to_matrix.cc
- Write up here → https://gitlab.cern.ch/emus/OSDFChargeFlavourAsymmCode/-/blob/master/sagitta/lester/DOCS/SagittaBias.pdf
- Nothing I've said or done here invalidates any of your existing work.
- I am not a calibration or muon expert.
- Happy to help if any of this is any use to you.

Backup

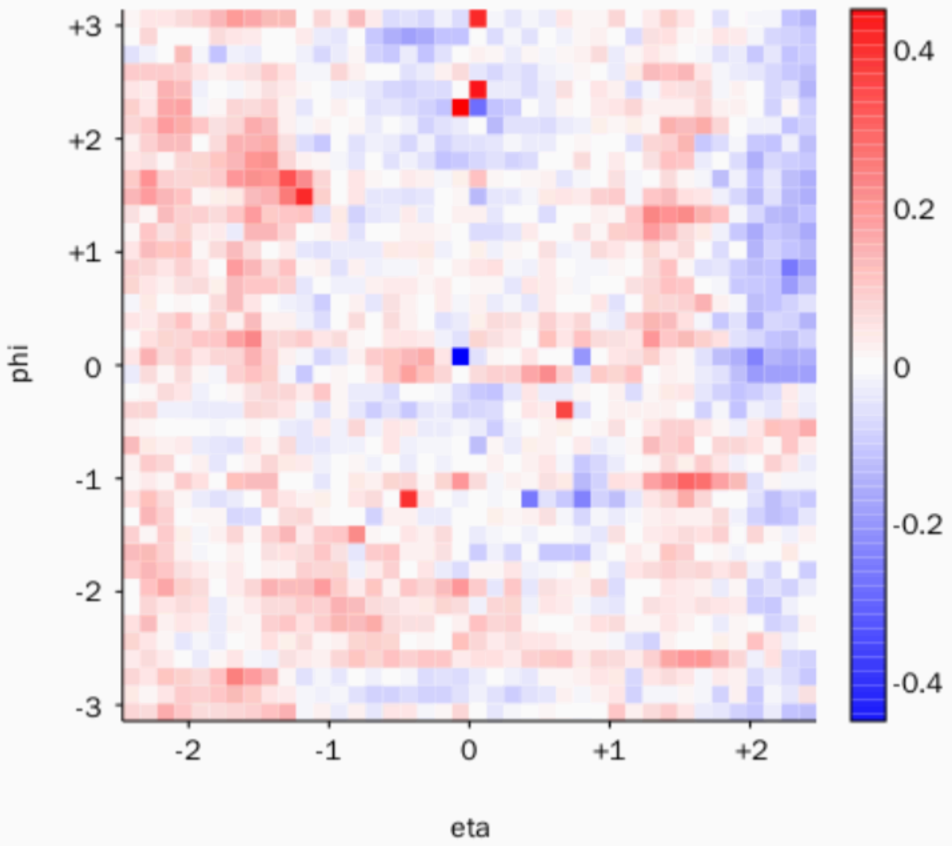
https://arxiv.org/abs/2007.07624



sagittaCorrection in Tev^-1. (paper_data15-without-sag-corr-and-blue-smiley)



sagittaCorrection in Tev^-1. (paper_data15-without-sag-corr-and-free-scale)



sagittaCorrection in Tev^-1. (paper_data15-without-sag-corr)