$$
\begin{aligned}
&+1024\, p_2.p_3\; p_1.p_6\; p_5.p_3\; m_4^2/m_3^2\\
&+2048\, p_2.p_3\; p_1.p_7\; p_5.p_3\; p_6.p_7\; m_4^2/(m_3^2\; m_7^2)\\
&+1024\, p_2.p_3\; p_1.p_3\; p_4.p_5\; p_4.p_6/m_3^2\\
&+2048\, p_2.p_3\; p_1.p_3\; p_4.p_5\; p_4.p_7\; p_6.p_7/(m_3^2\; m_7^2)\\
&+2048\, p_2.p_3\; p_1.p_3\; p_4.p_6\; p_4.p_3\; p_5.p_3/m_3^4\\
&+4096\, p_2.p_3\; p_1.p_3\; p_4.p_7\; p_4.p_3\; p_5.p_3\; p_6.p_7/(m_3^4\; m_7^2)\\
&-512\, p_2.p_3\; p_1.p_3\; p_5.p_6\; m_4^2/m_3^2\\
&-1024\, p_2.p_3\; p_1.p_3\; p_5.p_7\; p_6.p_7\; m_4^2/(m_3^2\; m_7^2)\\
&-2048\, p_2.p_3\; p_1.p_3\; p_5.p_3\; p_6.p_7\; p_7.p_3\; m_4^2/(m_3^4\; m_7^2)\\
&-1024\, p_2.p_3\; p_1.p_3\; p_5.p_3\; p_6.p_3\; m_4^2/m_3^4. \qquad (78)
\end{aligned}
$$

C   q lfar Invariant Mass Distribution

The work in this section was all done with Chris Lester.

Here we attempt to find an analytical expression for the quark and far lepton invariant mass distribution. As the invariant involves two particles which are maximally far apart in the chain in this case, it is a more involved calculation.

We define angles θ, ϕ and ψ such that $p_q = (c, c\cos\theta, c\sin\theta, 0)$ in the rest frame of the Z^* and $p_{lfar} = (p, p\sin\psi\cos\phi, p\sin\psi\sin\phi, p\cos\psi)$ in the rest frame of the l^*. These definitions give us the following expression for the total phase space cross section up to an overall constant (all equations in this file are only defined up to overall constants):

$$\sigma = \int_{-1}^{1} \mathrm{d}(\cos\theta) \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{+1} \mathrm{d}(\cos\psi). \qquad (79)$$

Upon boosting p_{lfar} into the rest frame of the Z^*, we get the following definition for the q lfar invariant mass squared:

$$m_{qlfar}^2 = -2cp\gamma(-1 + \beta\cos\theta + (-\beta + \cos\theta)\cos\phi\sin\psi + \gamma^{-1}\sin\theta\sin\phi\sin\psi), \qquad (80)$$

where β is the speed of the boost equal to $(M_{Z^*}^2 - M_{l^*}^2)/(M_{Z^*}^2 + M_{l^*}^2)$ (where we have taken the l^* to be travelling along the x-axis in the rest frame of the Z^*) and $\gamma = 1/\sqrt{1-\beta^2}$ (natural units).

Now, from (79), we can write

$$\frac{\mathrm{d}I}{\mathrm{d}m} = \int_{-1}^{1} \mathrm{d}(\cos\theta) \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{+1} \mathrm{d}(\cos\psi)\; \delta(m_{qlfar}^2 - m^2) \qquad (81)$$

where m^2 is a variable representing fixed m_{qlfar}^2. However, it is actually easier to calculate

$$I = \int_{-1}^{1} \mathrm{d}(\cos\theta) \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{+1} \mathrm{d}(\cos\psi)\Theta(m^2 - m_{qlfar}^2) \qquad (82)$$

and then differentiate to get the required distribution function.

As the only ψ dependence in $m^2_{ql^{far}}$ is a linear dependence on $\sin\psi$, we will look at that integral first. Changing variables from $\cos\psi$ to $y = \sin\psi$, we can write (82) as

$$\begin{aligned} I &= \int_{-1}^{1} \mathrm{d}(\cos\theta) \int_0^{2\pi} \mathrm{d}\phi \left(\int_0^1 \mathrm{d}y \frac{y}{\sqrt{1-y^2}} - \int_1^0 \mathrm{d}y \frac{y}{\sqrt{1-y^2}} \right) \Theta(A+By) \\ &= 2\int_{-1}^{1} \mathrm{d}(\cos\theta) \int_0^{2\pi} \mathrm{d}\phi \int_0^1 \mathrm{d}y \frac{y}{\sqrt{1-y^2}} \Theta(A+By), \end{aligned} \tag{83}$$

where we have defined

$$\begin{aligned} A &= m^2 - 2rpQ, \\ Q &= \gamma(1-\beta\cos\theta), \\ B &= 2rpQ\cos(\phi-\lambda(\theta)), \\ \lambda(\theta) &= \tan^{-1}\left(\frac{\sin\theta}{\gamma\cos\theta - \gamma\beta}\right). \end{aligned} \tag{84}$$

We also define the scaled invariant mass squared to be $k^2 = m^2/m^2_{max}$ where $m_{max} = 4rp\gamma(1+\beta)$, such that $k \in [0,1]$.

Using the relation $\Theta(A+By) = \Theta(y + A/B)\Theta(B) + \Theta(-y - A/B)\Theta(-B)$, we can split (83) into:

$$\begin{aligned} \frac{I}{2} &= \int_{A>0} \mathrm{d}(\cos\theta) \int_{B>0} \mathrm{d}\phi \, 1 + \int_{A<0} \mathrm{d}(\cos\theta) \int_{B>0} \mathrm{d}\phi \sqrt{1-Y^2} \\ &\quad + \int_{A>0} \mathrm{d}(\cos\theta) \int_{B<0} \mathrm{d}\phi \, 1 - \int_{A>0} \mathrm{d}(\cos\theta) \int_{B<0} \mathrm{d}\phi \sqrt{1-Y^2} \\ &= I_5 \quad + \quad I_6 \quad + \quad I_7 \quad + \quad I_8 \end{aligned} \tag{85}$$

where $Y = \min(1, -\frac{A}{B})$, (note that it only features in the integrals where A and B have opposite sign and is a relic of the Heaviside function).

At this point it is helpful to define a new variable $\hat{c}$ where

$$\hat{c} = \begin{cases} +1 & \frac{R^2}{2} > k^2 & A > 0 \\ \frac{1-2k^2(1+\beta)}{\beta} & \frac{R^2}{2} \le k^2 \le \frac{1}{2} & A = 0 \text{ for } \cos\theta = \hat{c} \\ -1 & k^2 > \frac{1}{2} & A < 0 \end{cases}, \tag{86}$$

where $R^2 = \frac{m^2_{l^*}}{m^2_{Z^*}} = \frac{1-\beta}{1+\beta}$. While this looks obscure, it simply defines the regions in which A is positive or negative. Using the definition of A in (84), $A > 0$ for $\cos\theta > \hat{c}$ and vice versa.

First consider the integrals without square roots:

$$I_5 + I_7 = \int_{A>0} \mathrm{d}(\cos\theta) \int_0^{2\pi} \mathrm{d}\phi \, 1 = 2\pi \int_{\hat{c}}^{1} \mathrm{d}(\cos\theta) \, 1 = 2\pi(1-\hat{c}). \tag{87}$$

Now look at the remaining two integrals which depend on Y:

$$I_6+I_8 = \int_{A<0} \mathrm{d}(\cos\theta) \int_{\lambda-\pi/2}^{\lambda+\pi/2} \mathrm{d}\phi\sqrt{1-Y^2} - \int_{A>0} \mathrm{d}(\cos\theta) \int_{\lambda+\pi/2}^{\lambda+3\pi/2} \mathrm{d}\phi\sqrt{1-Y^2}. \tag{88}$$

It is helpful here to define a variable $\tilde{c}$ where

$$\tilde{c} \quad = \quad \begin{cases} +1 & k^2 < R^2 \qquad \frac{|A|}{2rp\gamma Q} < 1 \\ \frac{1-k^2(1+\beta)}{\beta} & R^2 \leq k^2 \leq 1 \qquad \frac{|A|}{2rp\gamma Q} < 1 \text{ for } \cos\theta < \tilde{c} \end{cases} . \tag{89}$$

This is used to define the regions where Y takes different values. $Y = -\frac{A}{B}$ for $\cos\theta < \tilde{c}$, and equals 1 otherwise. There is also an additional constraint on ϕ which defines these regions which will be implemented through the following change of variable from ϕ to $\mu(\phi)$. (Note that when $Y = 1$, the integrand is zero so we are only interested in the section where $Y \neq 1$). Define

$$\mu(\phi) = \tan^{-1}\left(\frac{\sin\phi}{\sqrt{\cos^2\phi - a^2}}\right) \tag{90}$$

where $a = \frac{|A|}{B} < 1$ from above, and this covers the range $|\cos\phi| > a$. With this change, the integrand becomes $\left(1 - \frac{\sec^2\mu}{\left(\frac{1}{a}\right)^2 + \tan^2\mu}\right)$ sign$(\cos\phi)$. The limits of I_6 are from $-\pi/2$ to $\pi/2$ and of I_8 are from $\pi/2$ to $-\pi/2$. So,

$$\begin{aligned} I_6 &= \int_{A<0,\cos\theta<\tilde{c}} \mathrm{d}(\cos\theta) \int_{-\pi/2}^{\pi/2} \mathrm{d}\mu\, 1 - \frac{\sec^2\mu}{\left(\frac{1}{a}\right)^2 + \tan^2\mu} \\ &= \int_{A<0,\cos\theta<\tilde{c}} \mathrm{d}(\cos\theta)\ \pi(1-a) \\ &= \pi\int_{-1}^{\min(\hat{c},\tilde{c})} \mathrm{d}(\cos\theta)\ 1 - \left(1 - \frac{m^2}{2rp\gamma(1-\beta\cos\theta)}\right) \\ &= \pi\left[\frac{-m^2}{2rp\gamma\beta}\log(1-\beta\cos\theta)\right]_{-1}^{\hat{c}} \\ &= \frac{-m^2\pi}{2rp\gamma\beta}\log\left(\frac{1-\beta\hat{c}}{1+\beta}\right), \end{aligned} \tag{91}$$

where we have used $A < 0$ to give $a = -A/2rpQ$, and (on careful inspection) that $\min(\hat{c},\tilde{c})=\hat{c}$.

I_8 was originally defined with a minus sign, has picked up a minus from the sign of $\cos\phi$ and another one from reversing the order of the limits to give

$$\begin{aligned} I_8 &= -\int_{A>0,\cos\theta<\tilde{c}} \mathrm{d}(\cos\theta) \int_{-\pi/2}^{\pi/2} \mathrm{d}\mu\, 1 - \frac{\sec^2\mu}{\left(\frac{1}{a}\right)^2 + \tan^2\mu} \\ &= -\int_{\hat{c}}^{\tilde{c}} \mathrm{d}(\cos\theta)\ \pi(1-a) \end{aligned}$$

$$\begin{aligned}
&= -\pi \int_{\hat{c}}^{\tilde{c}} \mathrm{d}(\cos\theta)\ 1 - \left(\frac{m^2}{2rp\gamma(1-\beta\cos\theta)} - 1\right) \\
&= -\pi \left[2\cos\theta + \frac{m^2}{2rp\gamma\beta}\log(1-\beta\cos\theta)\right]_{\hat{c}}^{\tilde{c}} \\
&= -\pi \left(2(\tilde{c}-\hat{c}) + \frac{m^2}{2rp\gamma\beta}\log\frac{1-\beta\tilde{c}}{1-\beta\hat{c}}\right). \qquad (92)
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{I}{2} &= I_5 + I_6 + I_7 + I_8 \\
&= 2\pi(1-\hat{c}) + 2\pi(\hat{c}-\tilde{c}) - \frac{m^2}{2rp\gamma\beta}\log\frac{1-\beta\tilde{c}}{1+\beta} \\
&= 2\pi(1-\tilde{c}) - \frac{m^2}{2rp\gamma\beta}\log\frac{1-\beta\tilde{c}}{1+\beta}. \qquad (93)
\end{aligned}$$

Note that this is independent of $\hat{c}$ and so the boundaries in values of $\hat{c}$, which may have led to different behaviour in each region, effectively vanish. This is not the case for the boundaries in $\tilde{c}$. For $k^2 < R^2$, $\tilde{c}$ is a constant (in fact, $+1$), while for $R^2 \leq k^2 \leq 1$, $\tilde{c}$ depends linearly on k^2. So the distribution in k^2 space looks like:

$$\frac{\mathrm{d}I}{\mathrm{d}k^2} = \begin{cases} -\frac{2\pi(1+\beta)}{\beta}\log(R^2) & k^2 \leq R^2 \\ -\frac{2\pi(1+\beta)}{\beta}\log(k^2) & k^2 > R^2 \end{cases} \qquad (94)$$

on substitution of $\tilde{c}$ and R. Note that R^2 and k^2 lie between 0 and 1 and so this is always positive.

This translates to a distribution in k space of:

$$\frac{\mathrm{d}I}{\mathrm{d}k} = \begin{cases} -\frac{8\pi(1+\beta)}{\beta}k\log(R) & k \leq R \\ -\frac{8\pi(1+\beta)}{\beta}k\log(k) & k > R \end{cases}. \qquad (95)$$

This is then appropriately normalised to 1. I am currently working on doing a similar integral for the UED and MSSM angular distributions, but this is considerably harder still as it involves integrating over complicated functions of the angles.

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