

Why Do We Have Three Families ?

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- We have many particles;
 $(5^* + 10 + 1) \times 3$ + a Higgs + Gauge Fields
48 quarks/leptons

Why do we have so many particles ?

String theories \rightarrow many many fields

But, we can not explain why we have three families of quarks and leptons

Composite Bound States

~ 1970-1980

If quarks and leptons are composite states, we may explain naturally why we have so many quarks and leptons

Composite quarks and leptons

It is however very difficult to generate massless composite fermions by strong dynamics.

But, we know an example for massless composite states in QCD which are Nambu-Goldstone bosons.

Massless composite bosons are easily generated

Supersymmetry give us a solution !!!

If we have **SUSY**, the NG bosons have always fermion partners which are nothing but massless composite fermions

We may identify those with quarks/ leptons

Buchmuller, Peccei, Love , Yanagida (1982)

A solution to a fundamental problem

Family Problem

- Why the quarks/leptons are $\mathbf{5}^* + \mathbf{10}$ of SU(5) ?
- Why we have three families ?
- What determines their masses ?

Quasi Nambu-Goldstone Fermions

Buchmuller, Love, Peccei and Yanagida (1982)

- NG bosons are always accompanied by massless fermions in supersymmetric non-linear sigma model ; $G \rightarrow H$
- Properties and number of NG bosons are determined by a given G/H
- We identify the quasi NG fermions with the observed quarks and leptons

- NG chiral multiplets are given by G/H and hence for a given G/H we can determine properties and number of quasi NG fermions , that is, quarks and leptons

We can answer to one of the most fundamental questions in particle physics;

Why do we have three families ?

Search for G/H

- G/H must be Kahler manifold in SUSY theories
- $SU(6)/SU(5) \times U(1) \rightarrow$ NG multiplet = **5***
- $SO(10)/SU(5) \times U(1) \rightarrow$ NG multiplet = **10**
- $E_6/SO(10) \times U(1) \rightarrow$ NG multiplet = **16** of $SO(10)$; **16 = 5* + 10 + 1** (one family)
- Exceptional groups are very interesting to have family structure !

• $E_7/SU(5) \times U(1)^3 \rightarrow$

$$\text{NG multiplets} = 3 \times (5^* + 10 + 1) + 5$$

Three families, Kugo and Yanagida (1983)

$E_8/SU(5) \times U(1)^4 \rightarrow$

$$\text{NG multiplets} = 4 \times (5^* + 10 + 1) + 1 \times (5 + 10^* + 1) + \dots$$

Three families !

*We concluded the maximal number of families
is 3 !!!*

(E_8 is the maximal exceptional group)

SUSY Non-Linear Sigma Model

- G/H must be a Kahler (complex) manifold



Nambu-Goldstone multiplets are Chiral

The simplest example is $CP^1 = SU(2)/U(1)$

The NG multiplet is $\phi(+1)$

We do not have $\phi(-1)$

SU(2) generators: T, X^+, X^-

$$[T, X^+] = X^+, [T, X^-] = -X^-, [X^+, X^-] = 2T$$

One NG chiral multiplet ; $\phi(+1)$

$$[T, \phi] = +\phi, [X^+, \phi] = \phi \psi, [X^-, \phi] = 1$$

SU(2) invariant Kahler potential ;

$$K = \log(1 + \phi \psi^*)$$

$$K \rightarrow K + \phi + \phi^*$$

Integration of θ shows invariance !

E₇ has 133 generators;

$$T^{i_j} (63) + E_{\{ijkl\}} (70) \text{ of } \mathbf{SU(8)}$$

Consider the Kahler manifold $E_7 / \mathbf{SU(5) \times SU(3) \times U(1)}$

The broken generators are

$T^{a_i} (\mathbf{5^*, 3})$; $E_{\{abij\}} (\mathbf{10, 3^*})$; $E_{\{ijkl\}} (\mathbf{5, 1})$ and their conjugates

NG multiplets are

$$\phi(\mathbf{5^*, 3, +2}) + \phi(\mathbf{10, 3^*, +1}) + \phi(\mathbf{5, 1, +3})$$

We should add a $\mathbf{X(5^*, 1)}$ multiplet to cancel non-linear sigma model anomalies

We have massless three families of quarks and leptons !!!

A larger manifold $E_7/SU(5) \times U(1) \times U(1) \times U(1)$

$$E_7/E_6 \times U(1) \rightarrow \text{NG multiplets} = 5^* + 5^* + 10 + 1 + 1 + 5$$

$$E_6/SO(10) \times U(1) \rightarrow \text{NG multiplets} = 5^* + 10 + 1$$

$$SO(10)/SU(5) \times U(1) \rightarrow \text{NG multiplets} = 10$$

We have three families of $5^* + 10 + 1$!!!

1 is the right-handed neutrino

Mass hierarchy

Explicit breaking of E_7 gives Yukawa couplings;

Suppose **hierarchy** in the explicit breaking as
 $E_7 \rightarrow E_6 \rightarrow SO(10) \rightarrow SU(5)$

We obtain the mass hierarchy as

$$m_t : m_c : m_u = 1 : \epsilon^2 : \epsilon^4$$
$$m_b : m_s : m_d = 1 : \epsilon : \epsilon^3$$

We have a large neutrino mixing !

Is the NG hypothesis consistent ?

Why E_7 ?

Is the NG hypothesis consistent ?

- The squarks and sleptons are all massless at the GUT scale
- The Higgs multiplets are NOT NG multiplets and they have soft SUSY breaking masses of the order of $m_{3/2} \sim 100$ TeV
- Higgs loop diagrams give negative soft masses² for squarks and sleptons (tachyonic)
- ***Our vacuum is no longer stable !!***

If the soft SUSY breaking masses² of Higgs bosons are negative, the masses² of squarks and sleptons are positive !!!

Yin, Yokozaki (2016)

Higgs bosons have positive masses²

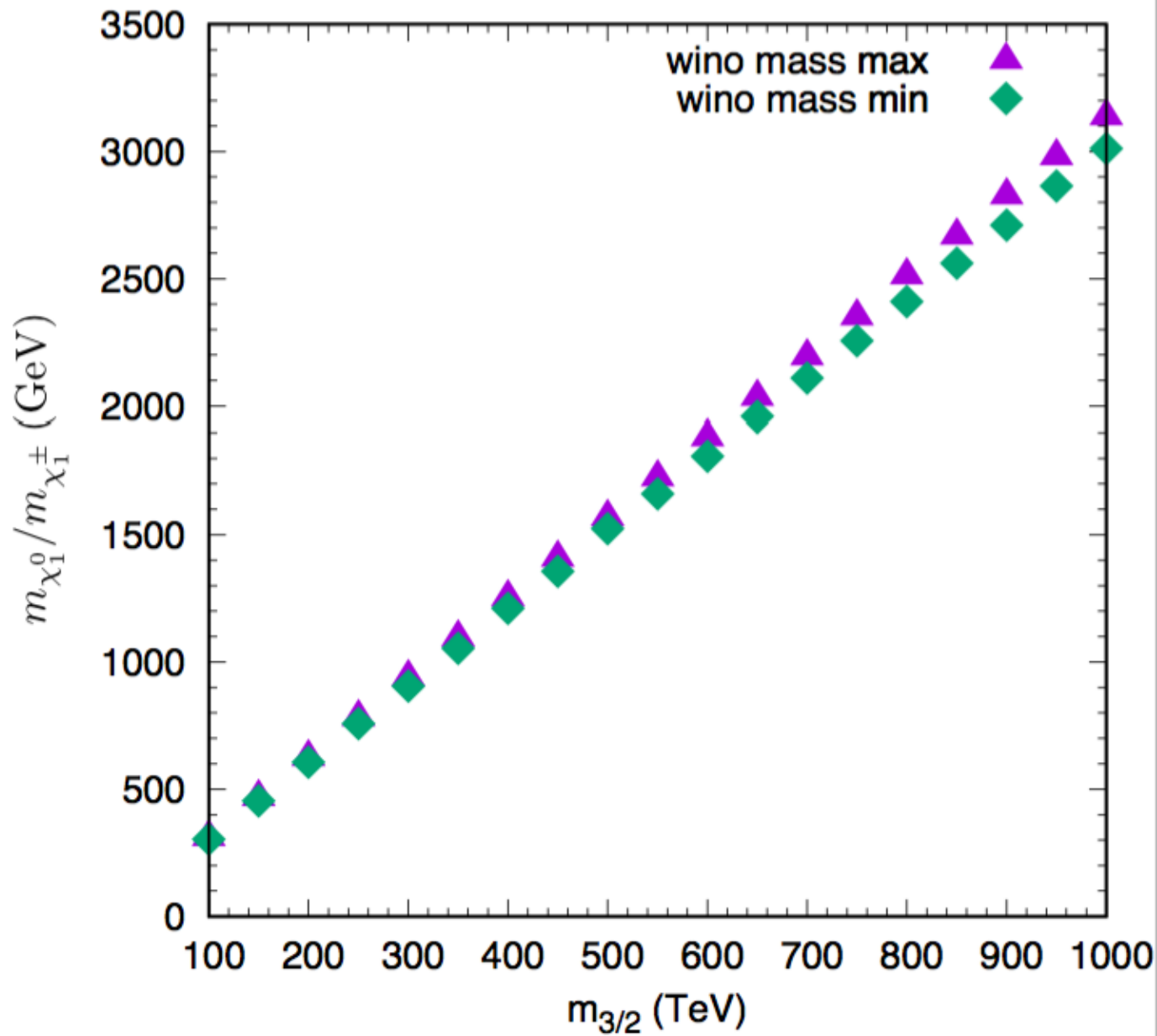
$$= |\mu|^2 + m^2(\text{soft}) > 0$$

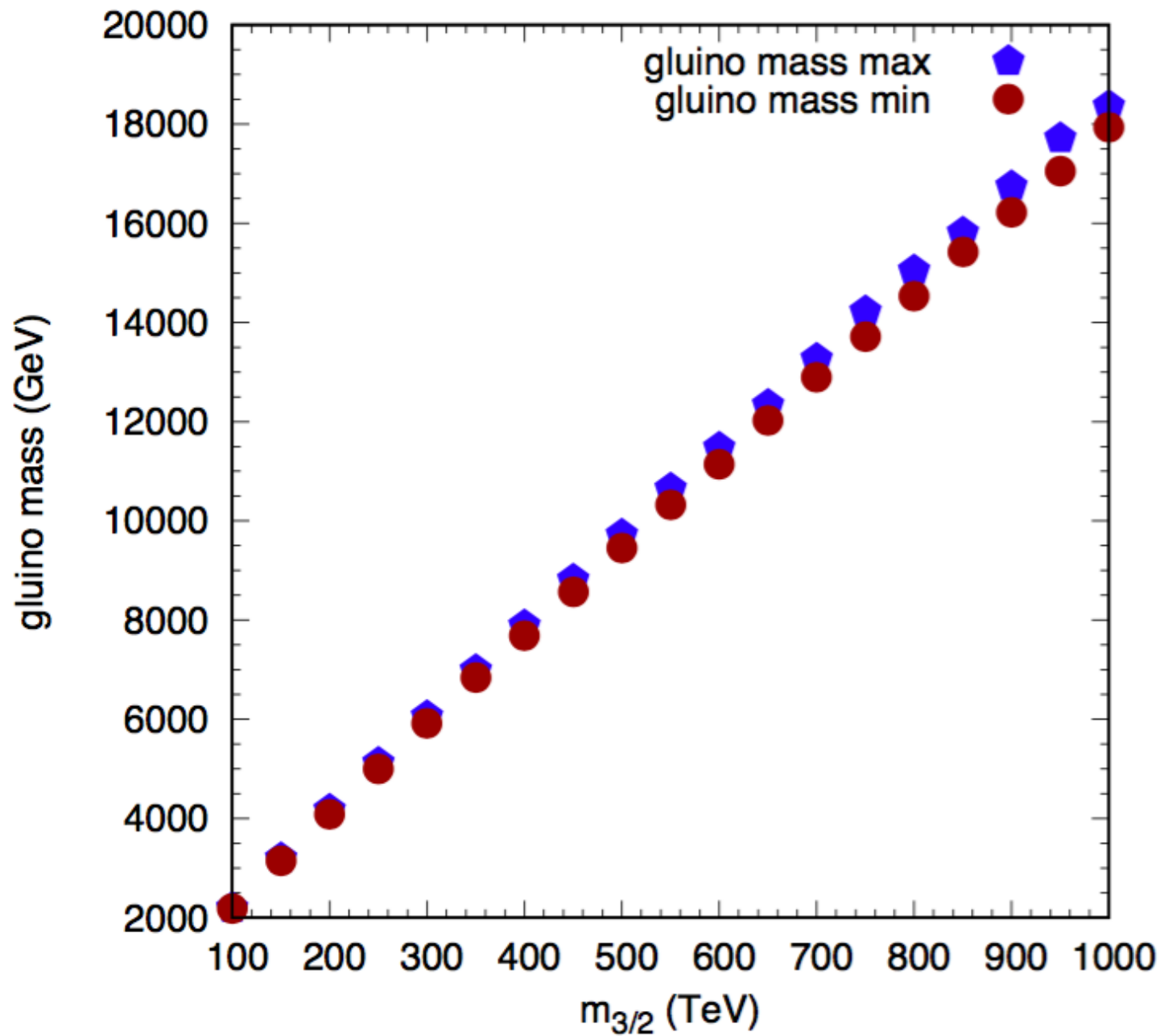
Our Vacuum is Stable !!!

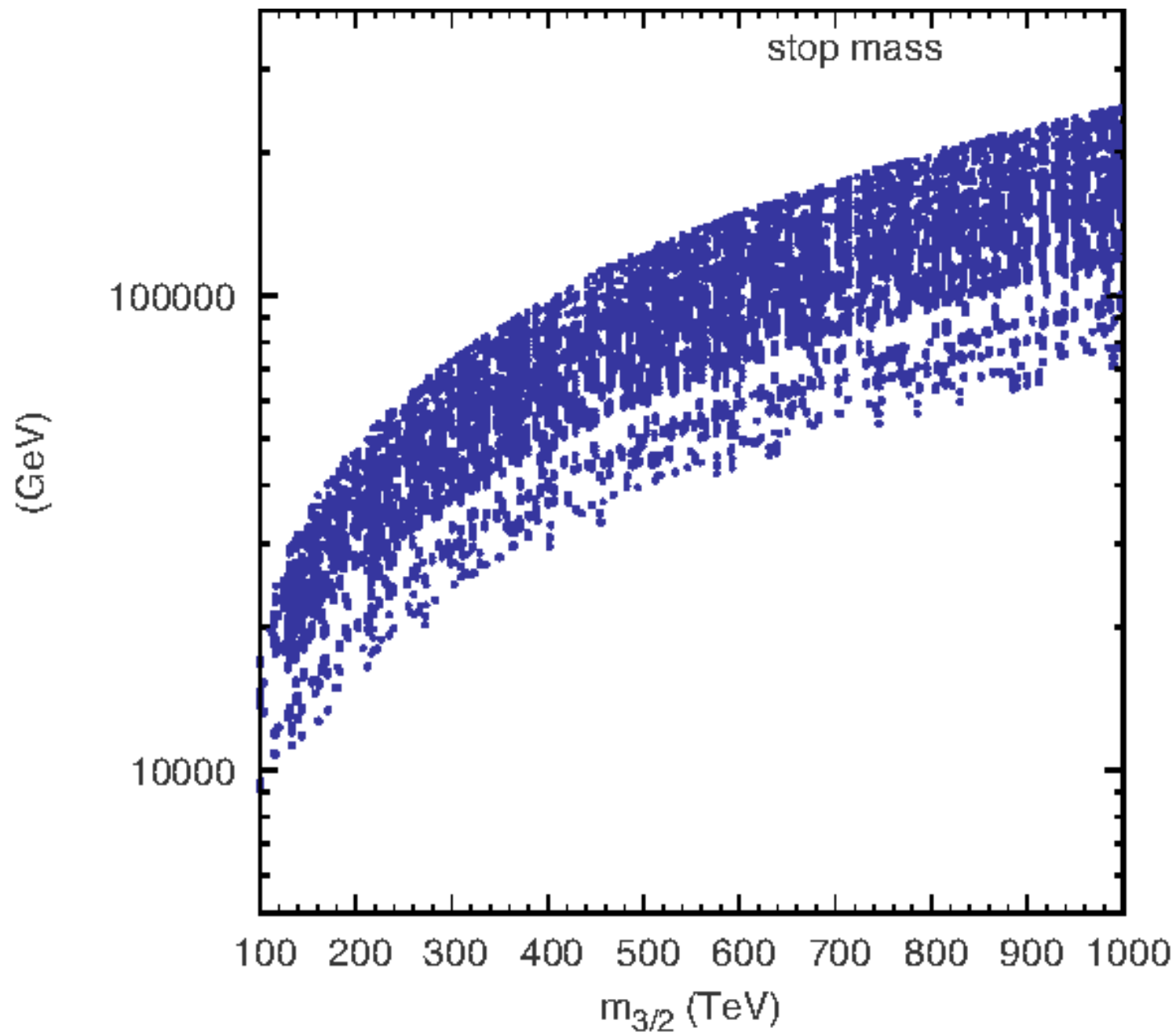
Stop mass = $O(10)$ TeV explaining the Higgs boson mass = 125 GeV

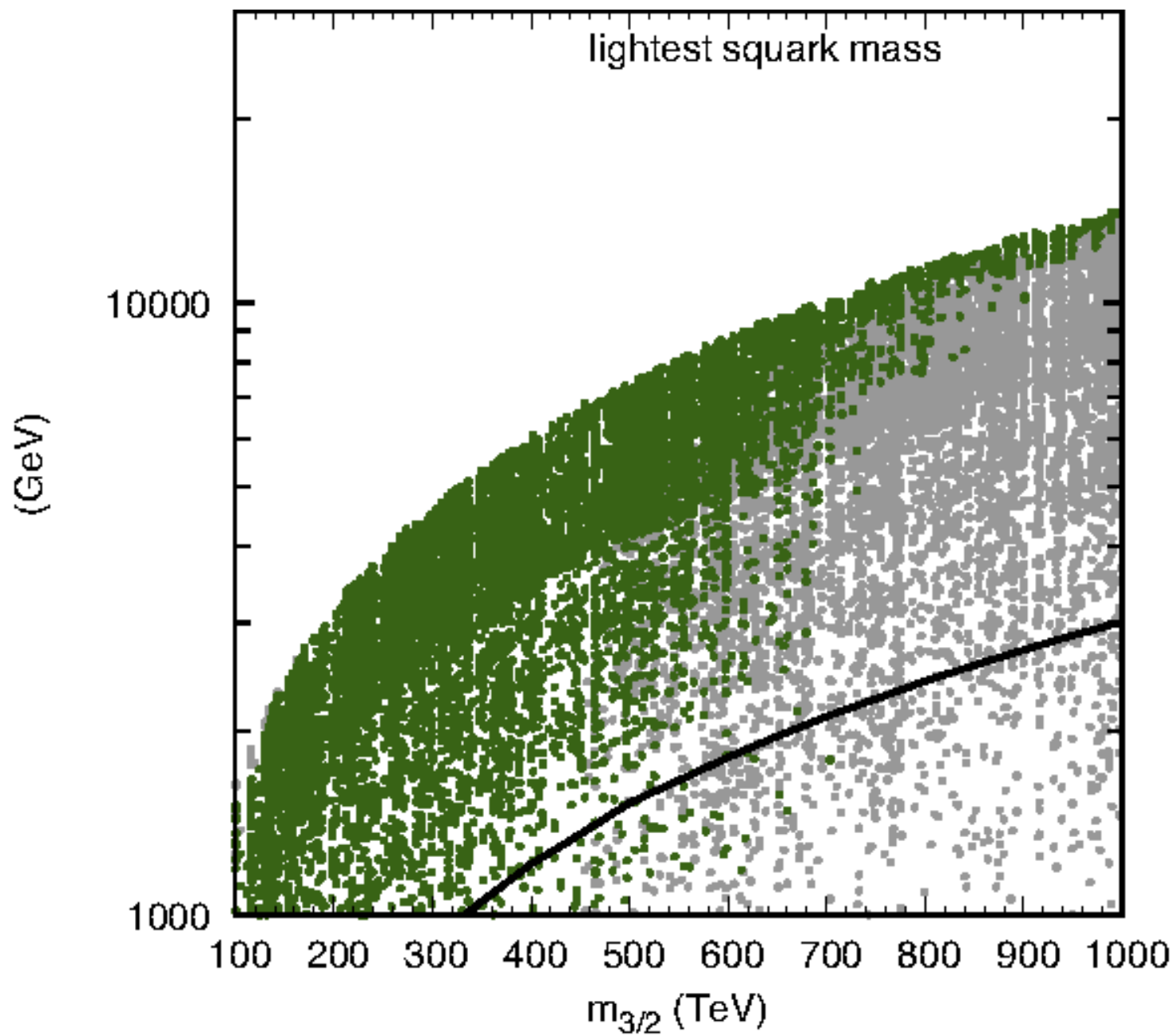
Squark (u,d,s,c) and slepton masses = $O(1)$ TeV, since their Yukawa couplings are small

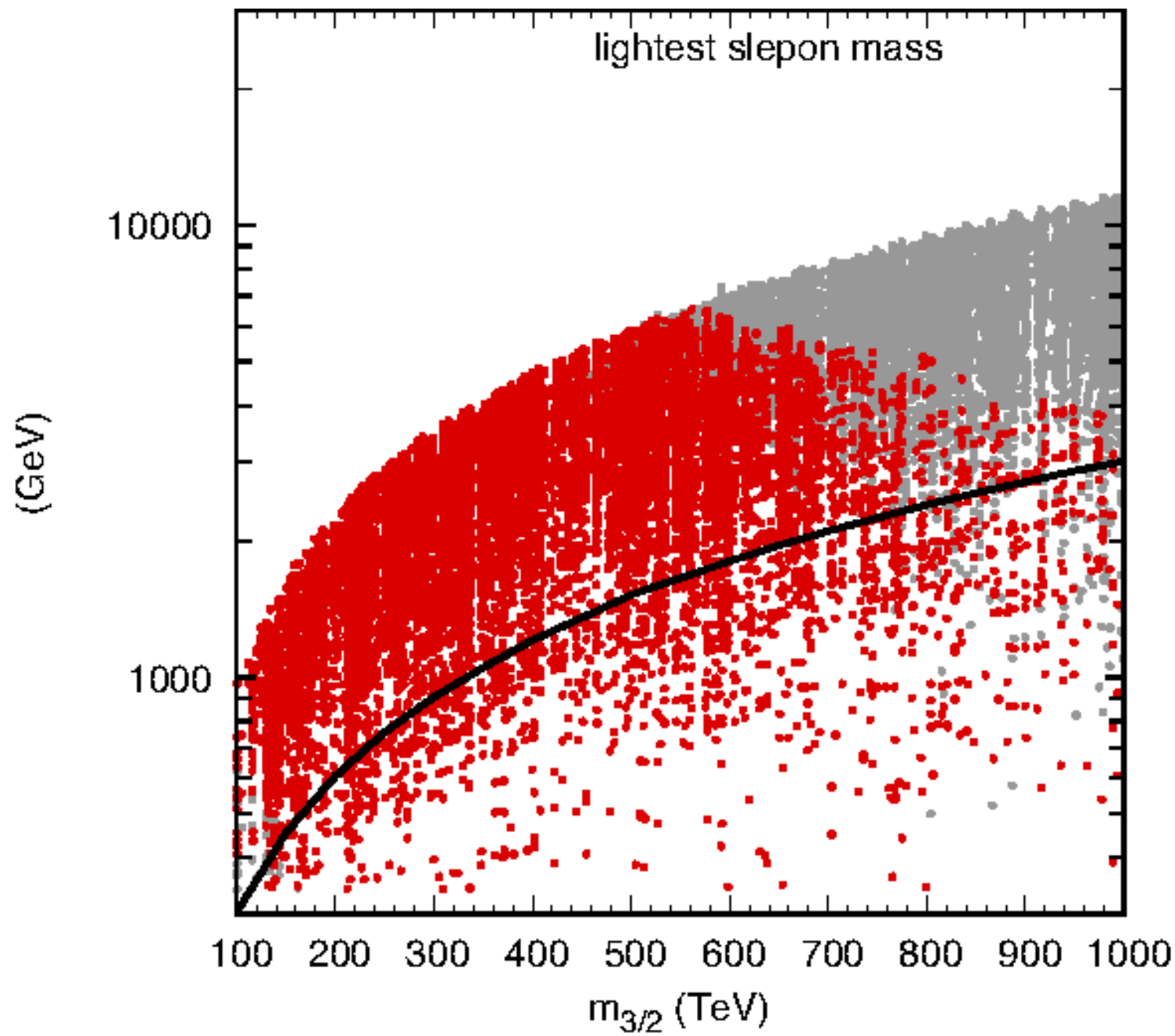
There is a parameter region where we can explain the muon $g-2$ anomaly











The Nambu-Goldstone hypothesis for squarks and sleptons is consistent with all observations so far

Yanagida, Yin, Yokozaki (2016)

Why Does Nature Choose E_7 ?

N=8 Supergravity

Gravity multiplet; one graviton (2), 8 gravitinos (3/2), 28 vector bosons (1)
56 Majorana spinors (1/2), 70 real scalar boson (0)

70 scalar boson = Nambu-Goldston bosons on $E_{7,7}/SU(8)$

Cremmer, Julia (1978)
De Wit, Nicolai (1981)

The maximal subgroup of E_7 is $SU(8)$:

$$E_7 \text{ generators (133)} = T^{ij} (63) + E_{\{l,j,k,l\}} (70)$$


SU(8) generators (i,j=1-8)

$E_7/SU(8)$ has 70 NG bosons !!

This hidden $E_{7,7}$ may be the origin of our effective E_7 ?

When $N=8 \rightarrow N=1$ SUSY , G/H must be a Kahler manifold
But, $E_7/SU(8)$ is NOT a Kahler manifold

We need rethinking

*N=8 supergravity has a local $SO(8)$ symmetry
and a hidden local $SU(8)$ symmetry* Nicolai (1982)

Let us assume some of the symmetries survive the breaking of
the $N=8$ supergravity down to $N=1$ supergravity

Assume $[SU(2) \times SU(2)] \times SU(8)$

A subgroup of $SO(8)$



Preon Model

Consider eight $SU(2)$ -doublet preons Q^i_a , ; $i=1-8$ and $a=1,2$
and eight $SU(2)'$ -doublet preons Q'^j_b ; $J=1-8$ and $b=1,2$

Here we have a global $SU(8) \times SU(8)'$

Consider Mesons; $M^{ij} = Q^i Q^j$ and $M'_{ij} = Q'_i Q'_j$
and superpotential $W = M^{ij} M'_{ij}$

We have a global $SU(8)$

Consider the strong coupling limit of the $SU(2) \times SU(2)$ gauge theory which has infrared fixed points

Seiberg (1996)

On the fixed point we have an enhanced global symmetry that is E_7 !!!

Dimofte, Gaiotto (2012)

This may be the origin of our E_7

8 fundamental preons Q and $\{\bar{Q}\}$

The theory has an IR fixed point, on which we have an enhanced symmetry E_7

Quarks and Leptons can be identified with massless quasi-NG fermions, which are bound states of the preons

The presence of $SU(8)$ may be a crucial in $N=8$ Supergravity

conclusion

- The higgs mass 125 GeV suggests high scale SUSY..... $m_{3/2}=100-300$ TeV, $m_{sq}=O(100)$ TeV and $m_{gluino}=2-6$ TeV
- But, NG hypothesis for squarks and sleptons still survive from all experimental data
- This suggests that m_{sq} in the 1st and 2d generations = 1-4 TeV and $m_{gluino}=2-6$ TeV which may be tested in future LHC