Diboson production at the LHC:
Precision phenomenology beyond NNLO QCD

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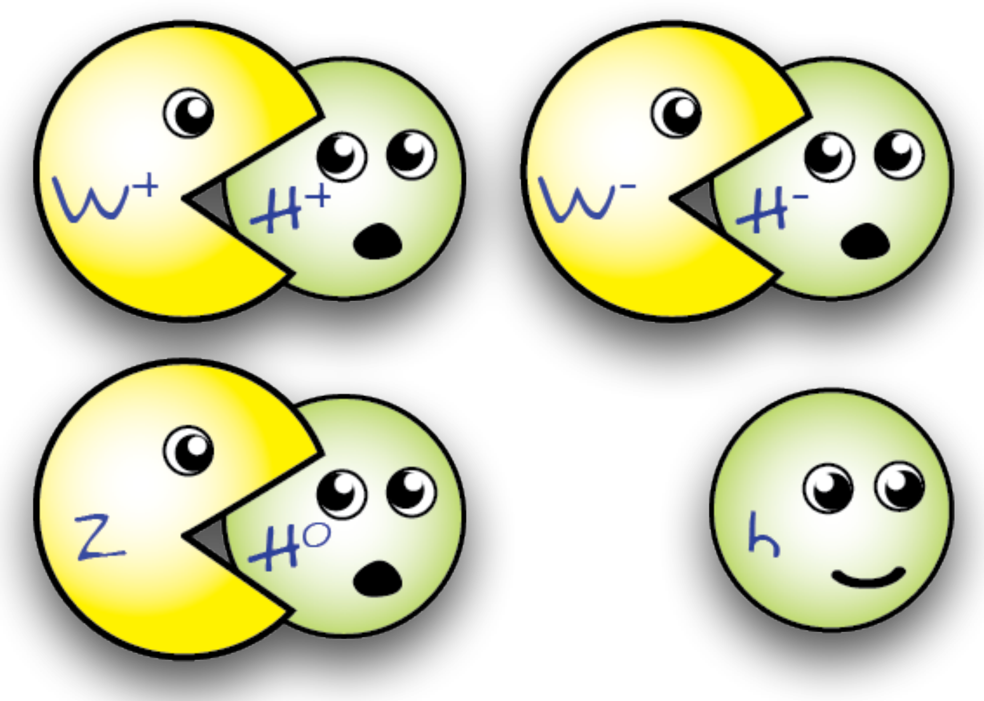


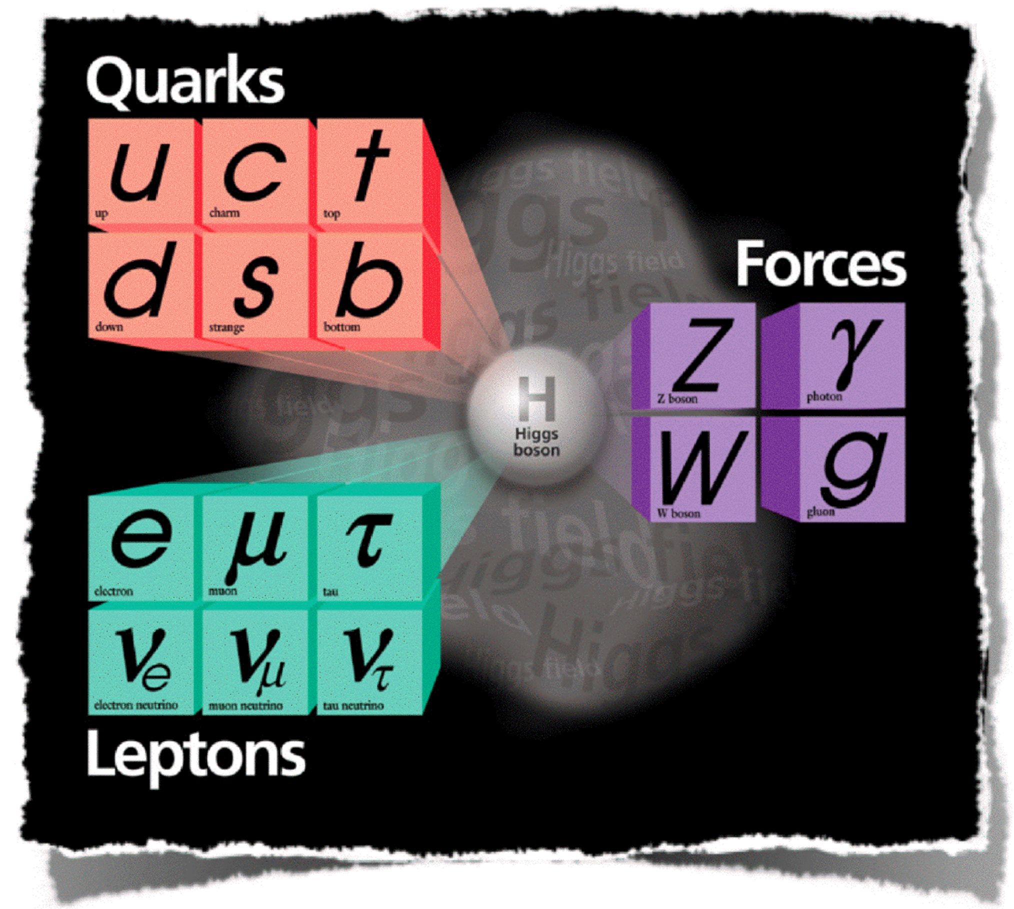
Cavendish-DAMTP seminar

Cambridge (UK), February 27th, 2020

Standard Model: A successful theory

- Theoretically sound description of fundamental interactions between elementary particles

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \not{D} \psi + \bar{\psi}_i y_{ij} \psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi)$$



Standard Model: A successful theory

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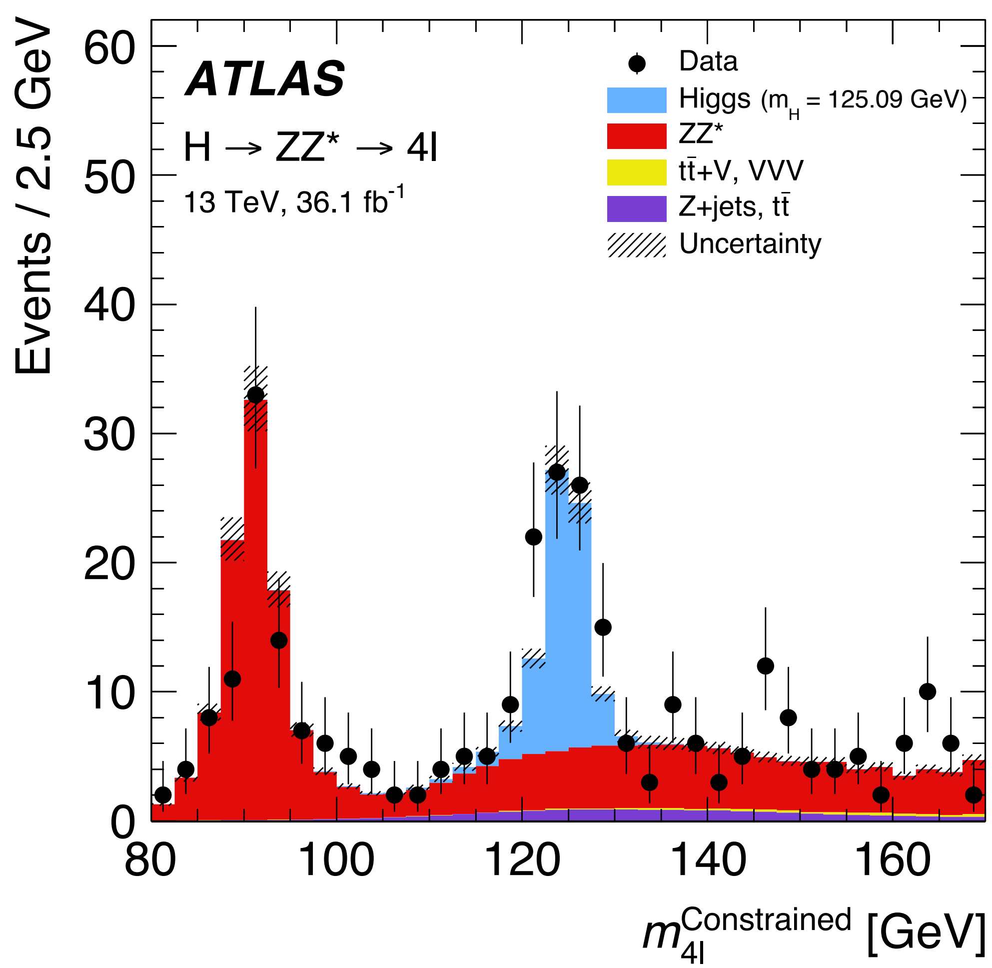
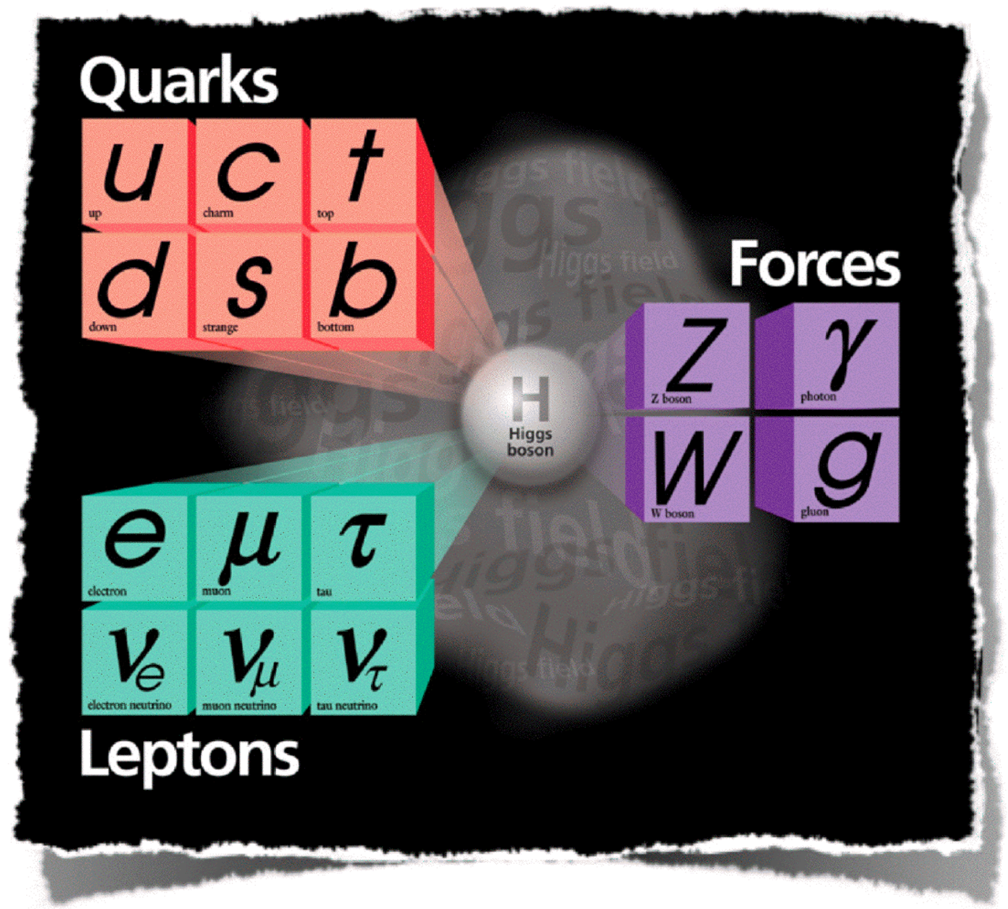
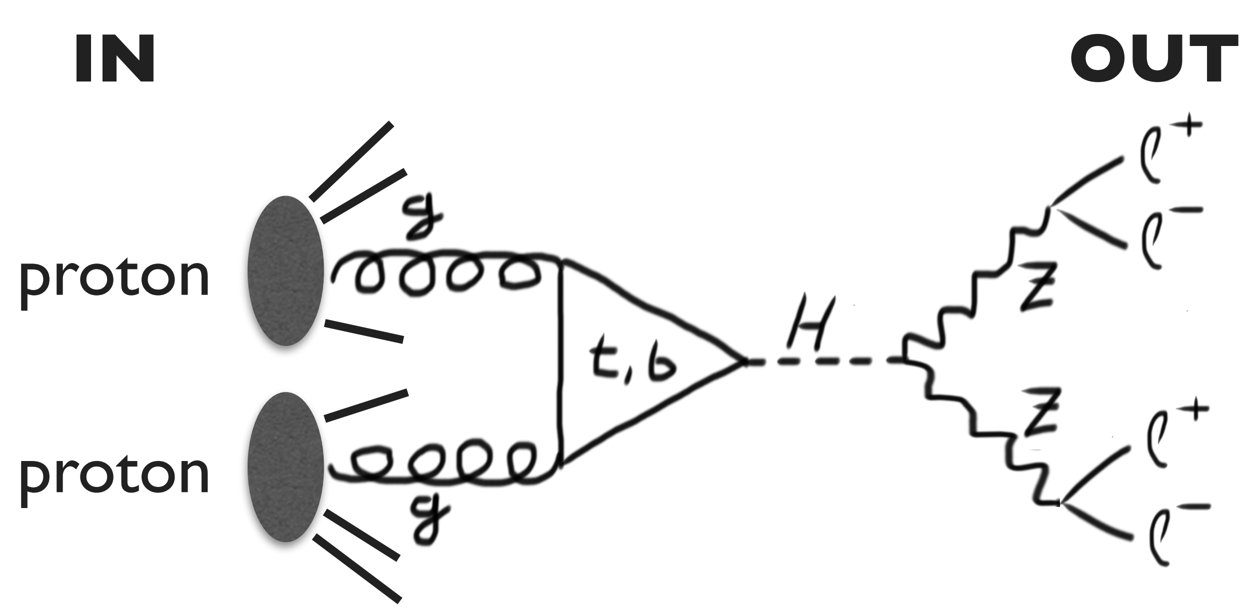
- LHC **discovered** Higgs particle in 2012

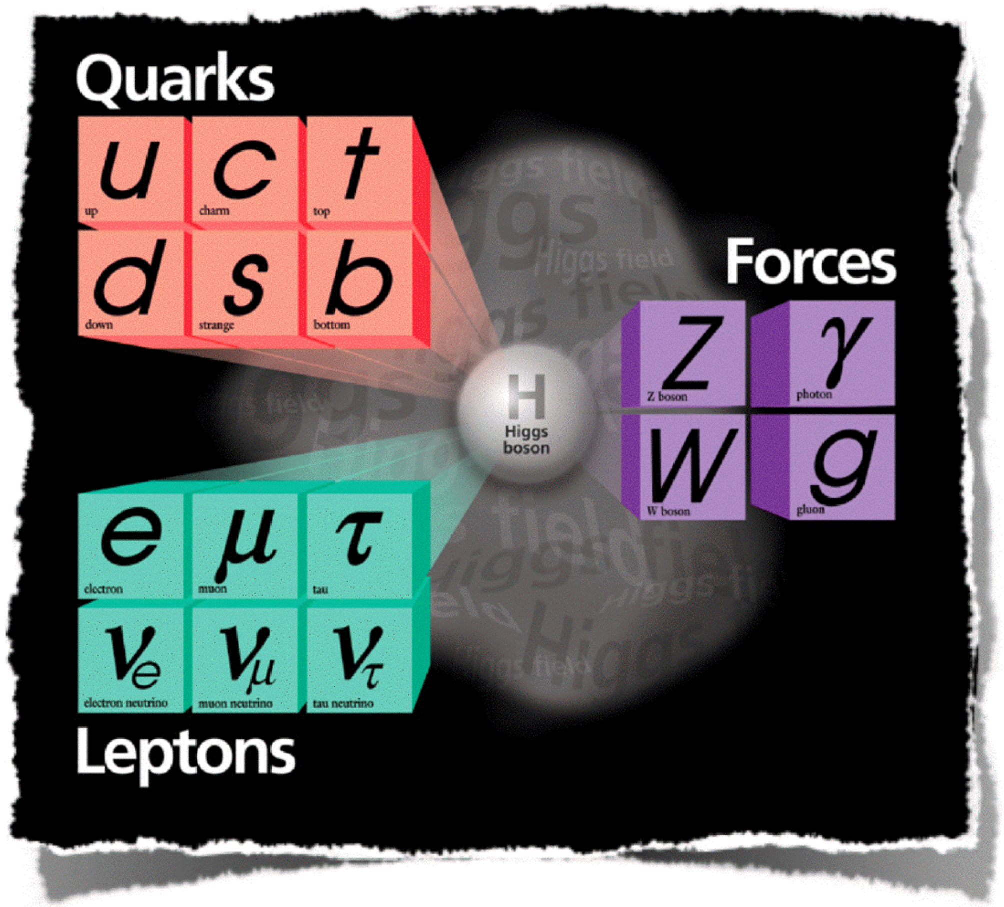




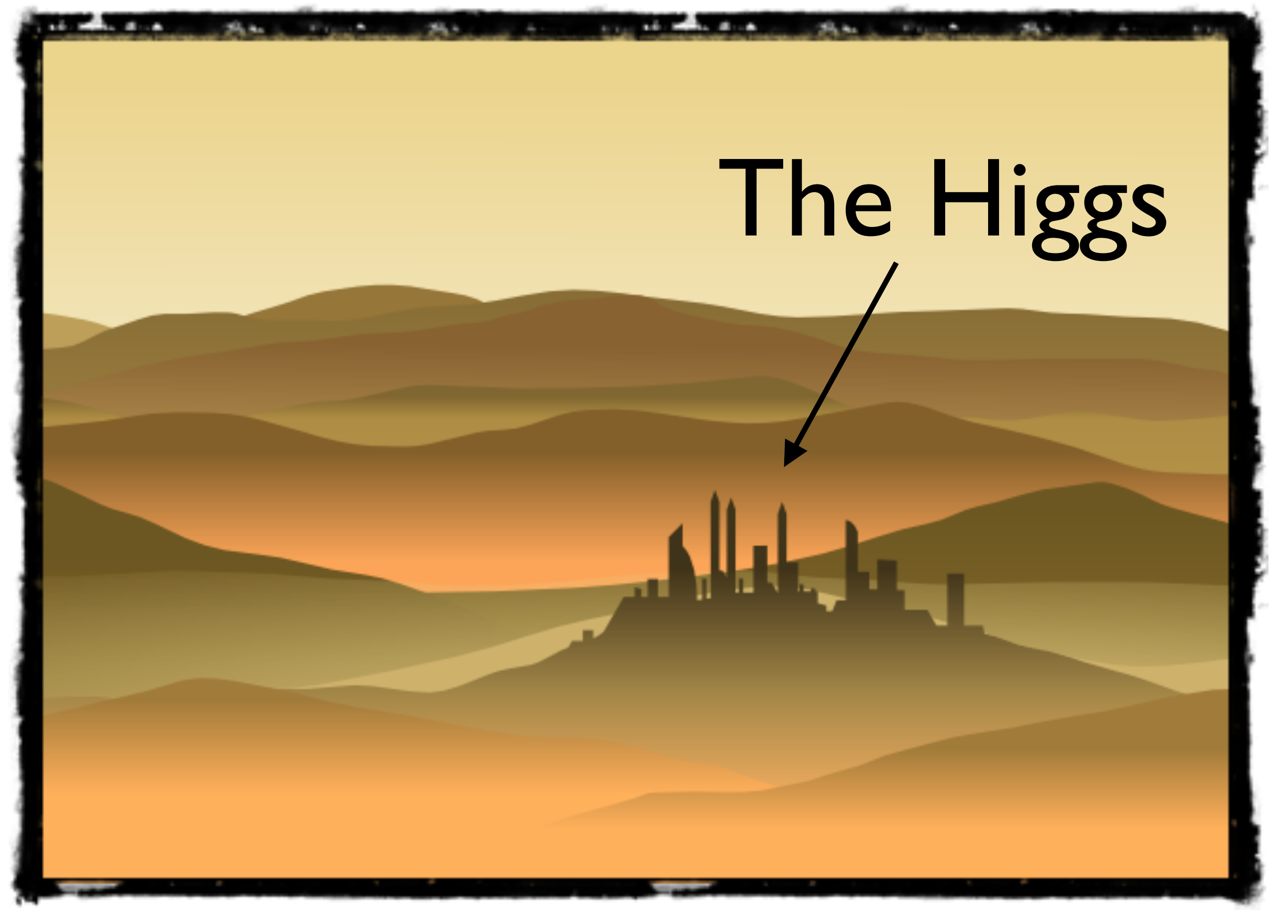


Standard Model: A successful theory

- Theoretically sound description of fundamental interactions between elementary particles
- LHC **discovered** Higgs particle in 2012
- **BUT: No ultimate theory** of nature
  - → incomplete (gravity, dark matter, ...)
  - → theoretical issues (mass hierarchies, fine-tuning, ...)

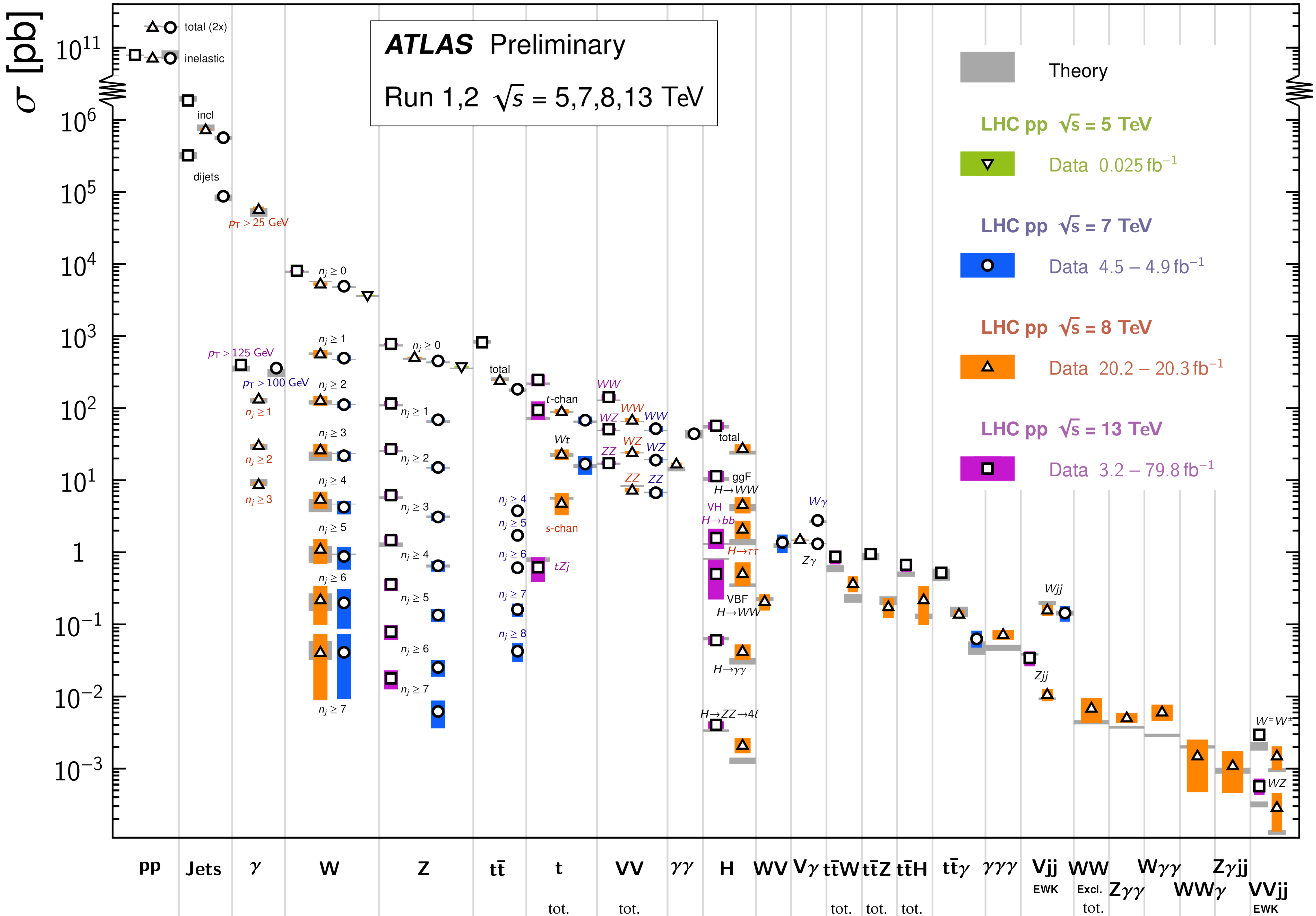


Physics beyond the SM expected at the TeV scale


The Higgs

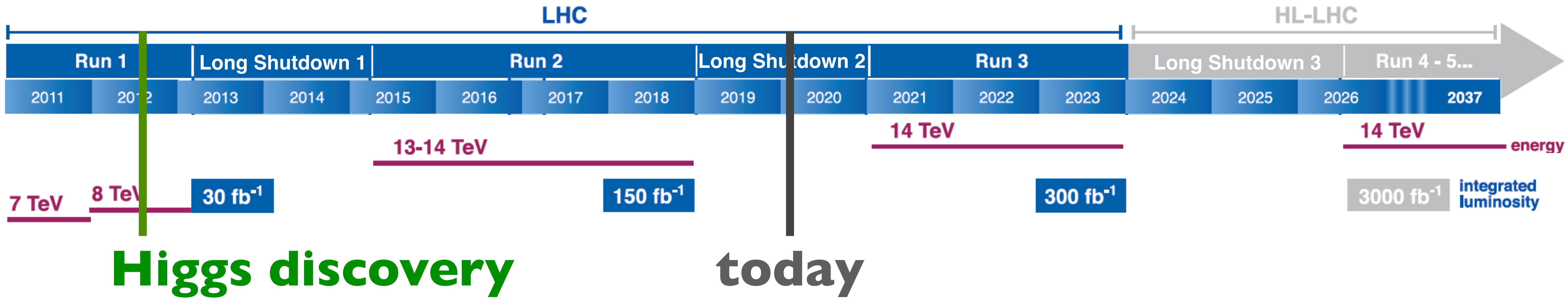
Standard Model Production Cross Section Measurements

Status: March 2019



LHC Physics after Higgs discovery

- LHC will remain major particle-physics experiment for $\gtrsim$15 years



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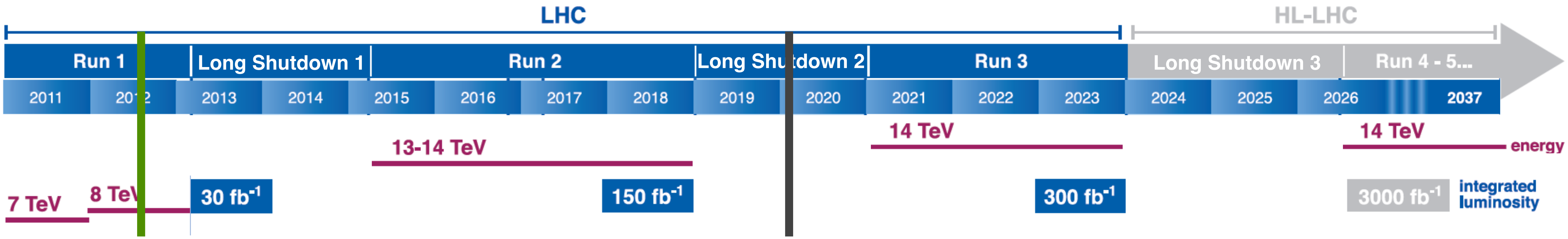

- Primary goals since 2012:
  - study Higgs properties → compatible with SM Higgs Boson
  - discover ***New Physics*** → **no direct evidence**

LHC Physics after Higgs discovery

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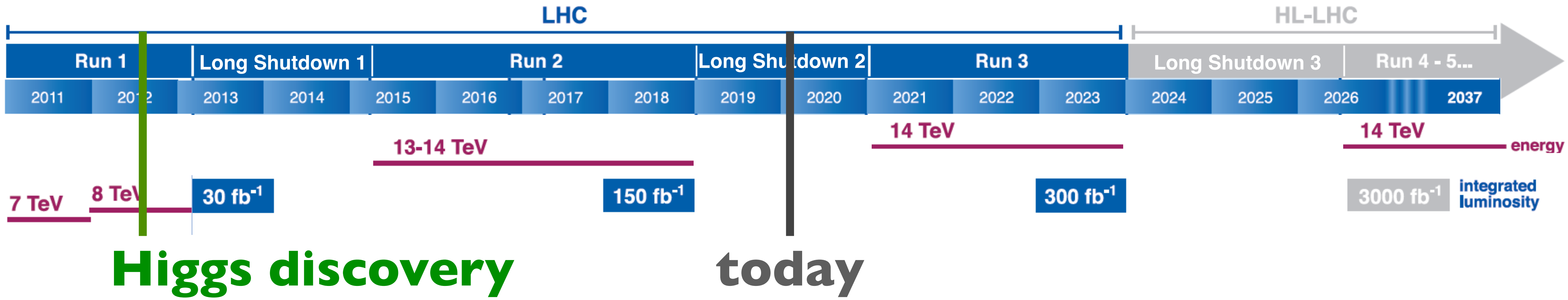

- Primary goals since 2012:
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  - discover ***New Physics*** → **no direct evidence**

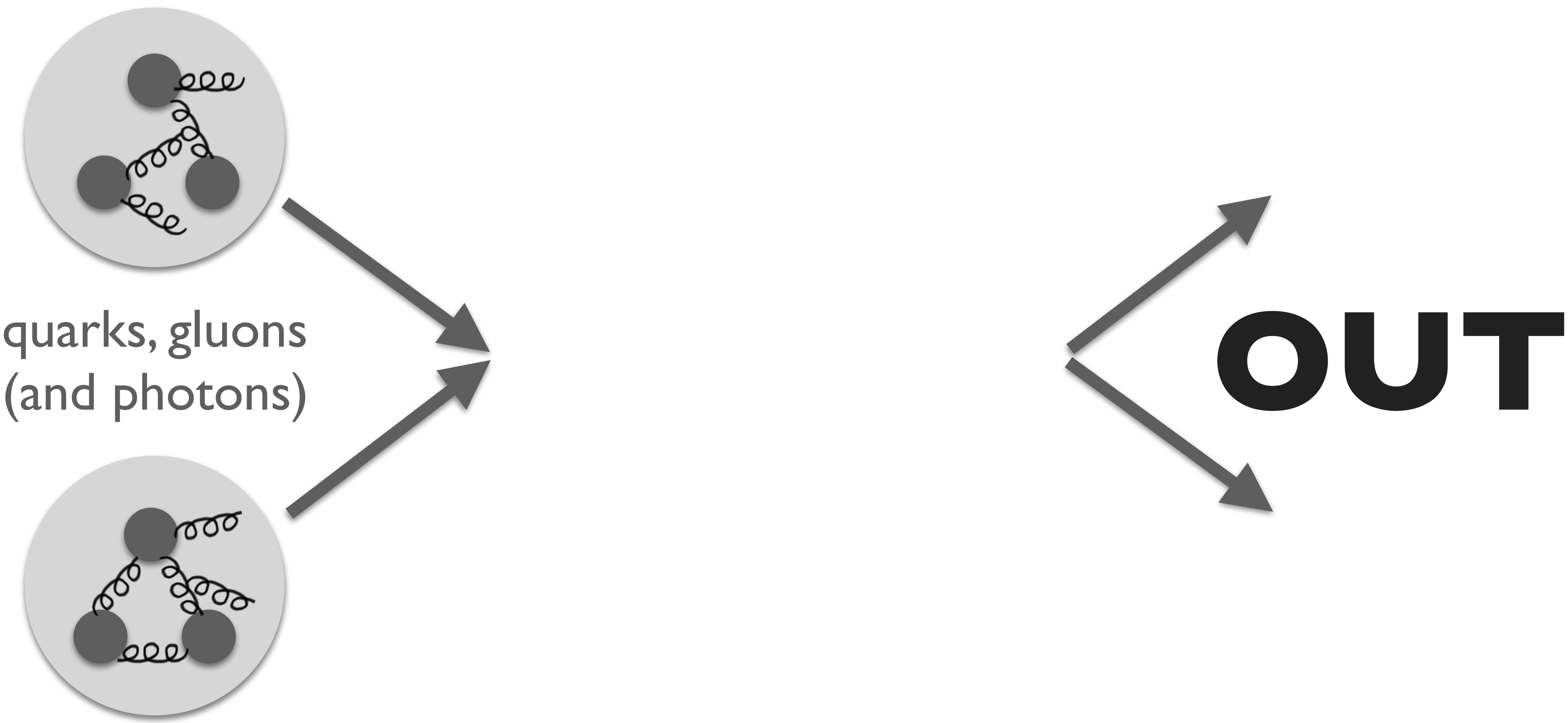
Precision era of the LHC has begun

(chance to probe New Physics)

LHC collisions: small SM deviations

LHC collisions: small SM deviations

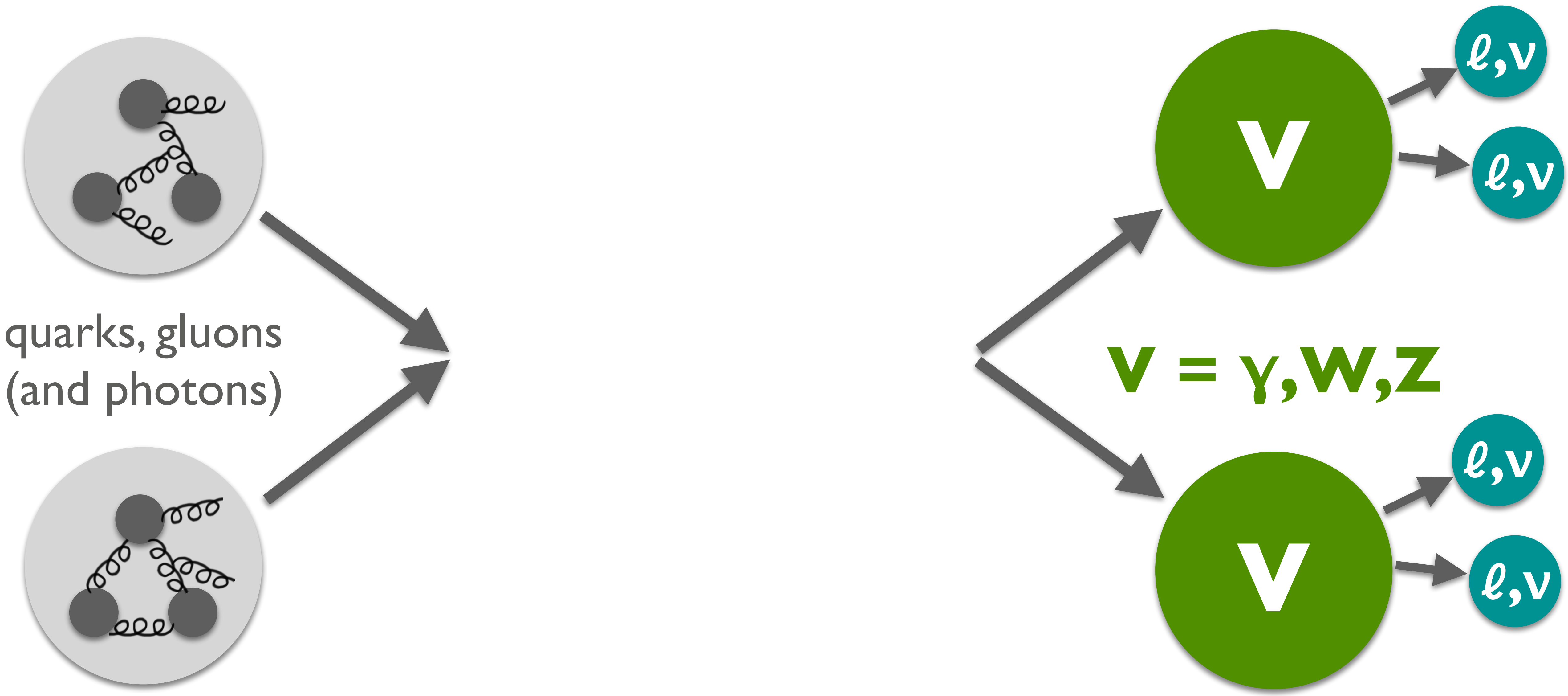
proton-proton collisions



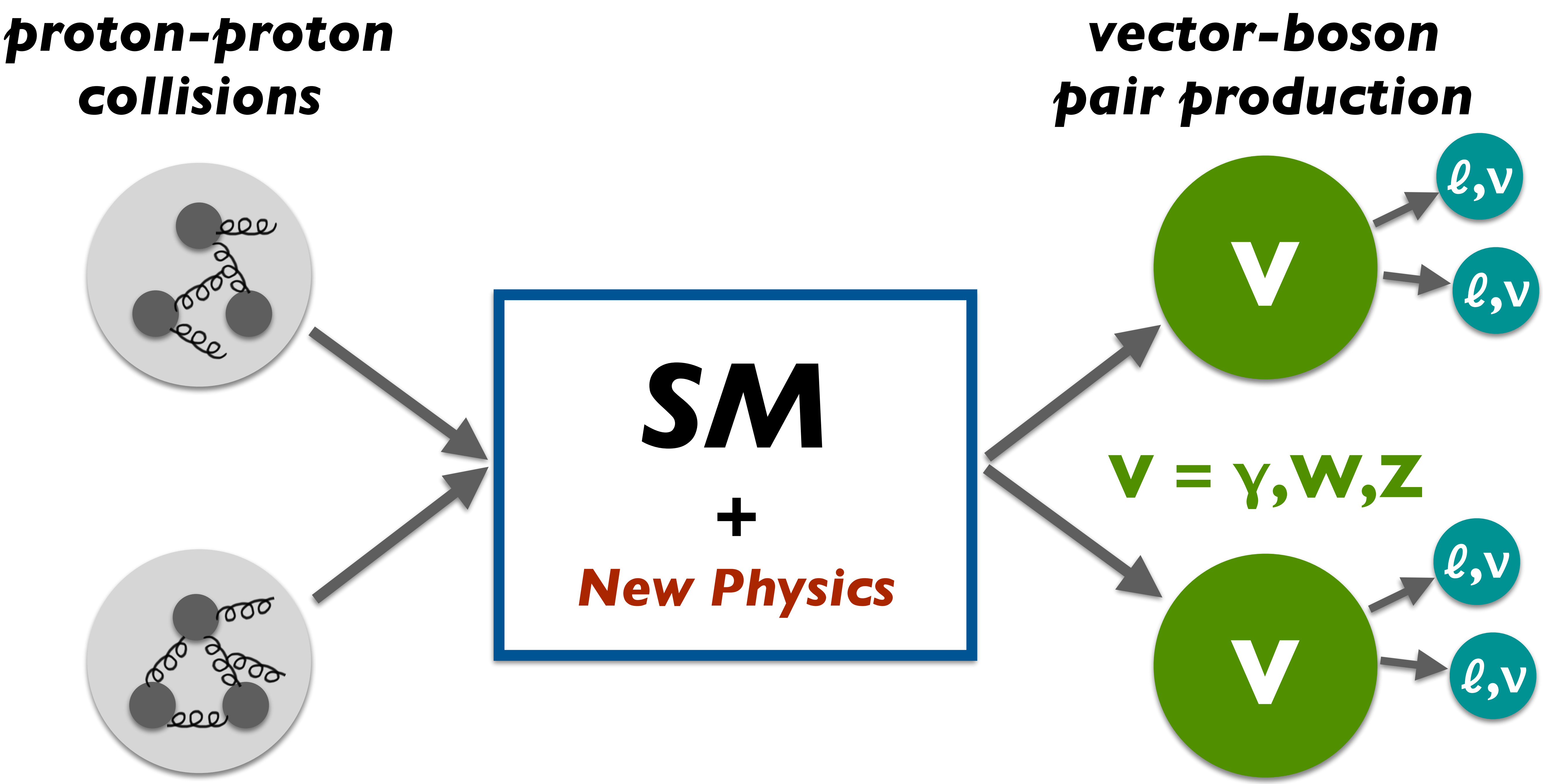
LHC collisions: small SM deviations

proton-proton collisions

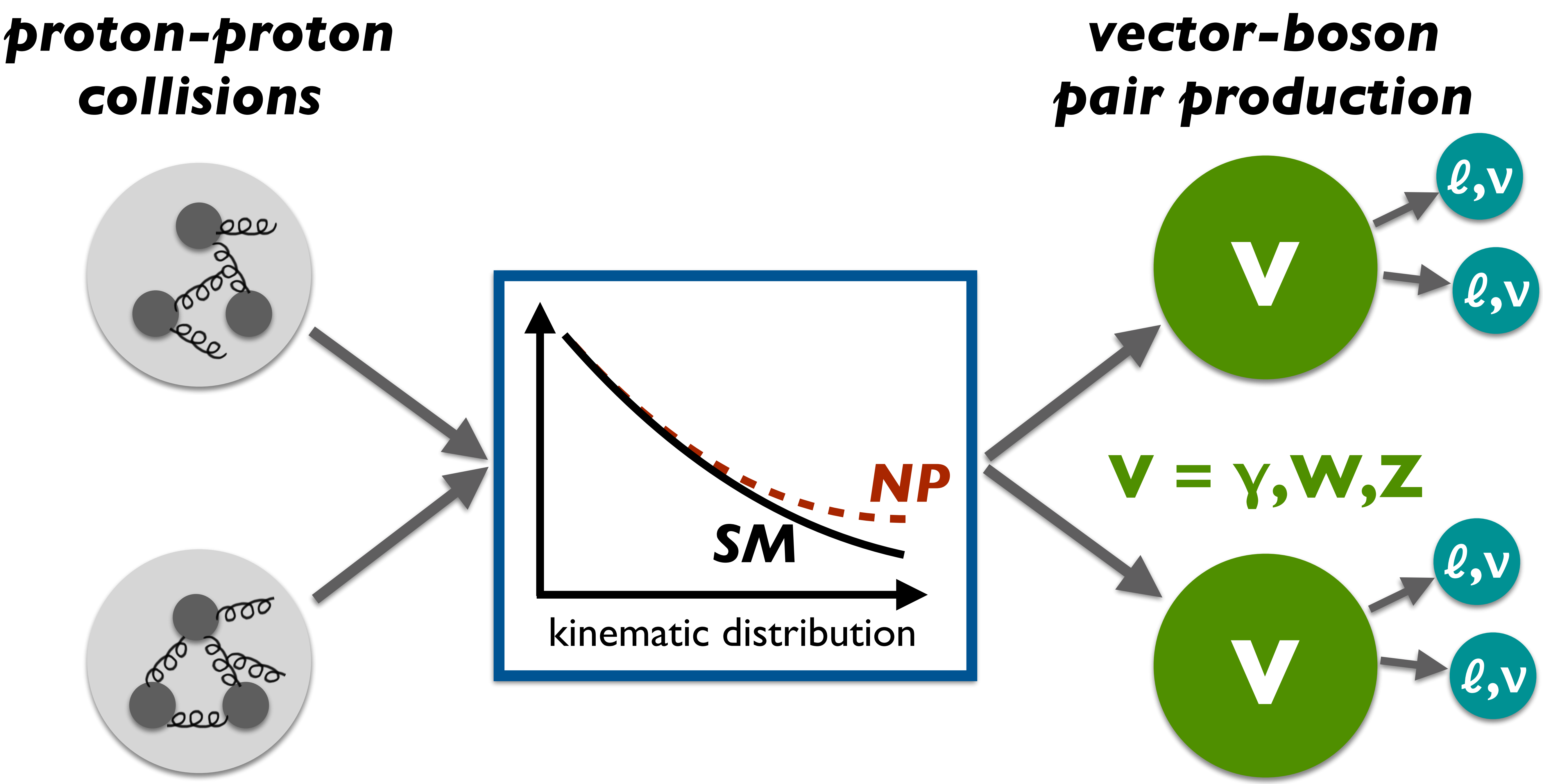
vector-boson pair production



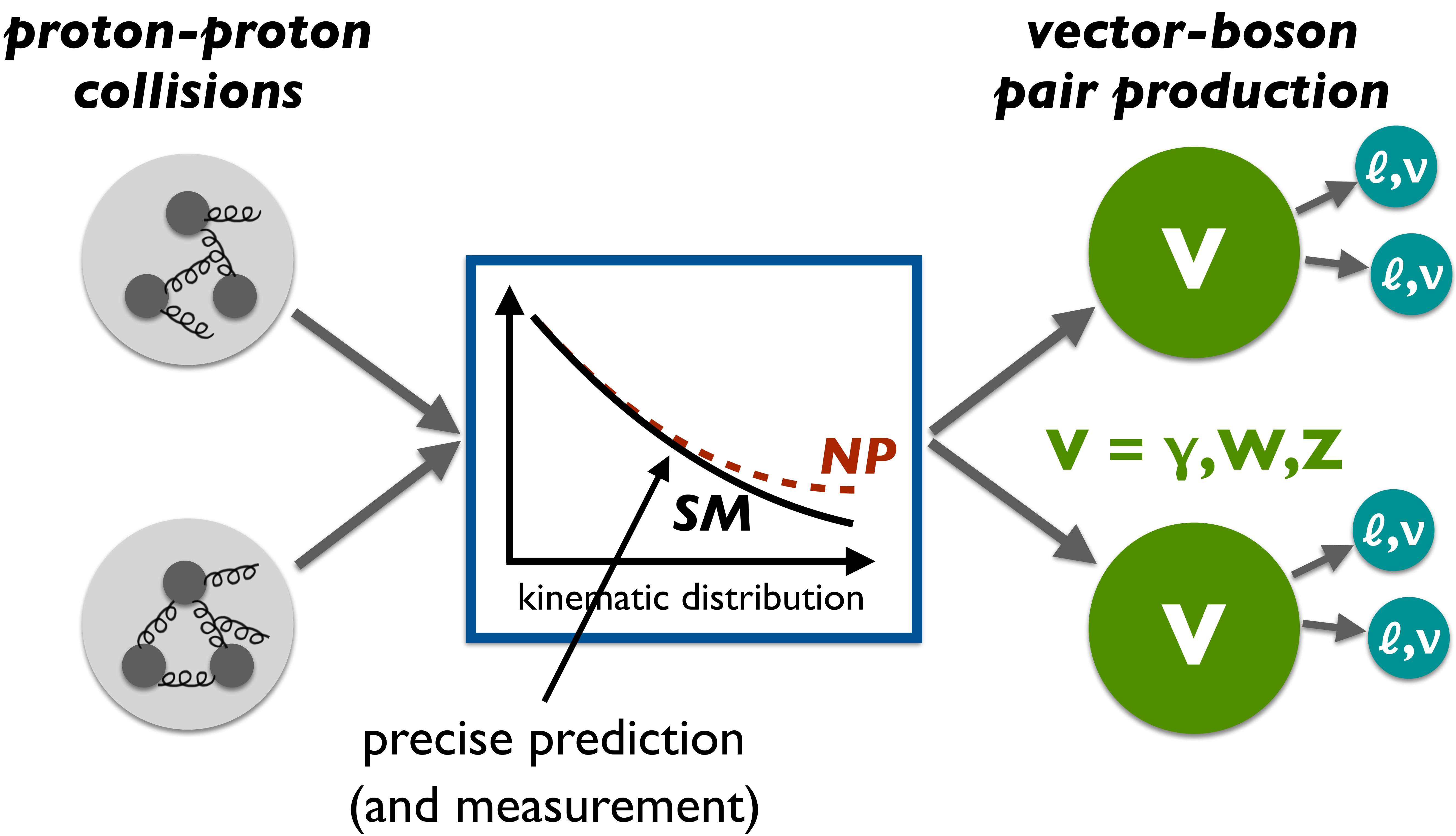
LHC collisions: small SM deviations



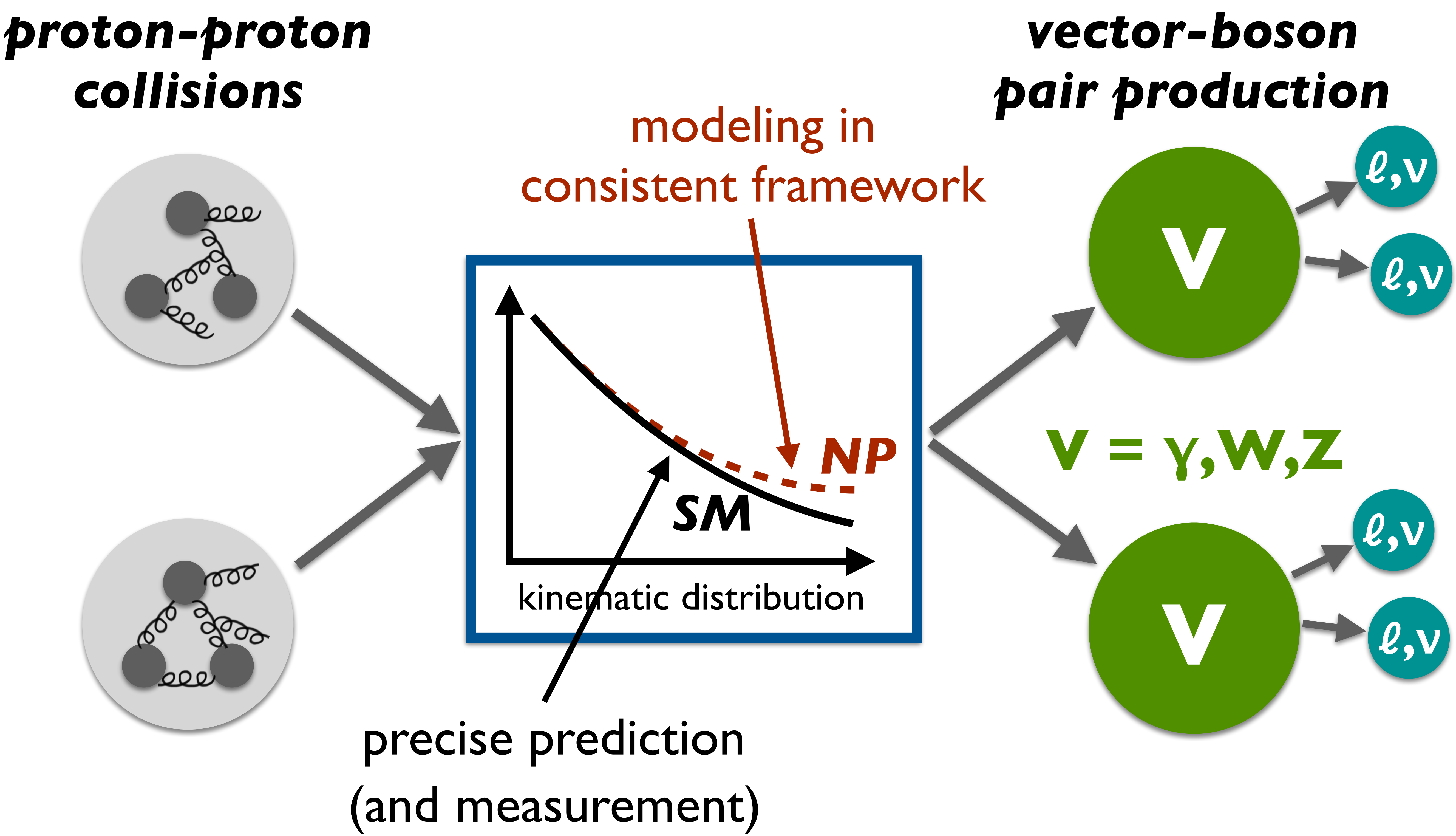
LHC collisions: small SM deviations



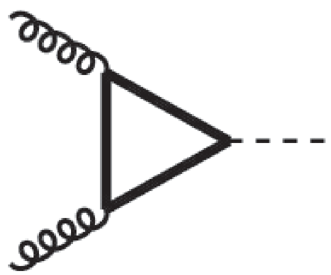
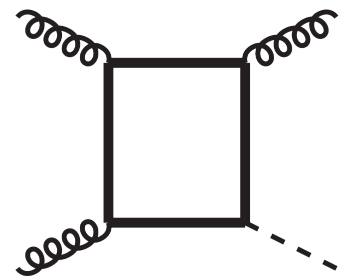
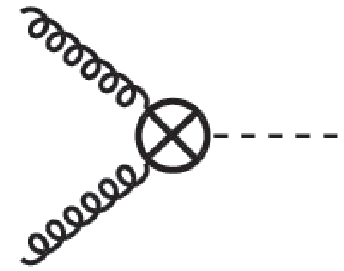
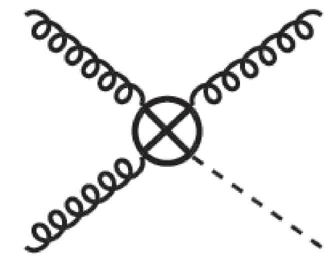
LHC collisions: small SM deviations



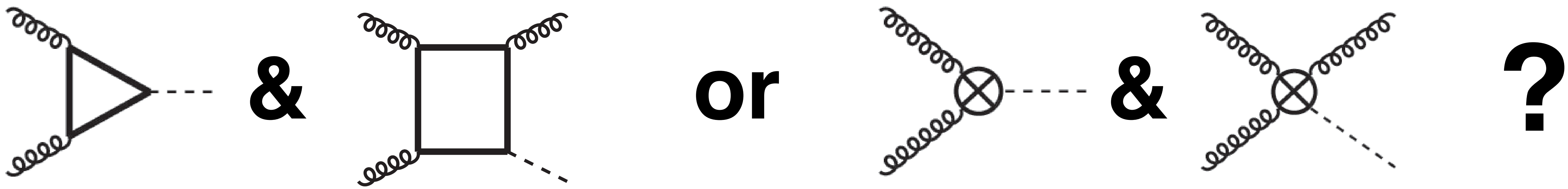
LHC collisions: small SM deviations

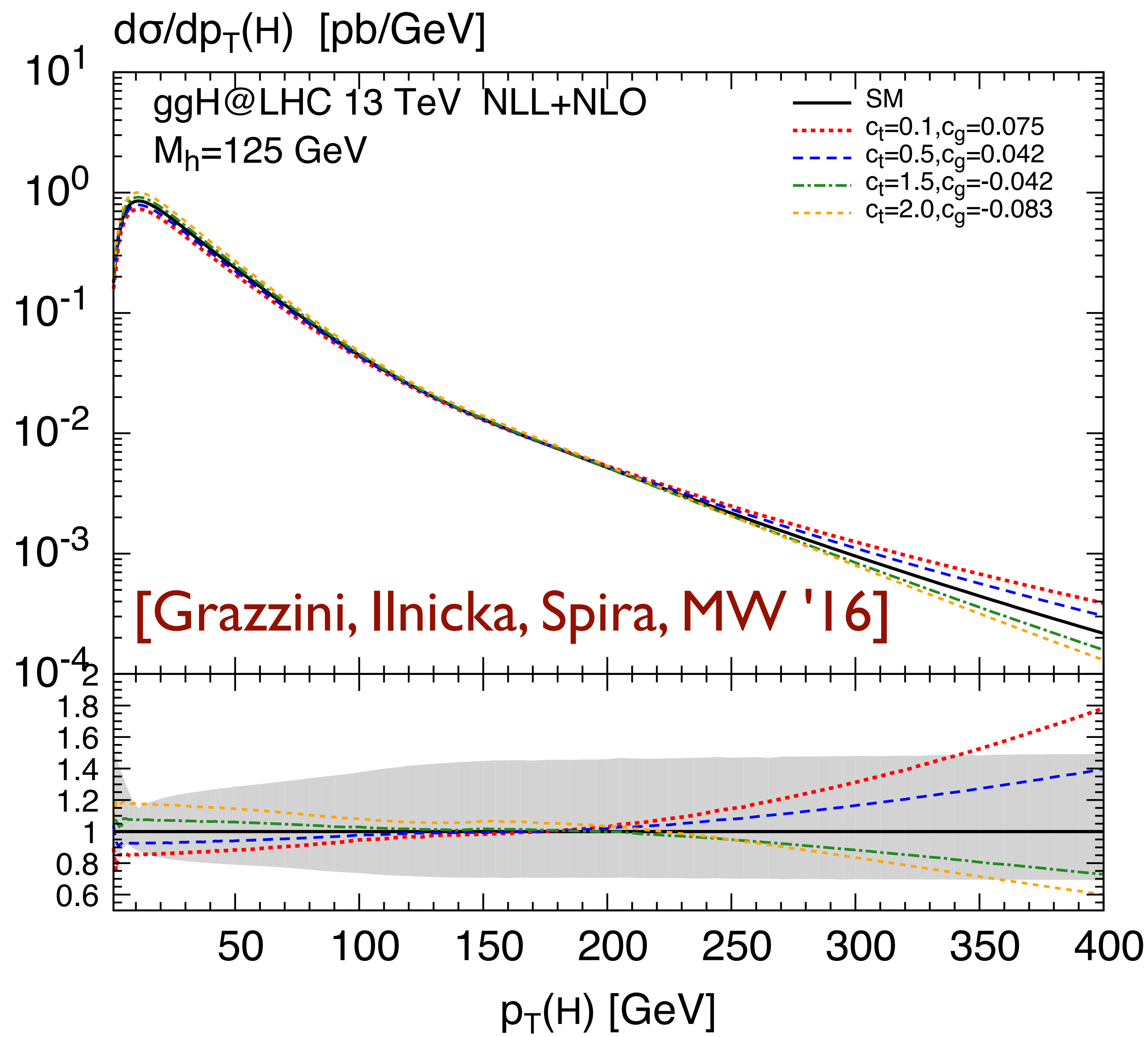


Example: Higgs coupling to gluons

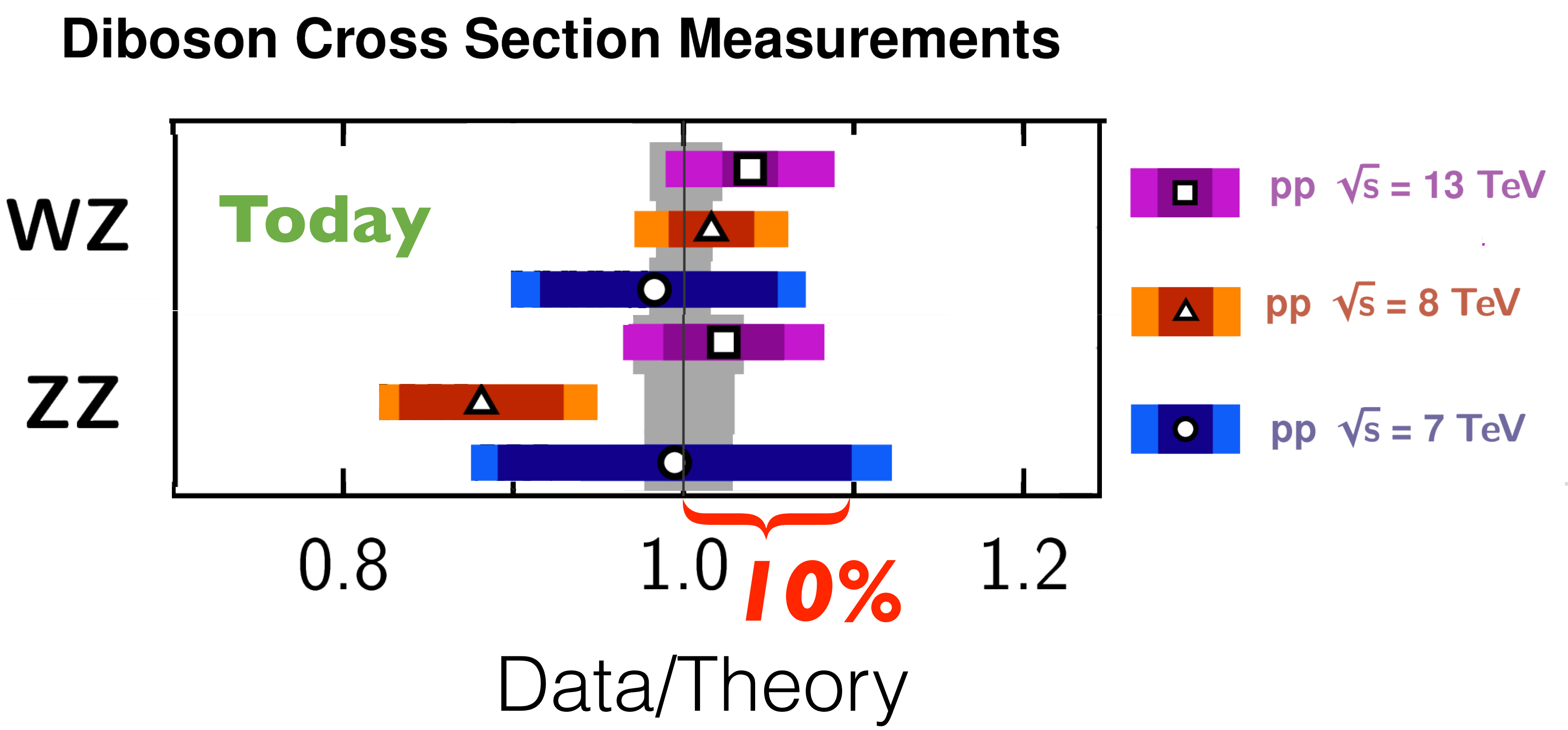
 **&**  **or**  **&**  **?**

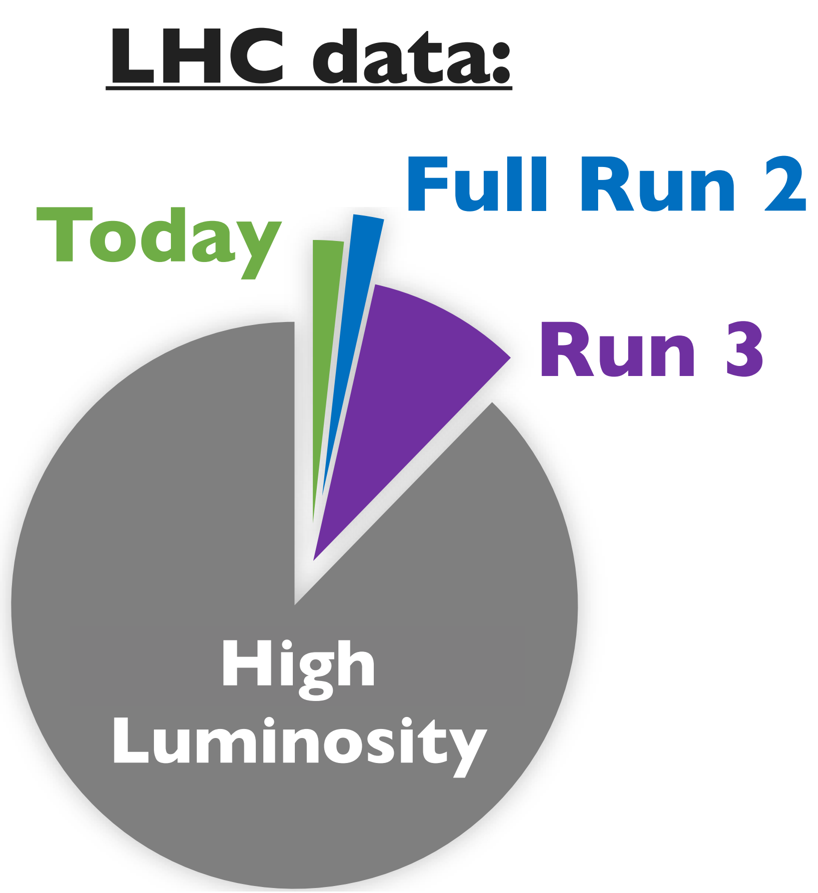
Example: Higgs coupling to gluons





Precision at the LHC





Experiment demands $\mathcal{O}$(1%) theoretical precision

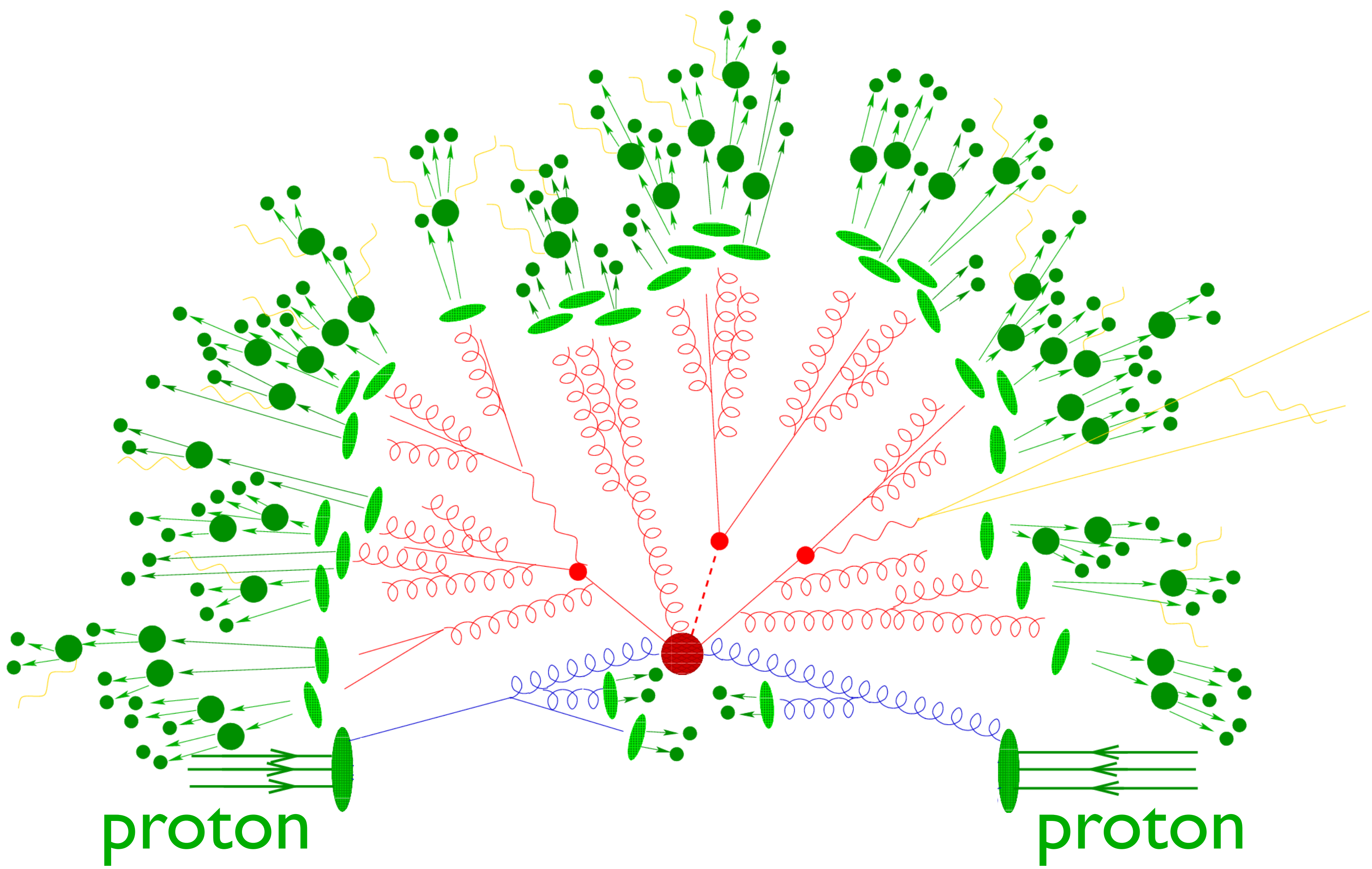
This talk

1. VV production at the LHC
   - NNLO QCD ⊕ NLO EW ⊕ loop-induced gg NLO QCD

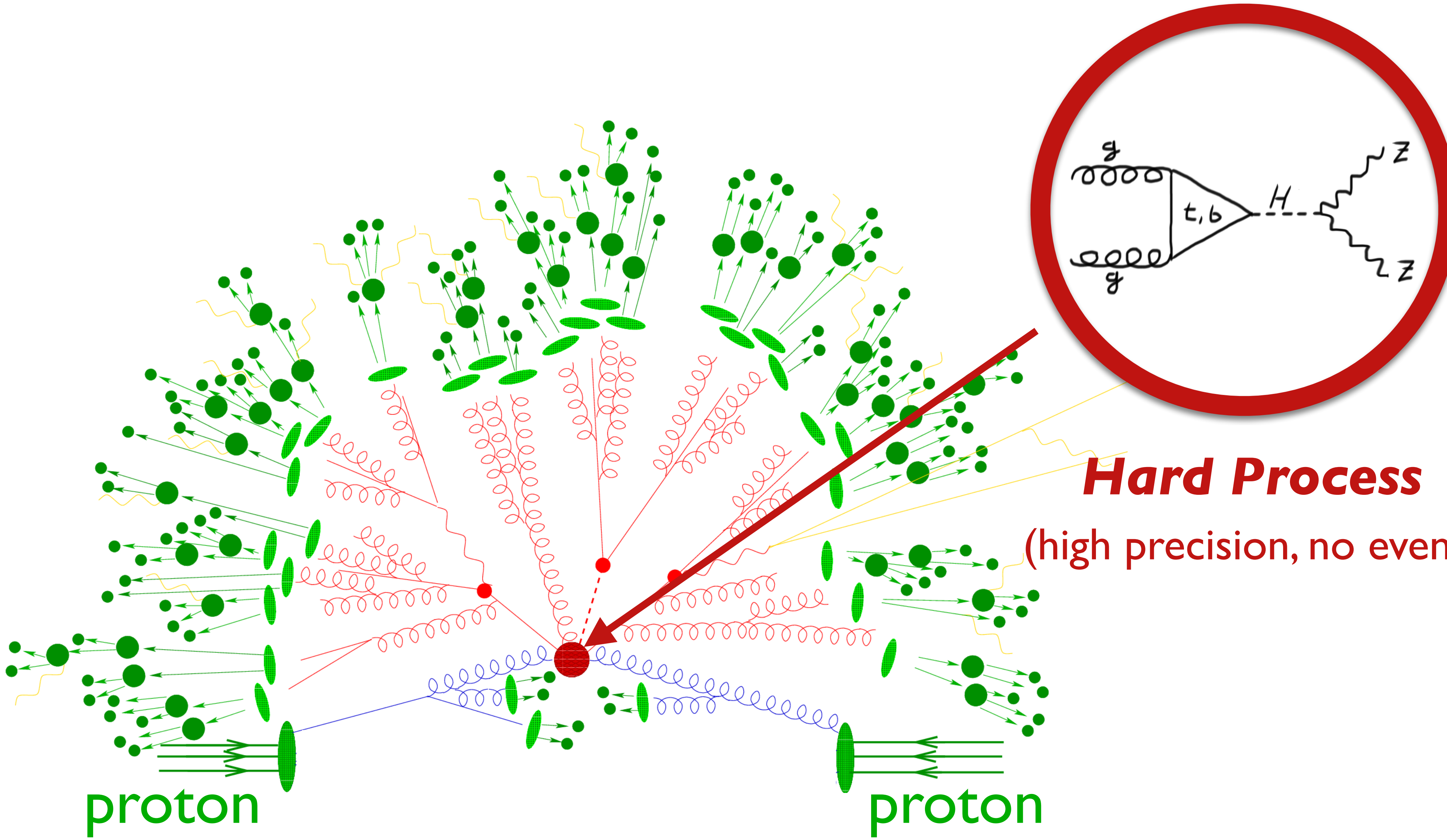
2. NNLO QCD + multi-differential resummation
   - p_T of color singlets at N3LL
   - jet-veto veto resummation at NNLL
   - double-differential resummation

3. NNLO+PS matching
   - MiNLO+reweighting
   - NNLO+PS for WW production
   - Novel approach

LHC event



LHC event



Hard Process

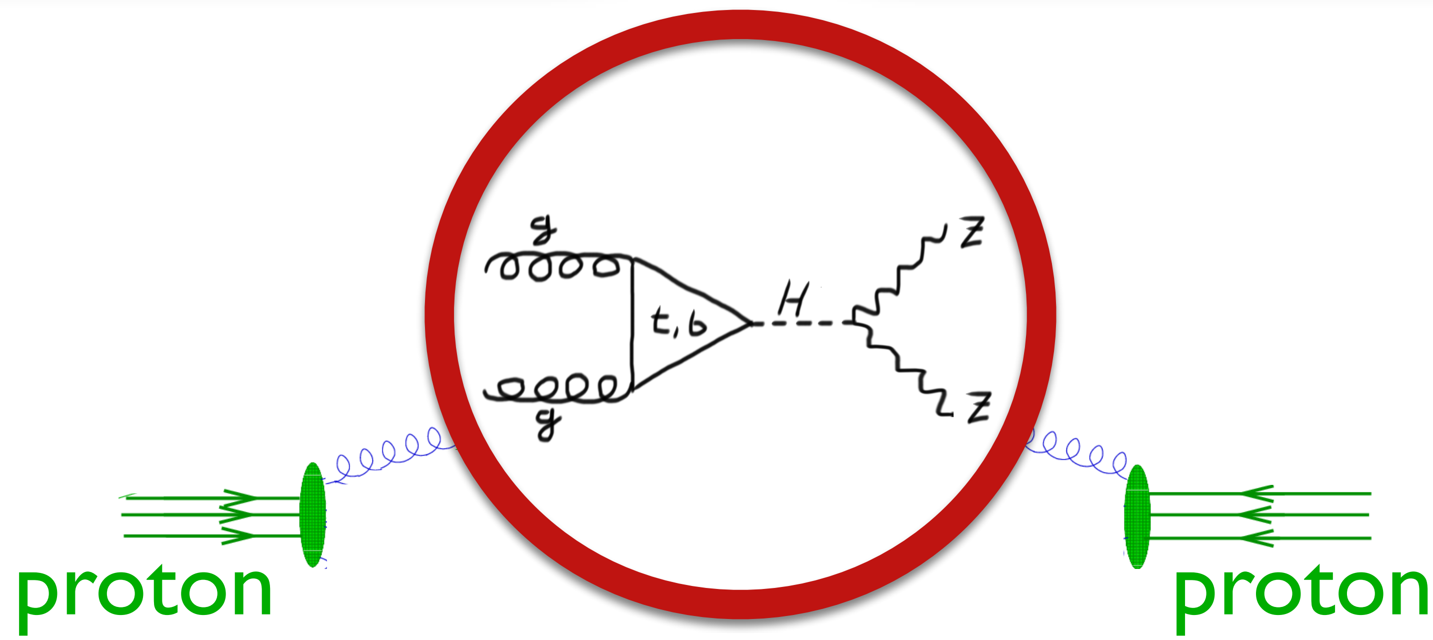
(high precision, no event)

Perturbation Theory

$$\sigma \sim \underbrace{\underbrace{\sigma_{\rm LO} \cdot (1 + \alpha}_{\rm NLO} + \alpha^2}_{\rm NNLO} + \ldots)$$

<u>Uncertainties:</u>
($\alpha \sim 0.118$)

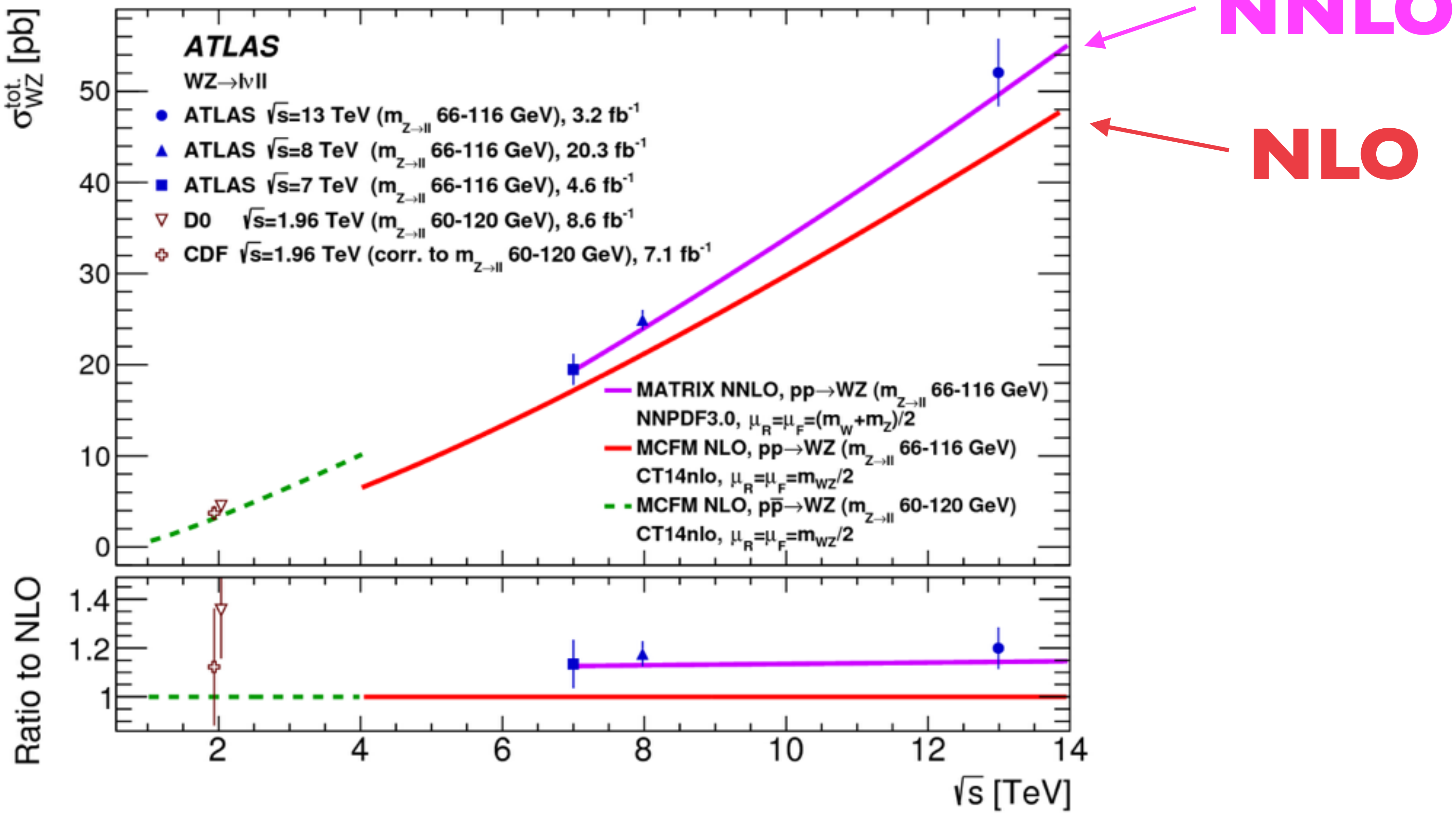
LO ~ $\mathcal{O}(100\%)$
NLO ~ $\mathcal{O}(10\%)$
NNLO ~ $\mathcal{O}(1\%)$



Hard Process

Importance of QCD corrections
(example WZ)

[Grazzini, Kallweit, Rathlev, MW '16]



NNLO crucial for accurate description of data

MATRIX

[Grazzini, Kallweit, MW '17]

Public, automated NNLO framework

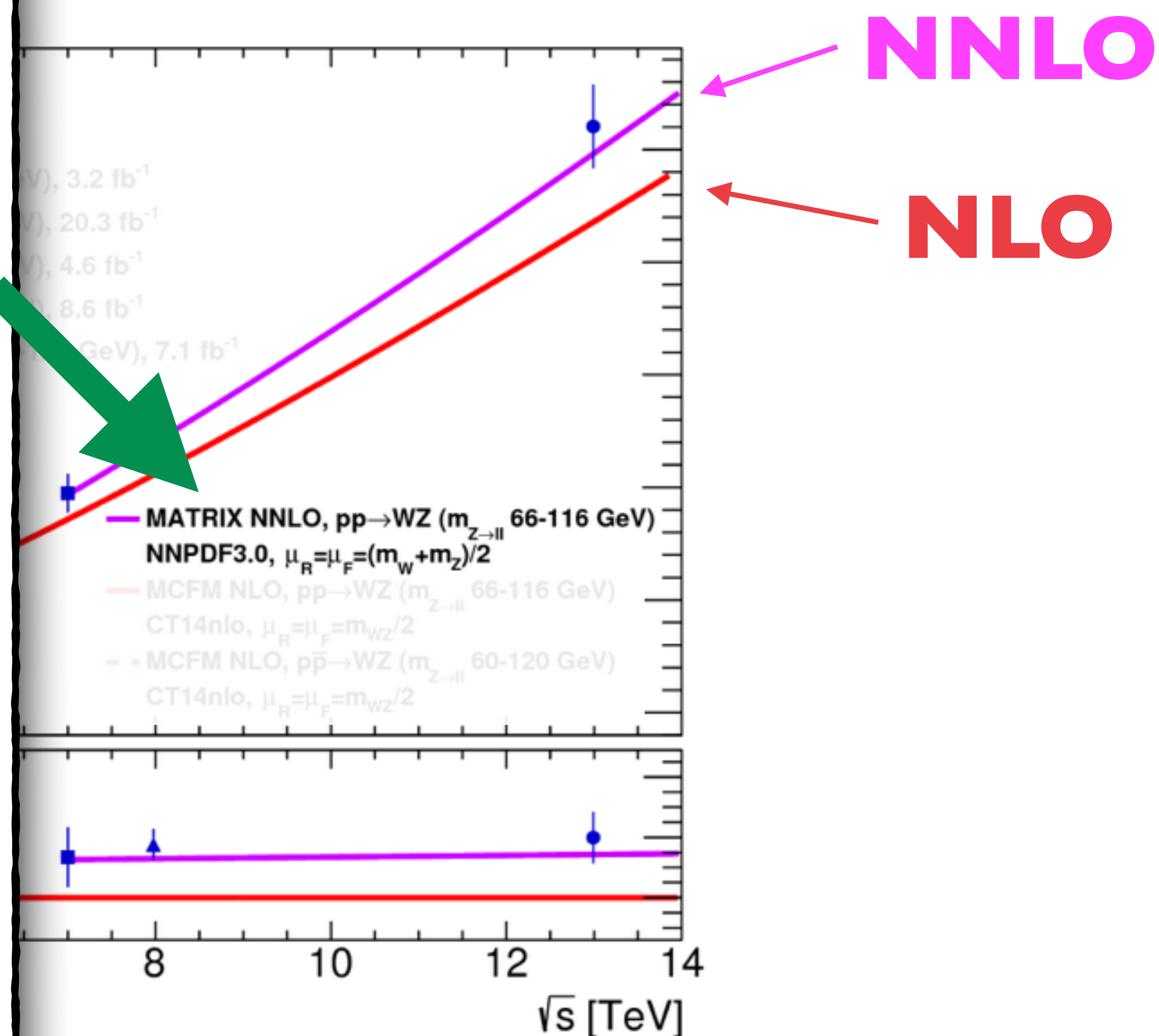
Large number of processes:

process		status	comment
$pp\to Z/\gamma^*(\to\ell\ell/\nu\nu)$		✓	validated analytically + FEWZ
$pp\to W(\to\ell\nu)$	single boson processes	✓	validated with FEWZ, NNLOjet
$pp\to H$		✓	validated analytically (by SusHi)
$pp\to\gamma\gamma$		✓	validated with 2γNNLO
$pp\to Z\gamma\to\ell\ell\gamma$	photon processes	✓	[Grazzini, Kallweit, Rathlev '15]
$pp\to Z\gamma\to\nu\nu\gamma$		✓	[Grazzini, Kallweit, Rathlev '15]
$pp\to W\gamma\to\ell\nu\gamma$		✓	[Grazzini, Kallweit, Rathlev '15]
$pp\to ZZ$		✓	[Cascioli et al. '14]
$pp\to ZZ\to\ell\ell\ell\ell$		✓	[Grazzini, Kallweit, Rathlev '15], [Kallweit, **MW** '18]
$pp\to ZZ\to\ell\ell\ell'\ell'$		✓	[Grazzini, Kallweit, Rathlev '15], [Kallweit, **MW** '18]
$pp\to ZZ\to\ell\ell\nu'\nu'$		✓	[Kallweit, **MW** '18]
$pp\to ZZ/WW\to\ell\ell\nu\nu$	massive diboson processes	✓	[Kallweit, **MW** '18]
$pp\to WW$		✓	[Gehrmann et al. '14]
$pp\to WW\to\ell\nu\,\ell'\nu'$		✓	[Grazzini, Kallweit, Pozzorini, Rathlev, **MW** '16]
$pp\to WZ$		✓	[Grazzini, Kallweit, Rathlev, **MW** '16]
$pp\to WZ\to\ell\nu\ell\ell$		✓	[Grazzini, Kallweit, Rathlev, **MW** '17]
$pp\to WZ\to\ell'\nu'\ell\ell$		✓	[Grazzini, Kallweit, Rathlev, **MW** '17]
$pp\to HH$		(✓)	not in public release

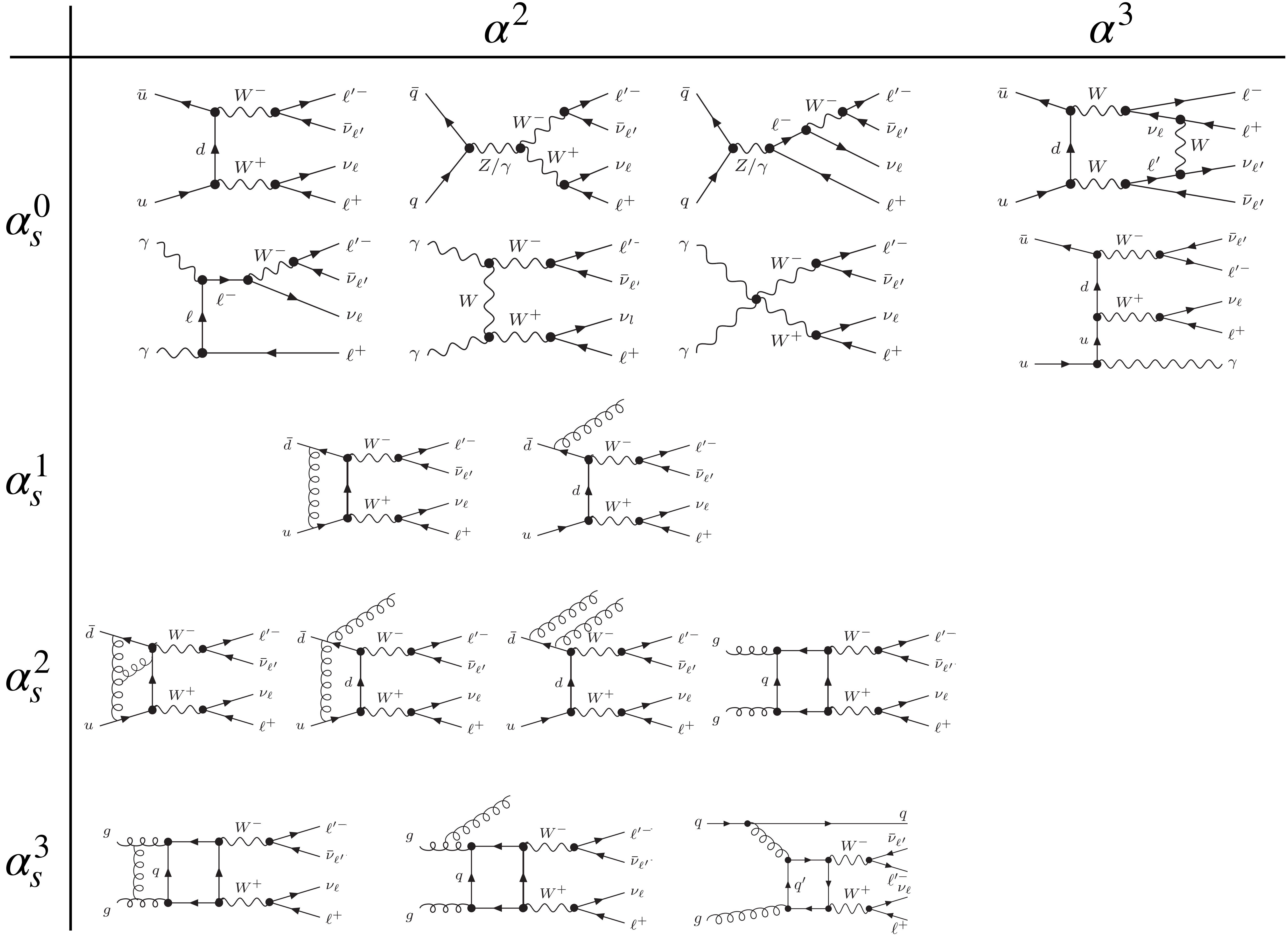
QCD corrections

ple WZ)

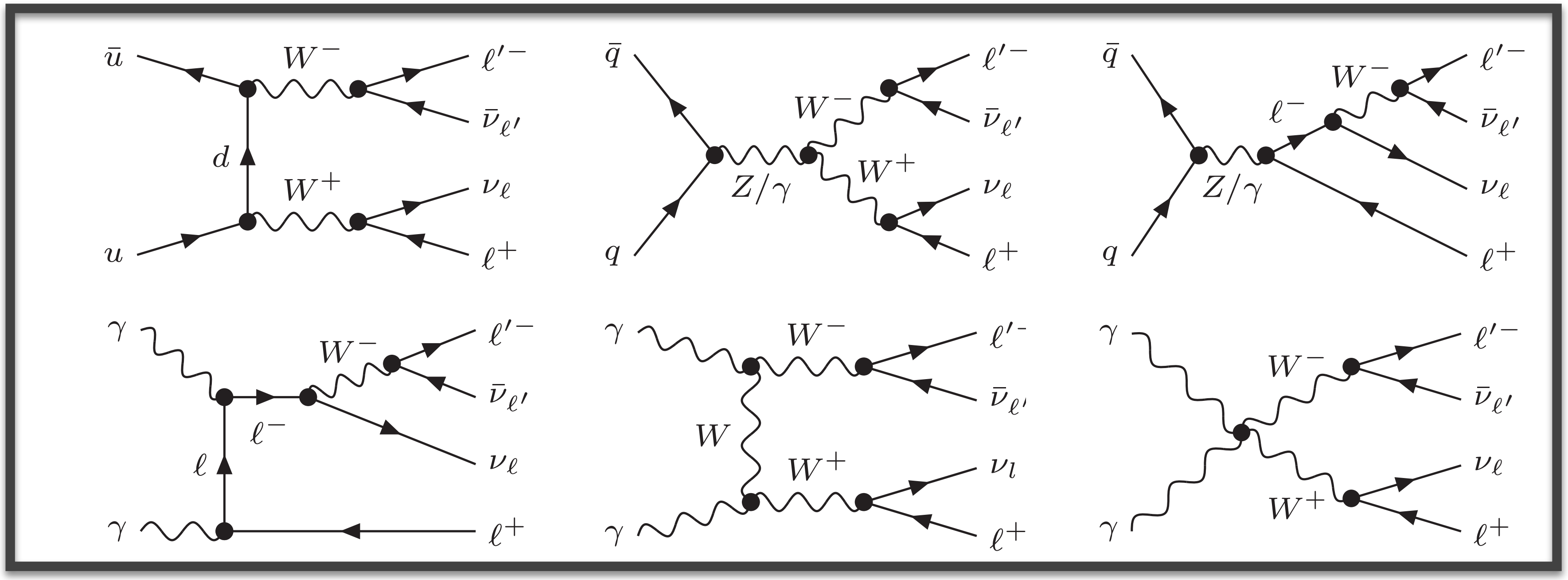
[Grazzini, Kallweit, Rathlev, MW '16]

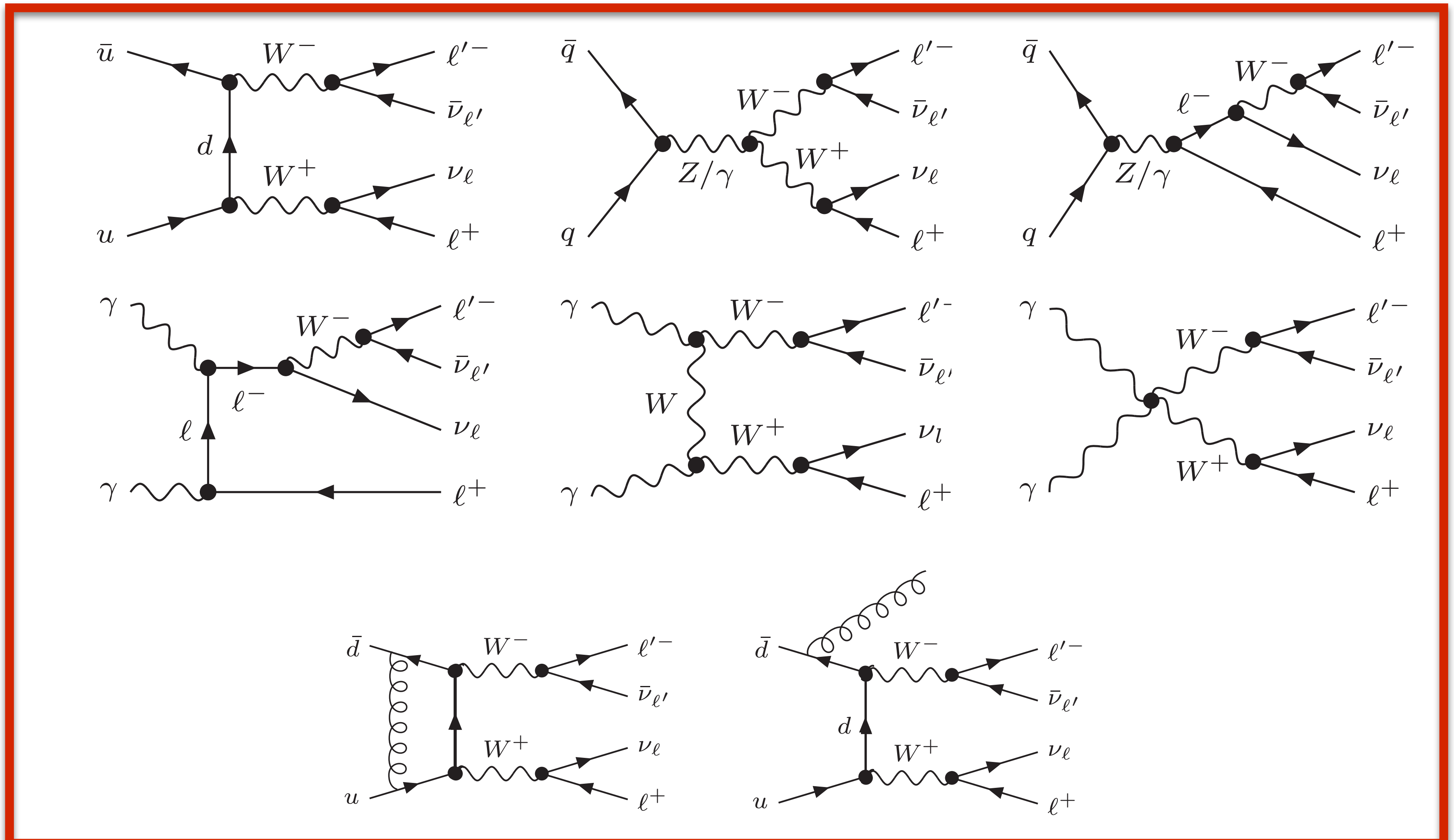


rate description of data

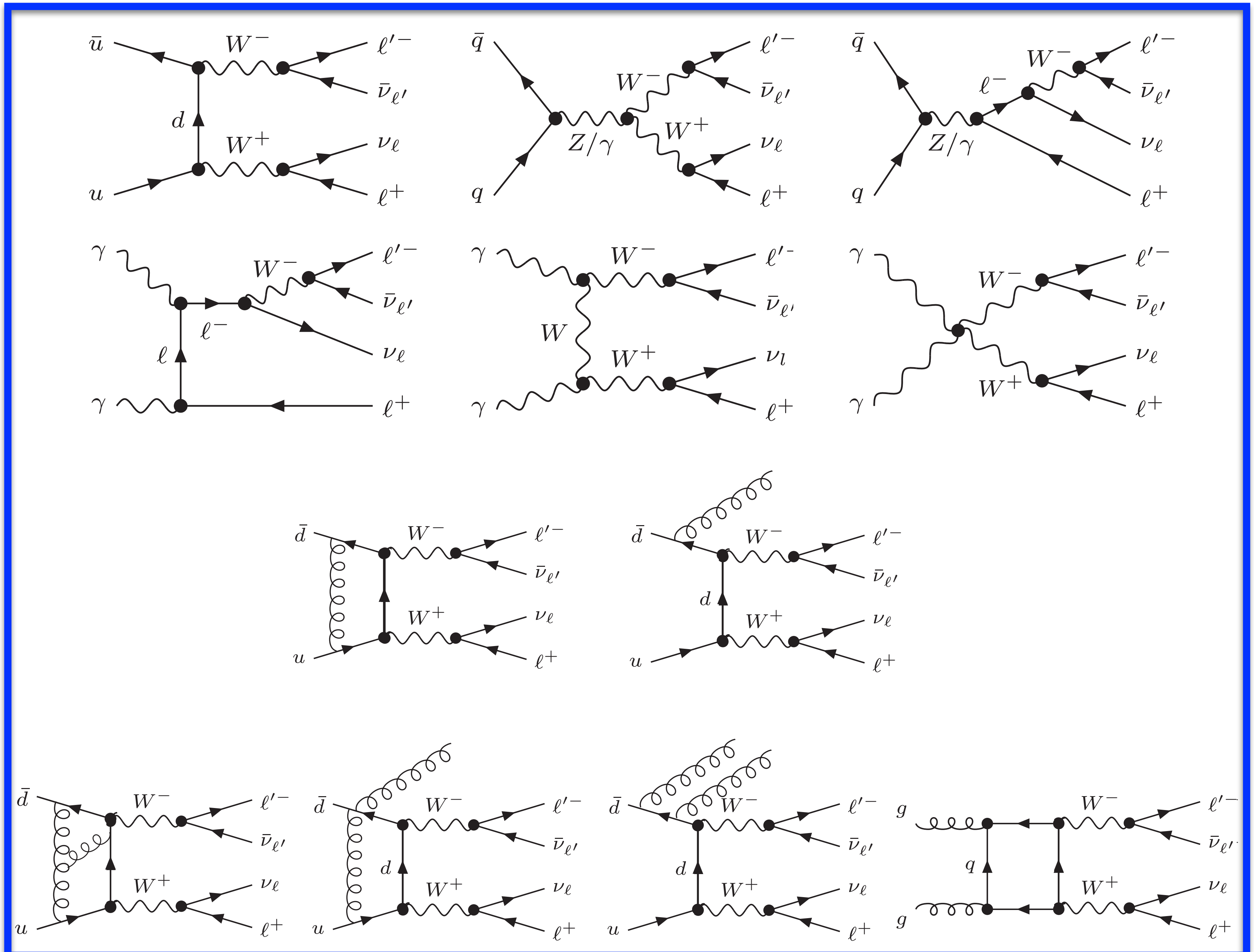


α^2
α^3
α_s^0
α_s^1
α_s^2
α_s^3

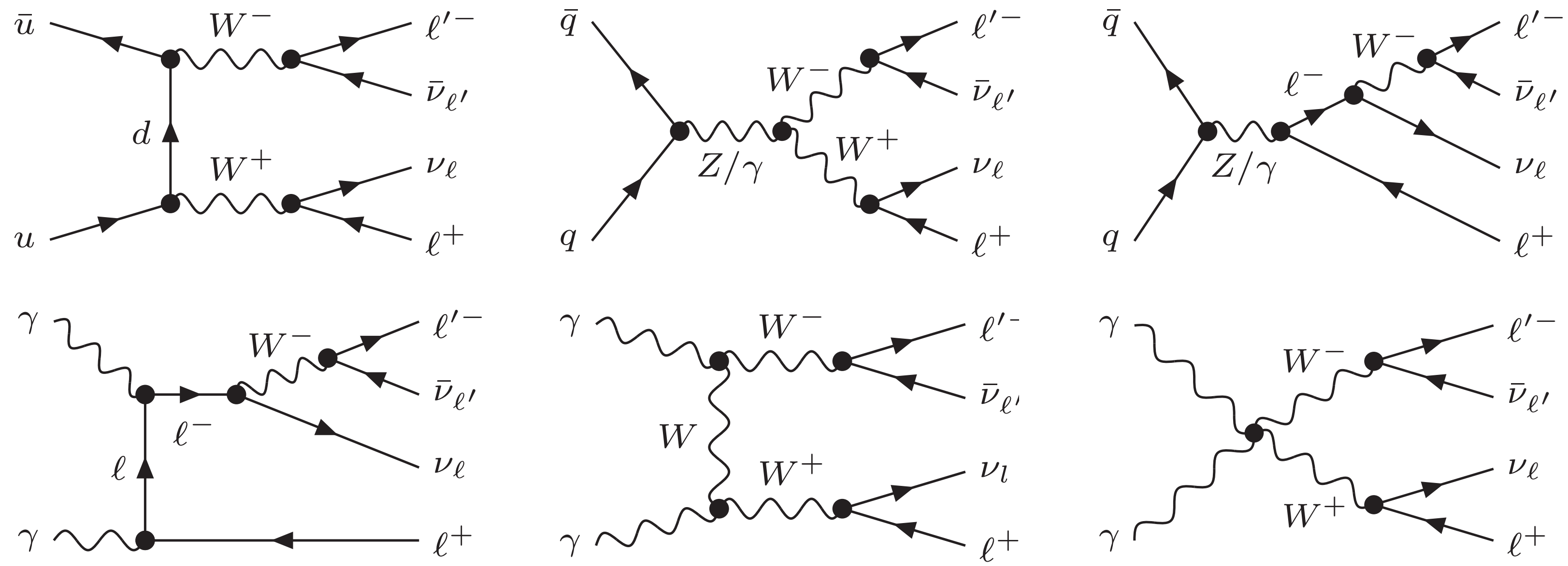
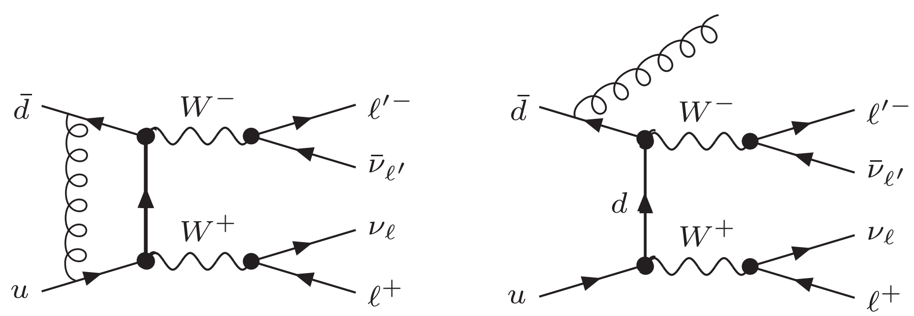
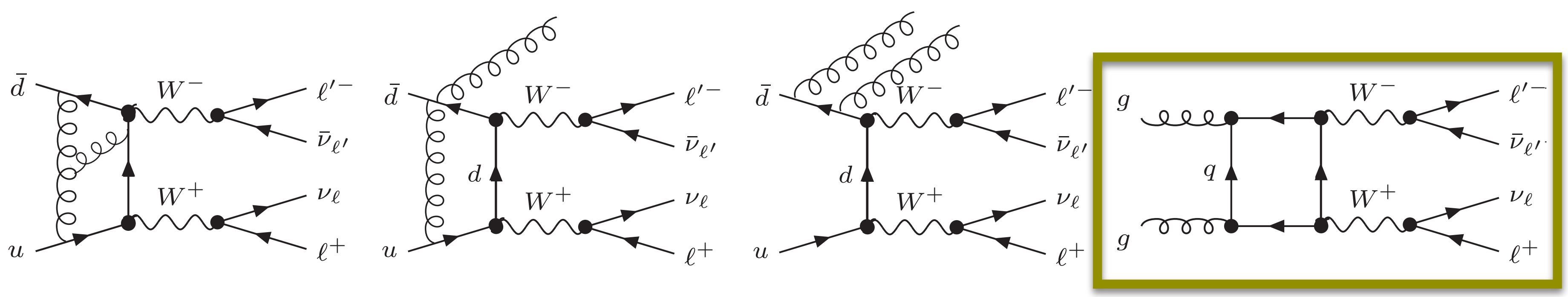
	α^2	α^3
α_s^0		**LO**
α_s^1		
α_s^2		
α_s^3		

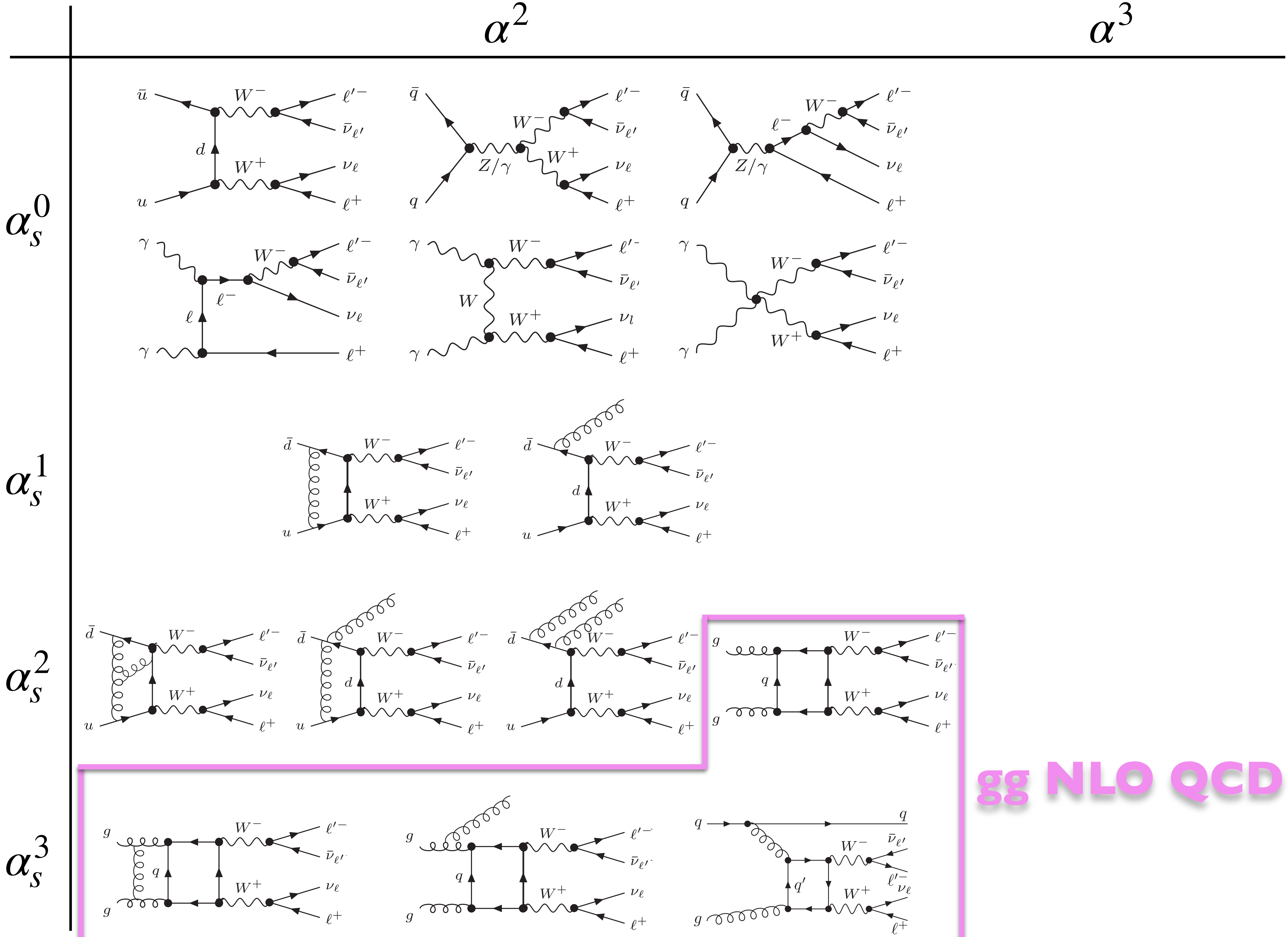
	α^2	α^3
α_s^0		**NLO QCD**
α_s^1		
α_s^2		
α_s^3		

	α^2	α^3
α_s^0		
α_s^1		
α_s^2		
α_s^3		

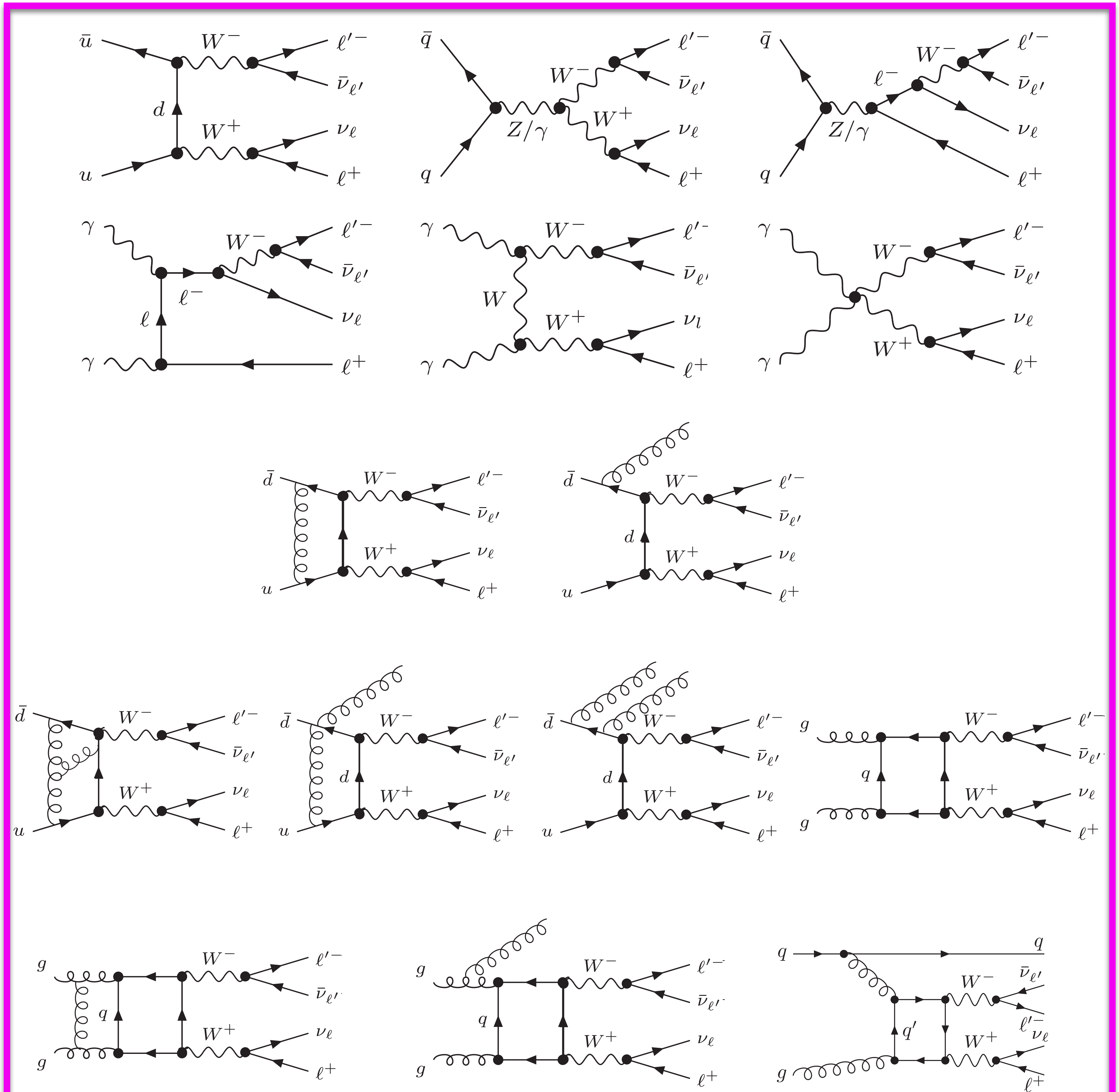


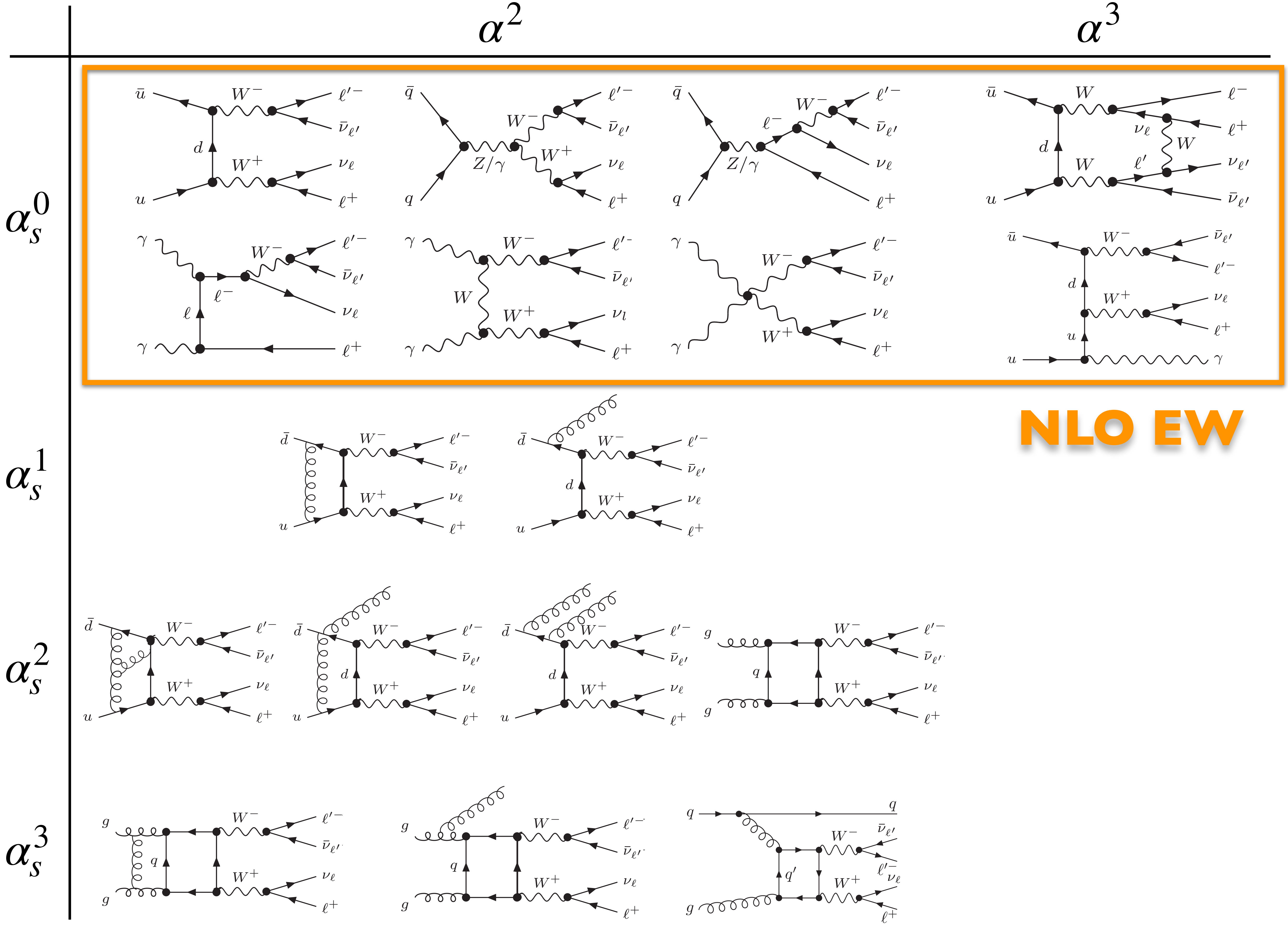
NNLO QCD

	α^2	α^3
α_s^0		
α_s^1		
α_s^2	 **gg LO QCD**	
α_s^3		


α^2
α^3
α_s^0
α_s^1
α_s^2
α_s^3
gg NLO QCD

	α^2	α^3
α_s^0		
α_s^1		**nNNLO QCD**
α_s^2		
α_s^3		




α^2
α^3
α_s^0
α_s^1
α_s^2
α_s^3
NLO EW

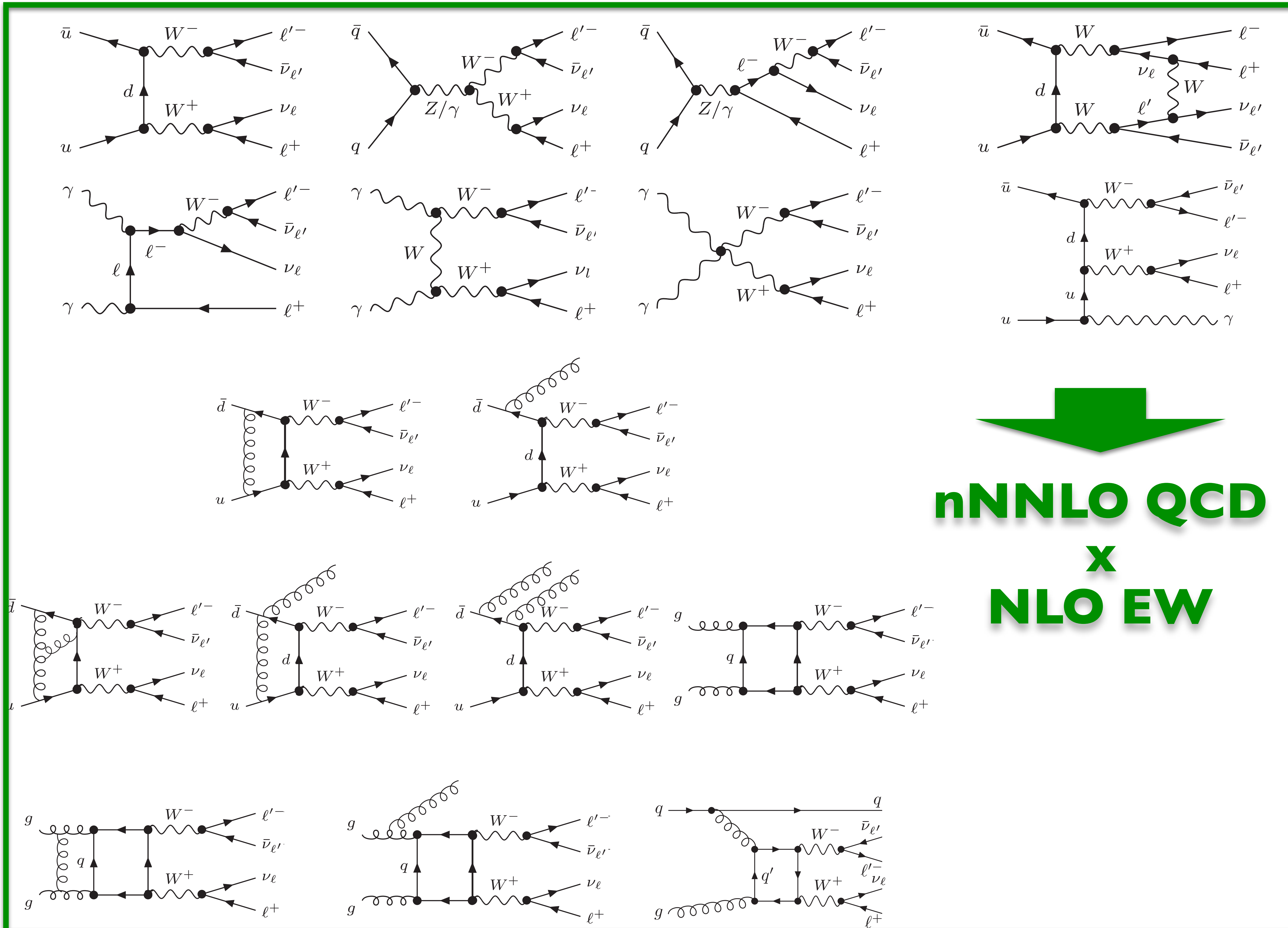
α^2 α^3

α_s^0

α_s^1

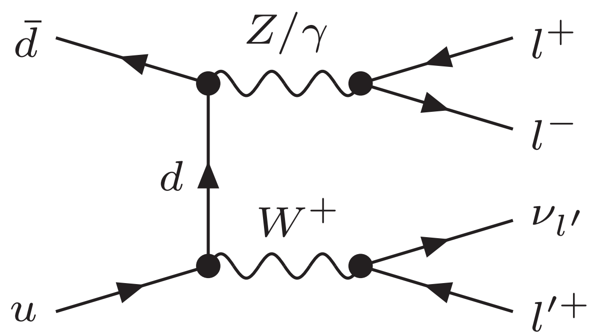
α_s^2

α_s^3



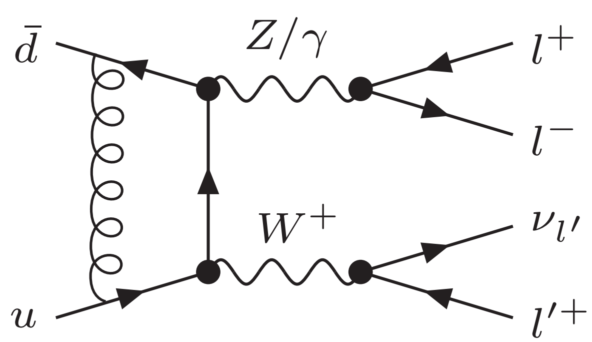
NLO corrections through subtraction

LO
(pp→X)

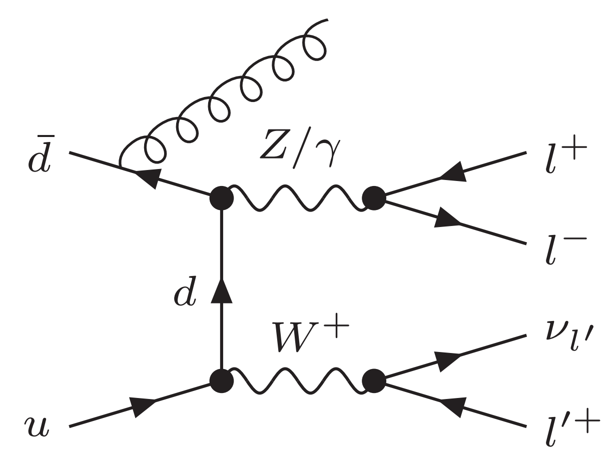


$\rightarrow \mathrm{d}\sigma^{\mathrm{B}}$

NLO
(pp→X)



$\rightarrow \mathrm{d}\sigma^{\mathrm{V}}$

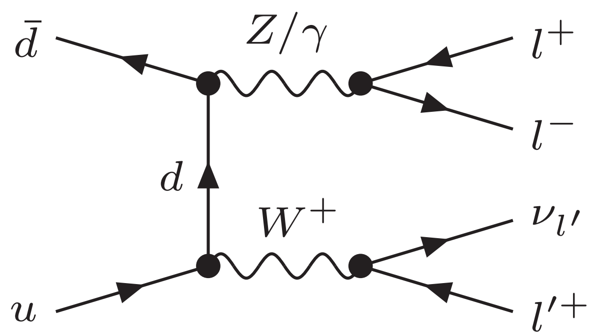


$\rightarrow \mathrm{d}\sigma^{\mathrm{R}}$

$$\sigma_{\mathrm{NLO}} = \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{B}} + \int_{\Phi_{\mathrm{B+1}}} \mathrm{d}\sigma^{\mathrm{R}} + \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{V}}$$

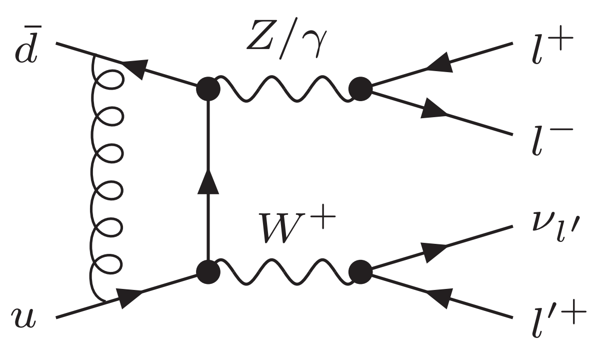
NLO corrections through subtraction

LO (pp→X)

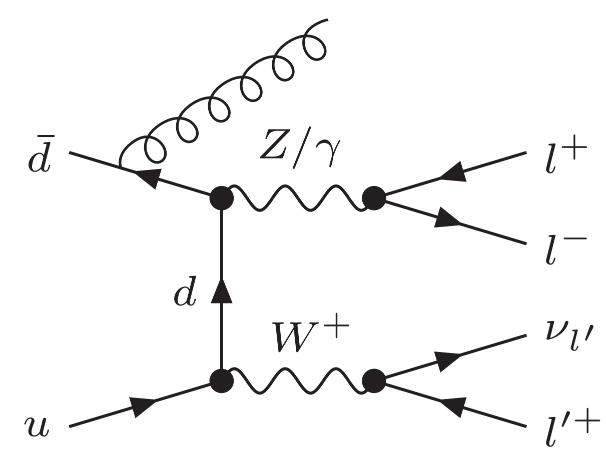


$\rightarrow \mathrm{d}\sigma^{\mathrm{B}}$

NLO (pp→X)



$\rightarrow \mathrm{d}\sigma^{\mathrm{V}}$



$\rightarrow \mathrm{d}\sigma^{\mathrm{R}}$

$\mathrm{d}\sigma^{\mathrm{S}}$: subtraction term

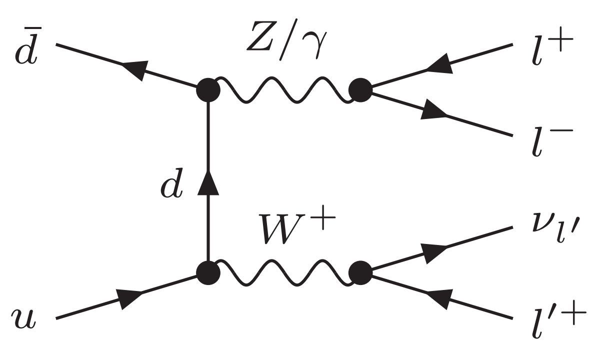
- → CS [Catani, Seymour '96]
- → FKS [Frixione, Kunszt, Signer '96]
- → Antenna [Gehrmann et al. '05]
- → ...

$$\sigma_{\mathrm{NLO}} = \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{B}} + \int_{\Phi_{\mathrm{B+1}}} \mathrm{d}\sigma^{\mathrm{R}} + \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{V}}$$

$$= \int_{\Phi_{\mathrm{B}}} \mathrm{d}\sigma^{\mathrm{B}} + \int_{\Phi_{\mathrm{B+1}}} \left(\mathrm{d}\sigma^{\mathrm{R}} - \mathrm{d}\sigma^{\mathrm{S}}\right) + \int_{\Phi_{\mathrm{B}}} \left(\mathrm{d}\sigma^{\mathrm{V}} + \int_{1} \mathrm{d}\sigma^{\mathrm{S}}\right)$$

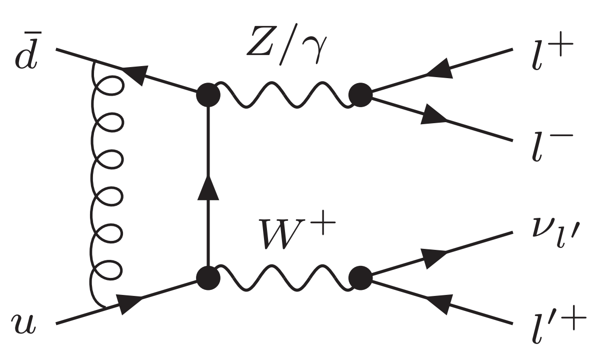
NNLO through X+jet at NLO + Slicing

LO (pp→X)

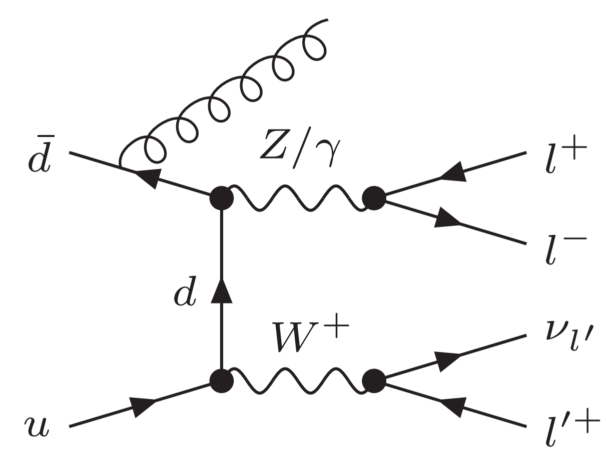


$\to \mathrm{d}\sigma^{\mathrm{B}}$

NLO (pp→X)

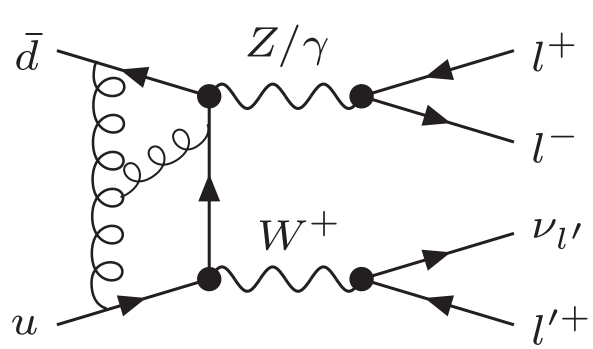


$\to \mathrm{d}\sigma^{\mathrm{V}}$

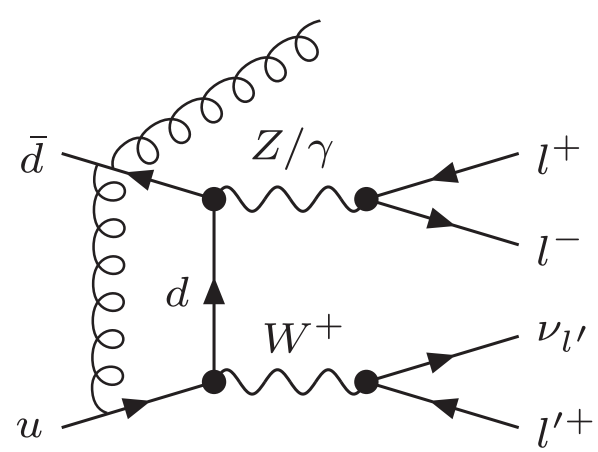


$\to \mathrm{d}\sigma^{\mathrm{R}}$

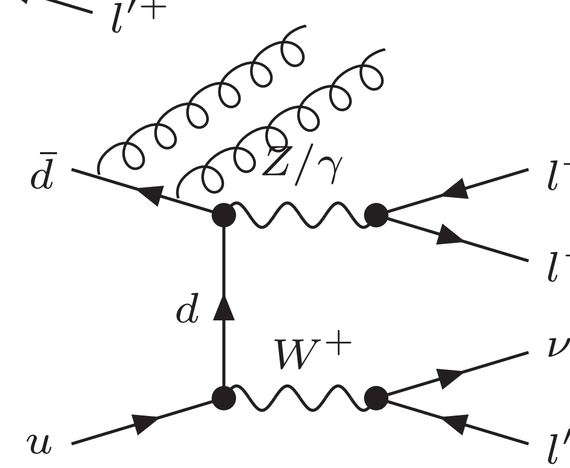
NNLO (pp→X)



$\to \mathrm{d}\sigma^{\mathrm{VV}}$



$\to \mathrm{d}\sigma^{\mathrm{RV}}$



$\to \mathrm{d}\sigma^{\mathrm{RR}}$

NNLO through X+jet at NLO + Slicing

$$\sigma_{\mathrm{NLO}}^{\mathrm{X+jet}} = \int_{\Phi_{\mathrm{RV}}} \mathrm{d}\sigma^{\mathrm{RV}} + \int_{\Phi_{\mathrm{RV}+1}} \left(\mathrm{d}\sigma^{\mathrm{RR}} - \mathrm{d}\sigma^{\mathrm{S}}\right) + \int_{\Phi_{\mathrm{RV}}} \left(\mathrm{d}\sigma^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma^{\mathrm{S}}\right)$$

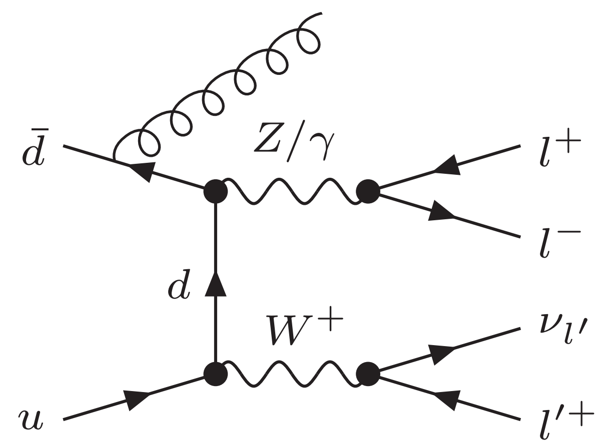
$\mathrm{d}\sigma^{\mathrm{S}}$: subtraction term

- → CS [Catani, Seymour '96]
- → FKS [Frixione, Kunszt, Signer '96]
- → Antenna [Gehrmann et al. '05]
- → ...

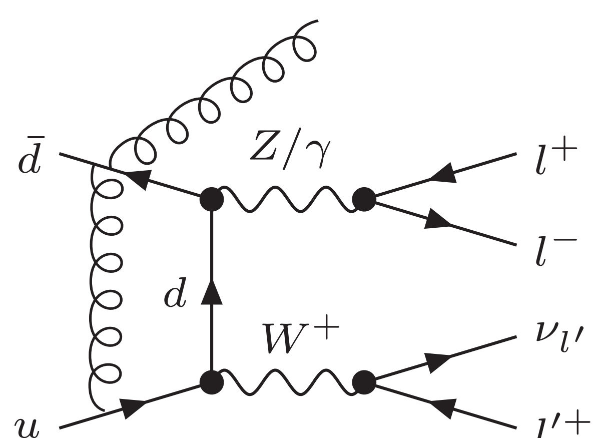
~~LO~~
~~(pp→X)~~

~~N~~LO
(pp→X+jet)

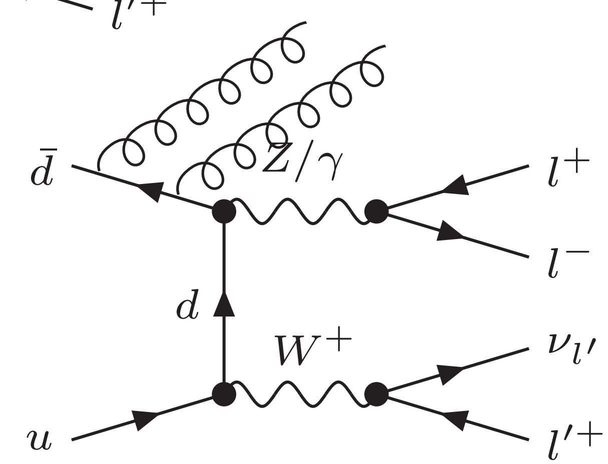
~~N~~NLO
(pp→X+jet)



$\rightarrow \mathrm{d}\sigma^{\mathrm{R}}$



$\rightarrow \mathrm{d}\sigma^{\mathrm{RV}}$

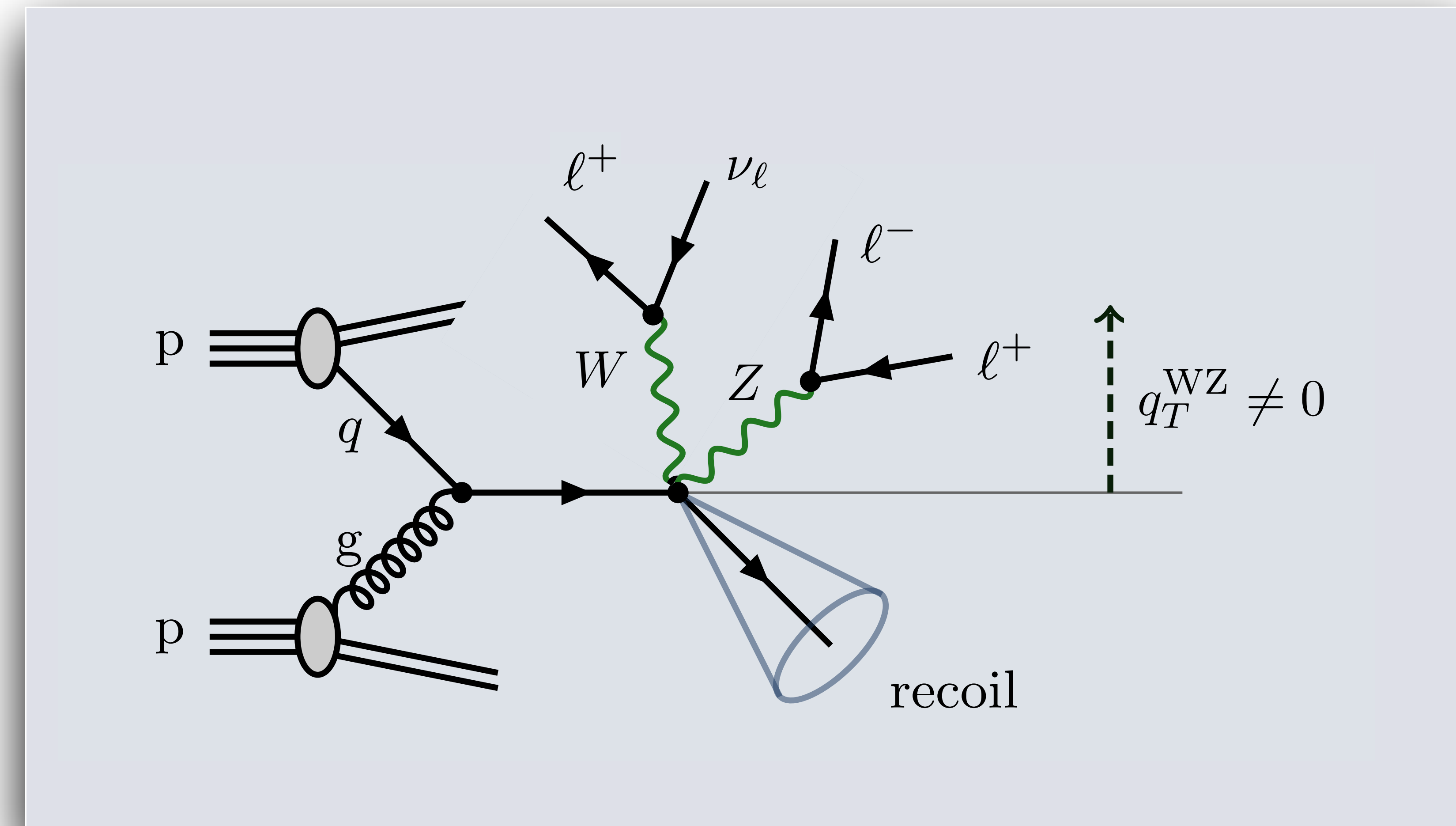


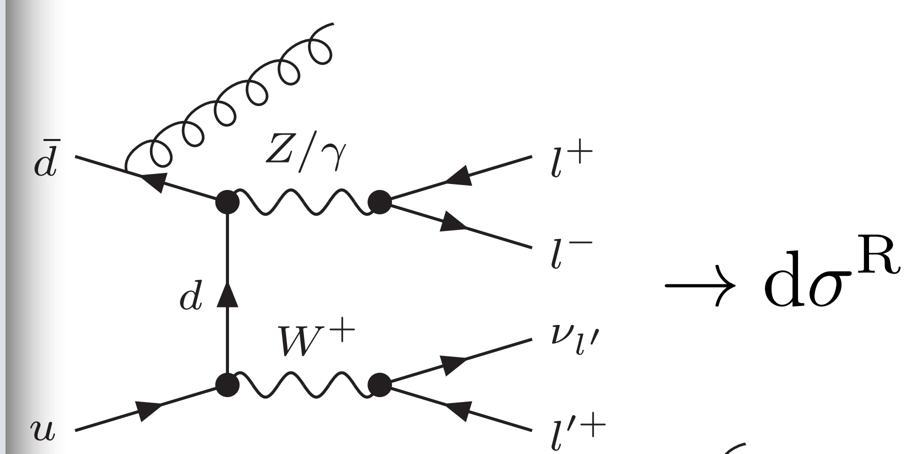
$\rightarrow \mathrm{d}\sigma^{\mathrm{RR}}$

NNLO through X+jet at NLO + Slicing

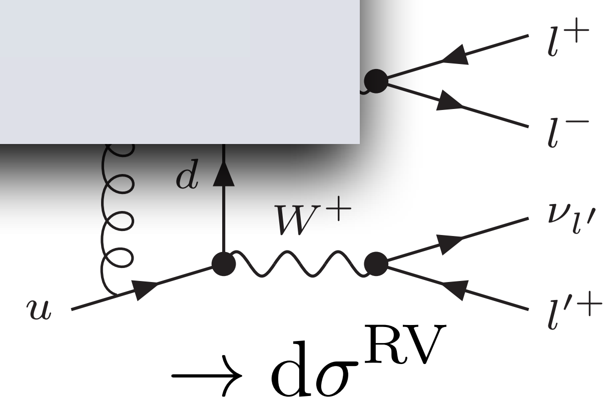
$$\sigma^{\mathrm{X+jet}}_{\mathrm{NLO}} = \left[\int_{\Phi_{\mathrm{RV}}} \mathrm{d}\sigma^{\mathrm{RV}} + \int_{\Phi_{\mathrm{RV+1}}} \left(\mathrm{d}\sigma^{\mathrm{RR}} - \mathrm{d}\sigma^{\mathrm{S}}\right) + \int_{\Phi_{\mathrm{RV}}} \left(\mathrm{d}\sigma^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma^{\mathrm{S}}\right)\right]_{\frac{q_T}{Q} \equiv r > r_{\mathrm{cut}}}$$

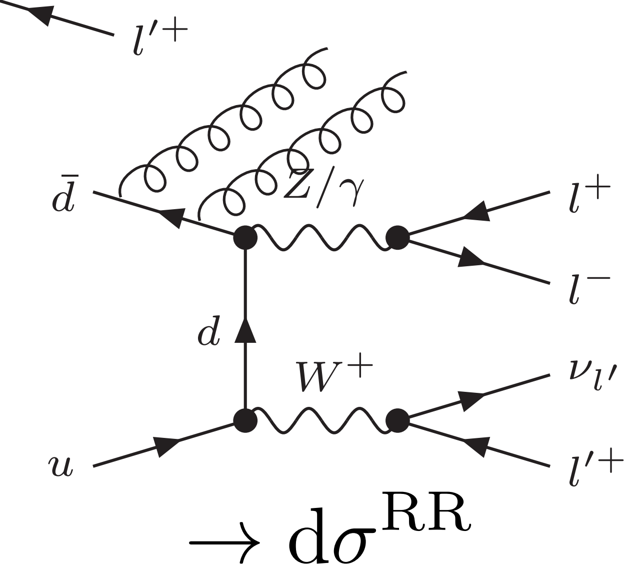
$$\xrightarrow{r_{\mathrm{cut}} \ll 1} \left[A \cdot \log^4(r_{\mathrm{cut}}) + B \cdot \log^3(r_{\mathrm{cut}}) + C \cdot \log^2(r_{\mathrm{cut}}) + D \cdot \log(r_{\mathrm{cut}})\right] \otimes \mathrm{d}\sigma^{\mathrm{B}}$$





NNLO
(pp→X+jet)





NNLO through X+jet at NLO + Slicing

$$\sigma^{\mathrm{X+jet}}_{\mathrm{NLO}} = \left[\int_{\Phi_{\mathrm{RV}}} \mathrm{d}\sigma^{\mathrm{RV}} + \int_{\Phi_{\mathrm{RV+1}}} \left(\mathrm{d}\sigma^{\mathrm{RR}} - \mathrm{d}\sigma^{\mathrm{S}}\right) + \int_{\Phi_{\mathrm{RV}}} \left(\mathrm{d}\sigma^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma^{\mathrm{S}}\right)\right]_{\frac{q_T}{Q} \equiv r > r_{\mathrm{cut}}}$$

$$\xrightarrow{r_{\mathrm{cut}} \ll 1} \left[A \cdot \log^4(r_{\mathrm{cut}}) + B \cdot \log^3(r_{\mathrm{cut}}) + C \cdot \log^2(r_{\mathrm{cut}}) + D \cdot \log(r_{\mathrm{cut}})\right] \otimes \mathrm{d}\sigma^{\mathrm{B}}$$

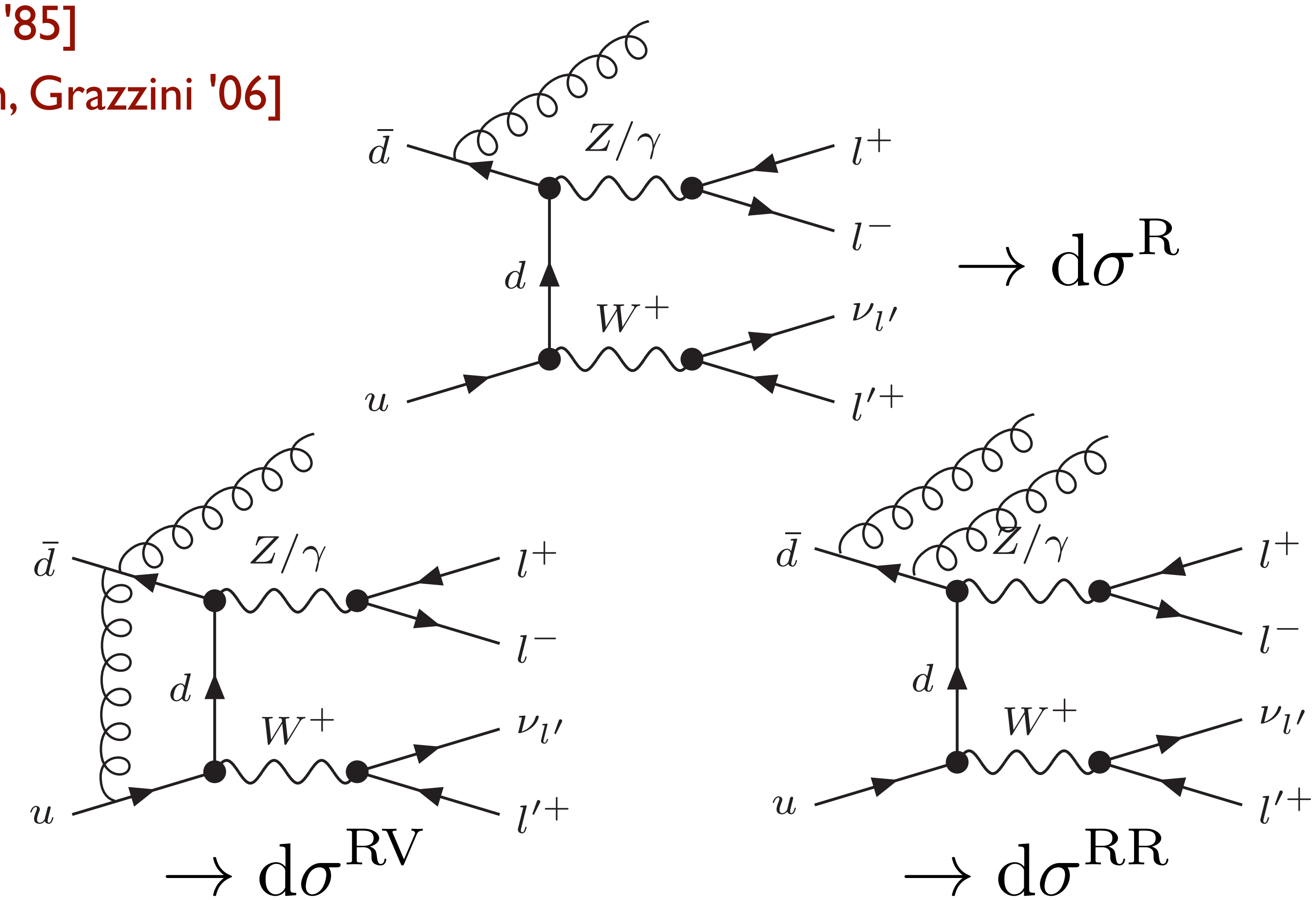
$$= \int_{r > r_{\mathrm{cut}}} \left[d\sigma^{(\mathrm{res})}\right]_{\mathrm{f.o.}} \equiv \Sigma_{\mathrm{NNLO}}(r_{\mathrm{cut}}) \otimes \mathrm{d}\sigma^{\mathrm{B}}$$

[Collins, Soper, Sterman '85]
[Bozzi, Catani, de Florian, Grazzini '06]

~~LO~~ (pp→X)

~~N~~LO (pp→X+jet)

~~N~~NLO (pp→X+jet)



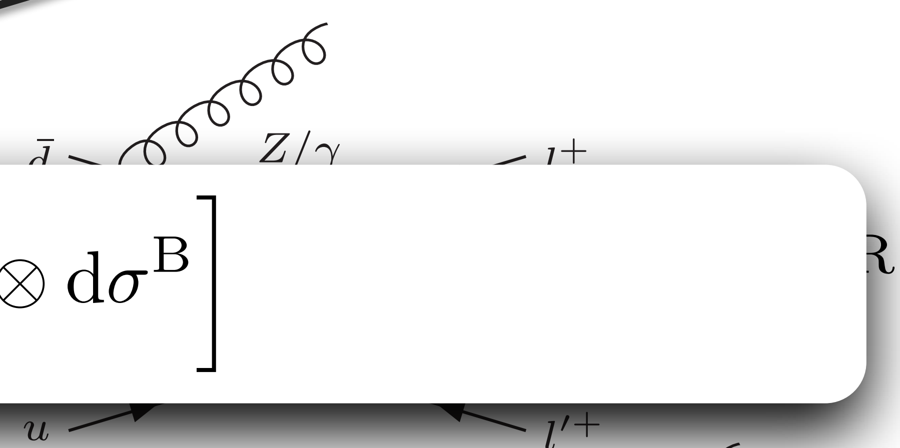
NNLO through X+jet at NLO + Slicing

$$\sigma_{\rm NLO}^{\rm X+jet} = \left[\int_{\Phi_{\rm RV}} {\rm d}\sigma^{\rm RV} + \int_{\Phi_{\rm RV+1}} ({\rm d}\sigma^{\rm RR} - {\rm d}\sigma^{\rm S}) + \int_{\Phi_{\rm RV}} \left({\rm d}\sigma^{\rm RV} + \int_1 {\rm d}\sigma^{\rm S}\right)\right]_{\frac{q_T}{Q} \equiv r > r_{\rm cut}}$$

$$\xrightarrow{r_{\rm cut} \ll 1} \left[A \cdot \log^4(r_{\rm cut}) + B \cdot \log^3(r_{\rm cut}) + C \cdot \log^2(r_{\rm cut}) + D \cdot \log(r_{\rm cut})\right] \otimes {\rm d}\sigma^{\rm B}$$

$$= \int_{r > r_{\rm cut}} \left[d\sigma^{\rm (res)}\right]_{\rm f.o.} \equiv \Sigma_{\rm NNLO}(r_{\rm cut}) \otimes {\rm d}\sigma^{\rm B}$$

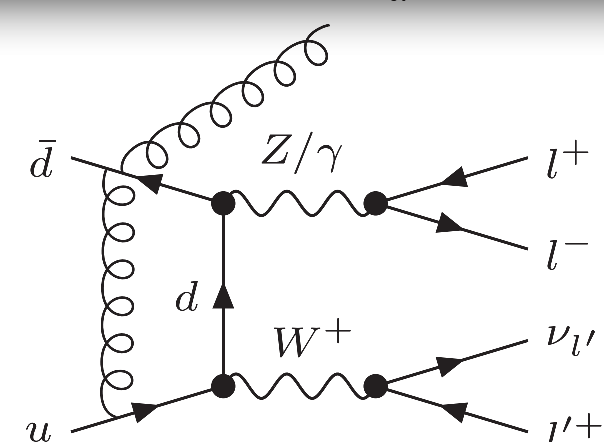
~~LO (pp→X)~~



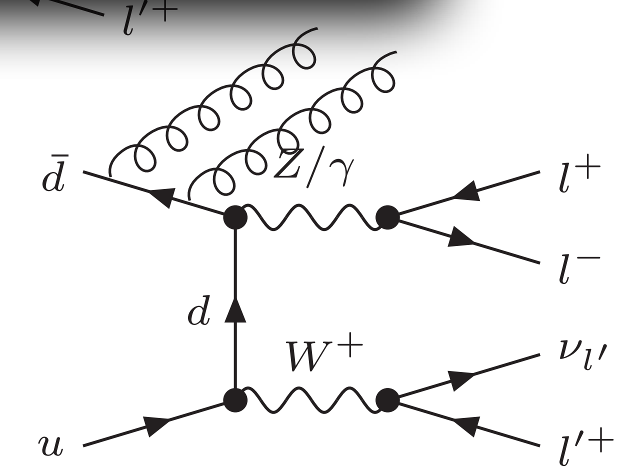
$${\rm d}\sigma_{\rm NNLO}^{\rm X} = \left[{\rm d}\sigma_{\rm NLO}^{\rm X+jet}\Big|_{r > r_{\rm cut}} - \Sigma_{\rm NNLO}(r_{\rm cut}) \otimes {\rm d}\sigma^{\rm B}\right]$$

~~N~~ (pp→X+jet)

~~N~~NLO (pp→X+jet)



$\to {\rm d}\sigma^{\rm RV}$



$\to {\rm d}\sigma^{\rm RR}$

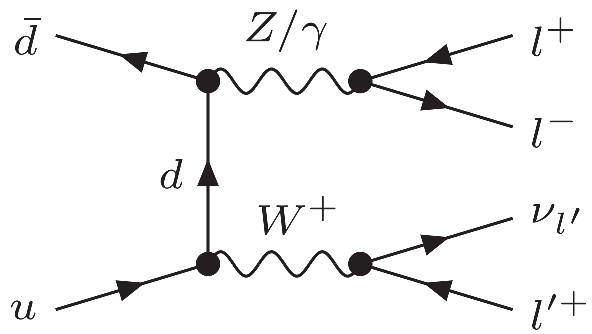
NNLO through X+jet at NLO + Slicing

$$\mathrm{d}\sigma^{\mathrm{X}}_{\mathrm{NNLO}} = \left[\mathrm{d}\sigma^{\mathrm{X+jet}}_{\mathrm{NLO}} \Big|_{r > r_{\mathrm{cut}}} - \Sigma_{\mathrm{NNLO}}(r_{\mathrm{cut}}) \otimes \mathrm{d}\sigma^{\mathrm{B}} \right] +$$

q_T subtraction

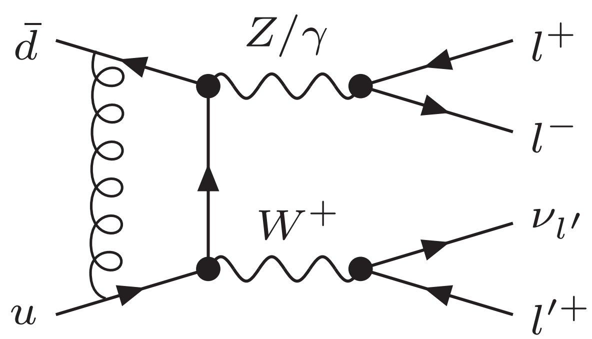
[Catani, Grazzini '07]

LO (pp→X)

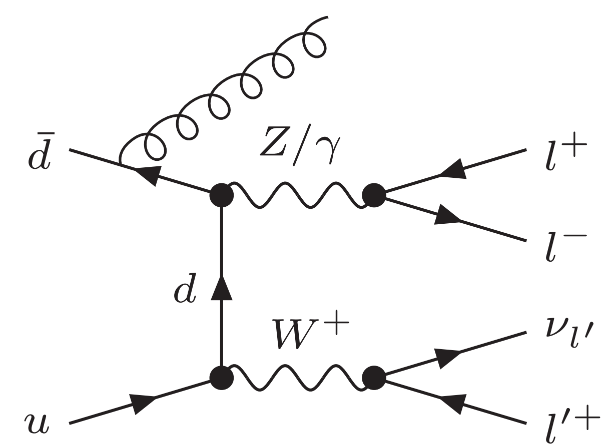


$\rightarrow \mathrm{d}\sigma^{\mathrm{B}}$

NLO (pp→X)

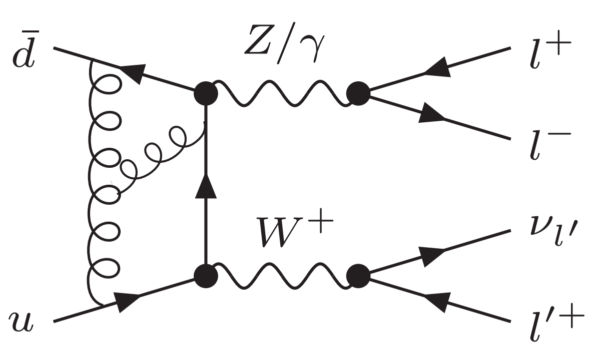


$\rightarrow \mathrm{d}\sigma^{\mathrm{V}}$

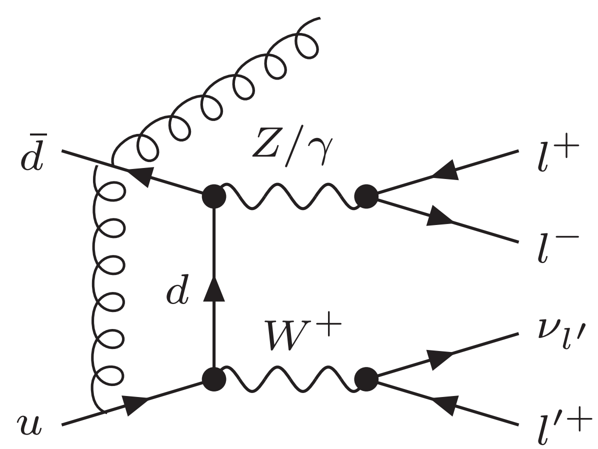


$\rightarrow \mathrm{d}\sigma^{\mathrm{R}}$

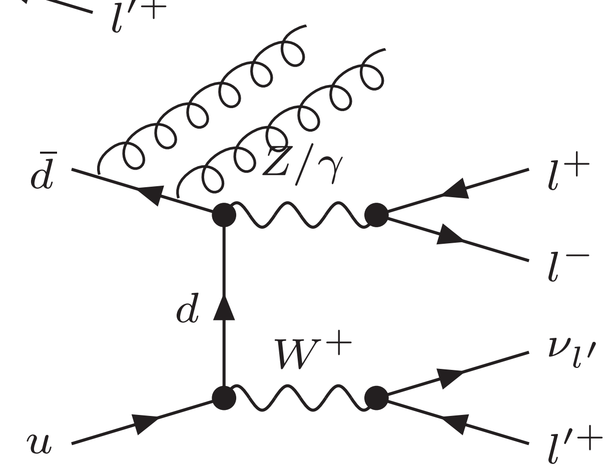
NNLO (pp→X)



$\rightarrow \mathrm{d}\sigma^{\mathrm{VV}}$



$\rightarrow \mathrm{d}\sigma^{\mathrm{RV}}$



$\rightarrow \mathrm{d}\sigma^{\mathrm{RR}}$

NNLO through X+jet at NLO + Slicing

$$\mathrm{d}\sigma^{\mathrm{X}}_{\mathrm{NNLO}} = \left[\mathrm{d}\sigma^{\mathrm{X+jet}}_{\mathrm{NLO}} \Big|_{r > r_{\mathrm{cut}}} - \Sigma_{\mathrm{NNLO}}(r_{\mathrm{cut}}) \otimes \mathrm{d}\sigma^{\mathrm{B}} \right] + \mathcal{H}_{\mathrm{NNLO}} \otimes \mathrm{d}\sigma^{\mathrm{B}}$$

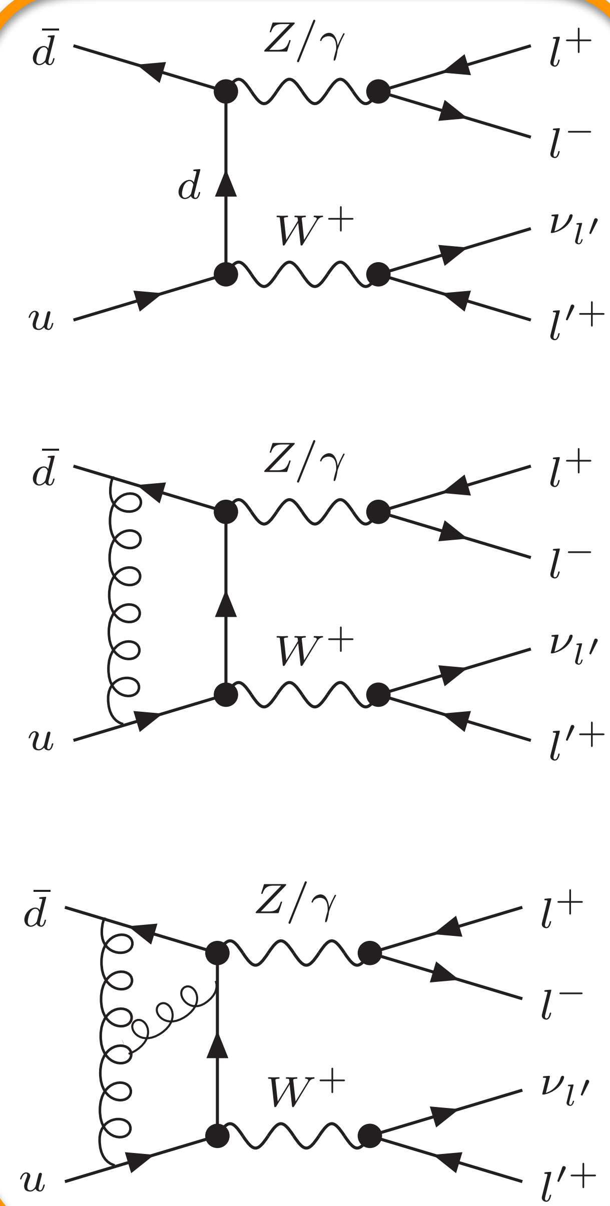
q_T subtraction

[Catani, Grazzini '07]

LO (pp→X)

NLO (pp→X)

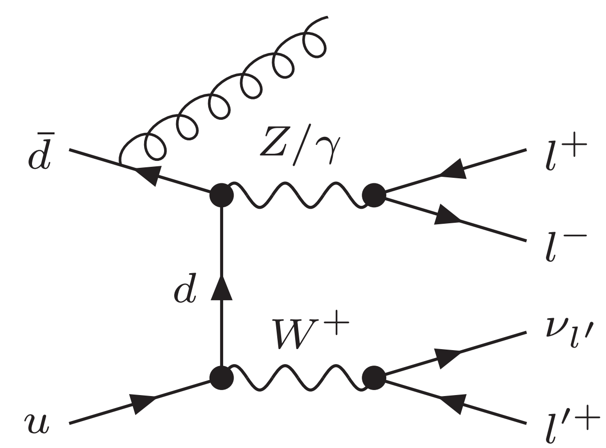
NNLO (pp→X)



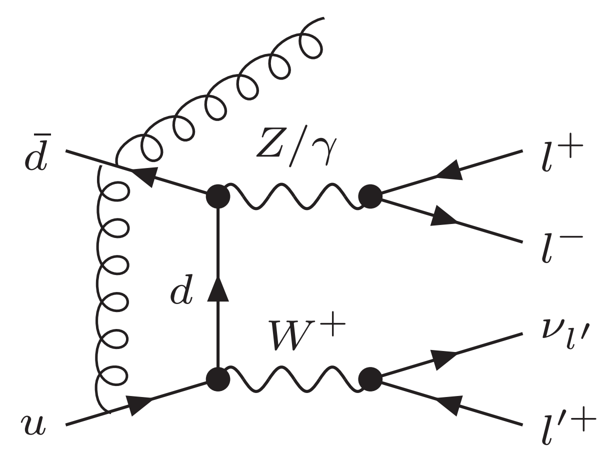
$\rightarrow \mathrm{d}\sigma^{\mathrm{B}}$

$\rightarrow \mathrm{d}\sigma^{\mathrm{V}}$

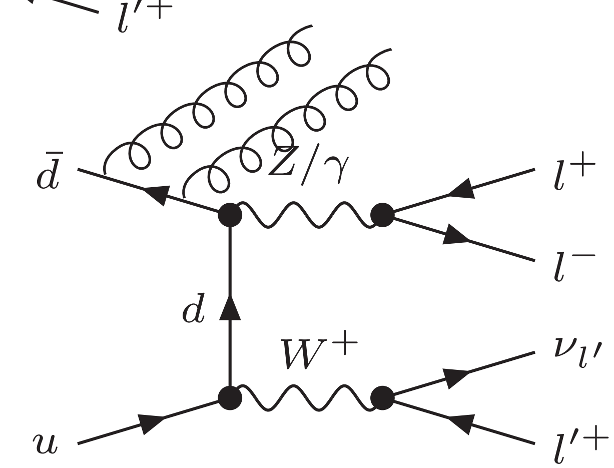
$\rightarrow \mathrm{d}\sigma^{\mathrm{VV}}$



$\rightarrow \mathrm{d}\sigma^{\mathrm{R}}$



$\rightarrow \mathrm{d}\sigma^{\mathrm{RV}}$

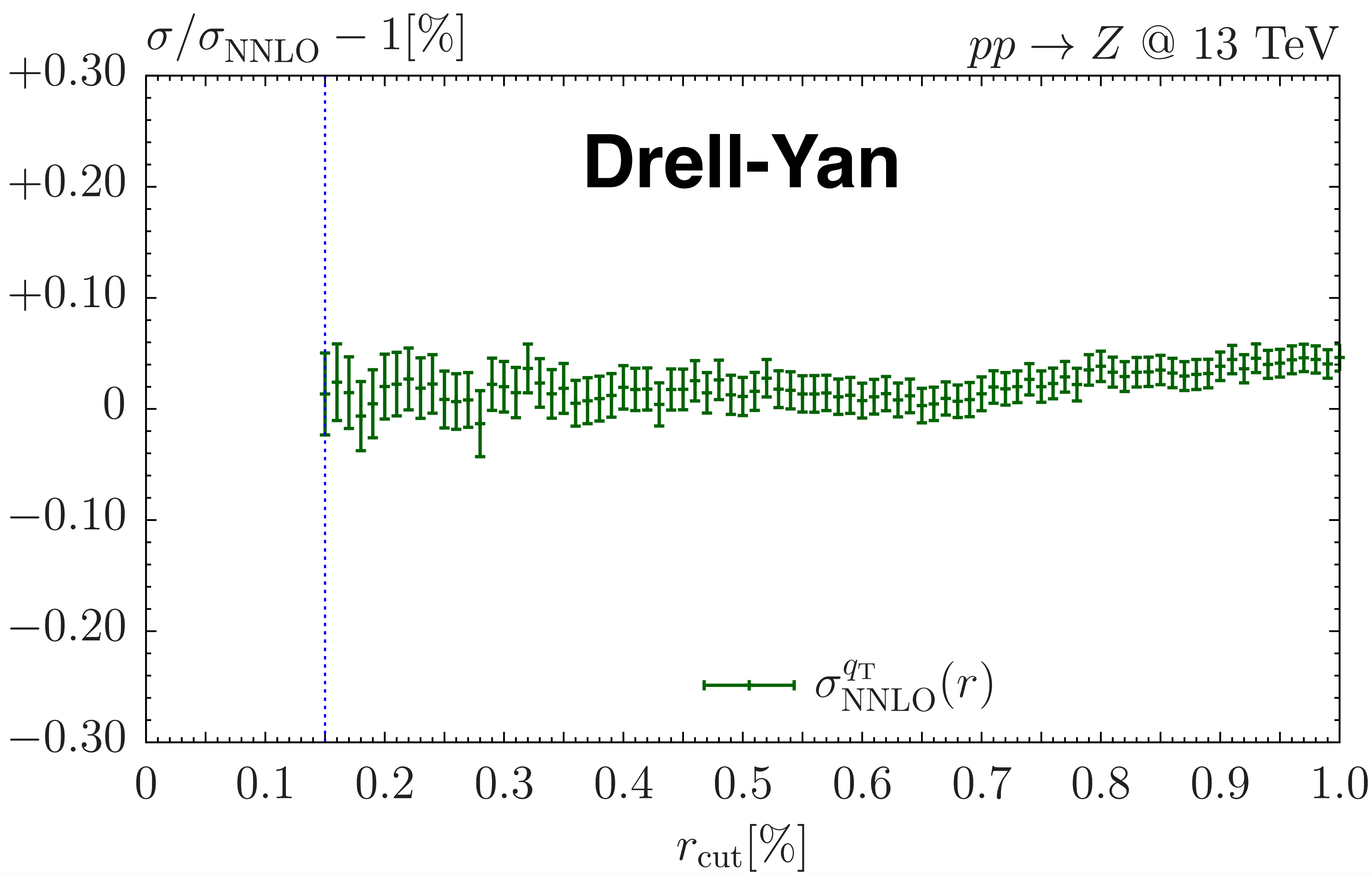


$\rightarrow \mathrm{d}\sigma^{\mathrm{RR}}$

$r_{cut}\to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]

automatically computed in every single MATRIX NNLO run

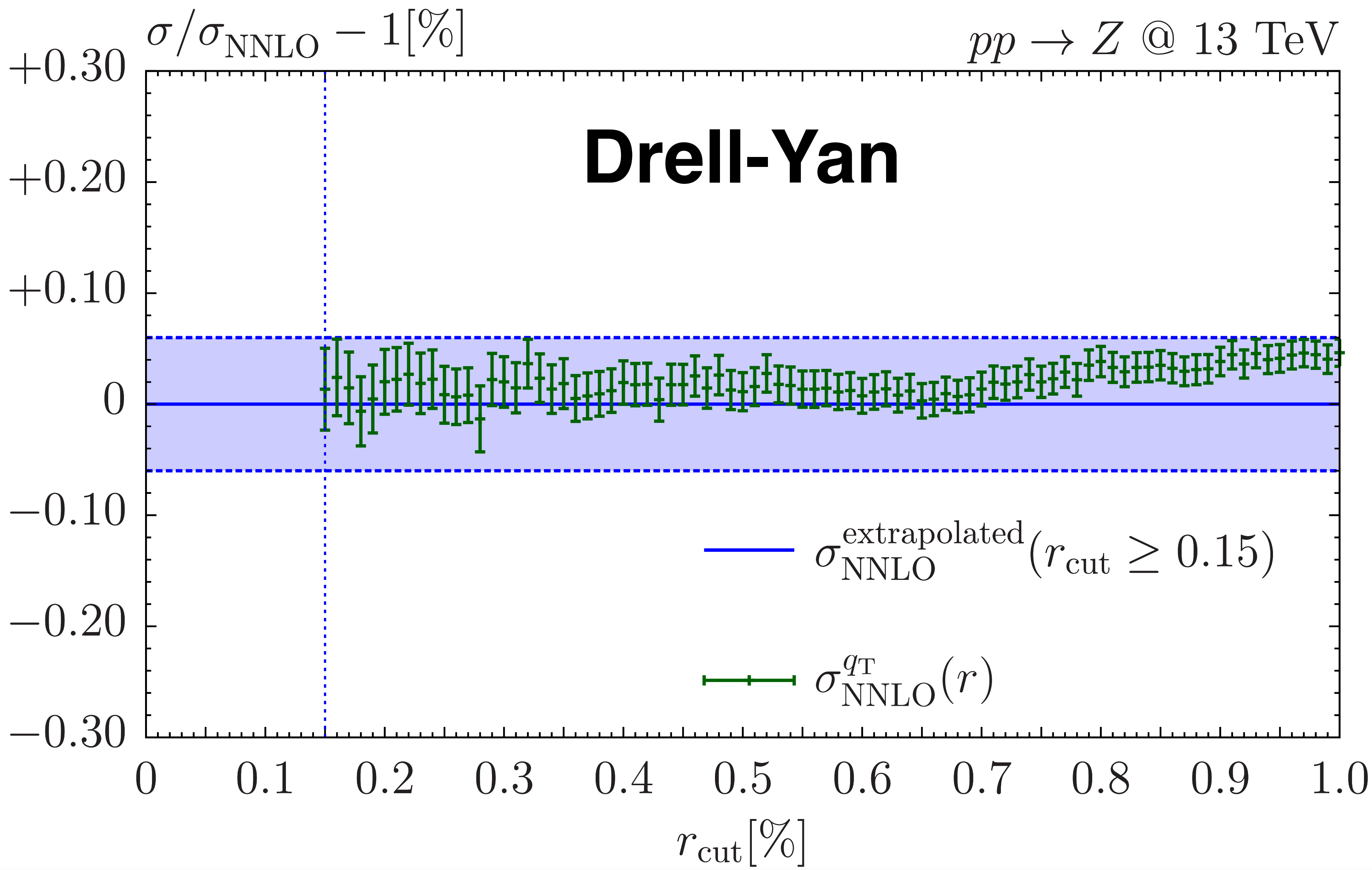


$$d\sigma^{\rm X}_{\rm NNLO} = \left[d\sigma^{\rm X+jet}_{\rm NLO}\Big|_{r>r_{\rm cut}} - \Sigma_{\rm NNLO}(r_{\rm cut})\otimes d\sigma^{\rm B}\right] + \mathcal{H}_{\rm NNLO}\otimes d\sigma^{\rm B}$$

$r_{cut} \to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]

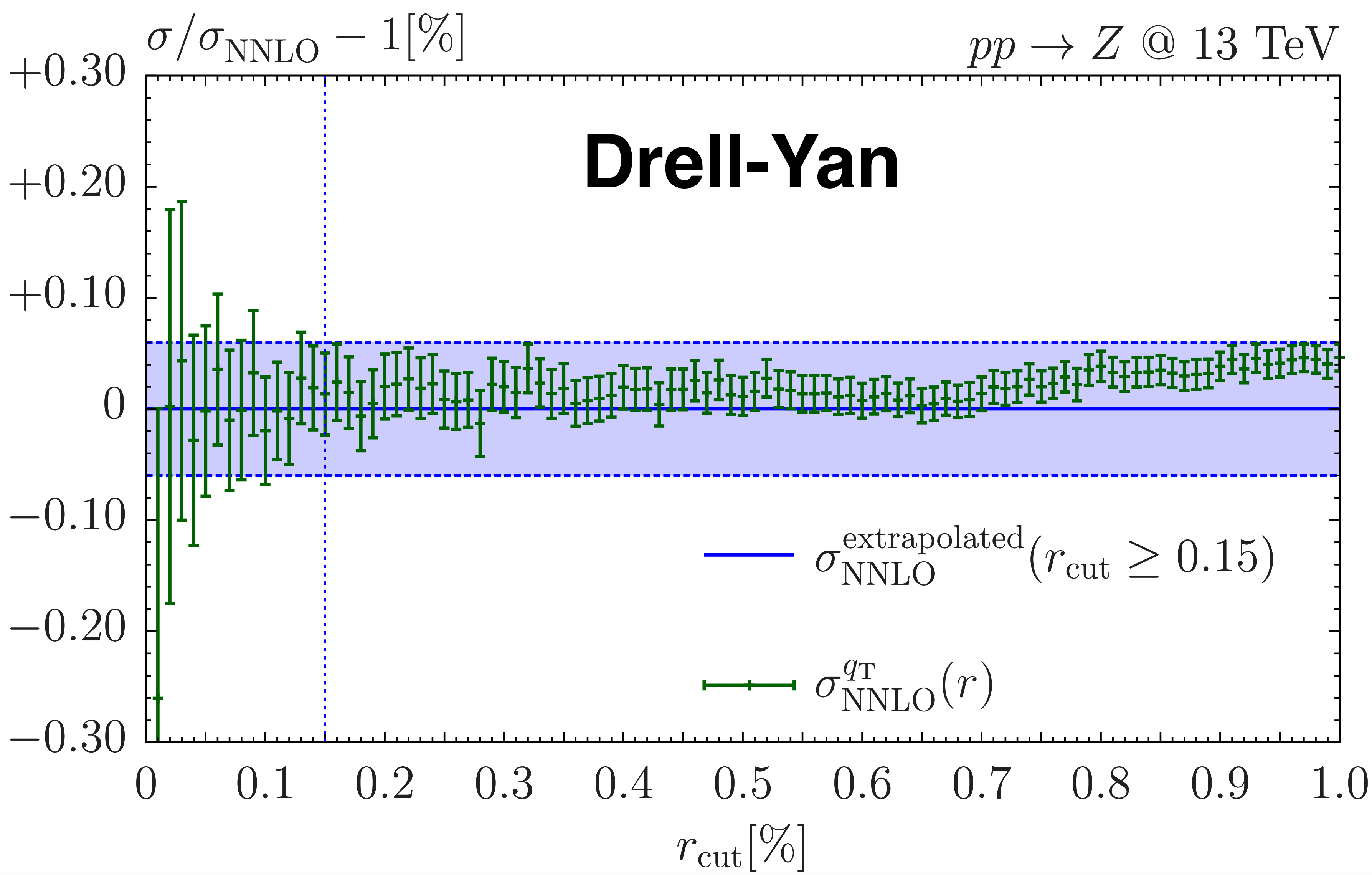
simple quadratic fit ($A * r_{cut}^2 + B * r_{cut} + C$) to extrapolate to $r_{cut}=0$



$$d\sigma_{\text{NNLO}}^{\text{X}} = \left[d\sigma_{\text{NLO}}^{\text{X+jet}} \Big|_{r > r_{\text{cut}}} - \Sigma_{\text{NNLO}}(r_{\text{cut}}) \otimes d\sigma^{\text{B}} \right] + \mathcal{H}_{\text{NNLO}} \otimes d\sigma^{\text{B}}$$

$r_{cut} \to 0$ extrapolation in MATRIX

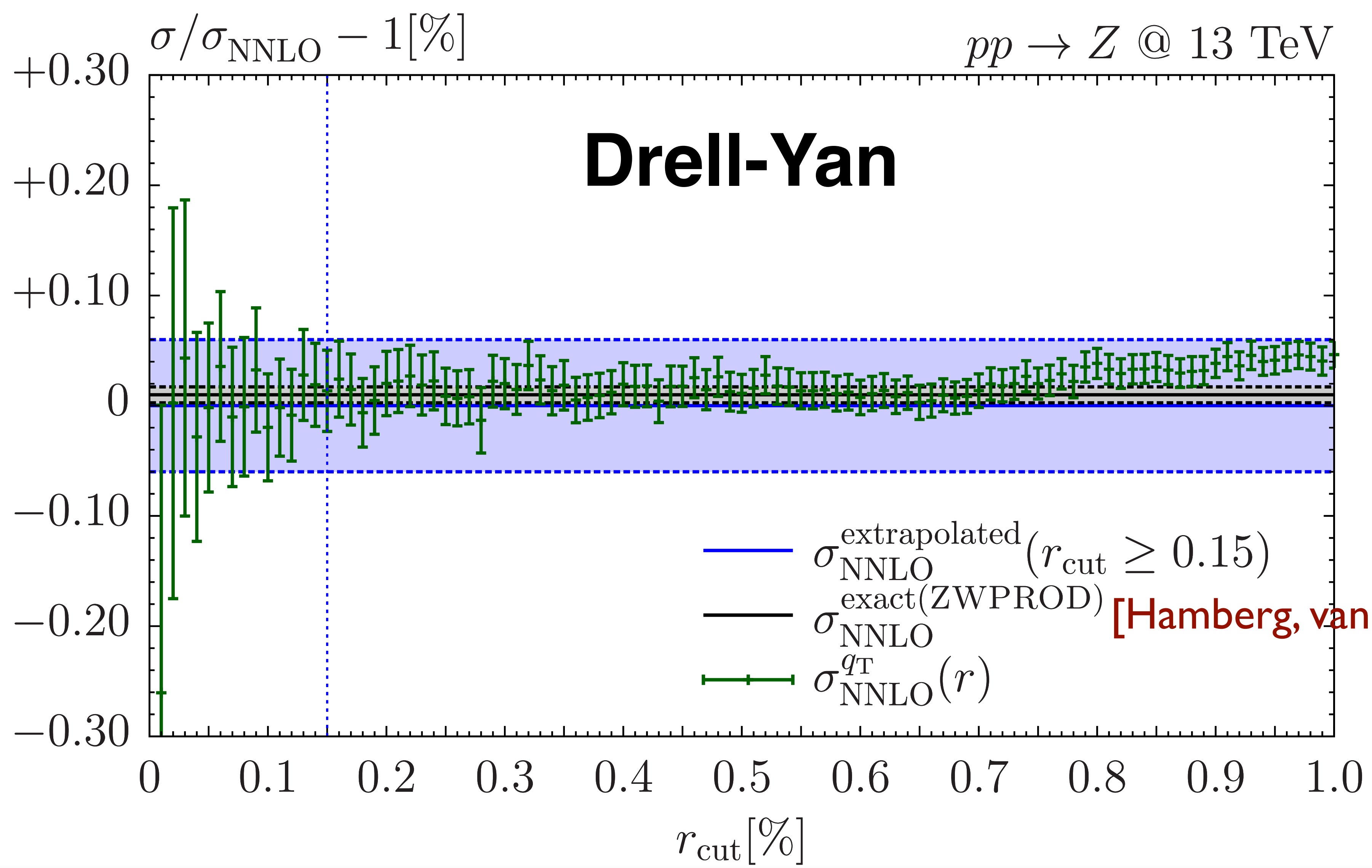
[Grazzini, Kallweit, MW '17]



$$\mathrm{d}\sigma^{\mathrm{X}}_{\mathrm{NNLO}} = \left[\mathrm{d}\sigma^{\mathrm{X+jet}}_{\mathrm{NLO}}\Big|_{r>r_{\mathrm{cut}}} - \Sigma_{\mathrm{NNLO}}(r_{\mathrm{cut}}) \otimes \mathrm{d}\sigma^{\mathrm{B}}\right] + \mathcal{H}_{\mathrm{NNLO}} \otimes \mathrm{d}\sigma^{\mathrm{B}}$$

r_{cut}→0 extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]

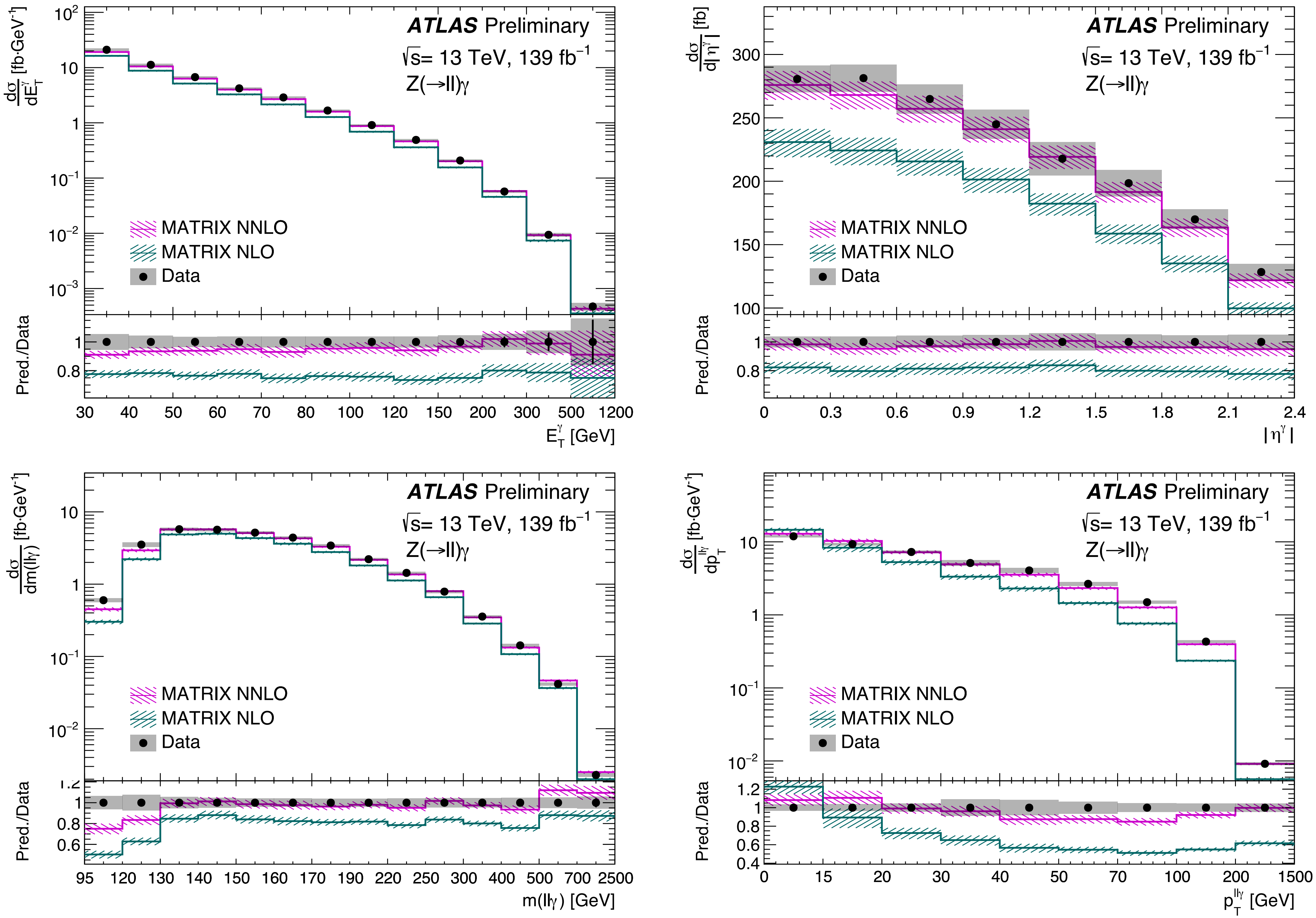


[Hamberg, van Neerven, Matsuura '91]

$$d\sigma^{\mathrm{X}}_{\mathrm{NNLO}} = \left[d\sigma^{\mathrm{X+jet}}_{\mathrm{NLO}}\Big|_{r > r_{\mathrm{cut}}} - \Sigma_{\mathrm{NNLO}}(r_{\mathrm{cut}}) \otimes d\sigma^{\mathrm{B}} \right] + \mathcal{H}_{\mathrm{NNLO}} \otimes d\sigma^{\mathrm{B}}$$

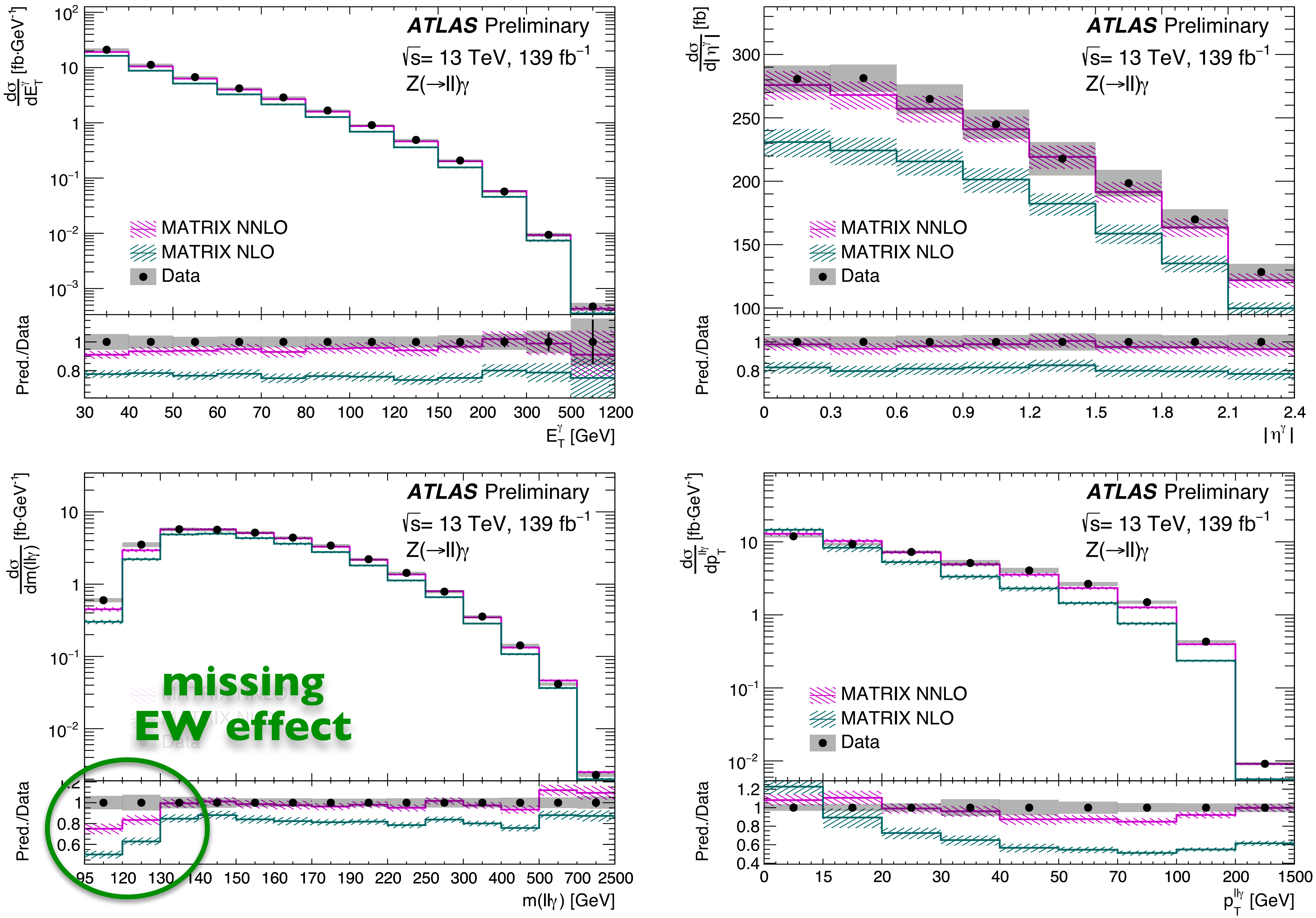
Recent Example: Zγ with 139 fb^{-1}

[ATLAS-CONF-2019-034]



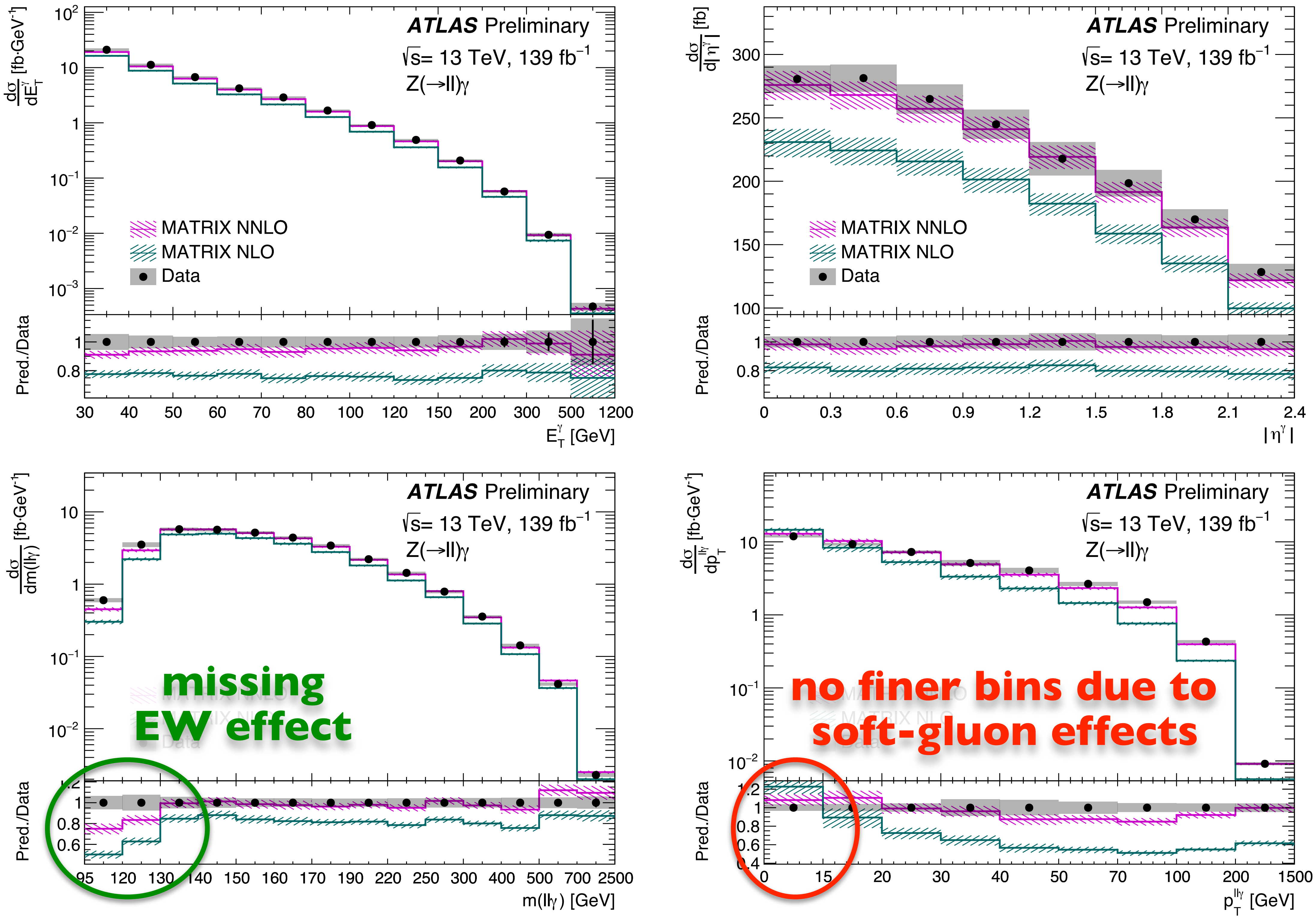
Recent Example: Zγ with 139 fb⁻¹

[ATLAS-CONF-2019-034]



Recent Example: Zγ with 139 fb^{-1}

[ATLAS-CONF-2019-034]



Importance of going beyond NNLO QCD

nNNLO QCD × **NLO EW**

ZZ → [Grazzini, Kallweit, MW, Yook '18]
WW → [Grazzini, Kallweit, MW, Yook '20]

ZZ, WW, WZ
→ [Grazzini, Kallweit, Lindert, Pozzorini, MW '19]

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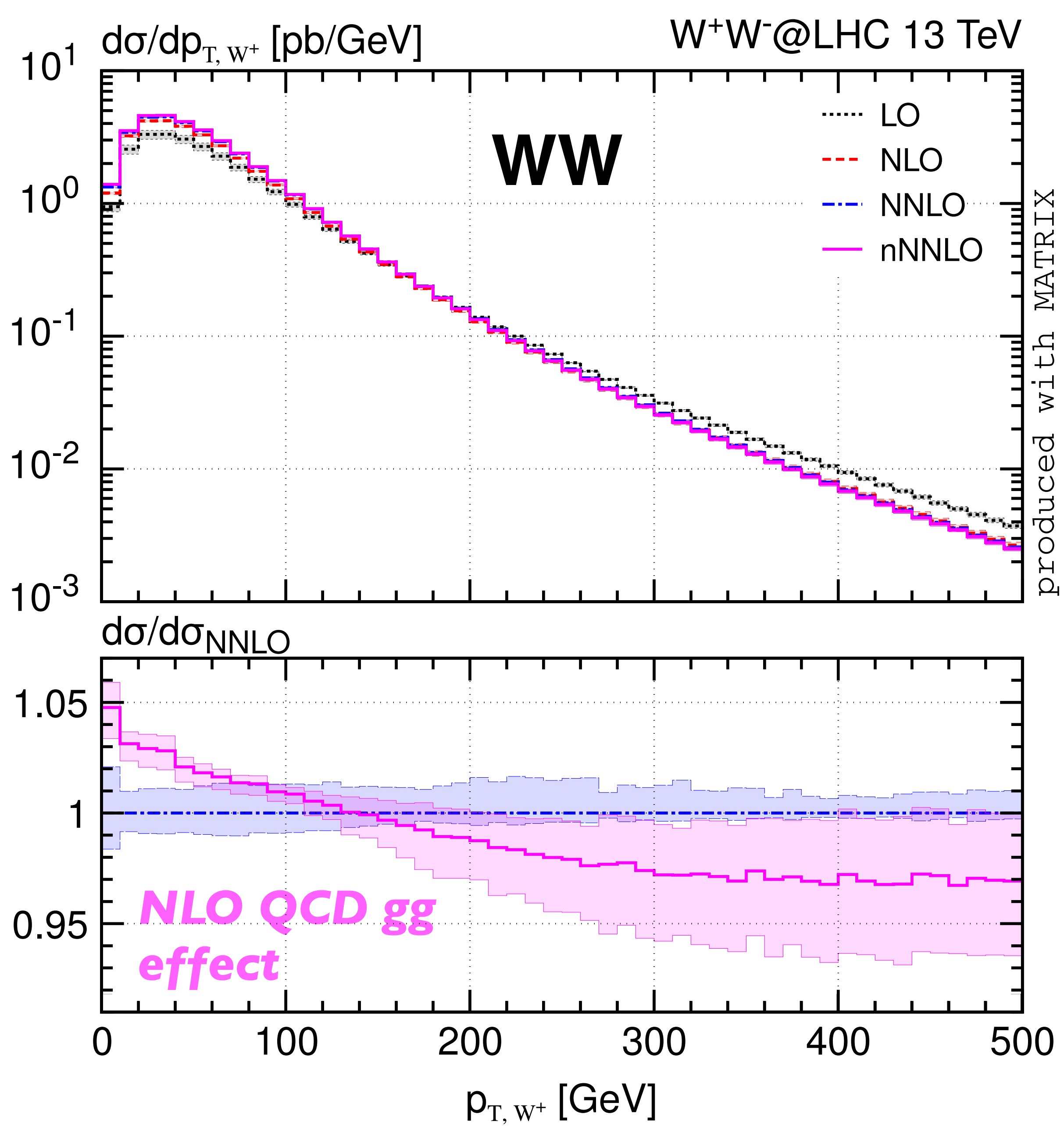

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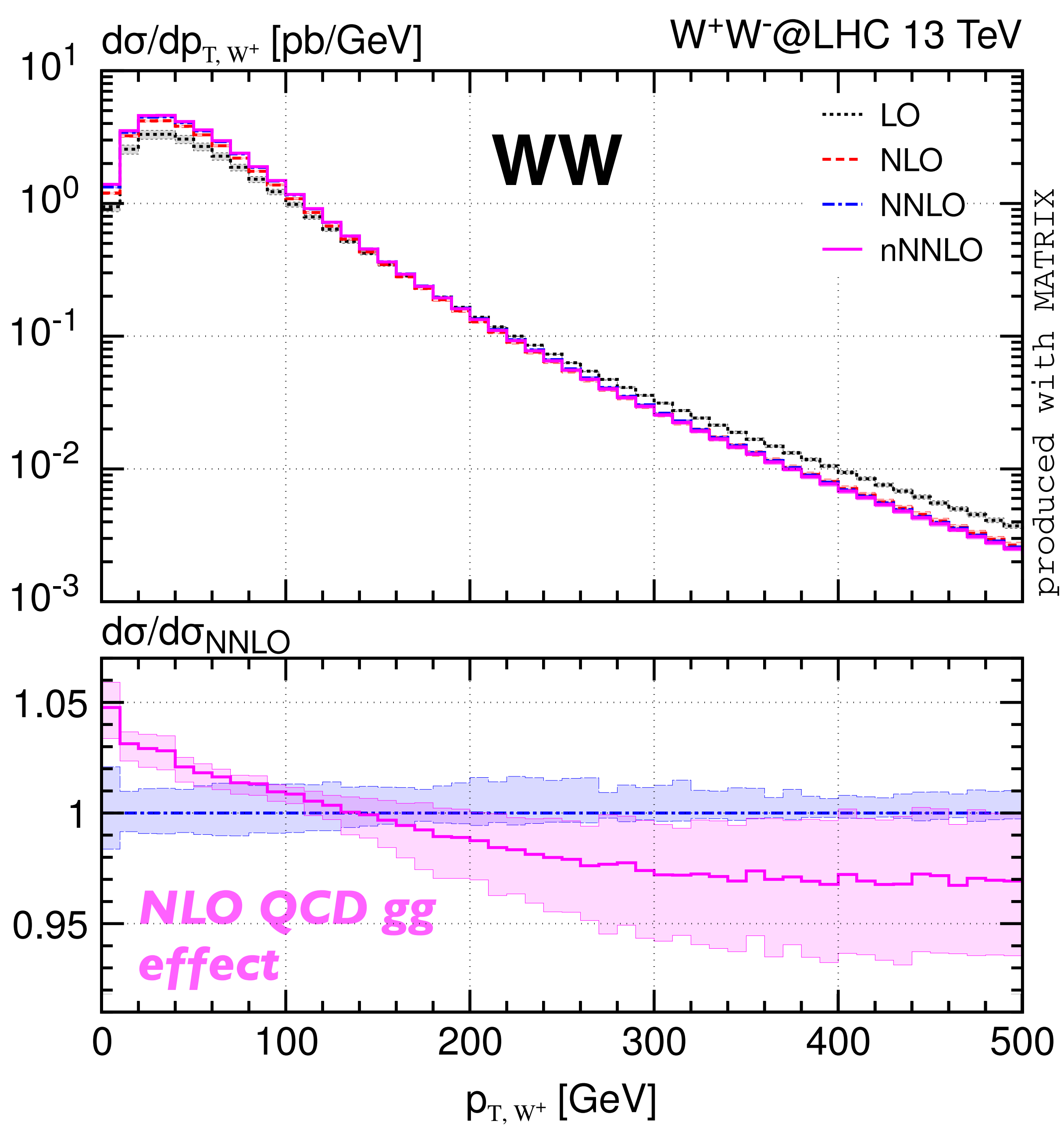
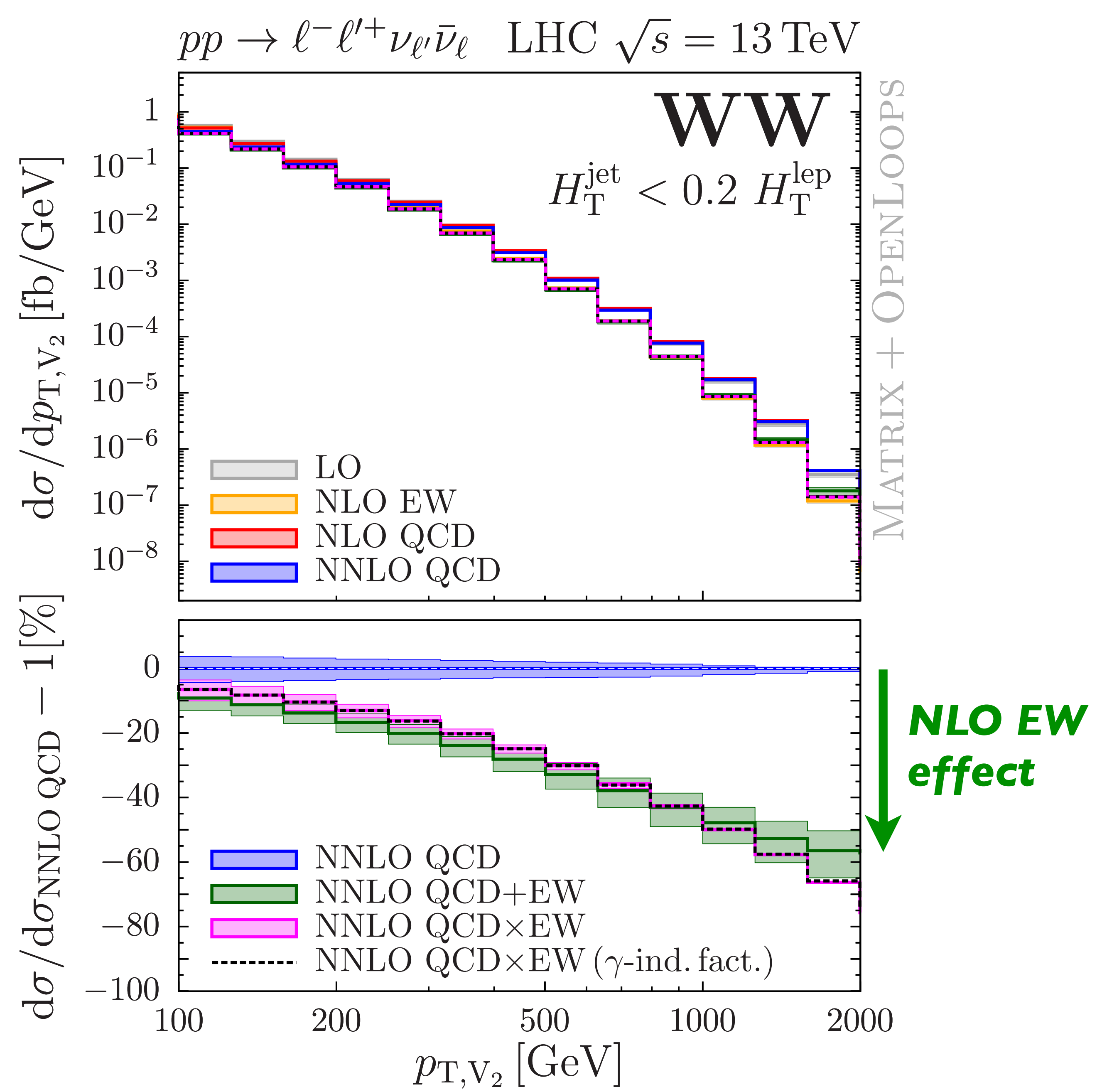
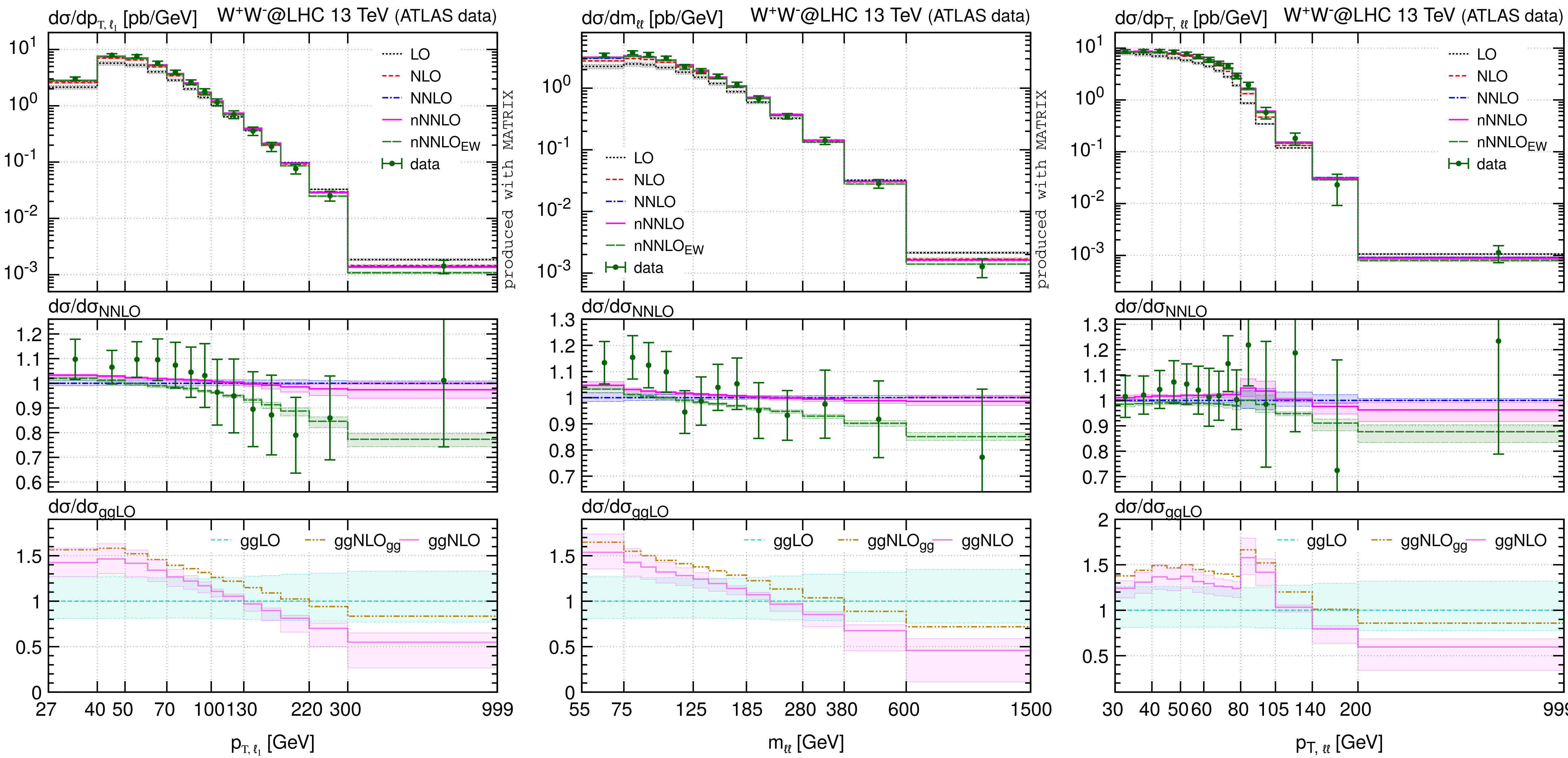




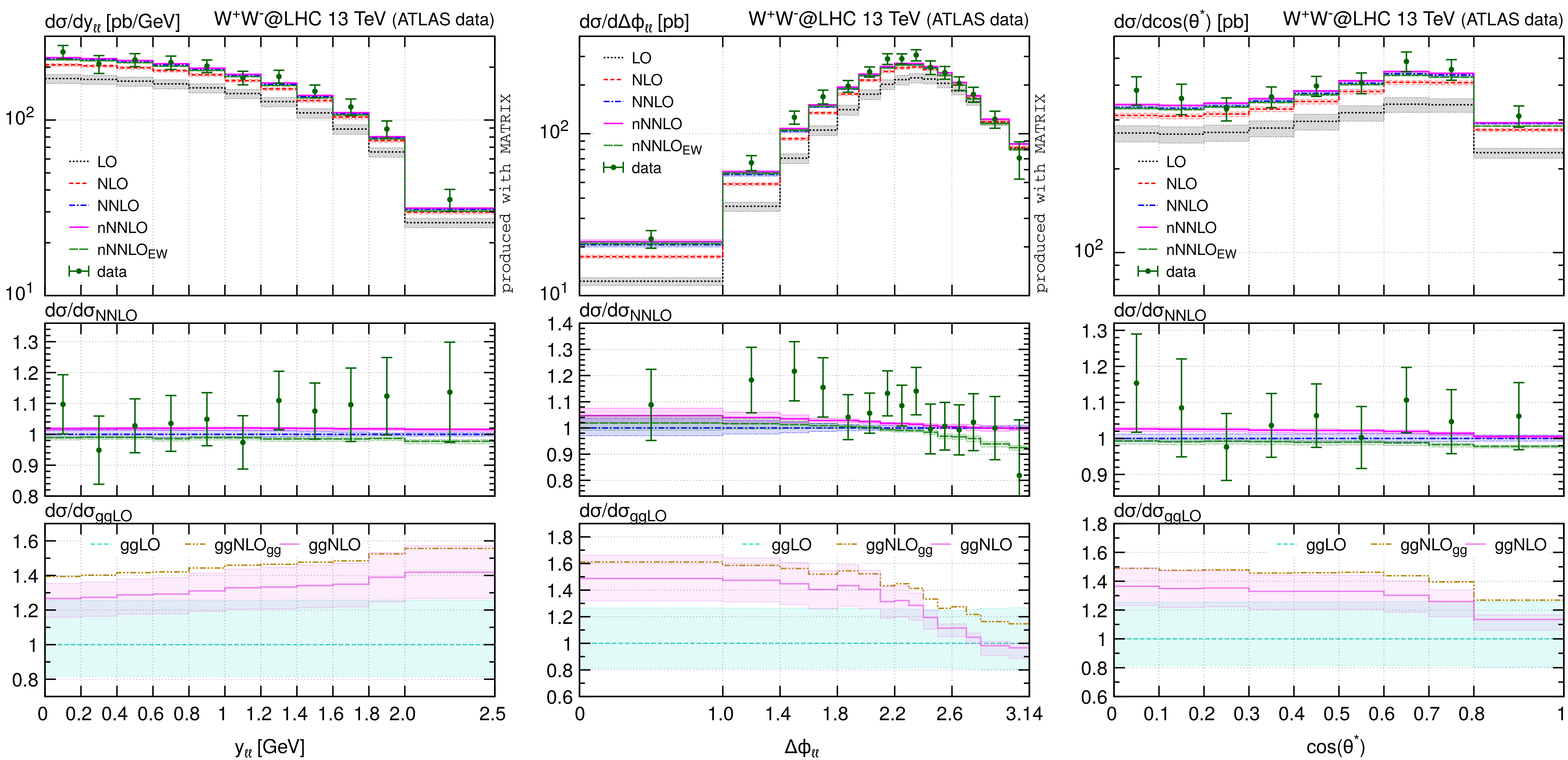
WW: nNNLO x NLO EW

[Grazzini, Kallweit, MW, Yook '20], [Grazzini, Kallweit, Linder, Pozzorini '19]



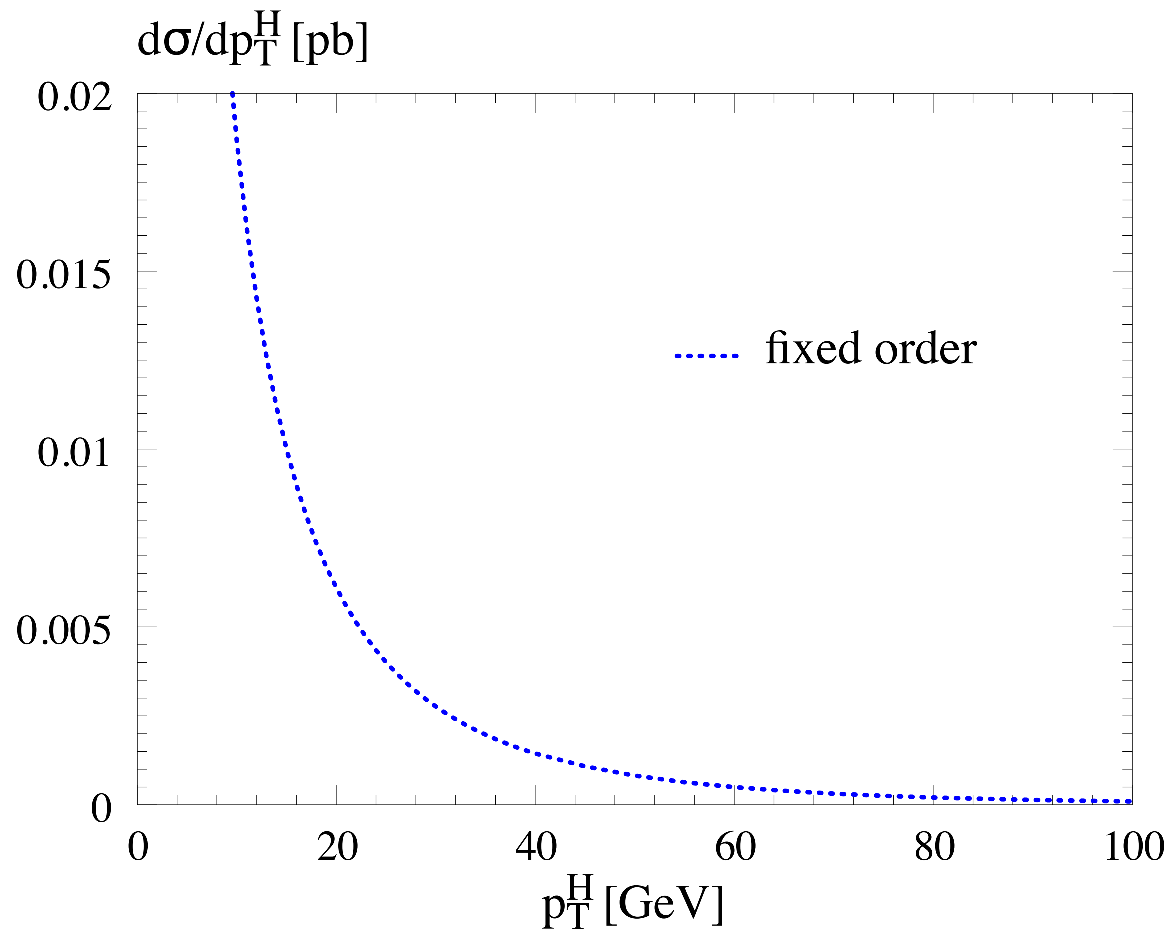
WW: nNNLO x NLO EW

[Grazzini, Kallweit, MW, Yook '20], [Grazzini, Kallweit, Linder, Pozzorini '19]



p_T resummation

- production of colorless particles (system $\mathcal{F}$, invariant mass M)
- problem: p_T distribution of $\mathcal{F}$ diverges at $p_T \to 0$

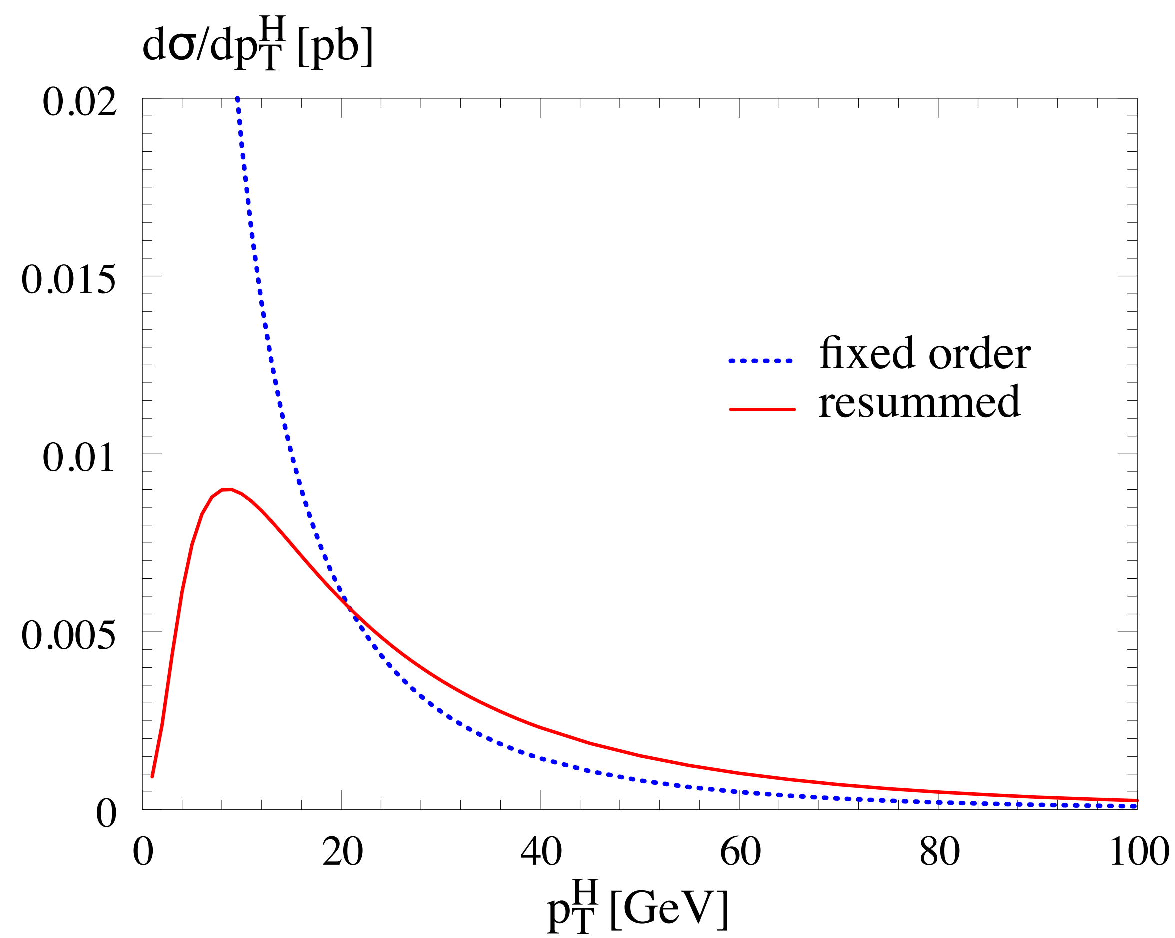


p_T resummation

- production of colorless particles (system $\mathcal{F}$, invariant mass M)
- problem: p_T distribution of $\mathcal{F}$ diverges at $p_T \to 0$
- reason: large logs $\ln p_T^2/M^2$ for $p_T \ll M$

$$\alpha_s : \quad \ln(p_T^2/M^2),\ \ln^2(p_T^2/M^2)$$
$$\alpha_s^2 : \quad \ln(p_T^2/M^2),\ \ln^2(p_T^2/M^2),\ \ln^3(p_T^2/M^2),\ \ln^4(p_T^2/M^2)$$
$$\dots$$

- solution: all order resummation



p_T resummation

- production of colorless particles (system $\mathcal{F}$, invariant mass M)
- problem: p_T distribution of $\mathcal{F}$ diverges at $p_T \to 0$
- reason: large logs $\ln p_T^2/M^2$ for $p_T \ll M$

$$\begin{aligned}
\alpha_s : &\quad \ln(p_T^2/M^2),\ \ln^2(p_T^2/M^2) \\
\alpha_s^2 : &\quad \ln(p_T^2/M^2),\ \ln^2(p_T^2/M^2),\ \ln^3(p_T^2/M^2),\ \ln^4(p_T^2/M^2) \\
\dots &
\end{aligned}$$

- solution: all order resummation [Collins, Soper, Sterman '85]

$$\frac{d\sigma^{(\text{res})}_{N_1,N_2}}{dp_T^2\, dy\, dM\, d\Omega} \sim \int db\, \frac{b}{2}\, J_0(b\, p_T)\, S(b, A, B)\, \mathcal{H}_{N_1,N_2}\, f_{N_1}\, f_{N_2}$$

$$S(A,B) = \exp\Big\{ \underbrace{\underbrace{\underbrace{L\, g^{(1)}(\alpha_s L)}_{LL} + g^{(2)}(\alpha_s L)}_{NLL} + \alpha_s g^{(3)}(\alpha_s L)}_{NNLL} + \alpha_s^2 \cdots \Big\}$$

MATRIX+RadISH framework

[Kallweit, Re, Rottoli, MW 'to appear]

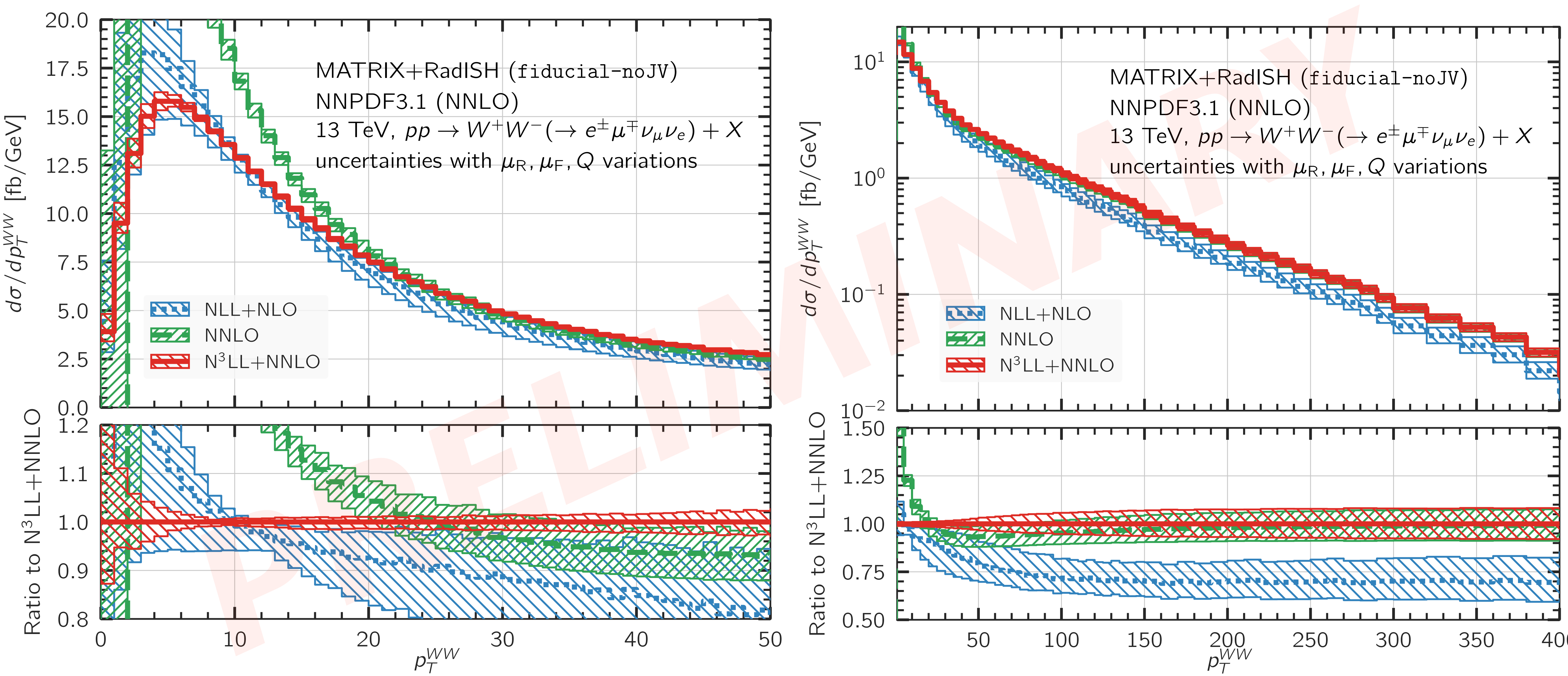
- ***General interface between MATRIX and RadISH codes:***
  - *all processes available in MATRIX (any color-singlet process possible where 2-loop known)*
  - *high-accuracy multi-differential resummation of various transverse observables*
  - *matching to NNLO QCD integrated cross section*

- ***MATRIX*** [Grazzini, Kallweit, MW '17]
  - *NNLO QCD, phase space, perturbative ingredients (amplitudes, coefficients, ...)*

- ***RadISH*** [Monni, Re, and Torrielli '16], [Bizon, Monni, Re, Rottoli, Torrielli '18], [Monni, Rottoli, Torrielli '19]
  - *resummation formalism in direct space (not in b-space)*
  - *numerical approach (like a semi-inclusive parton shower)*
  - *single-differential resummation* [Monni, Re, and Torrielli '16], [Bizon, Monni, Re, Rottoli, Torrielli '18]
  - *and double-differential resummation* [Monni, Rottoli, Torrielli '19]

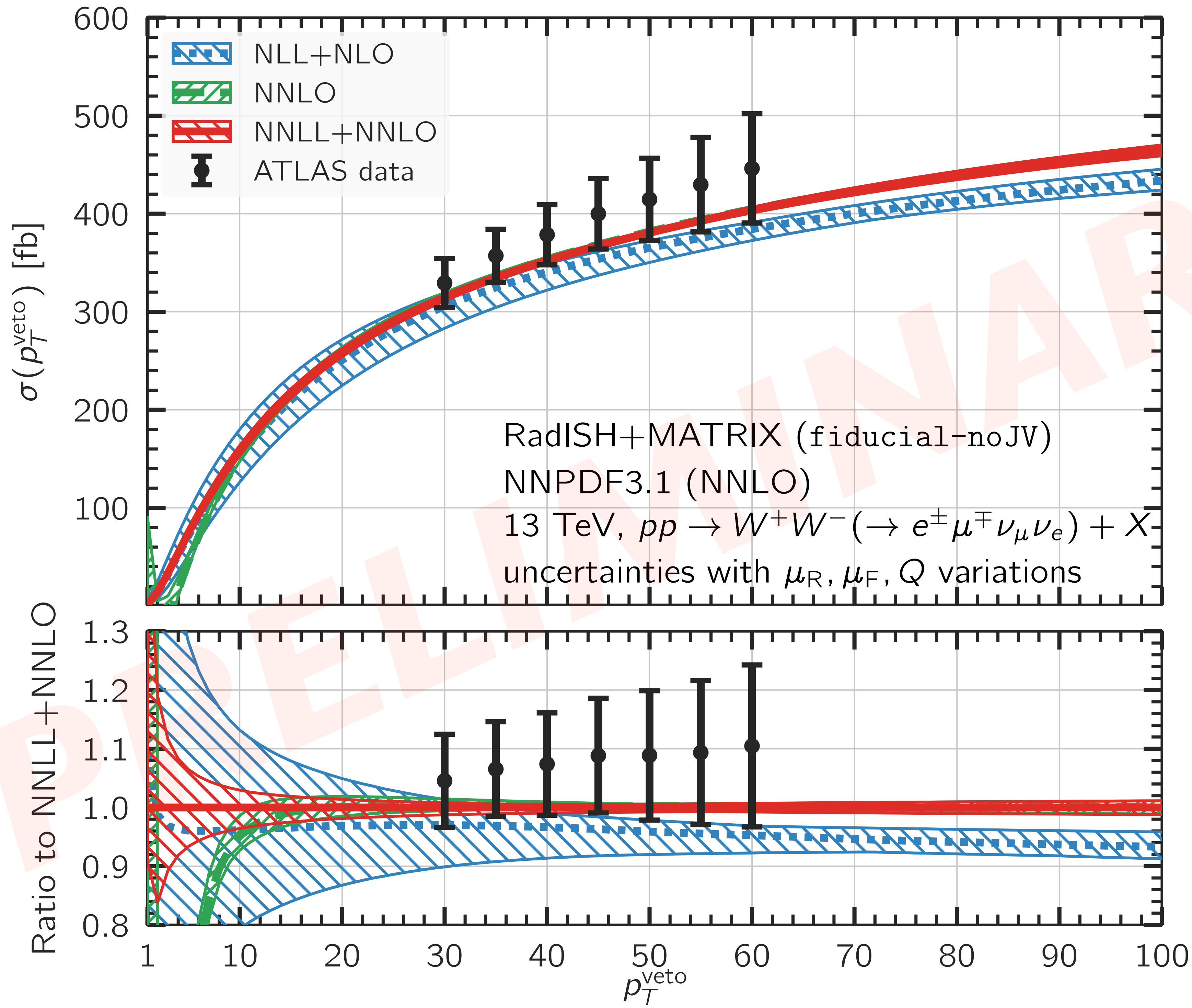
p_T-WW at N3LL+NNLO

[Kallweit, Re, Rottoli, MW 'to appear]



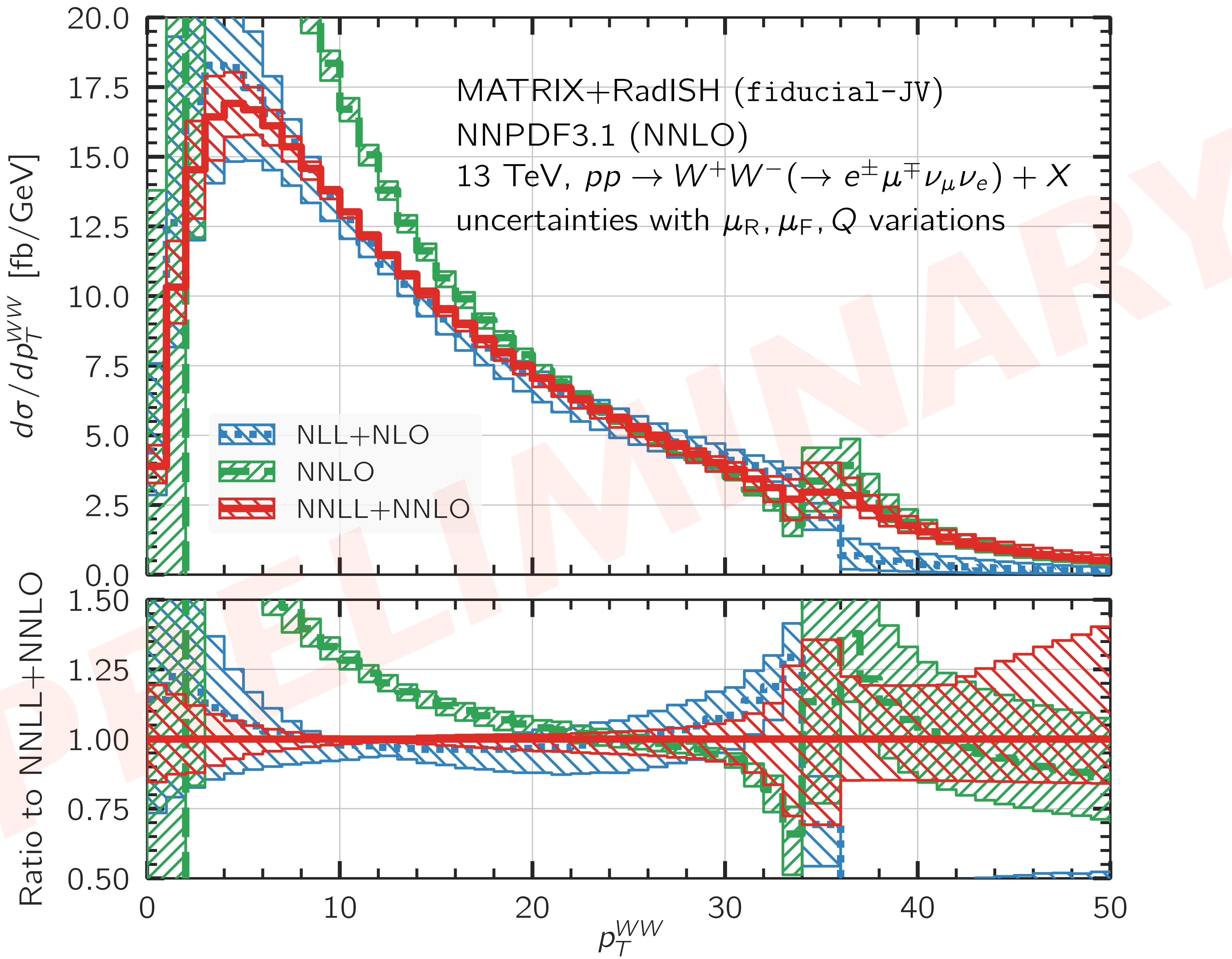
WW: Jet veto at NNLL+NNLO

[Kallweit, Re, Rottoli, MW 'to appear]

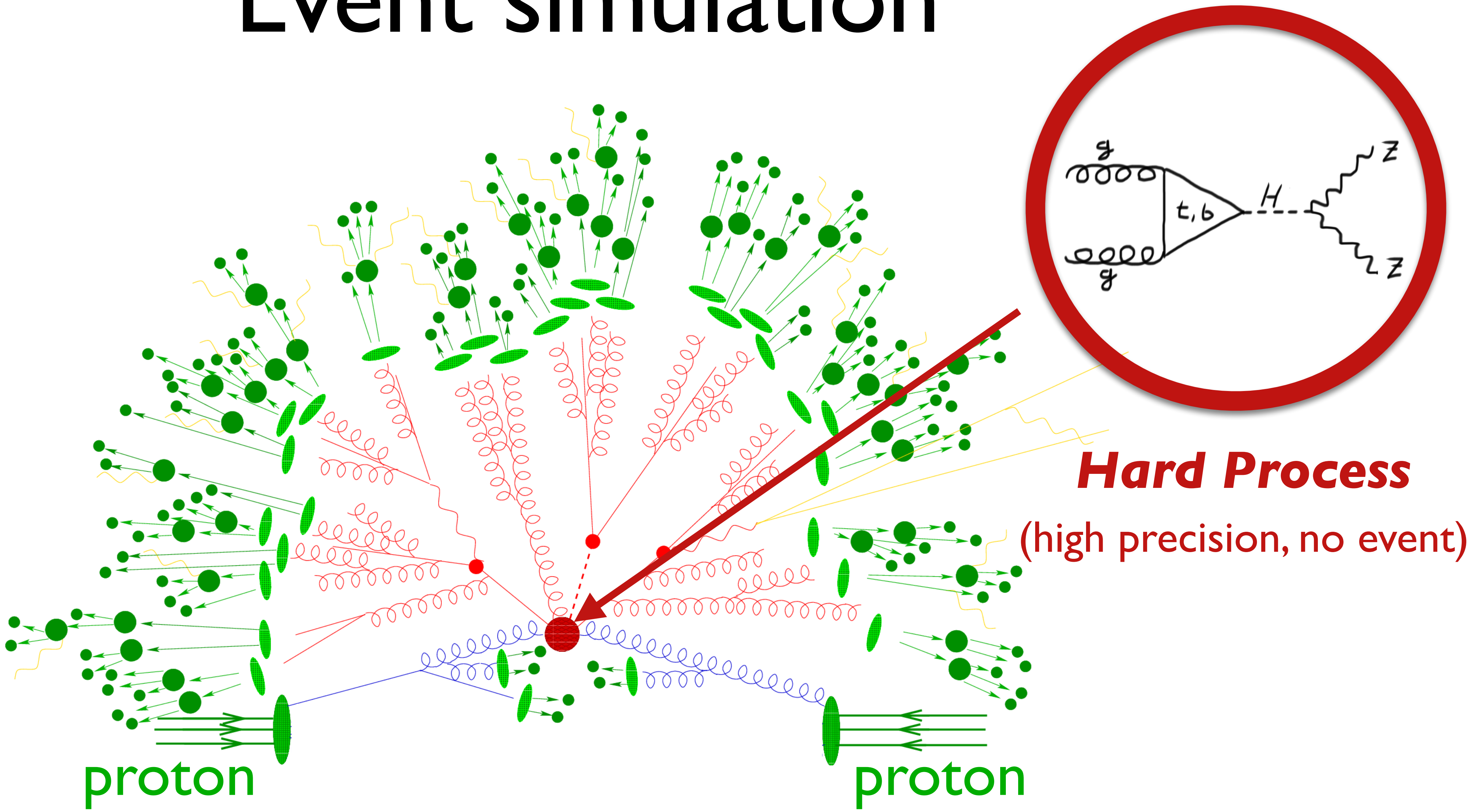


p_T-WW with jet veto at NNLL+NNLO

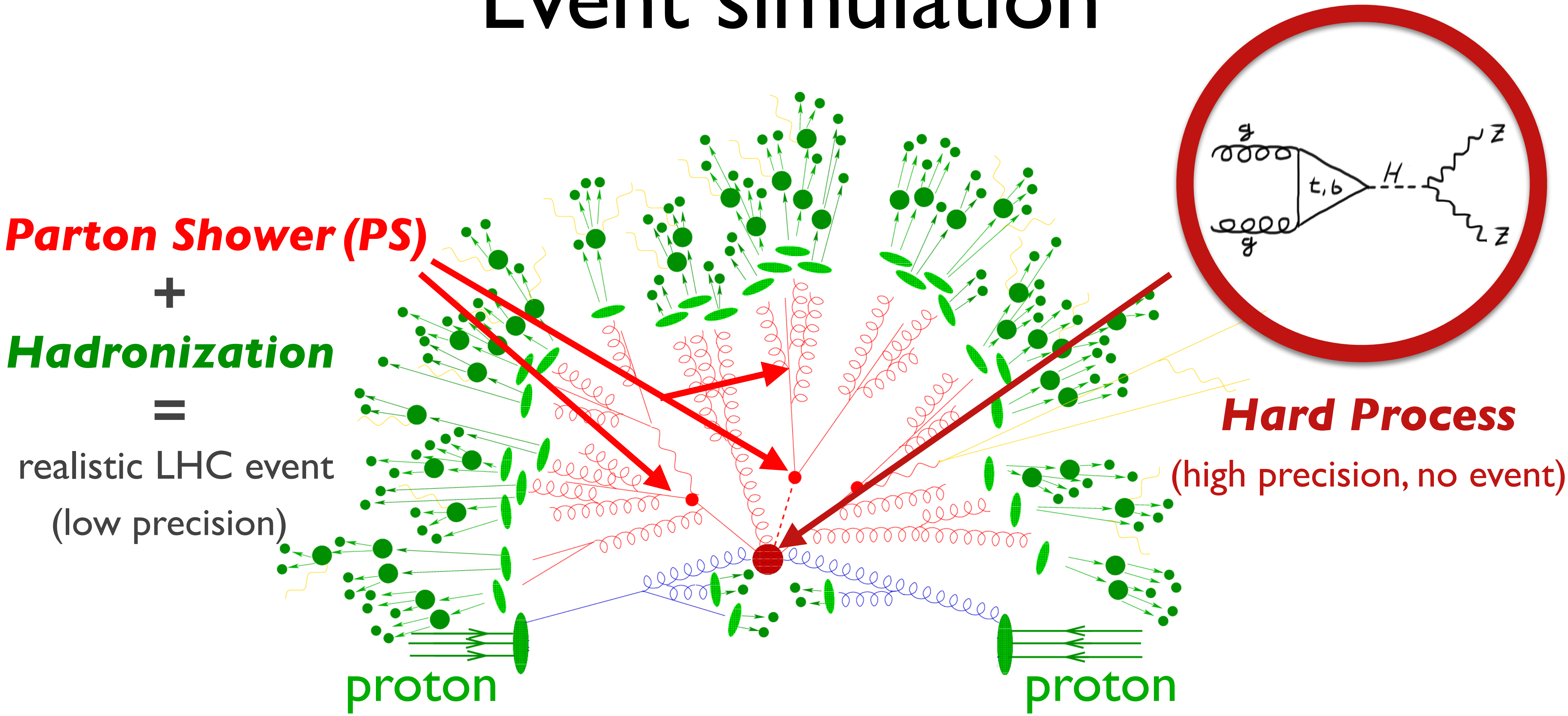
[Kallweit, Re, Rottoli, MW 'to appear]



Event simulation

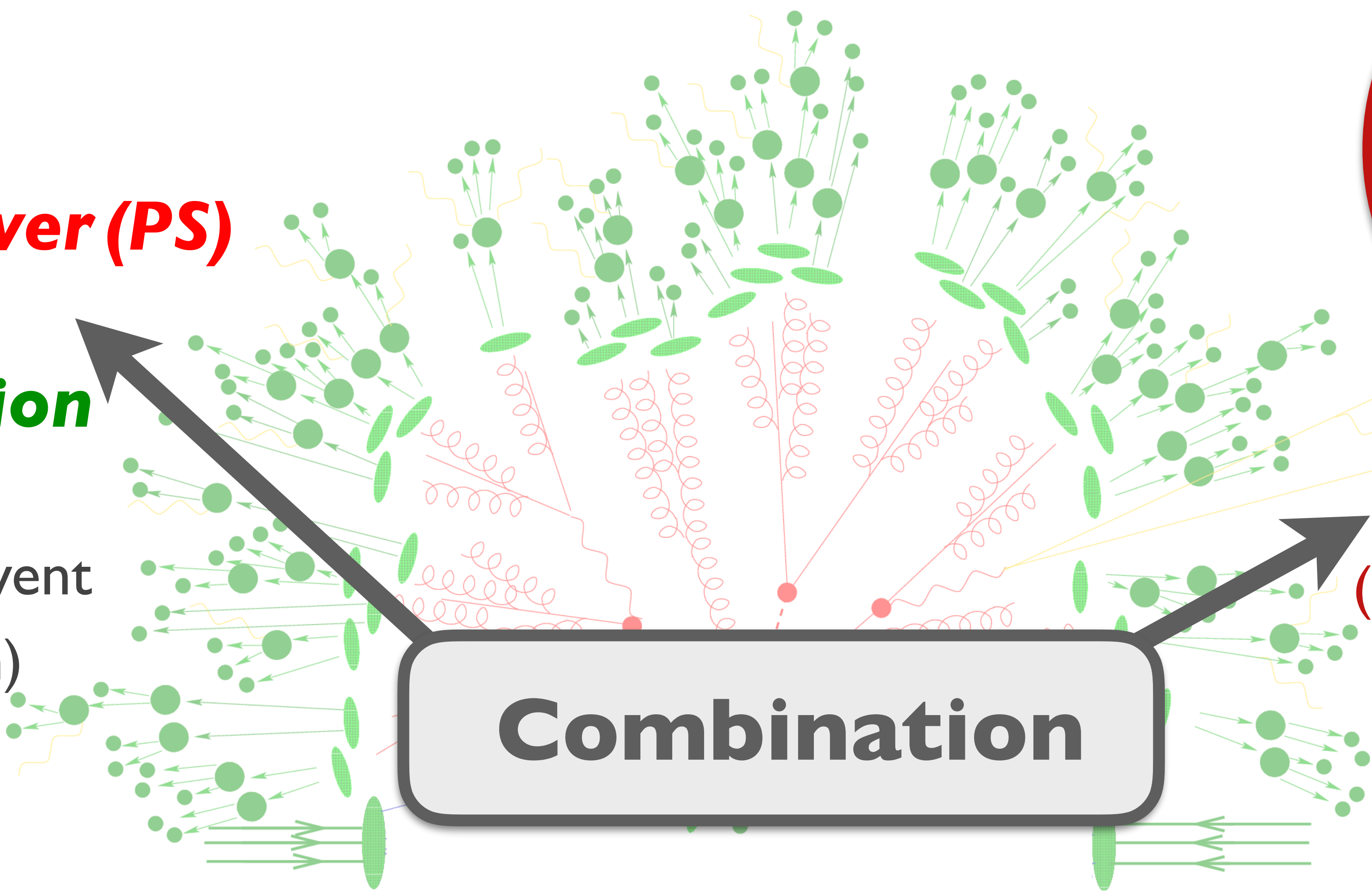


Event simulation

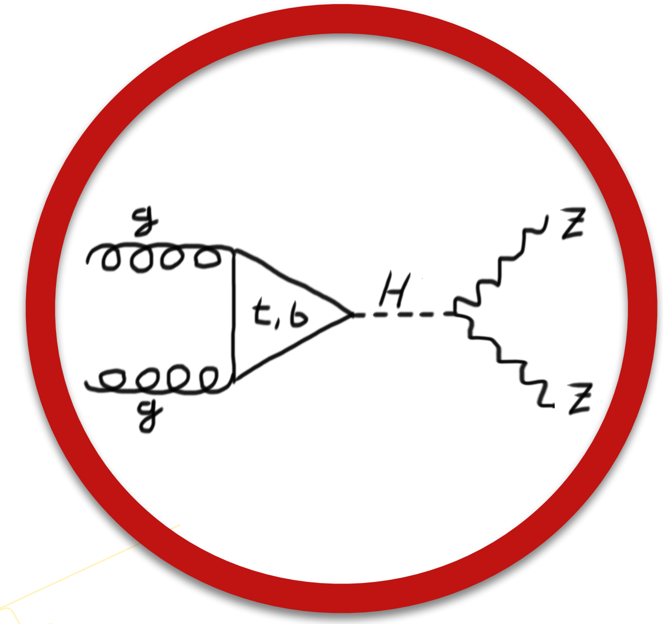


Event simulation

Parton Shower (PS)
+
Hadronization
=
realistic LHC event
(low precision)



Combination



Hard Process
(high precision, no event)

<u>*NLO+PS (~10%):*</u> *long-standing issue ➞ groundbreaking ~15 years; standard today*

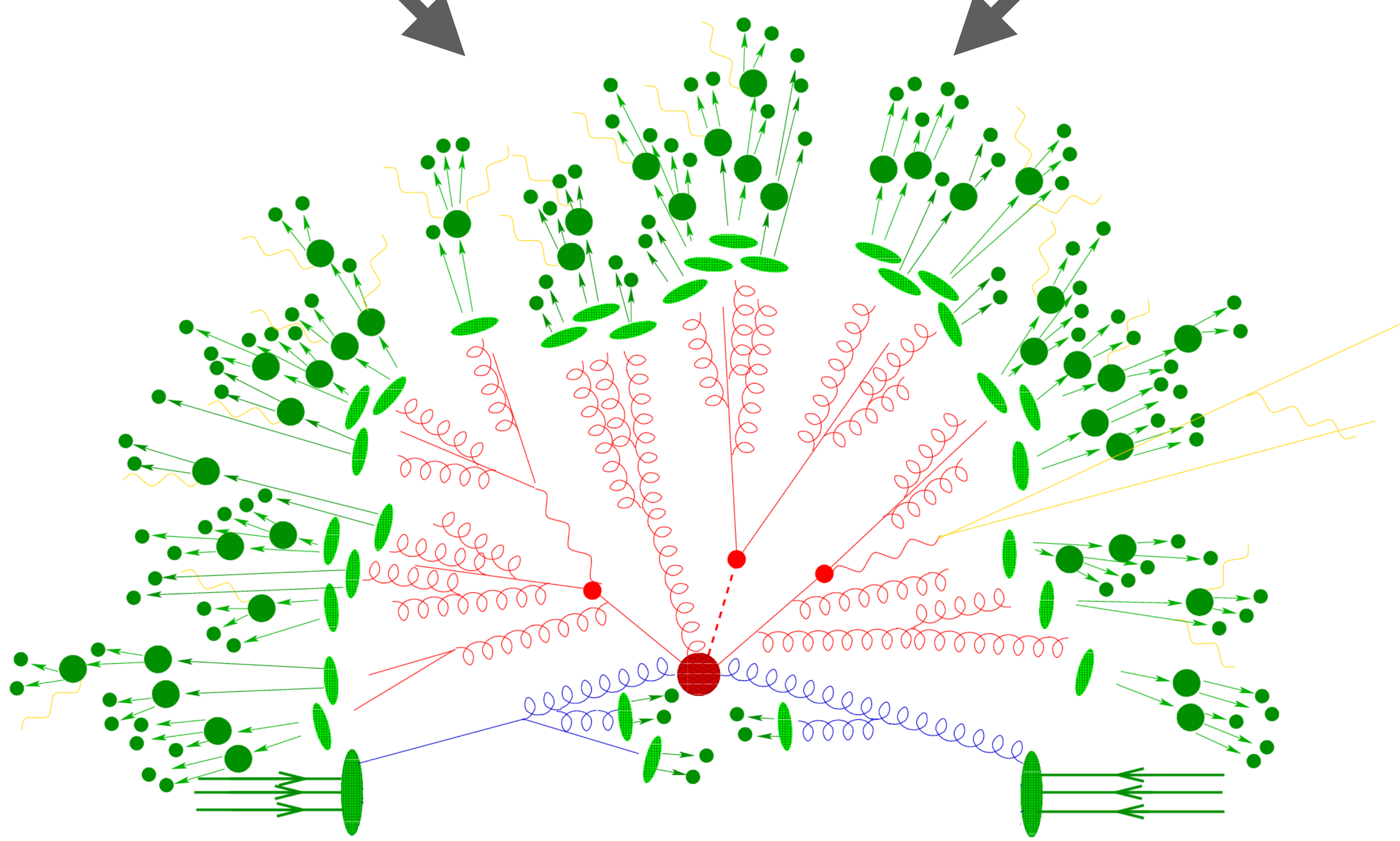
<u>*NNLO+PS(~1%):*</u> *extremely challenging; no general application to involved processes*

NNLO+PS

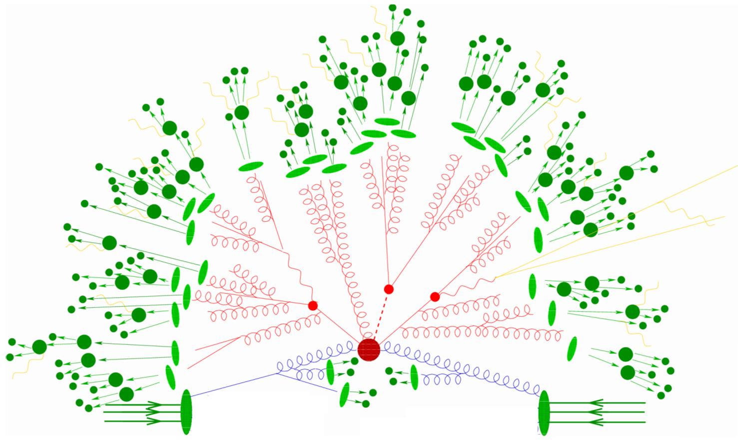
Experiment ⟷ **Theory high precision**

measurement

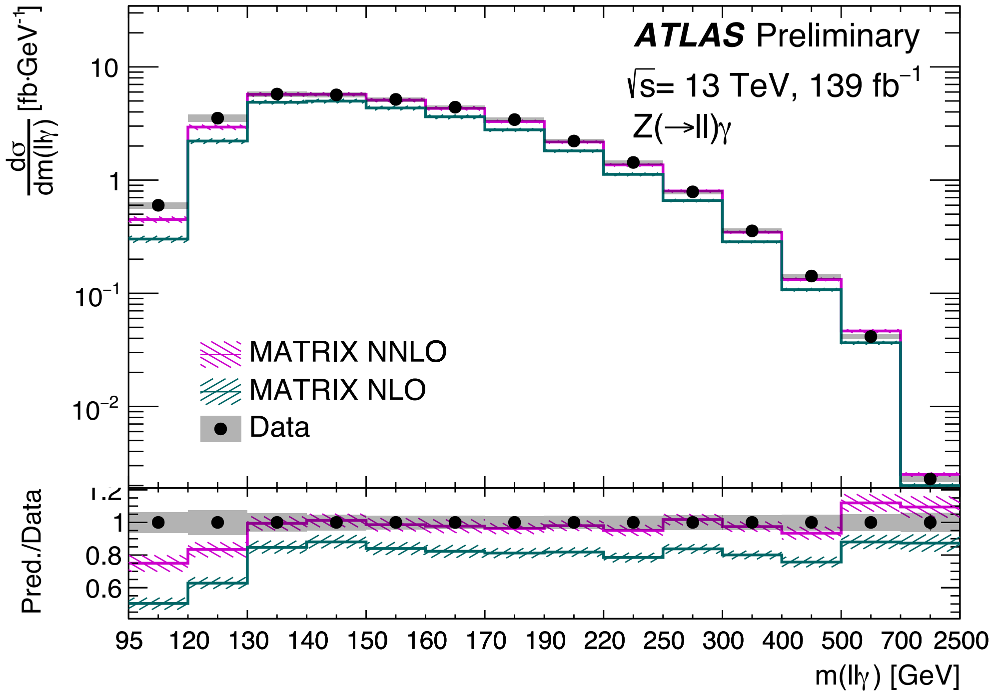
prediction



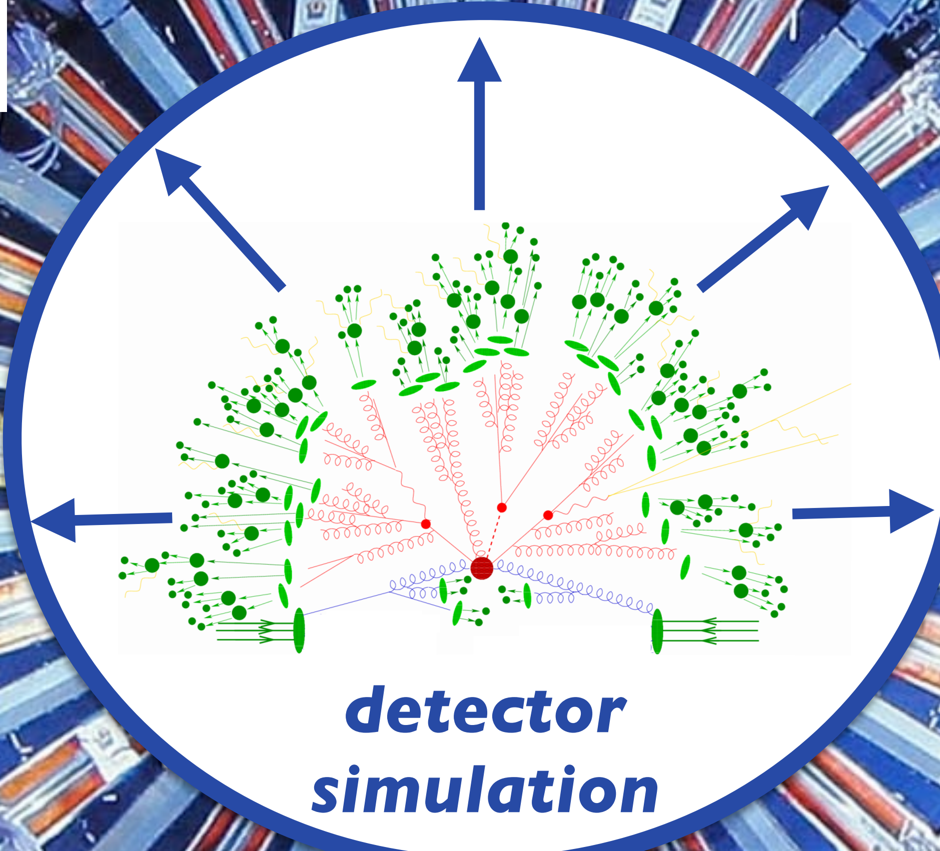
Why NNLO+PS?



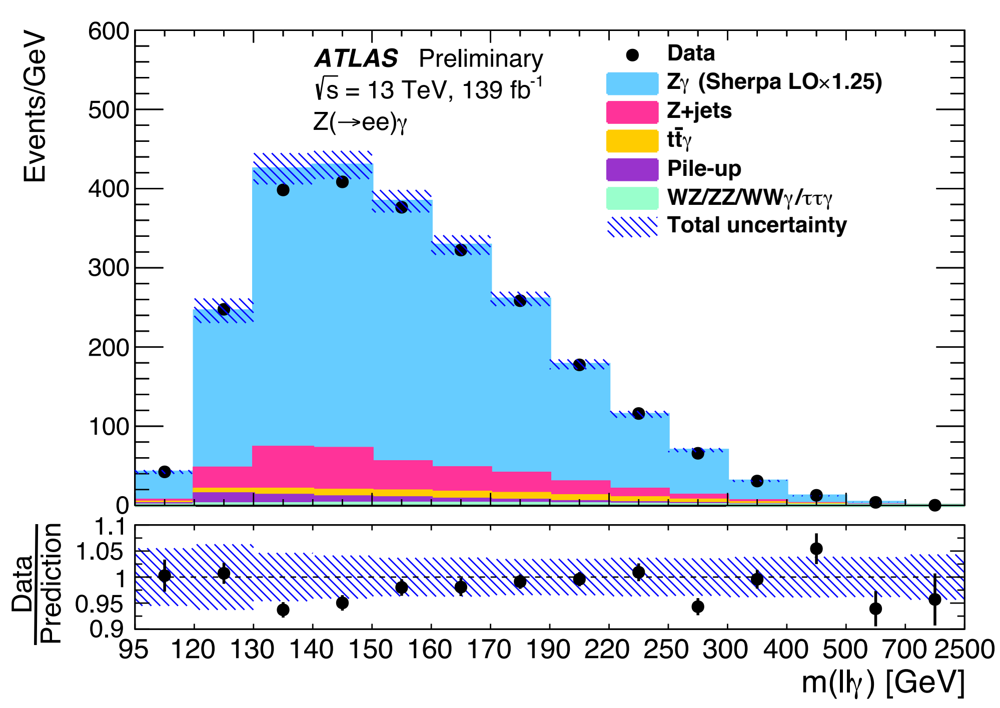
fiducial cross section



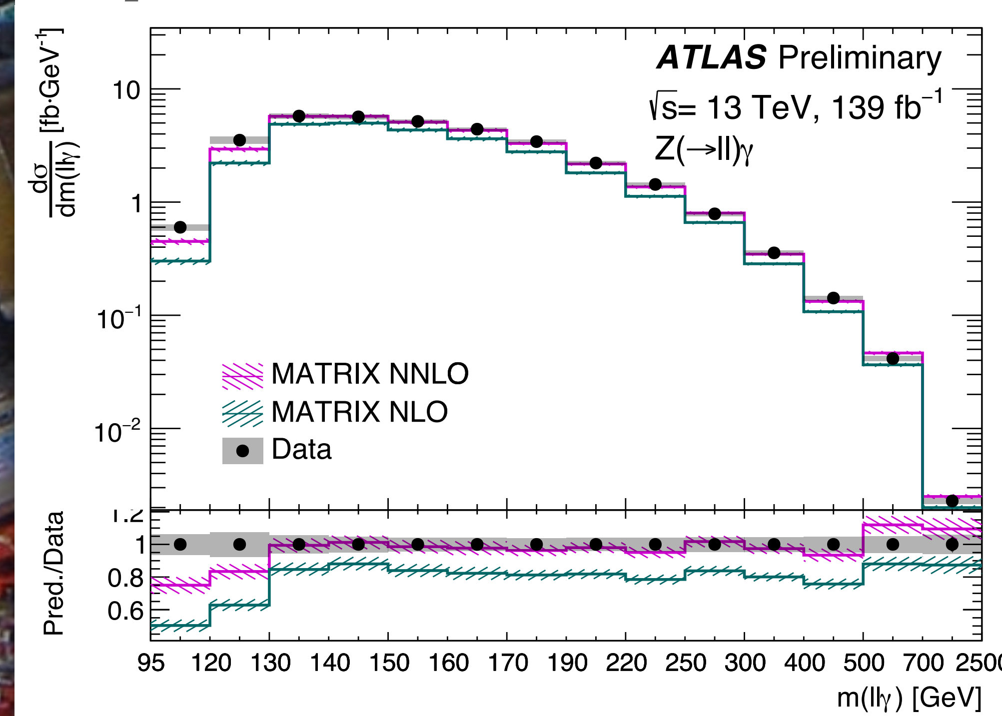
Why NNLO+PS?

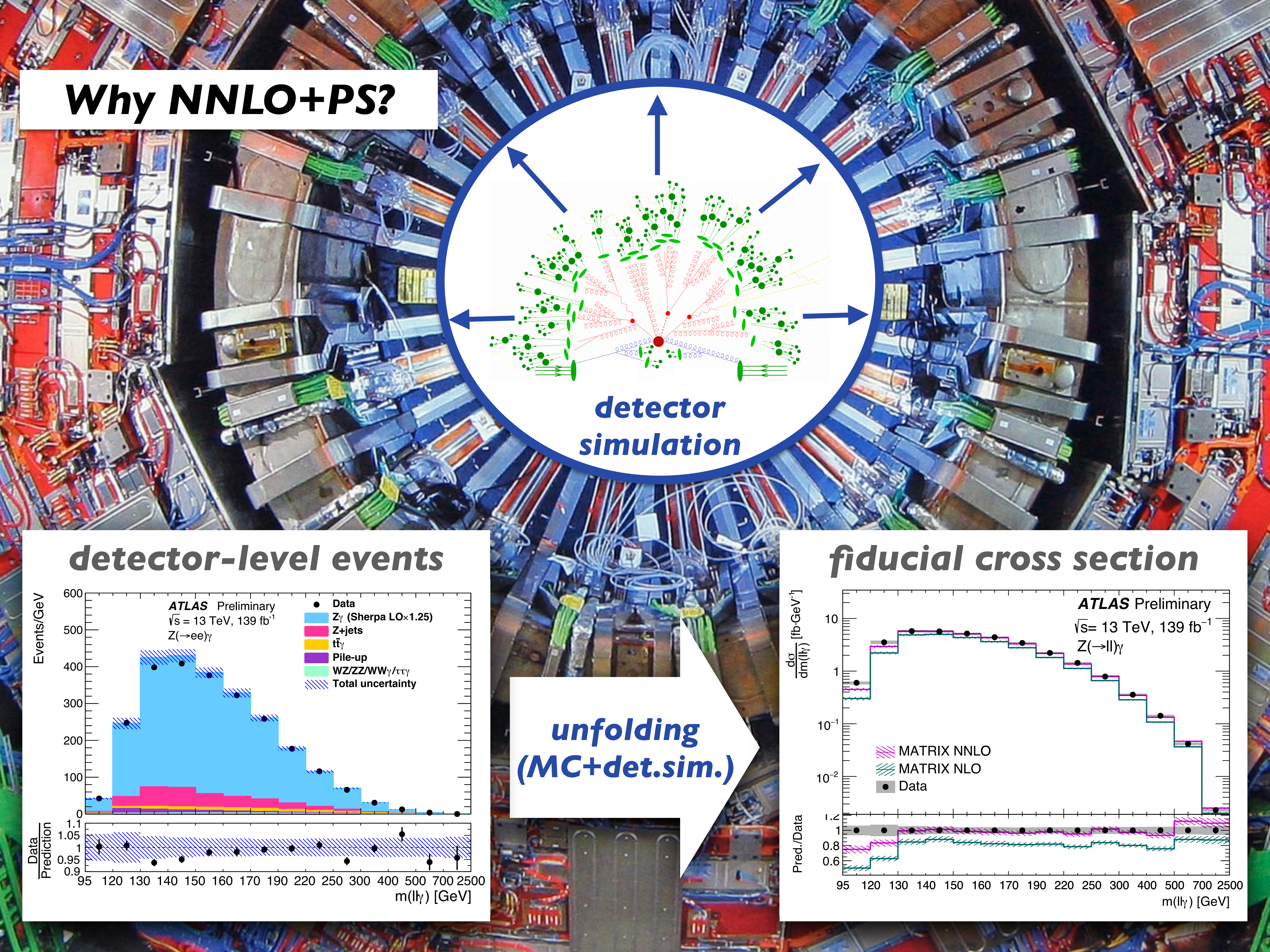


detector-level events



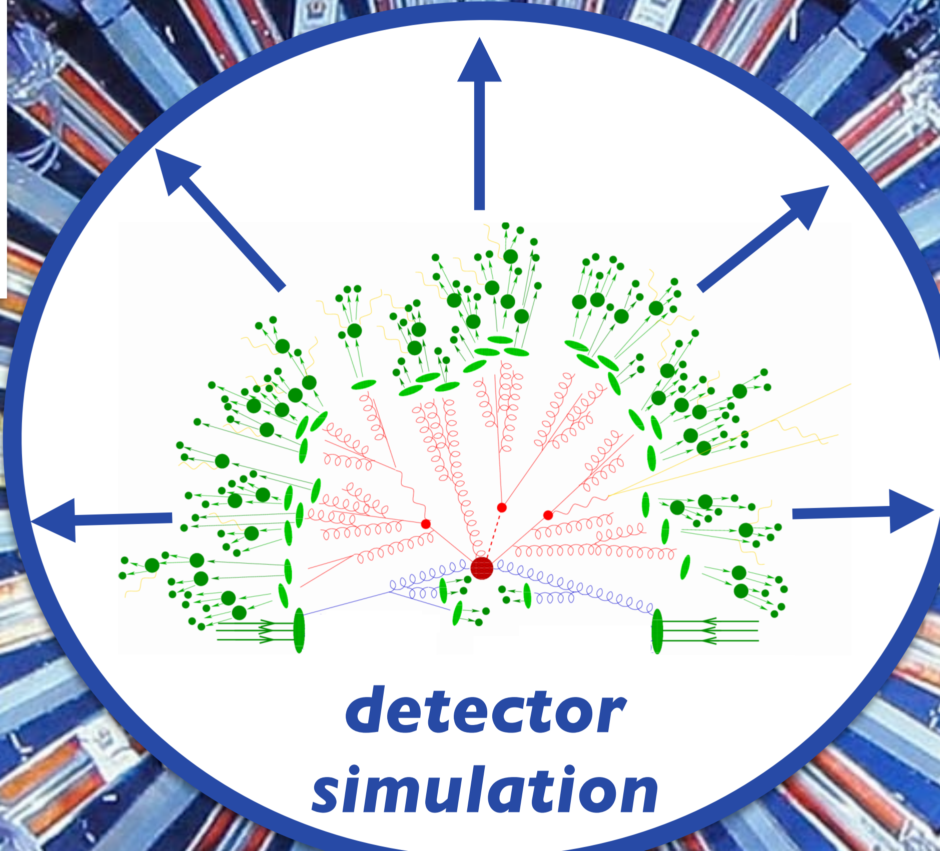
fiducial cross section



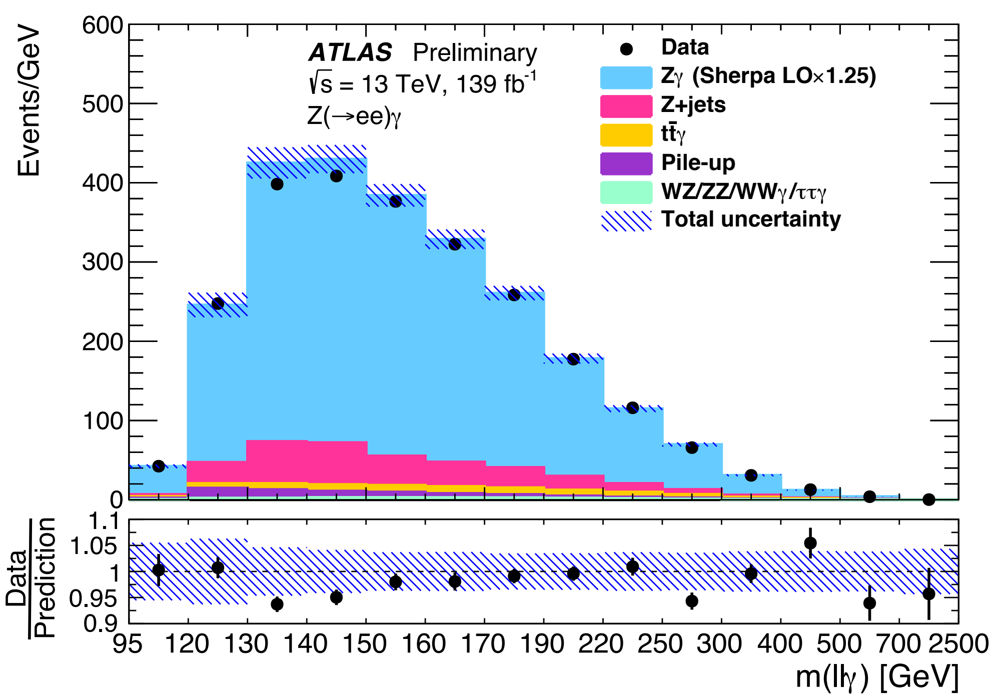

Why NNLO+PS?
detector simulation
detector-level events
Events/GeV
ATLAS Preliminary
√s = 13 TeV, 139 fb⁻¹
Z(→ee)γ
Data
Zγ (Sherpa LO×1.25)
Z+jets
tt̄γ
Pile-up
WZ/ZZ/WWγ/ττγ
Total uncertainty
Data / Prediction
m(llγ) [GeV]
unfolding (MC+det.sim.)
fiducial cross section
dσ/dm(llγ) [fb·GeV⁻¹]
ATLAS Preliminary
√s= 13 TeV, 139 fb⁻¹
Z(→ll)γ
MATRIX NNLO
MATRIX NLO
Data
Pred./Data
m(llγ) [GeV]

Why NNLO+PS?

1. comparison at event level
   (some analysis: no unfolding)

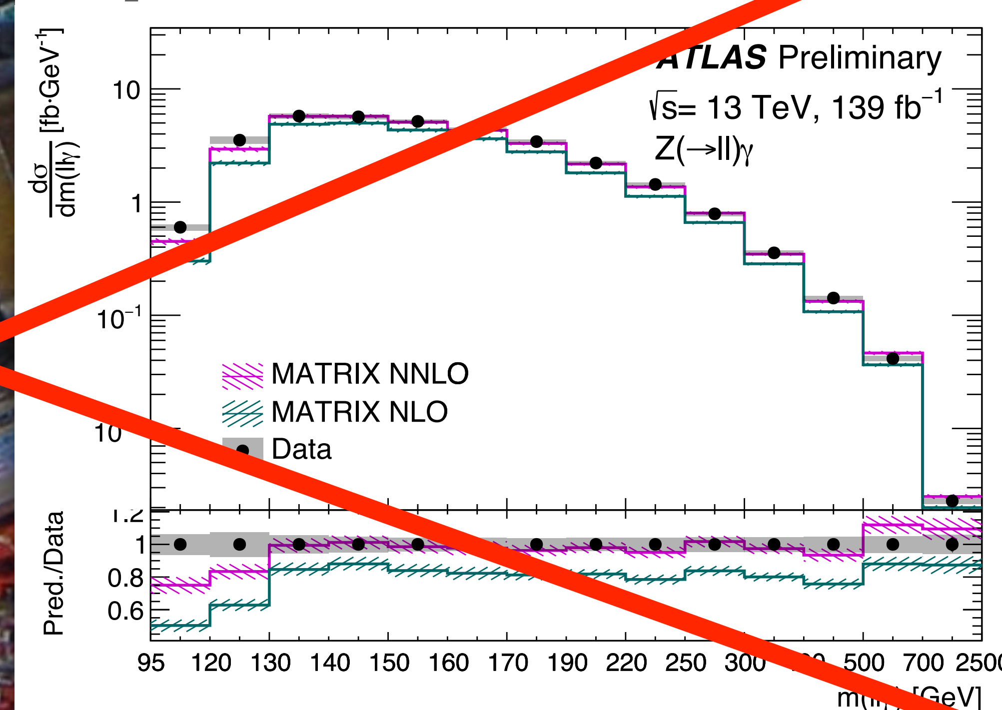


detector-level events



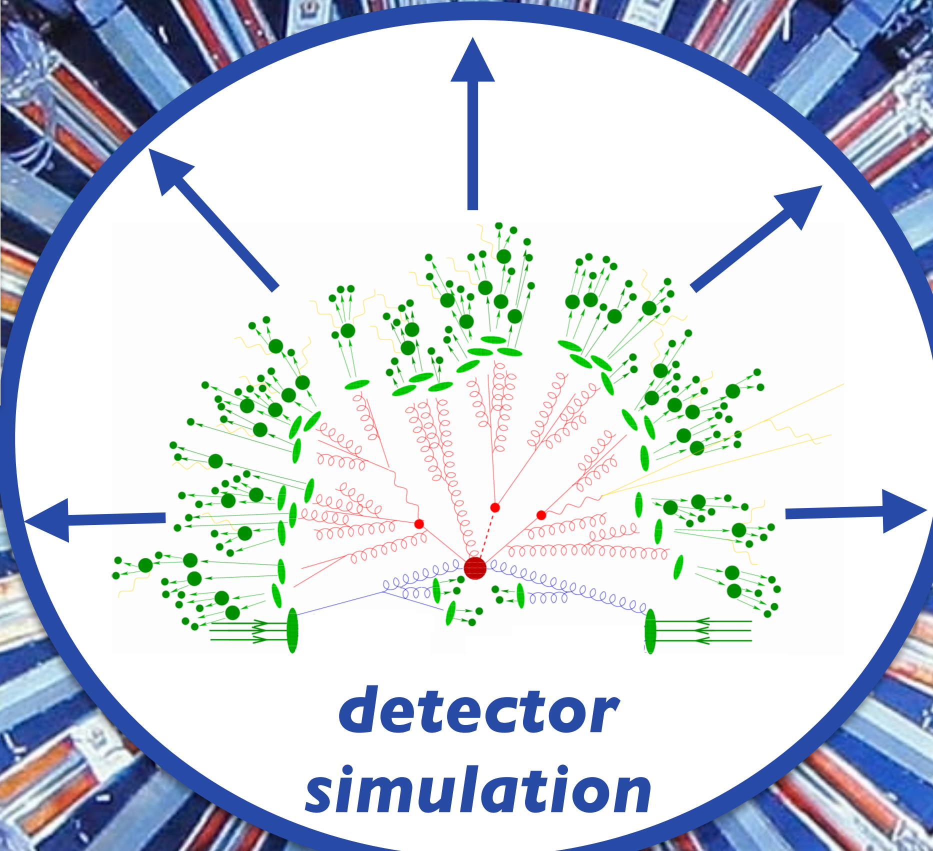
unfolding
(MC+det.sim.)

fiducial cross section

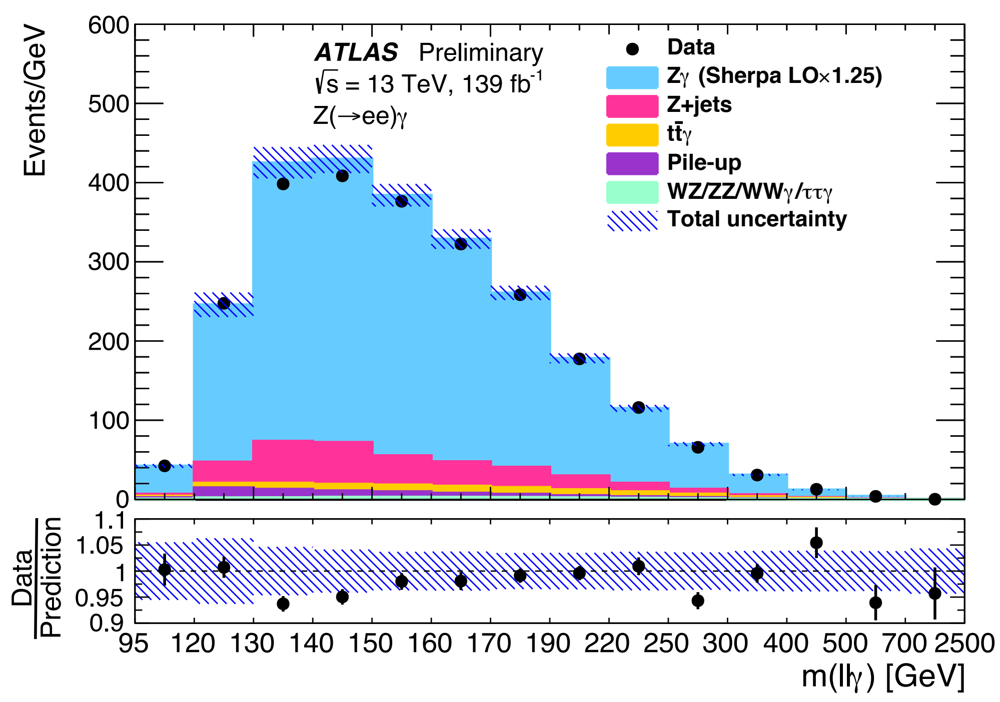


Why NNLO+PS?

1. comparison at event level
   (some analysis: no unfolding)
2. MC used for unfolding



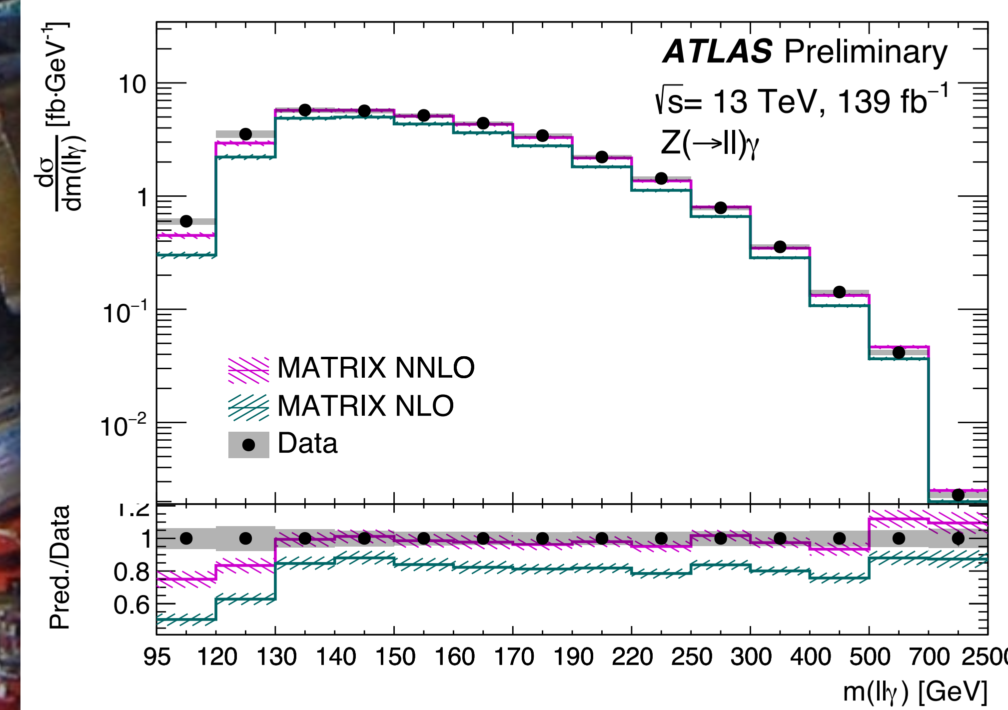
detector-level events



unfolding
(MC+det.sim.)

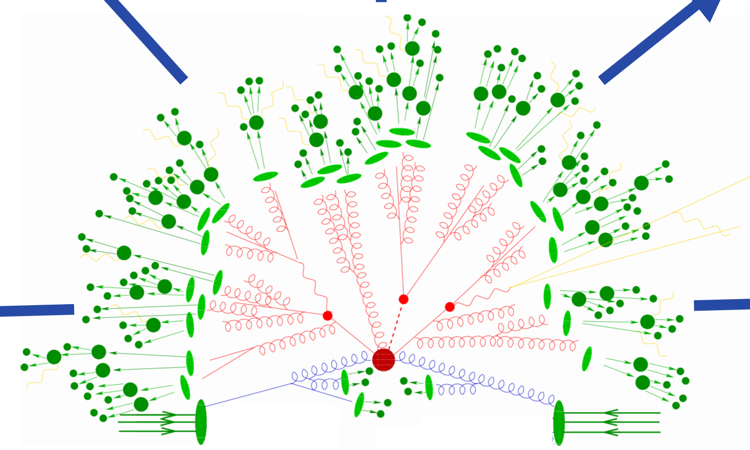
precision!

fiducial cross section



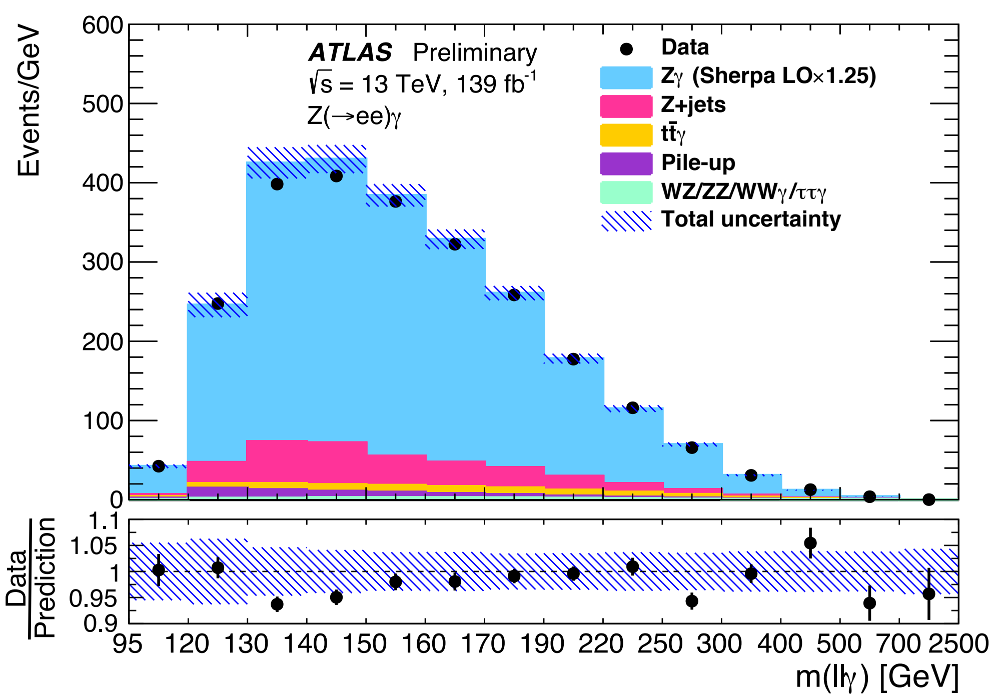
Why NNLO+PS?

1. comparison at event level (some analysis: no unfolding)
2. MC used for unfolding
3. some observables require shower resummation



detector simulation

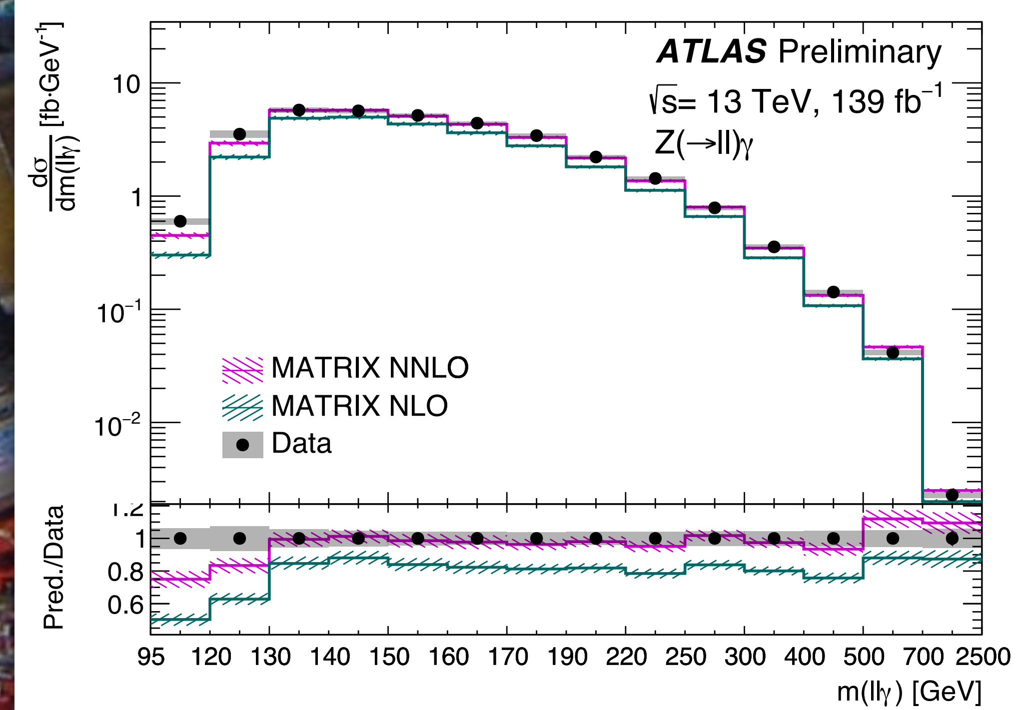
detector-level events



unfolding (MC+det.sim.)

precision!

fiducial cross section



NNLO+PS approaches

- ***MiNLO+reweighting*** [Hamilton, Nason, Zanderighi '12]
  - $pp \rightarrow H$ [Hamilton, Nason, Re, Zanderighi '13]
  - $pp \rightarrow \ell\ell$ (Z) [Karlberg, Hamilton, Zanderighi '14]
  - $pp \rightarrow \ell\ell H/\ell\nu H$ (ZH/WH) [Astill, Bizoń, Re, Zanderigh '16 '18]
  - $pp \rightarrow \ell\nu\ell'\nu'$ (WW) [Re, MW, Zanderighi '18]
- ***Geneva*** [Alioli, Bauer, Berggren, Tackmann, Walsh, Zuberi '13]
  - $pp \rightarrow \ell\ell$ (Z) [Alioli, Bauer, Berggren, Tackmann, Walsh '15]
  - $pp \rightarrow \ell\ell H/\ell\nu H$ (ZH/WH) [Alioli, Broggio, Kallweit, Lim, Rottoli '19]
- ***UNNLOPS*** [Höche, Prestel '14]
  - $pp \rightarrow H$ [Höche, Prestel '14]
  - $pp \rightarrow \ell\ell$ (Z) [Höche, Prestel '14]

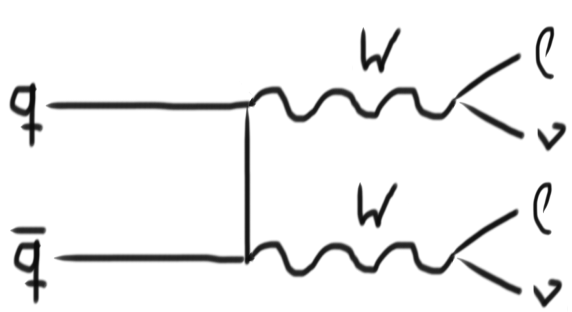
NNLO+PS approaches

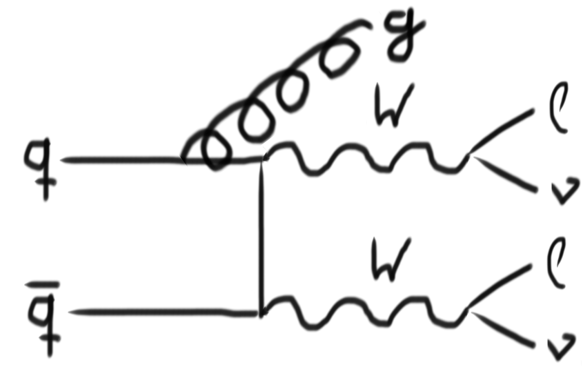
* ***MiNLO+~~reweighting~~*** ➠ **MiNNLO$_{PS}$** [Monni, Nason, Re, MW, Zanderighi '19]
  - $pp \to H$ [Hamilton, Nason, Re, Zanderighi '13]
  - $pp \to \ell\ell$ (Z) [Karlberg, Hamilton, Zanderighi '14]
  - $pp \to \ell\ell H/\ell\nu H$ (ZH/WH) [Astill, Bizoń, Re, Zanderigh '16 '18]
  - $pp \to \ell\nu\ell'\nu'$ (WW) [Re, MW, Zanderighi '18]
* ***Geneva*** [Alioli, Bauer, Berggren, Tackmann, Walsh, Zuberi '13]
  - $pp \to \ell\ell$ (Z) [Alioli, Bauer, Berggren, Tackmann, Walsh '15]
  - $pp \to \ell\ell H/\ell\nu H$ (ZH/WH) [Alioli, Broggio, Kallweit, Lim, Rottoli '19]
* ***UNNLOPS*** [Höche, Prestel '14]
  - $pp \to H$ [Höche, Prestel '14]
  - $pp \to \ell\ell$ (Z) [Höche, Prestel '14]

MiNLO+reweighting

	X	X+jet	X+2jets	X+nj (n>2)
XJ (NLO)	—	NLO	LO	—
XJ-MiNLO	NLO	NLO	LO	PS
X@NNLO	NNLO	NLO	LO	—
X@NNLOPS	NNLO	NLO	LO	PS

1. merge $pp \to WW$ *and* $pp \to WW$+jet *(both at NLO+PS)*





MiNLO+reweighting

	X	X+jet	X+2jets	X+nj (n>2)
XJ (NLO)	—	NLO	LO	—
XJ-MiNLO	NLO	NLO	LO	PS
X@NNLO	NNLO	NLO	LO	—
X@NNLOPS	NNLO	NLO	LO	PS

1. merge *pp → WW* *and* *pp → WW+jet* *(both at NLO+PS)*

- NLO (F+jet): $\frac{d\sigma_{FJ}^{(\mathrm{NLO})}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}} = \frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(1)} + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(2)}$

- MiNLO: $\frac{d\sigma}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}} = \exp[-S(p_{\mathrm{T}})]\left\{\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}[S(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(2)}\right\}$

$$S(p_{\mathrm{T}}) = 2\int_{p_{\mathrm{T}}}^{Q}\frac{dq}{q}\left(A(\alpha_s(q))\ln\frac{Q^2}{q^2} + B(\alpha_s(q))\right),$$

$$A(\alpha_s) = \sum_{k=1}^{2}\left(\frac{\alpha_s}{2\pi}\right)^k A^{(k)}, \quad B(\alpha_s) = \sum_{k=1}^{2}\left(\frac{\alpha_s}{2\pi}\right)^k B^{(k)},$$

MiNLO+reweighting

	X	X+jet	X+2jets	X+nj (n>2)
XJ (NLO)	—	NLO	LO	—
XJ-MiNLO	NLO	NLO	LO	PS
X@NNLO	NNLO	NLO	LO	—
X@NNLOPS	NNLO	NLO	LO	PS

1. merge *pp → WW* *and* *pp → WW+jet* *(both at NLO+PS)*

- NLO (F+jet): $$\frac{d\sigma_{FJ}^{(\mathrm{NLO})}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}} = \frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(1)} + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(2)}$$

- MiNLO: $$\frac{d\sigma}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}} = \exp[-S(p_{\mathrm{T}})]\left\{\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}[S(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(2)}\right\}$$

$$S(p_{\mathrm{T}}) = 2\int_{p_{\mathrm{T}}}^{Q}\frac{dq}{q}\left(A(\alpha_s(q))\ln\frac{Q^2}{q^2} + B(\alpha_s(q))\right),$$

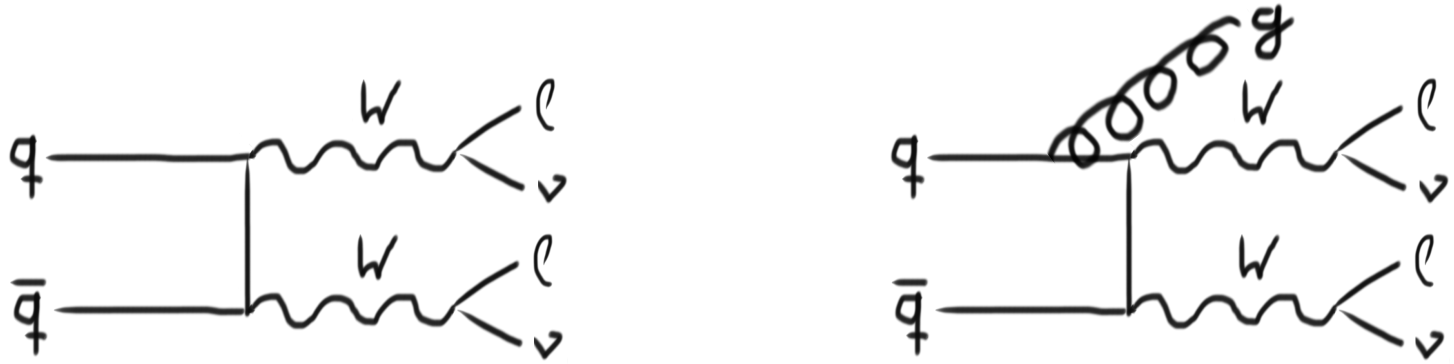
$$A(\alpha_s) = \sum_{k=1}^{2}\left(\frac{\alpha_s}{2\pi}\right)^k A^{(k)}, \quad B(\alpha_s) = \sum_{k=1}^{2}\left(\frac{\alpha_s}{2\pi}\right)^k B^{(k)},$$

μ_R and μ_F evaluated at p_T
→ resummation scheme choice
→ $B^{(2)}$ includes virtual amplitude to reach NLO accuracy

MiNLO+reweighting

	X	X+jet	X+2jets	X+nj (n>2)
XJ (NLO)	—	NLO	LO	—
XJ-MiNLO	NLO	NLO	LO	PS
X@NNLO	NNLO	NLO	LO	—
X@NNLOPS	NNLO	NLO	LO	PS

1. merge *pp → WW* *and* *pp → WW+jet* *(both at NLO+PS)*



MiNLO+reweighting

	X	X+jet	X+2jets	X+nj (n>2)
XJ (NLO)	—	NLO	LO	—
XJ-MiNLO	NLO	NLO	LO	PS
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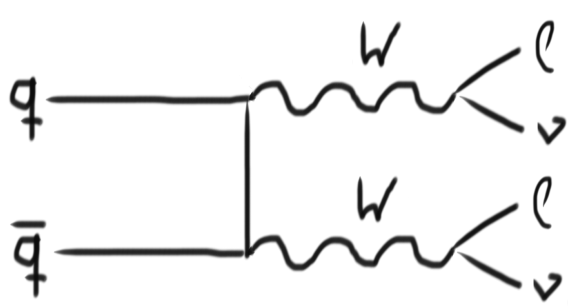
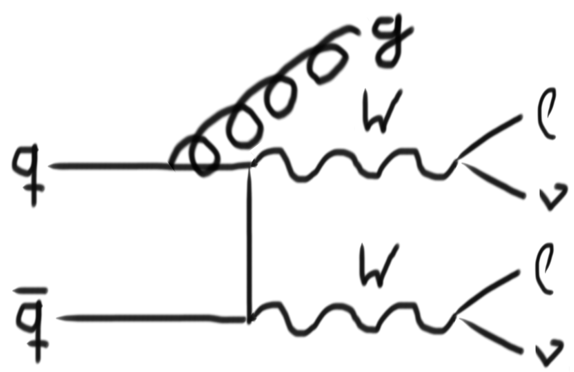
X@NNLOPS	NNLO	NLO	LO	PS

1. merge *pp → WW* *and* *pp → WW+jet* *(both at NLO+PS)*





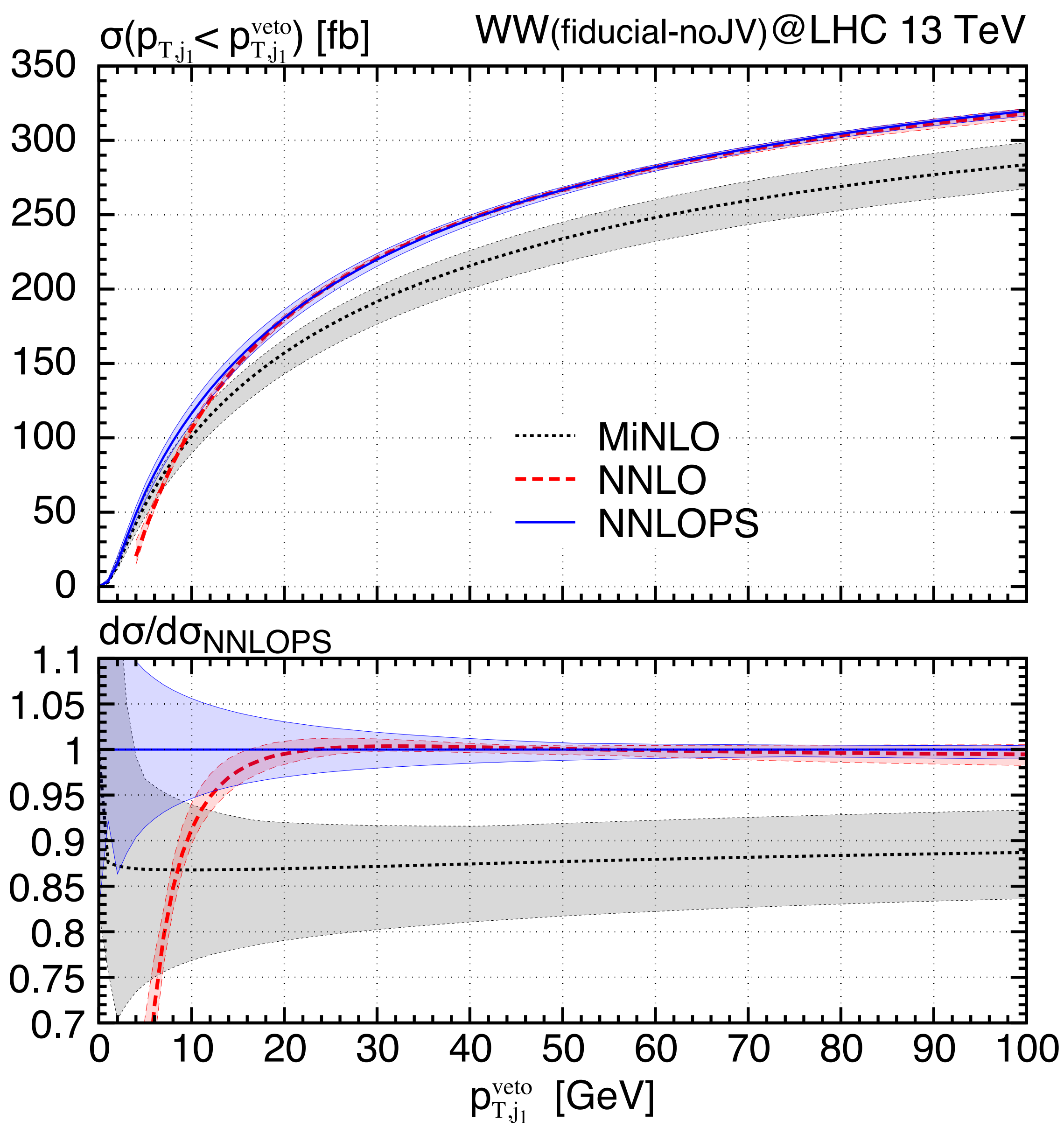
2. reweight to NNLO in born phase space

$$W(\Phi_B) = \frac{\left(\frac{d\sigma}{d\Phi_B}\right)_{\mathrm{NNLO}}}{\left(\frac{d\sigma}{d\Phi_B}\right)_{\mathrm{XJ-MiNLO'}}} = \frac{c_0 + c_1\alpha_S + c_2\alpha_S^2}{c_0 + c_1\alpha_S + d_2\alpha_S^2} \simeq 1 + \frac{c_2 - d_2}{c_0}\alpha_S^2 + \mathcal{O}(\alpha_S^3)$$

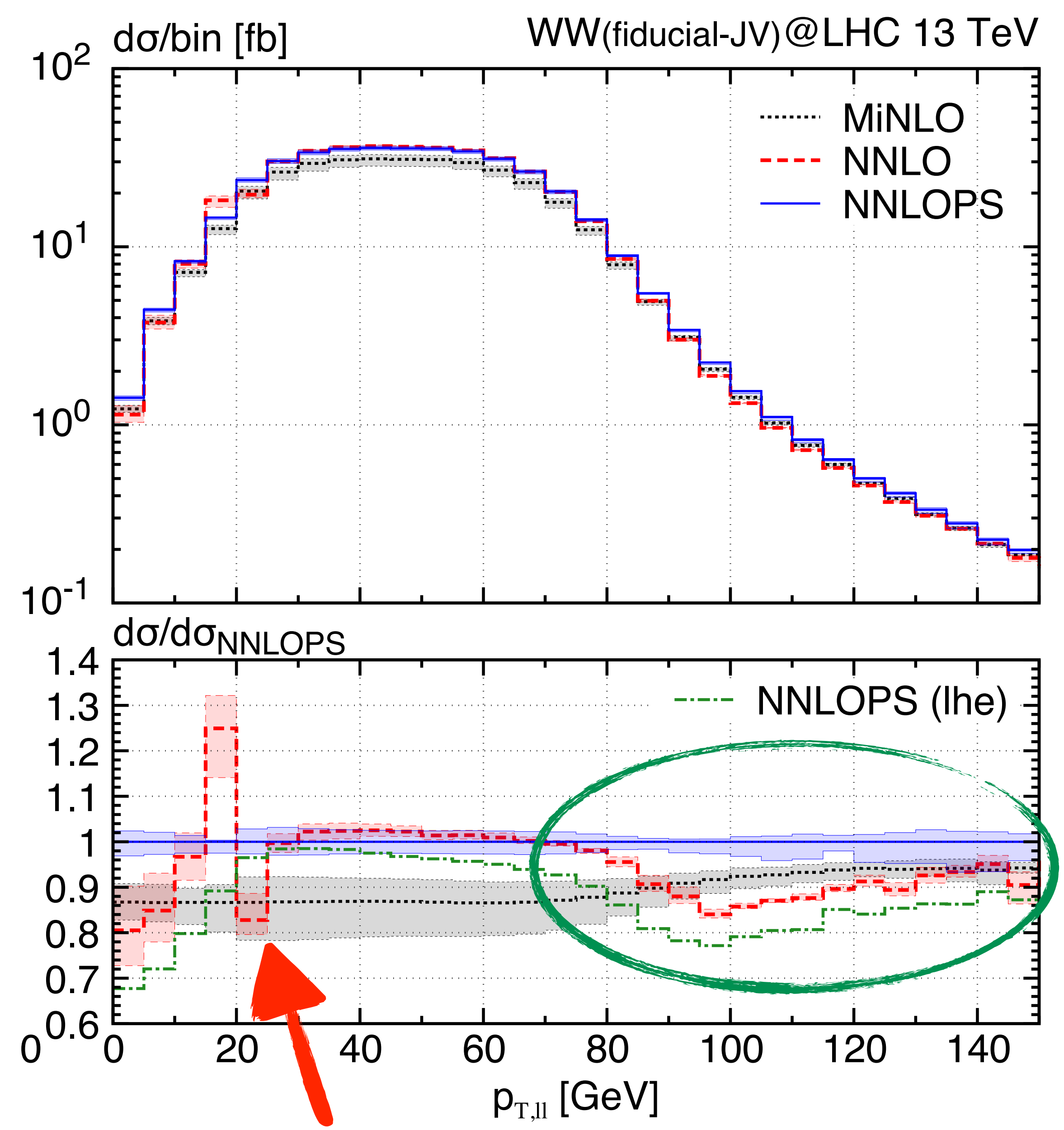
NNLO+PS for WW

[Re, MW, Zanderighi '18]

Jet veto



p_T of dilepton system



→ **NNLOPS physical down to $p_T = 0$**

→ **NNLOPS cures perturbative instabilities (p_T^{miss} cut)**

→ **NNLOPS induces additional shape effects**

The problem with reweighting

- 9D Born phase space: $$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{\mathrm{B}}} = \frac{\mathrm{d}^9\sigma}{\mathrm{d}p_{T,W^-}\,\mathrm{d}y_{WW}\,\mathrm{d}\Delta y_{W^+W^-}\,\mathrm{d}\cos\theta^{\mathrm{CS}}_{W^+}\,\mathrm{d}\phi^{\mathrm{CS}}_{W^+}\,\mathrm{d}\cos\theta^{\mathrm{CS}}_{W^-}\,\mathrm{d}\phi^{\mathrm{CS}}_{W^-}\,\mathrm{d}m_{W^+}\,\mathrm{d}m_{W^-}}$$

- approximation: m_W flat & CS angles [Collins, Soper '77] to convert to 81 3D moments

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{\mathrm{B}}} = \frac{9}{256\pi^2}\sum_{i=0}^{8}\sum_{j=0}^{8} AB_{ij}\, f_i(\theta^{\mathrm{CS}}_{W^-},\phi^{\mathrm{CS}}_{W^-})\, f_j(\theta^{\mathrm{CS}}_{W^+},\phi^{\mathrm{CS}}_{W^+})$$

$$f_0(\theta,\phi) = \left(1-3\cos^2\theta\right)/2\,,\quad f_1(\theta,\phi) = \sin 2\theta\cos\phi\,,\quad f_2(\theta,\phi) = (\sin^2\theta\cos 2\phi)/2\,,$$
$$f_3(\theta,\phi) = \sin\theta\cos\phi\,,\quad f_4(\theta,\phi) = \cos\theta\,,\quad f_5(\theta,\phi) = \sin\theta\sin\phi\,,$$
$$f_6(\theta,\phi) = \sin 2\theta\sin\phi\,,\quad f_7(\theta,\phi) = \sin^2\theta\sin 2\phi\,,\quad f_8(\theta,\phi) = 1+\cos^2\theta\,.$$

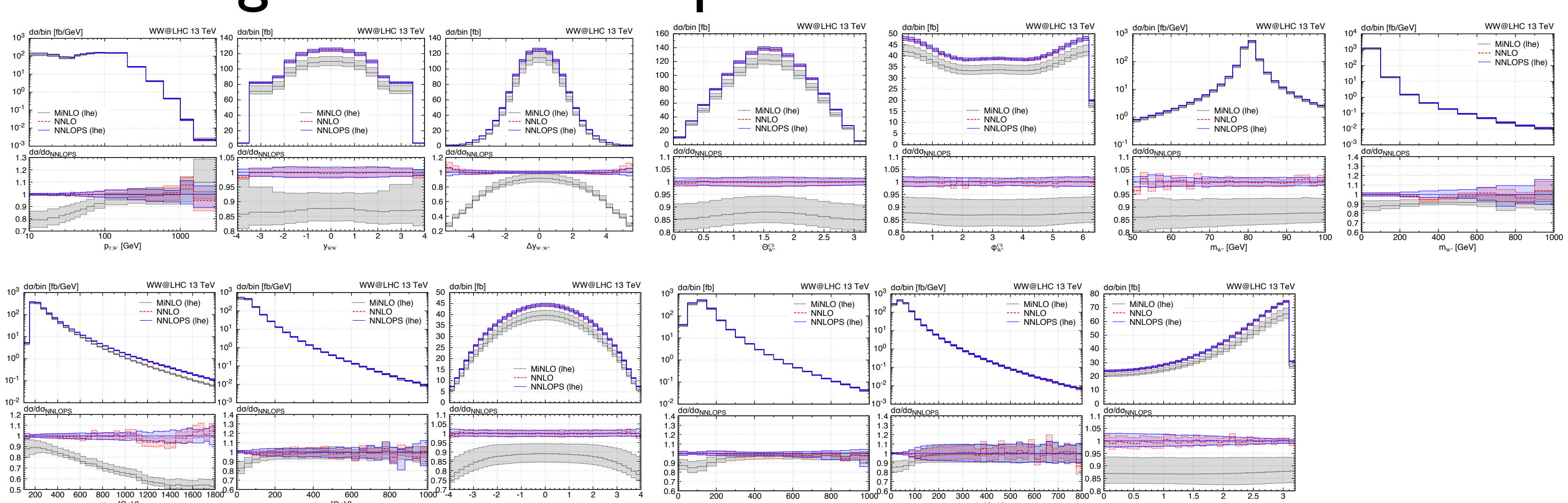
$$AB_{ij}(p_{T,W^-}, y_{WW}, \Delta y_{W^+W^-}) = \int \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_B}\, g_i(\theta^{\mathrm{CS}}_{W^-},\phi^{\mathrm{CS}}_{W^-})\, g_j(\theta^{\mathrm{CS}}_{W^+},\phi^{\mathrm{CS}}_{W^+})\,\mathrm{d}\cos\theta^{\mathrm{CS}}_{W^-}\,\mathrm{d}\phi^{\mathrm{CS}}_{W^-}\,\mathrm{d}\cos\theta^{\mathrm{CS}}_{W^+}\,\mathrm{d}\phi^{\mathrm{CS}}_{W^+}$$

- discrete binning limits applicability in less populated regions

$$\begin{aligned}
p_{T,W^-} &: \quad [0., 17.5, 25., 30., 35., 40., 47.5, 57.5, 72.5, 100., 200., 350., 600., 1000., 1500., \infty]\,;\\
y_{WW} &: \quad [-\infty, -3.5, -2.5, -2.0, -1.5, -1.0, -0.5, 0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.5, \infty]\,;\\
\Delta y_{W^+W^-} &: \quad [-\infty, -5.2, -4.8, -4.4, -4.0, -3.6, -3.2, -2.8, -2.4, -2.0, -1.6, -1.2,\\
&\qquad -0.8, -0.4, 0.0, 0.4, 0.8, 1.2, 1.6, 2.0, 2.4, 2.8, 3.2, 3.6, 4.0, 4.4, 4.8, 5.2, \infty]\,.
\end{aligned}$$

- reweighting still numerically intensive

- thorough validation required



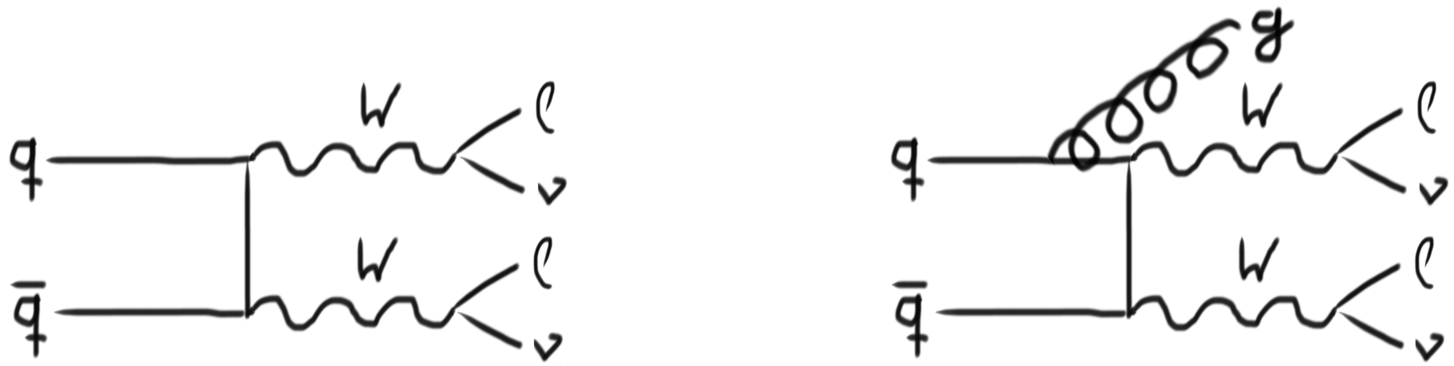
Issue in NNLOPS event production of experiments already for DY

New approach: MiNNLOPS

[Monni, Nason, Re, MW, Zanderighi '19]

	X	X+jet	X+2jets	X+nj (n>2)
XJ (NLO)	—	NLO	LO	—
XJ-MiNLO	NLO	NLO	LO	PS
X@NNLO	NNLO	NLO	LO	—
X@NNLOPS	NNLO	NLO	LO	PS

1. merge *pp → WW* *and* *pp → WW+jet* *(both at NLO+PS)*



2. reweight to NNLO in born phase space

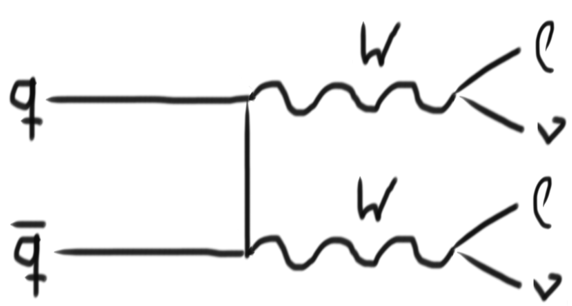
$$W(\Phi_B) = \frac{\left(\frac{d\sigma}{d\Phi_B}\right)_{\mathrm{NNLO}}}{\left(\frac{d\sigma}{d\Phi_B}\right)_{\mathrm{XJ-MiNLO'}}} = \frac{c_0 + c_1\alpha_S + c_2\alpha_S^2}{c_0 + c_1\alpha_S + d_2\alpha_S^2} \simeq 1 + \frac{c_2 - d_2}{c_0}\alpha_S^2 + \mathcal{O}(\alpha_S^3)$$

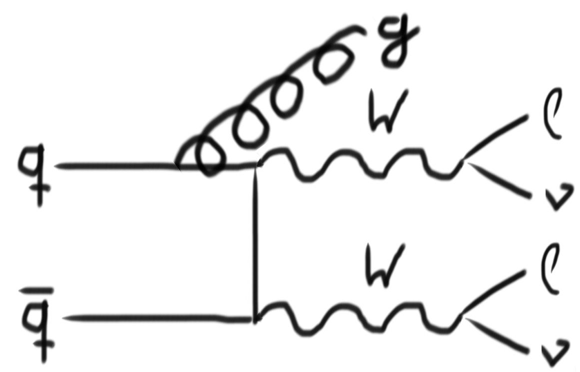
New approach: MiNNLO$_{\mathrm{PS}}$

[Monni, Nason, Re, MW, Zanderighi '19]

	X	X+jet	X+2jets	X+nj (n>2)
XJ (NLO)	—	NLO	LO	—
XJ-MiNLO	NLO	NLO	LO	PS
X@NNLO	NNLO	NLO	LO	—
X@NNLOPS	NNLO	NLO	LO	PS

1. merge *pp → WW* *and* *pp → WW+jet* *(both at NLO+PS)*





2. ~~reweight to NNLO in born phase space~~

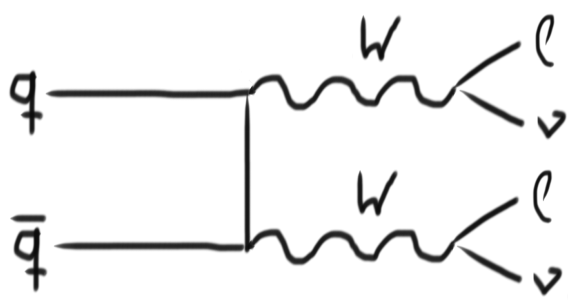
$$W(\Phi_B) = \frac{\left(\frac{d\sigma}{d\Phi_B}\right)_{\mathrm{NNLO}}}{\left(\frac{d\sigma}{d\Phi_B}\right)_{\mathrm{XJ-MiNLO}'}} = \frac{c_0 + c_1\alpha_S + c_2\alpha_S^2}{c_0 + c_1\alpha_S + d_2\alpha_S^2} \simeq 1 + \frac{c_2 - d_2}{c_0}\alpha_S^2 + \mathcal{O}(\alpha_S^3)$$

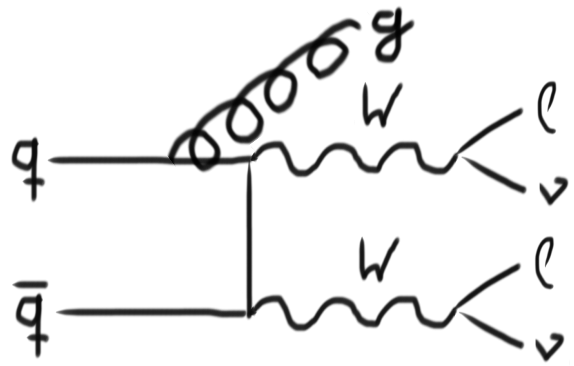
New approach: MiNNLO$_{PS}$

[Monni, Nason, Re, MW, Zanderighi '19]

	X	X+jet	X+2jets	X+nj (n>2)
XJ (NLO)	—	NLO	LO	—
XJ-MiNLO	NLO	NLO	LO	PS
X@NNLO	NNLO	NLO	LO	—
X@NNLOPS	NNLO	NLO	LO	PS

1. merge *pp → WW* *and* *pp → WW+jet* *(both at NLO+PS)*





2. add ***missing terms*** explicitly (from analytic all-order formula)

New approach: MiNNLOPS

[Monni, Nason, Re, MW, Zanderighi '19]

- NLO (F+jet): $$\frac{d\sigma_{FJ}^{(\mathrm{NLO})}}{d\Phi_F dp_T} = \frac{\alpha_s(p_T)}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_F dp_T}\right]^{(1)} + \left(\frac{\alpha_s(p_T)}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_F dp_T}\right]^{(2)}$$

- MiNLO: $$\frac{d\sigma}{d\Phi_F dp_T} = \exp[-S(p_T)]\left\{\frac{\alpha_s(p_T)}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_F dp_T}\right]^{(1)}\left(1+\frac{\alpha_s(p_T)}{2\pi}[S(p_T)]^{(1)}\right) + \left(\frac{\alpha_s(p_T)}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_F dp_T}\right]^{(2)}\right\}$$

$$S(p_T) = 2\int_{p_T}^{Q}\frac{dq}{q}\left(A(\alpha_s(q))\ln\frac{Q^2}{q^2} + B(\alpha_s(q))\right),$$

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- analytic all-order formula:

$$\frac{d\sigma}{d\Phi_{\mathrm{B}} dp_{\mathrm{T}}} = \frac{d}{dp_{\mathrm{T}}}\left\{\exp[-S(p_{\mathrm{T}})]\mathcal{L}(\Phi_{\mathrm{B}}, p_{\mathrm{T}})\right\} + R_f(p_{\mathrm{T}})$$

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[Monni, Nason, Re, MW, Zanderighi '19]

- NLO (F+jet): $$\frac{d\sigma_{FJ}^{(\mathrm{NLO})}}{d\Phi_{\mathrm{F}}dp_{\mathrm{T}}} = \frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}}dp_{\mathrm{T}}}\right]^{(1)} + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}}dp_{\mathrm{T}}}\right]^{(2)}$$

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counting:

$$\int_{\Lambda}^{Q} dp_{\mathrm{T}}\frac{1}{p_{\mathrm{T}}}\alpha_s^m(p_{\mathrm{T}})\ln^n\frac{p_{\mathrm{T}}}{Q}\exp(-S(p_{\mathrm{T}})) \approx \alpha_s^{m-\frac{n+1}{2}}(Q)$$

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$$\frac{d\sigma}{d\Phi_{\mathrm{B}}dp_{\mathrm{T}}} = \frac{d}{dp_{\mathrm{T}}}\left\{\exp[-S(p_{\mathrm{T}})]\mathcal{L}(\Phi_{\mathrm{B}},p_{\mathrm{T}})\right\} + R_f(p_{\mathrm{T}}) = \exp[-S(p_{\mathrm{T}})]\left\{D(p_{\mathrm{T}}) + \frac{R_f(p_{\mathrm{T}})}{\exp[-S(p_{\mathrm{T}})]}\right\}$$

New approach: MiNNLO$_{\mathrm{PS}}$

[Monni, Nason, Re, MW, Zanderighi '19]

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$$D(p_{\mathrm{T}}) \equiv -\frac{dS(p_{\mathrm{T}})}{dp_{\mathrm{T}}}\mathcal{L}(p_{\mathrm{T}}) + \frac{d\mathcal{L}(p_{\mathrm{T}})}{dp_{\mathrm{T}}}$$

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$$= \exp[-S(p_{\mathrm{T}})]\left\{\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}[S(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(2)} + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^3[D(p_{\mathrm{T}})]^{(3)} + \text{regular terms}\right\}$$

New approach: MiNNLOPS

[Monni, Nason, Re, MW, Zanderighi '19]

- NLO (F+jet): $\frac{d\sigma_{FJ}^{(\mathrm{NLO})}}{d\Phi_{\mathrm{F}}dp_{\mathrm{T}}} = \frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}}dp_{\mathrm{T}}}\right]^{(1)} + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}}dp_{\mathrm{T}}}\right]^{(2)}$

- MiNLO: $\frac{d\sigma}{d\Phi_{\mathrm{F}}dp_{\mathrm{T}}} = \boxed{\exp[-S(p_{\mathrm{T}})]\left\{\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}}dp_{\mathrm{T}}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}[S(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}}dp_{\mathrm{T}}}\right]^{(2)}\right\}}$

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$$= \underbrace{\boxed{\exp[-S(p_{\mathrm{T}})]\left\{\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}}dp_{\mathrm{T}}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}[S(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}}dp_{\mathrm{T}}}\right]^{(2)}}}_{\text{MiNLO}} + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^3[D(p_{\mathrm{T}})]^{(3)} + \text{regular terms}\right\}$$

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[Monni, Nason, Re, MW, Zanderighi '19]

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$$= \boxed{\exp[-S(p_{\mathrm{T}})]\left\{\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}}dp_{\mathrm{T}}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}[S(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}}dp_{\mathrm{T}}}\right]^{(2)}\right.} + \boxed{\left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^3[D(p_{\mathrm{T}})]^{(3)}} + \text{regular terms}\Big\}$$

MiNLO

missing terms for NNLO accuracy

MiNNLO$_{\text{PS}}$ practical implementation

[Monni, Nason, Re, MW, Zanderighi '19]

MiNNLO$_{\text{PS}}$ master formula

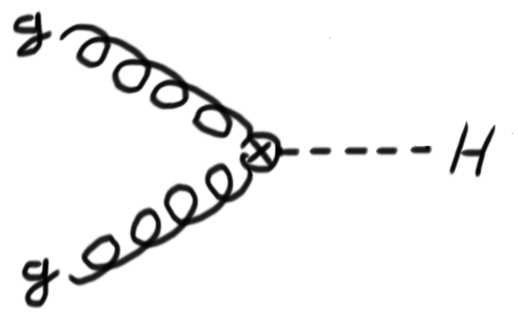
$$\frac{d\sigma}{d\Phi_{\rm FJ}} = \exp[-\tilde{S}(p_{\rm T})]\left\{\frac{\alpha_s(p_{\rm T})}{2\pi}\left[\frac{d\sigma_{\rm FJ}}{d\Phi_{\rm FJ}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\rm T})}{2\pi}[\tilde{S}(p_{\rm T})]^{(1)}\right) + \left(\frac{\alpha_s(p_{\rm T})}{2\pi}\right)^2\left[\frac{d\sigma_{\rm FJ}}{d\Phi_{\rm FJ}}\right]^{(2)} + \left(\frac{\alpha_s(p_{\rm T})}{2\pi}\right)^3 \underline{[D(p_{\rm T})]^{(3)}} F^{\rm corr}(\Phi_{\rm FJ})\right\}$$
$$\times\left\{\Delta_{\rm pwg}(\Lambda) + \int d\Phi_{\rm rad}\Delta_{\rm pwg}(p_{\rm T,rad})\frac{R(\Phi_{\rm FJ},\Phi_{\rm rad})}{B(\Phi_{\rm FJ})}\right\} + \mathcal{O}(\alpha_s^3)$$

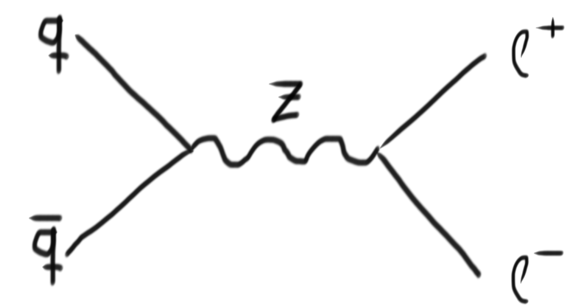
$$\underline{[D(p_{\rm T})]^{(3)}} = -\left[\frac{d\tilde{S}(p_{\rm T})}{dp_{\rm T}}\right]^{(1)}[\mathcal{L}(p_{\rm T})]^{(2)} - \left[\frac{d\tilde{S}(p_{\rm T})}{dp_{\rm T}}\right]^{(2)}[\mathcal{L}(p_{\rm T})]^{(1)} - \left[\frac{d\tilde{S}(p_{\rm T})}{dp_{\rm T}}\right]^{(3)}[\mathcal{L}(p_{\rm T})]^{(0)} + \left[\frac{d\mathcal{L}(p_{\rm T})}{dp_{\rm T}}\right]^{(3)}$$
$$= \frac{2}{p_{\rm T}}\left(A^{(1)}\ln\frac{Q^2}{{p_{\rm T}}^2} + B^{(1)}\right)[\mathcal{L}(p_{\rm T})]^{(2)} + \frac{2}{p_{\rm T}}\left(A^{(2)}\ln\frac{Q^2}{{p_{\rm T}}^2} + \tilde{B}^{(2)}\right)[\mathcal{L}(p_{\rm T})]^{(1)} + \frac{2}{p_{\rm T}}A^{(3)}\ln\frac{Q^2}{{p_{\rm T}}^2}[\mathcal{L}(p_{\rm T})]^{(0)} + \left[\frac{d\mathcal{L}(p_{\rm T})}{dp_{\rm T}}\right]^{(3)}$$

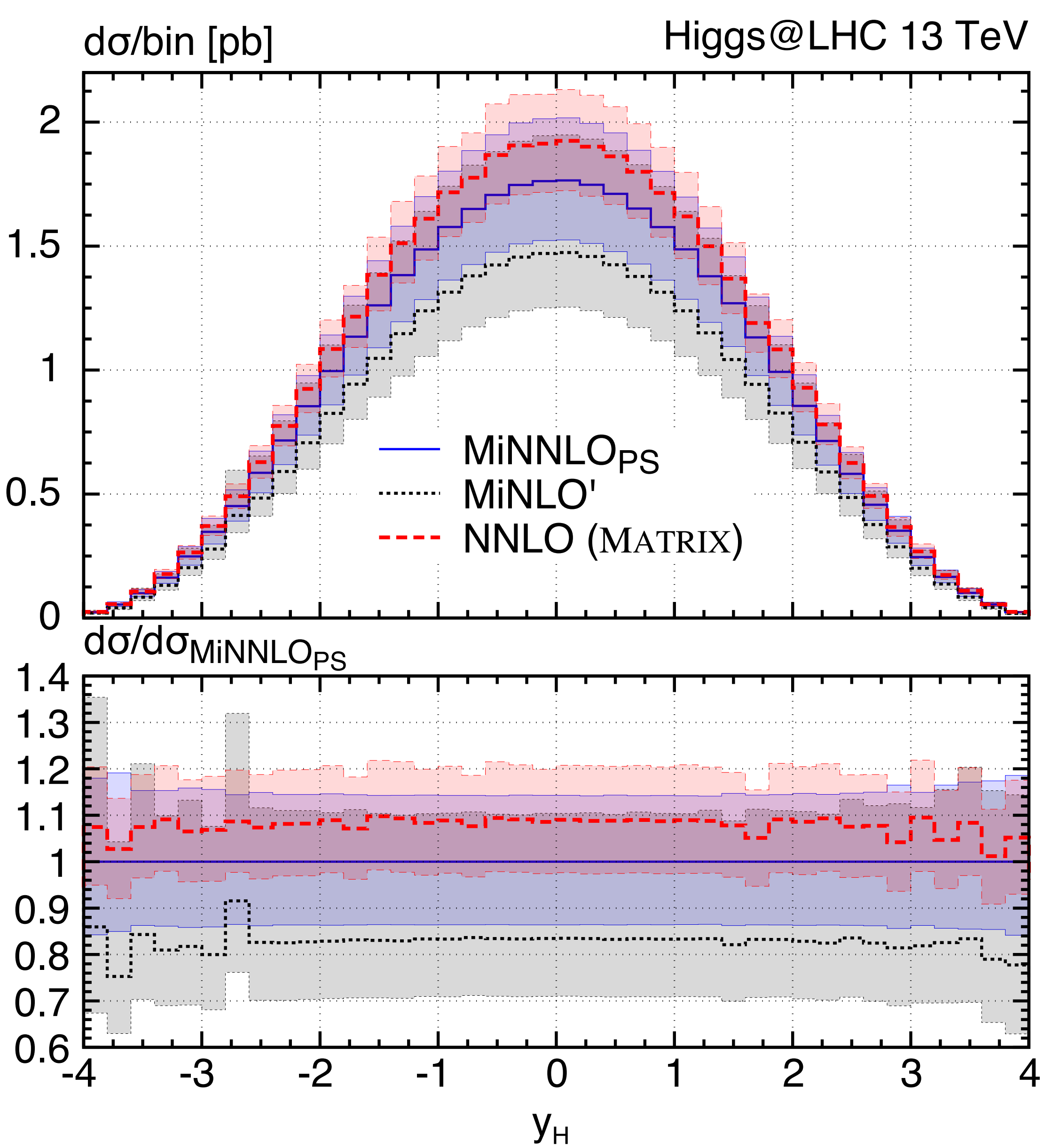
$$\mathcal{L}(k_{\rm T,1}) = \sum_{c,c'}\frac{d|M^{\rm F}|^2_{cc'}}{d\Phi_{\rm F}}\sum_{i,j}\left\{\left(\tilde{C}^{[a]}_{ci}\otimes f^{[a]}_i\right)\tilde{H}(k_{\rm T,1})\left(\tilde{C}^{[b]}_{c'j}\otimes f^{[b]}_j\right) + \left(G^{[a]}_{ci}\otimes f^{[a]}_i\right)\tilde{H}(k_{\rm T,1})\left(G^{[b]}_{c'j}\otimes f^{[b]}_j\right)\right\}$$

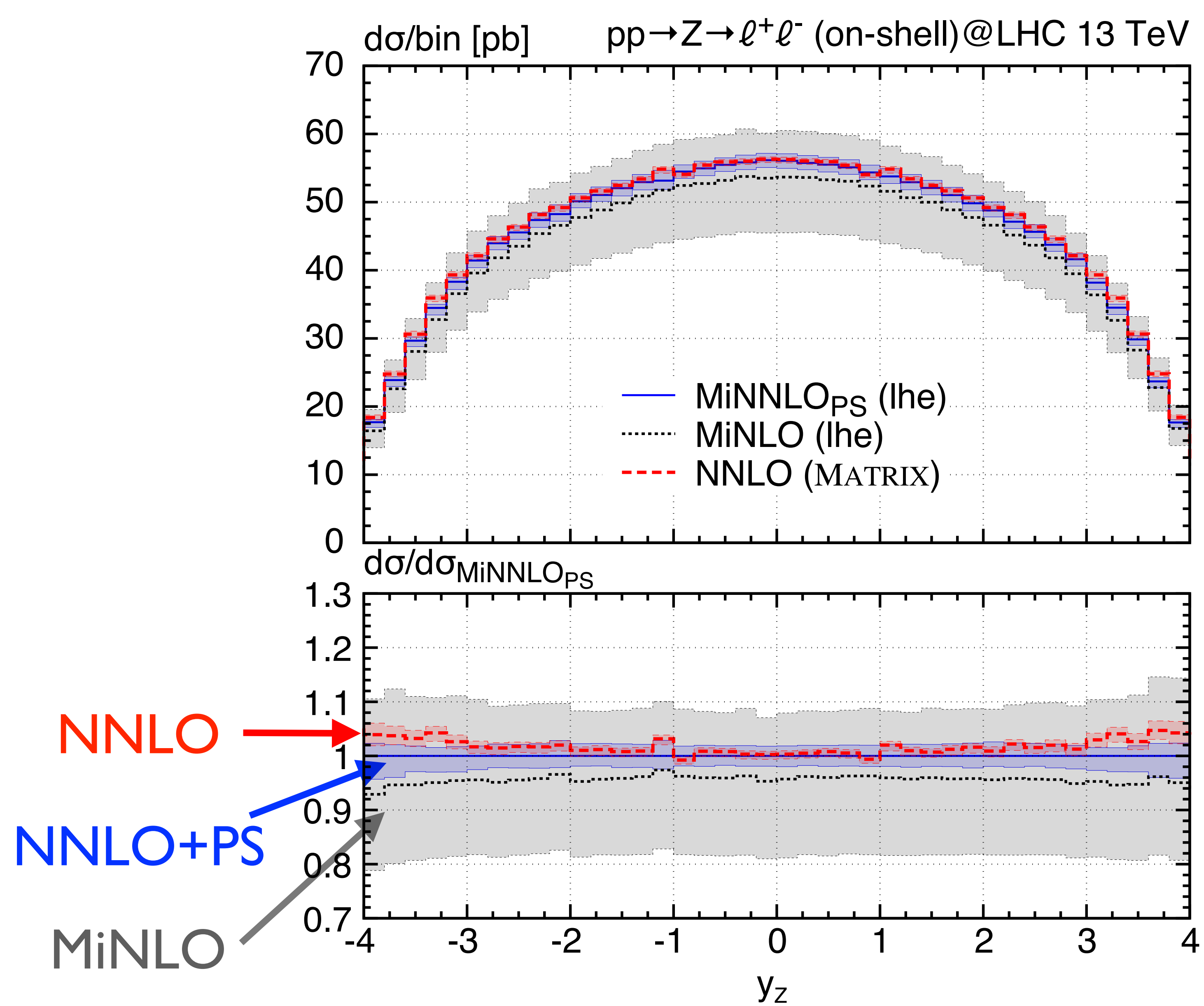
MiNNLOPS results

[Monni, Nason, Re, MW, Zanderighi '19]









MiNNLO$_{PS}$ features

[Monni, Nason, Re, MW, Zanderighi '19]

- **NO** reweighting (NNLO corrections directly evaluated during event generation)
- analytic Sudakov suppresses low-p_T region (integrable down to arbitrary low p_T)
  ➞ efficient event generation (only ~50% slower than MiNLO)
- **NO** merging scale (lower cut-off to switch from F+1-jet to F+0-jet NNLO)
- leading logarithmic accuracy of shower preserved (p_T ordered)

well suited for any color-singlet process, e.g. VV production

Conclusions

- **Diboson theory predictions under excellent control:**
  - ***NNLO QCD done!*** → publicly available within **MATRIX**
  - NLO QCD corrections for loop-induced gg contribution
  - Intriguing results for combination with NLO EW
- **MATRIX+RadISH: powerful resummation framework**
- **MiNNLO$_{\text{PS}}$: New NNLO+PS approach (no reweighting)**
- **Ongoing and future work:**
  - public MATRIX v2: NNLO QCD x NLO EW + gg NLO QCD
  - gg NLO QCD for Higgs interference in ZZ and WW
  - MiNNLO$_{\text{PS}}$ for diboson processes

FREE YOUR MIND.

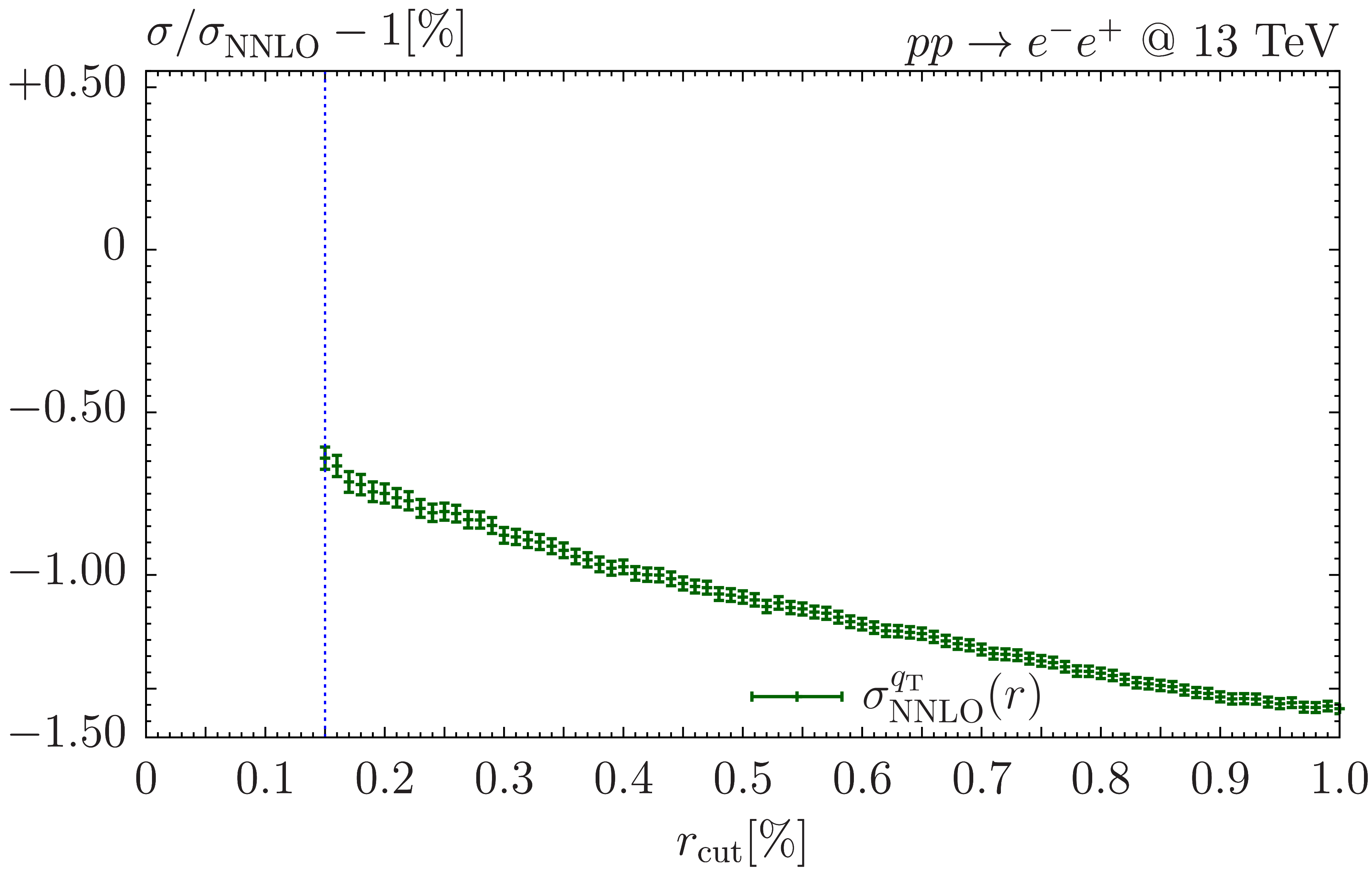
```
[wiesemann:~/different-branch-munich/MATRIX] ./matrix

        /-----------------------------------------------------------------------\
        | MATRIX: A fully-differential NNLO(+NNLL) process library               |
        |                                                                       |
        |                                                                       |
        |          Version: 1.0.0.release_candidate4      Aug 2017              |
        |                                                                       |
        | Munich -- the MUlti-chaNnel Integrator at swiss (CH) precision --     |
        | Automates qT-subtraction and Resummation to Integrate X-sections      |
        |                                                                       |
        | M. Grazzini                              (grazzini@physik.uzh.ch)     |
        | S. Kallweit                          (stefan.kallweit@cern.ch)        |
        | M. Wiesemann                        (marius.wiesemann@cern.ch)        |
        |-----------------------------------------------------------------------|
        | MATRIX is based on a number of different computations and tools      |
        | from various people and groups. Please acknowledge their efforts      |
        | by citing the list of references which is created with every run.     |
        \-----------------------------------------------------------------------/

<<MATRIX-MAKE>> This is the MATRIX process compilation.
<<MATRIX-READ>> Type process_id to be compiled and created. Type "list" to show
                available processes. Try pressing TAB for auto-completion. Type
                "exit" or "quit" to stop.
|============>> list
-------------------------------------------------------------------------------------
process_id    ||   process                          ||   description
-------------------------------------------------------------------------------------
pph21         >>   p p --> H                        >>   on-shell Higgs production
ppz01         >>   p p --> Z                        >>   on-shell Z production
ppw01         >>   p p --> W^-                      >>   on-shell W- production with CKM
ppwx01        >>   p p --> W^+                      >>   on-shell W+ production with CKM
ppeex02       >>   p p --> e^- e^+                  >>   Z production with decay
ppnenex02     >>   p p --> v_e^- v_e^+              >>   Z production with decay
ppenex02      >>   p p --> e^- v_e^+                >>   W- production with decay and CKM
ppexne02      >>   p p --> e^+ v_e^-                >>   W+ production with decay and CKM
ppaa02        >>   p p --> gamma gamma              >>   gamma gamma production
ppeexa03      >>   p p --> e^- e^+ gamma            >>   Z gamma production with decay
```

Back Up

$r_{cut}\to 0$ extrapolation in MATRIX

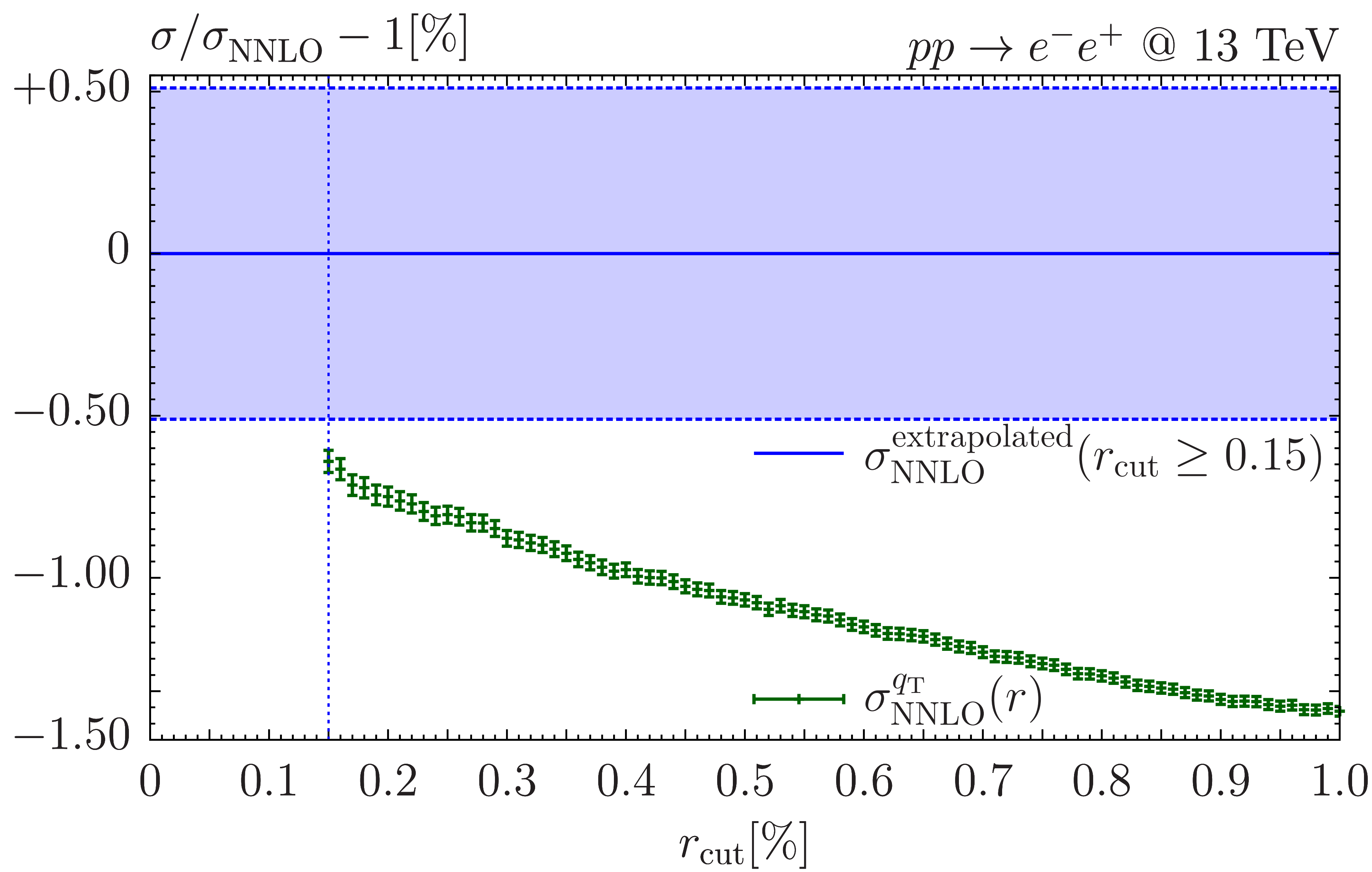
[Grazzini, Kallweit, MW '17]

dileptons with certain cuts (and photon final states) are special



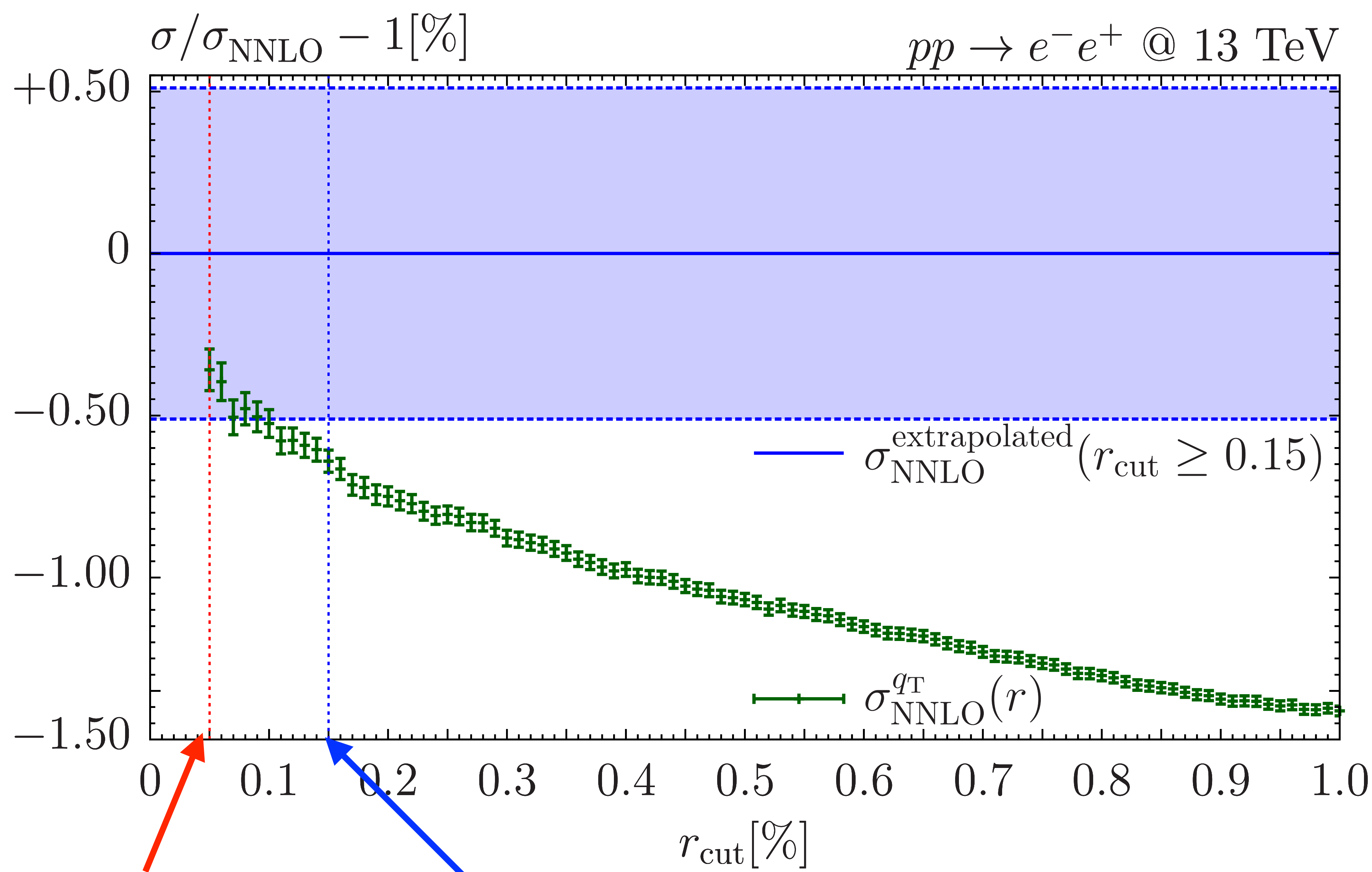
$r_{cut}\to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]



$r_{cut} \to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]

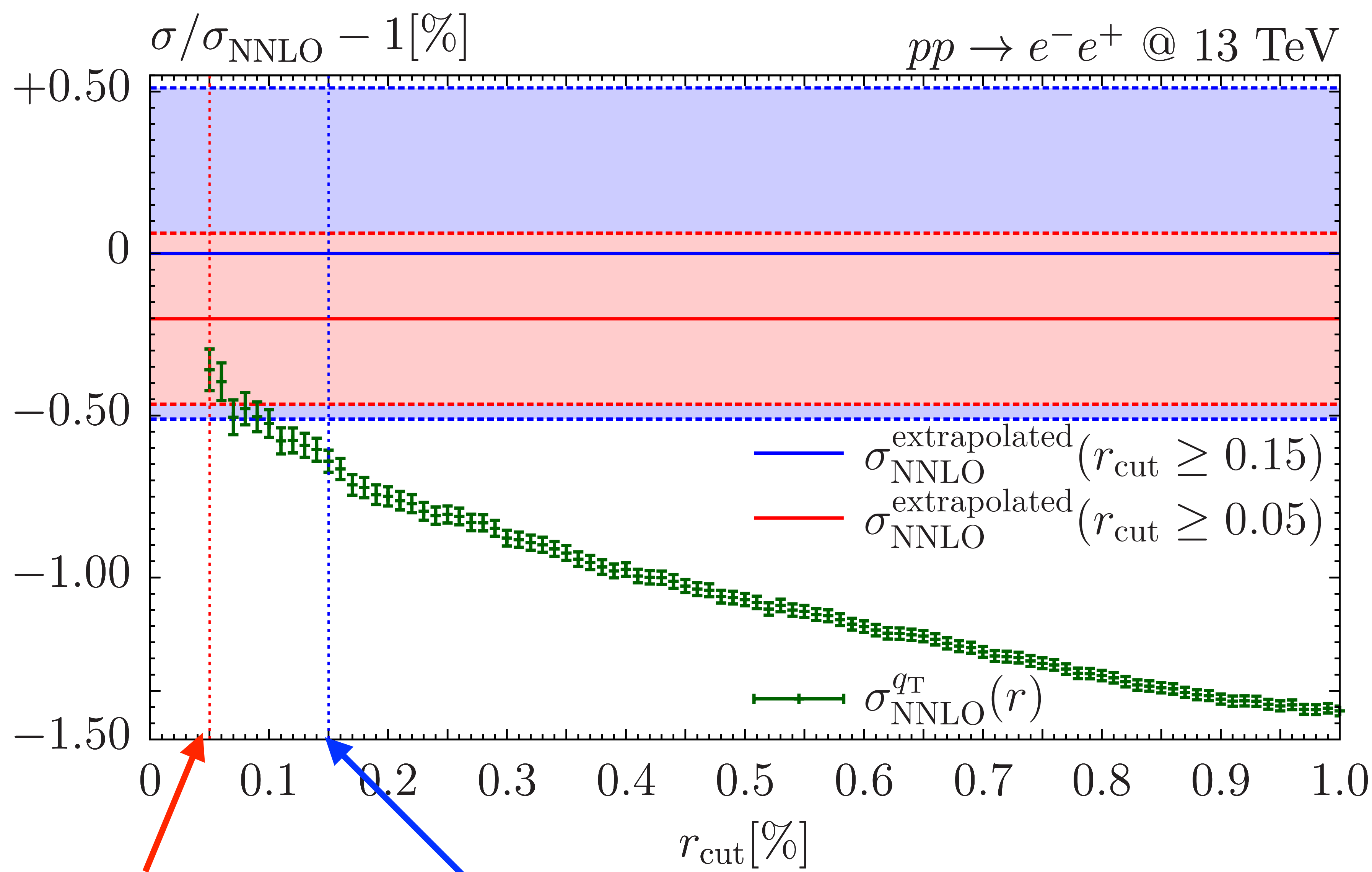


switch_qT_accuracy=1

switch_qT_accuracy=0

$r_{cut} \to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]

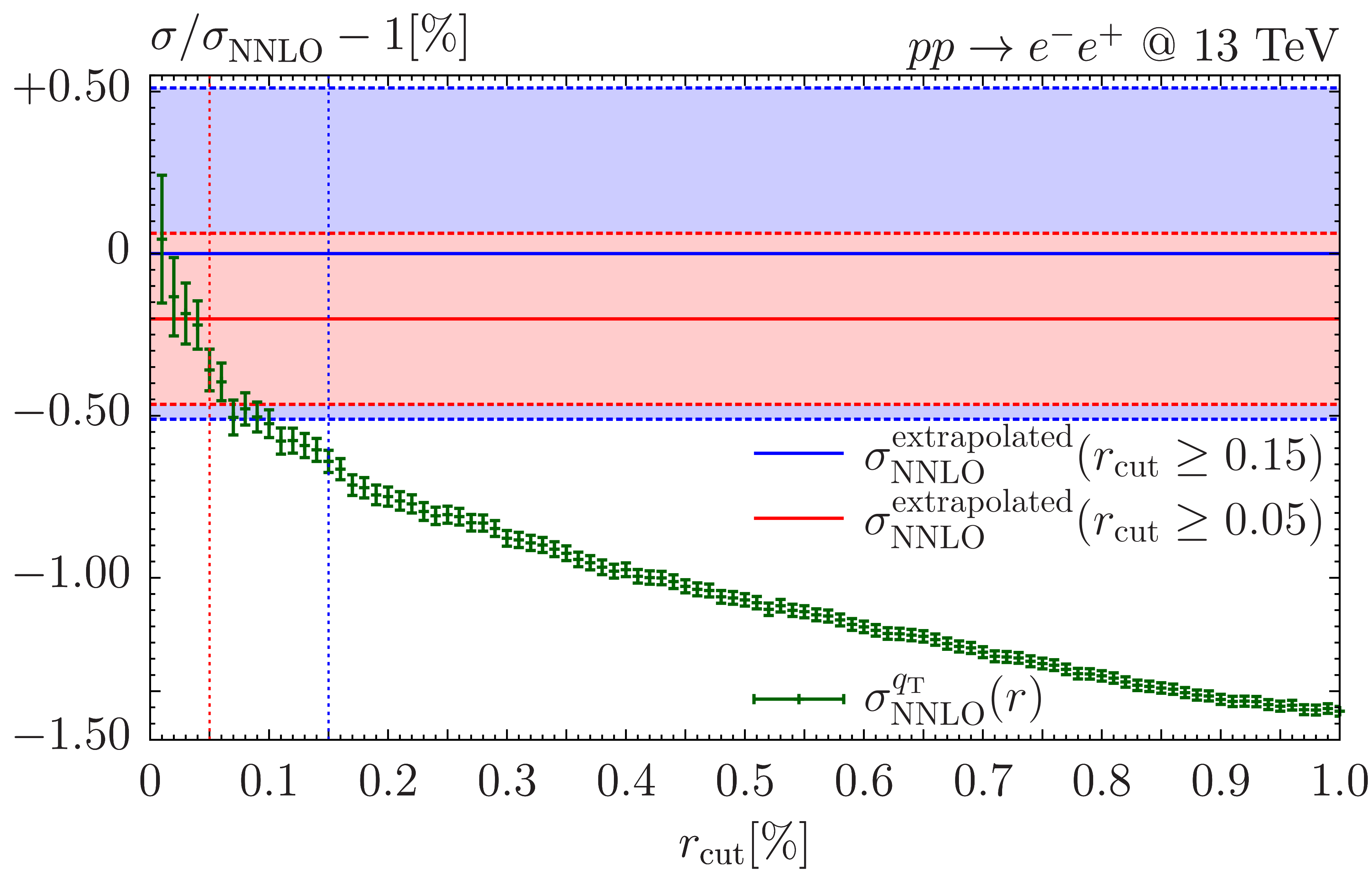


switch_qT_accuracy=1

switch_qT_accuracy=0

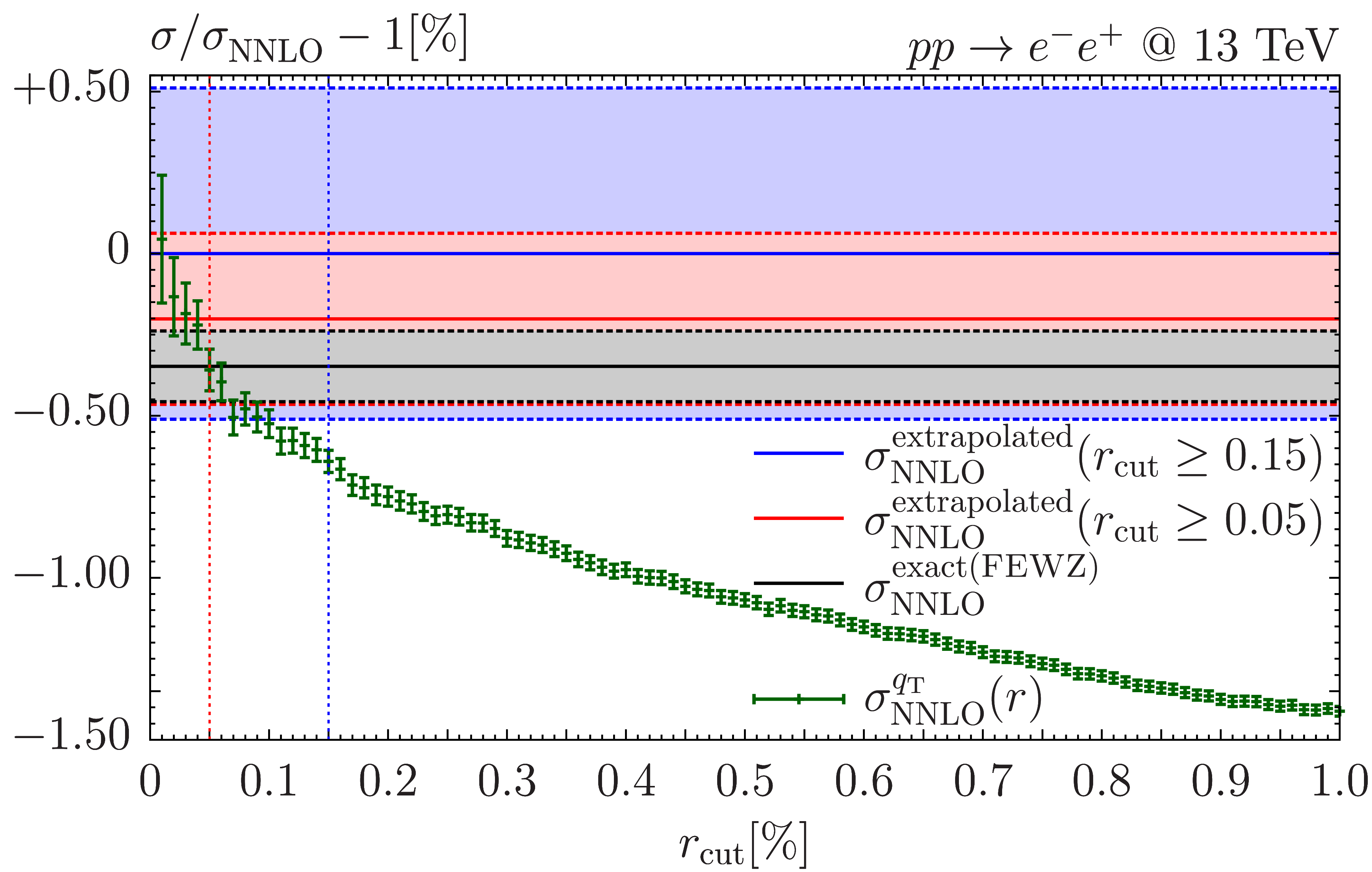
$r_{cut} \to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]



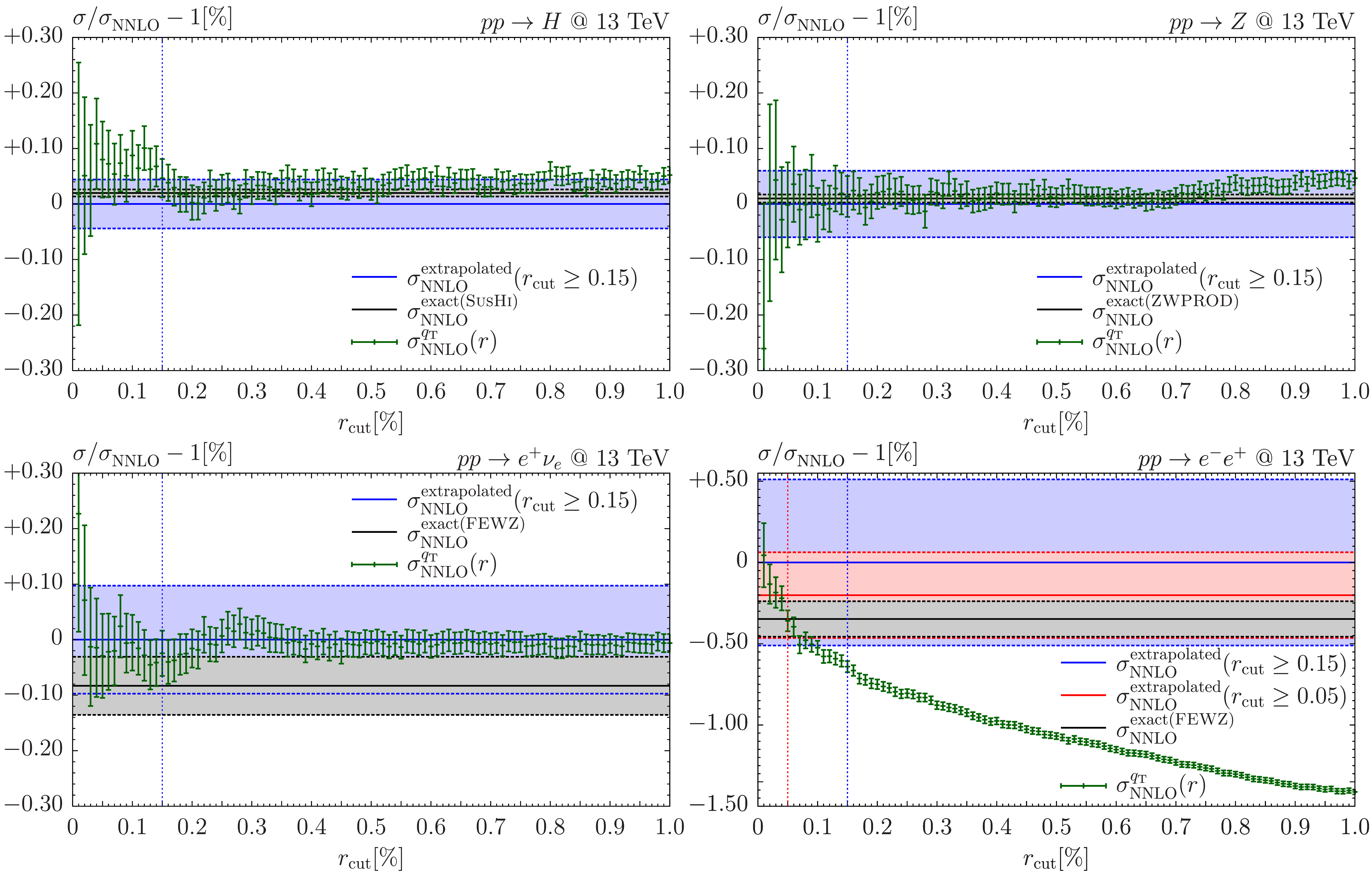
$r_{cut} \to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]



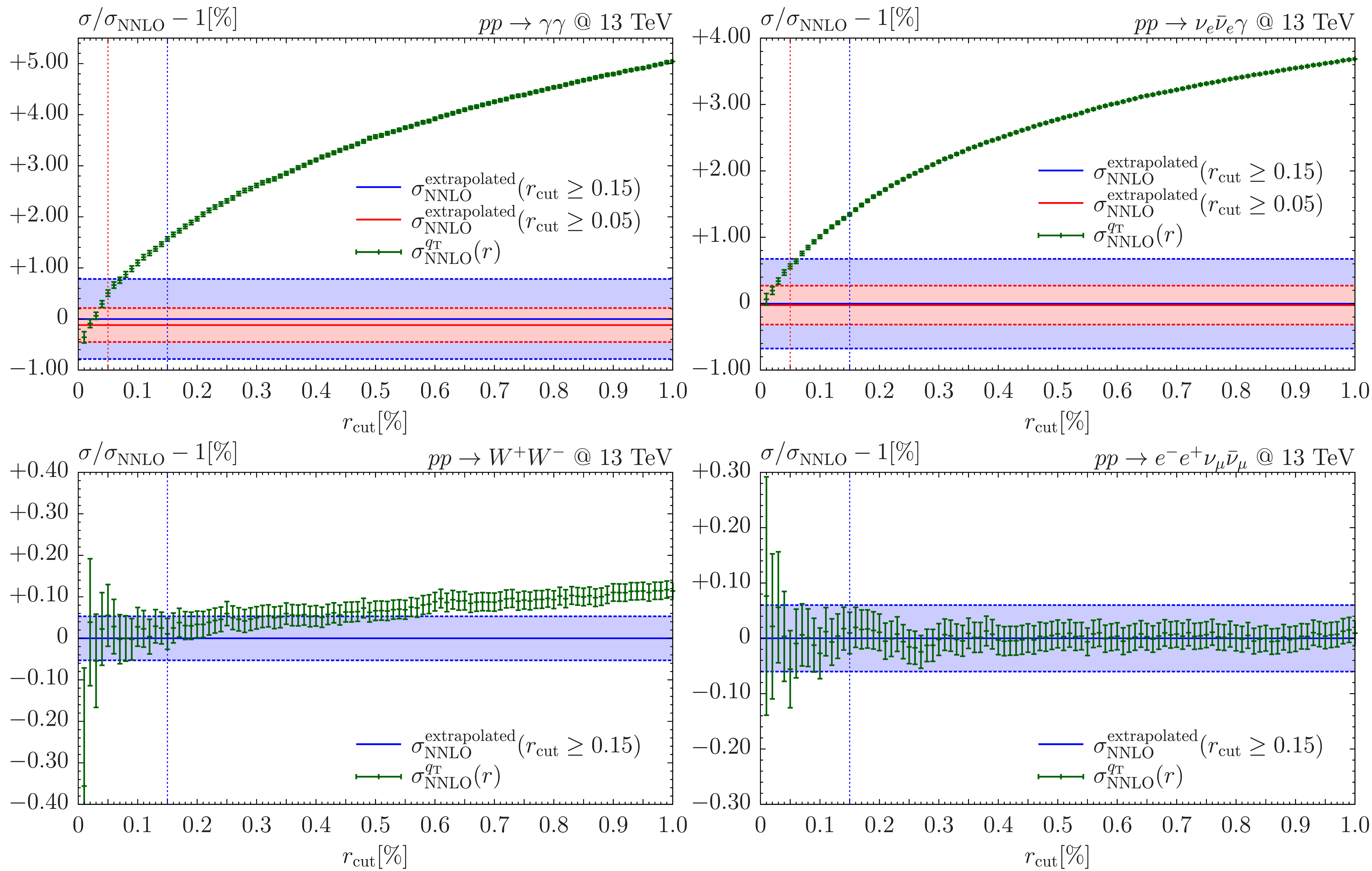
$r_{cut} \to 0$ extrapolation in MATRIX

[Grazzini, Kallweit, MW '17]



$r_{cut}\to 0$ extrapolation in MATRIX

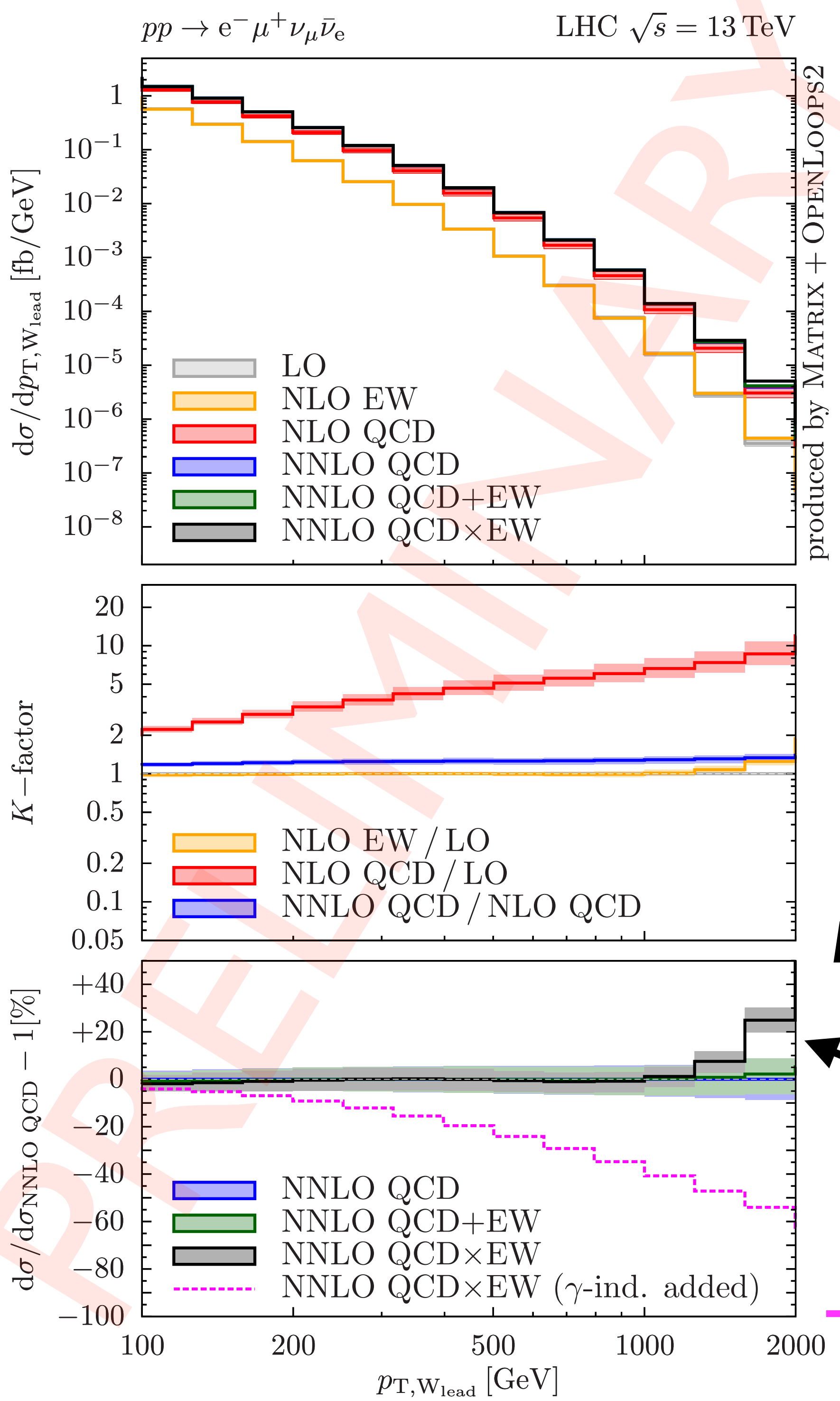
[Grazzini, Kallweit, MW '17]



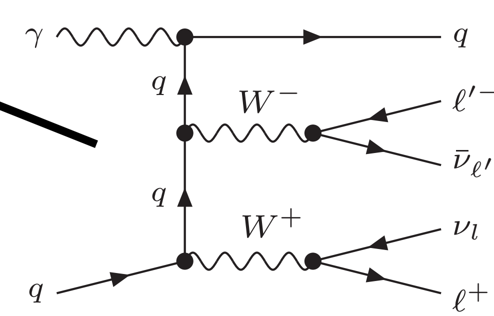
Combination: NNLO QCD and NLO EW

[Grazzini, Kallweit, Lindert, Pozzorini, MW 'to appear]

- *let's look in detail on one interesting aspect:* ***photon-induced + giant K-factor***



high p_T dominated by V+jet



~ $\ln^2(p_T/m_W)$

→ don't include γ in multiplicative combination!

Combination: NNLO QCD and NLO EW

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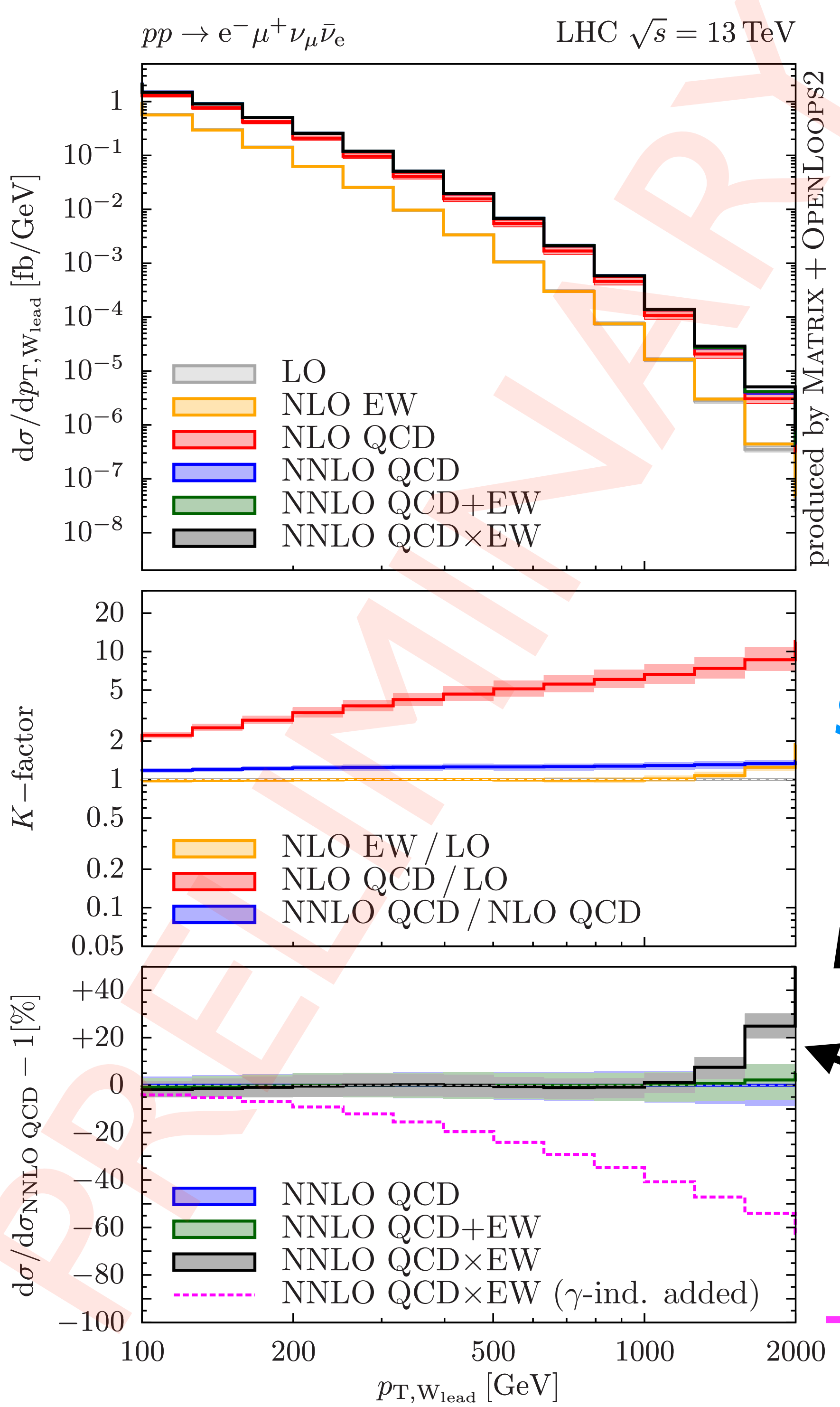
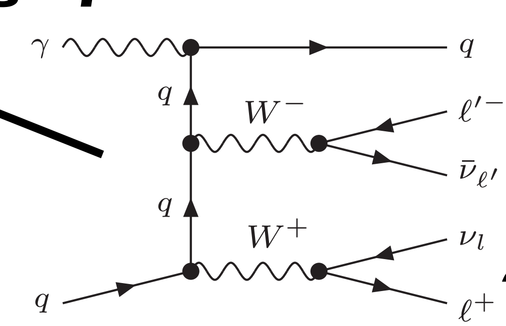
- *let's look in detail on one interesting aspect:* ***photon-induced + giant K-factor***



jet-veto ($H_{T,jet}$< 0.2 $H_{T,lep}$)

Sudakov suppression restored

high p_T dominated by V+jet



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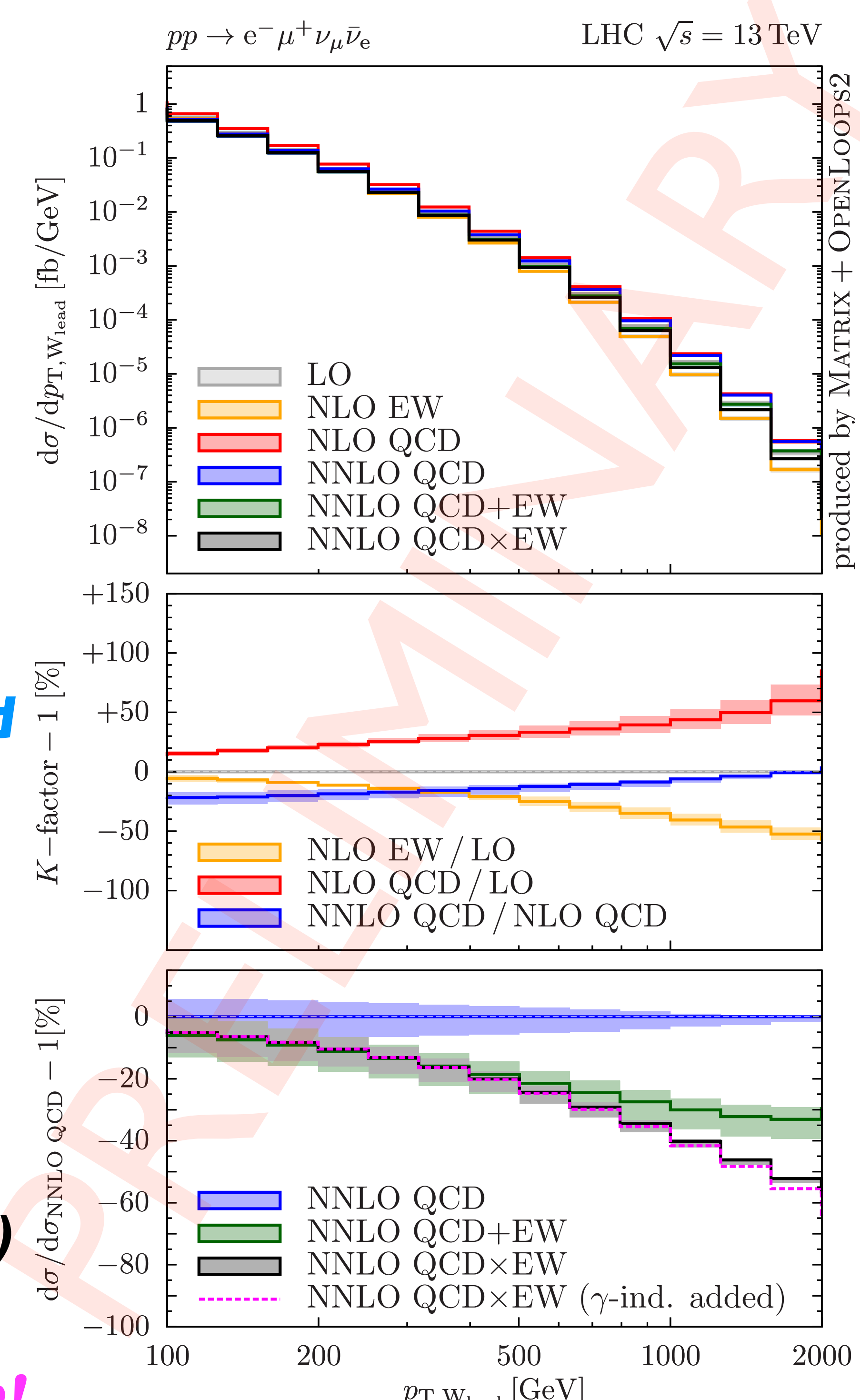


NNLOPS for WW

[Re, MW, Zanderighi '18]

Setup:

The remaining three variables and their binning chosen to be

$$\begin{aligned} p_{T,W^-} &: \quad [0., 17.5, 25., 30., 35., 40., 47.5, 57.5, 72.5, 100., 200., 350., 600., 1000., 1500., \infty]\,; \\ y_{WW} &: \quad [-\infty, -3.5, -2.5, -2.0, -1.5, -1.0, -0.5, 0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.5, \infty]\,; \\ \Delta y_{W^+W^-} &: \quad [-\infty, -5.2, -4.8, -4.4, -4.0, -3.6, -3.2, -2.8, -2.4, -2.0, -1.6, -1.2, \\ &\qquad -0.8, -0.4, 0.0, 0.4, 0.8, 1.2, 1.6, 2.0, 2.4, 2.8, 3.2, 3.6, 4.0, 4.4, 4.8, 5.2, \infty]\,. \end{aligned}$$

Cuts inspired by ATLAS 13 TeV study (1702.04519):

lepton cuts	$p_{T,\ell} > 25\,\text{GeV}, \quad \lvert\eta_\ell\rvert < 2.4, \quad m_{\ell^-\ell^+} > 10\,\text{GeV}$
lepton dressing	add photon FSR to lepton momenta with $\Delta R_{\ell\gamma} < 0.1$ *(our results do not include photon FSR, see text)*
neutrino cuts	$p_T^{\text{miss}} > 20\,\text{GeV}, \quad p_T^{\text{miss,rel}} > 15\,\text{GeV}$
jet cuts	anti-k_T jets with $R = 0.4$; $N_{\text{jet}} = 0$ for $p_{T,j} > 25\,\text{GeV}$, $\lvert\eta_j\rvert < 2.4$ and $\Delta R_{ej} < 0.3$; $N_{\text{jet}} = 0$ for $p_{T,j} > 30\,\text{GeV}$, $\lvert\eta_j\rvert < 4.5$ and $\Delta R_{ej} < 0.3$

NNLO uses the central scale

$$\mu_R = \mu_F = \mu_0 \equiv \frac{1}{2}\left(\sqrt{m^2_{e^-\bar\nu_e} + p^2_{T,e^-\bar\nu_e}} + \sqrt{m^2_{\mu^+\nu_\mu} + p^2_{T,\mu^+\nu_\mu}}\right)$$

All uncertainty bands are the envelop of 7-scales. In the NNLOPS scales in MiNLO and NNLO are varied in a correlated way

gg-channel not included in our study, as it can it is know at one-loop and can be added incoherently

NNLOPS for WW

[Re, MW, Zanderighi '18]

Validation:

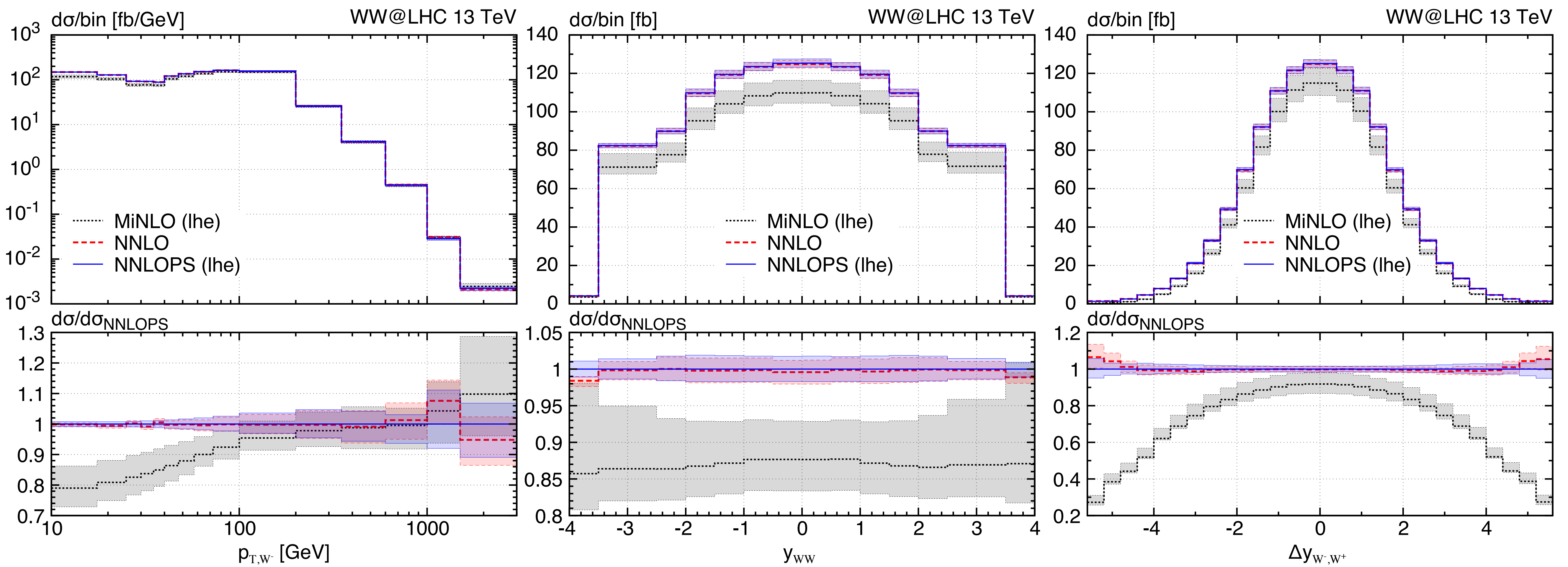
1. Total inclusive NNLO cross section reproduced by NNLOPS sample ✓
2. NNLO distributions for observables used for reweighting reproduced ✓
3. NNLO distributions for CS angles reproduced ✓
4. NNLO distributions for invariant masses of W's reproduced ✓
5. NNLO distributions for other Born-level observables reproduced ✓

NNLOPS for WW

[Re, MW, Zanderighi '18]

Validation at LHE level:

2. NNLO distributions for observables used for reweighting reproduced ✓

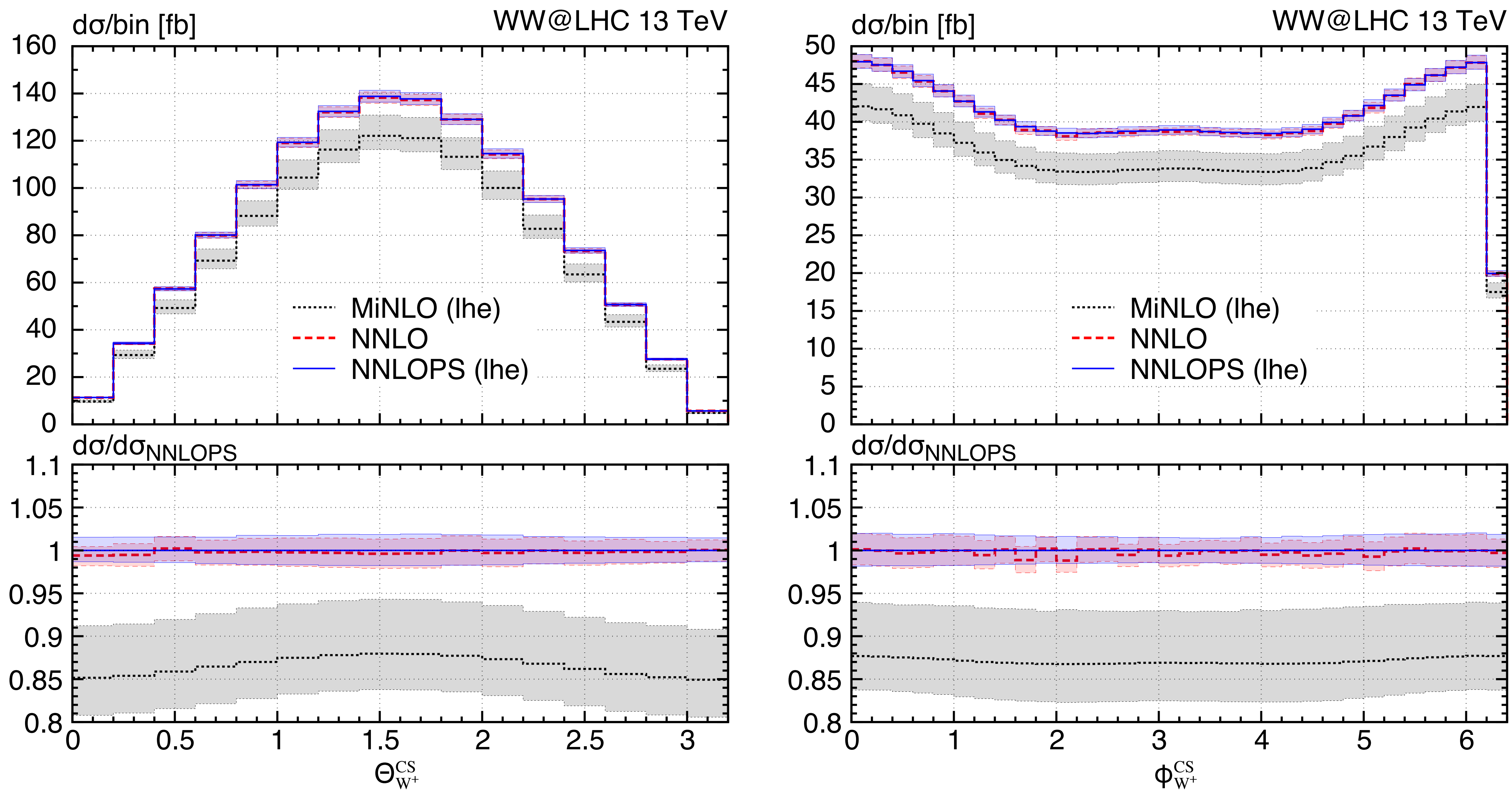


NNLOPS for WW

[Re, MW, Zanderighi '18]

Validation at LHE level:

3. NNLO distributions for CS angles reproduced ✓

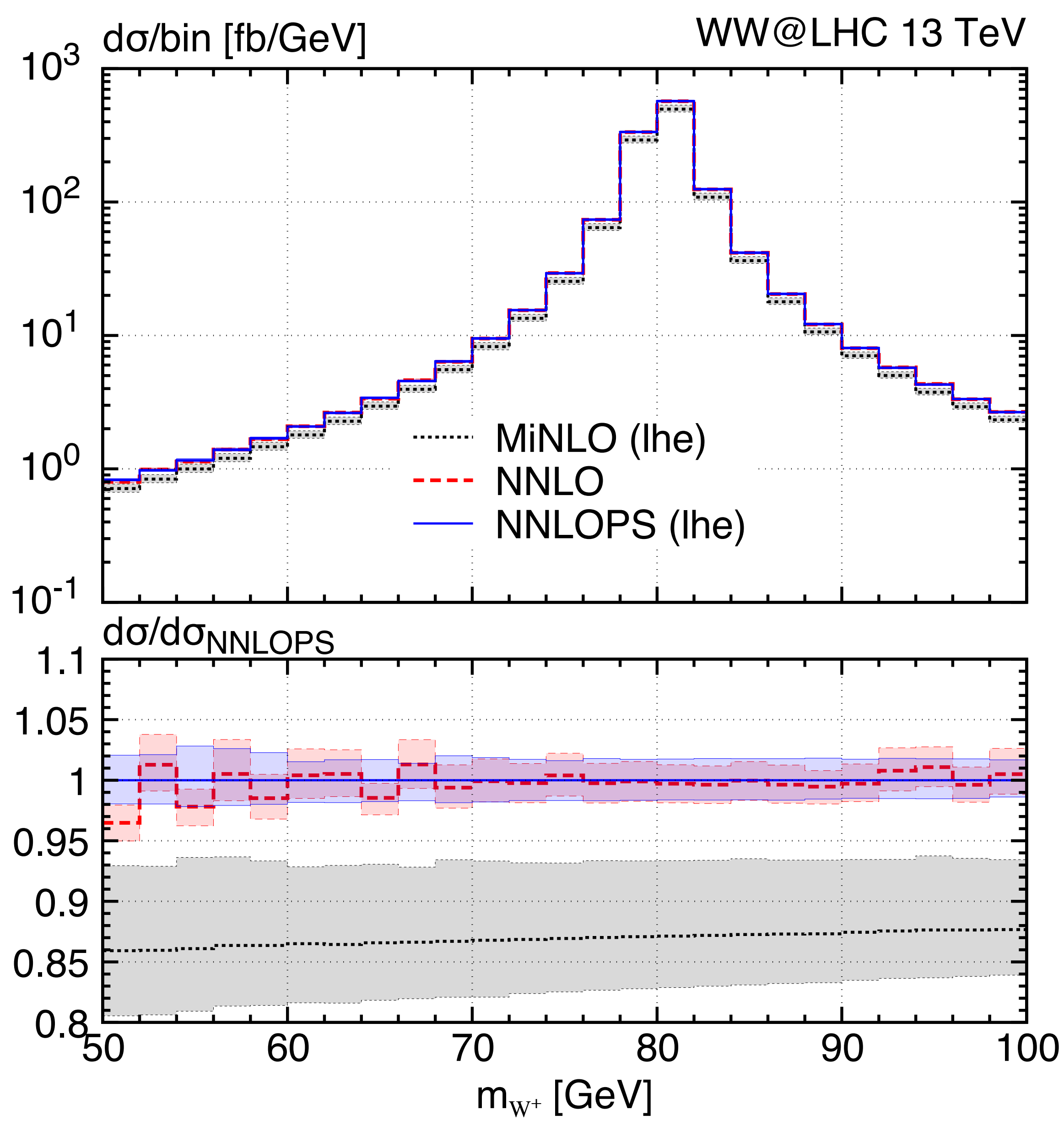


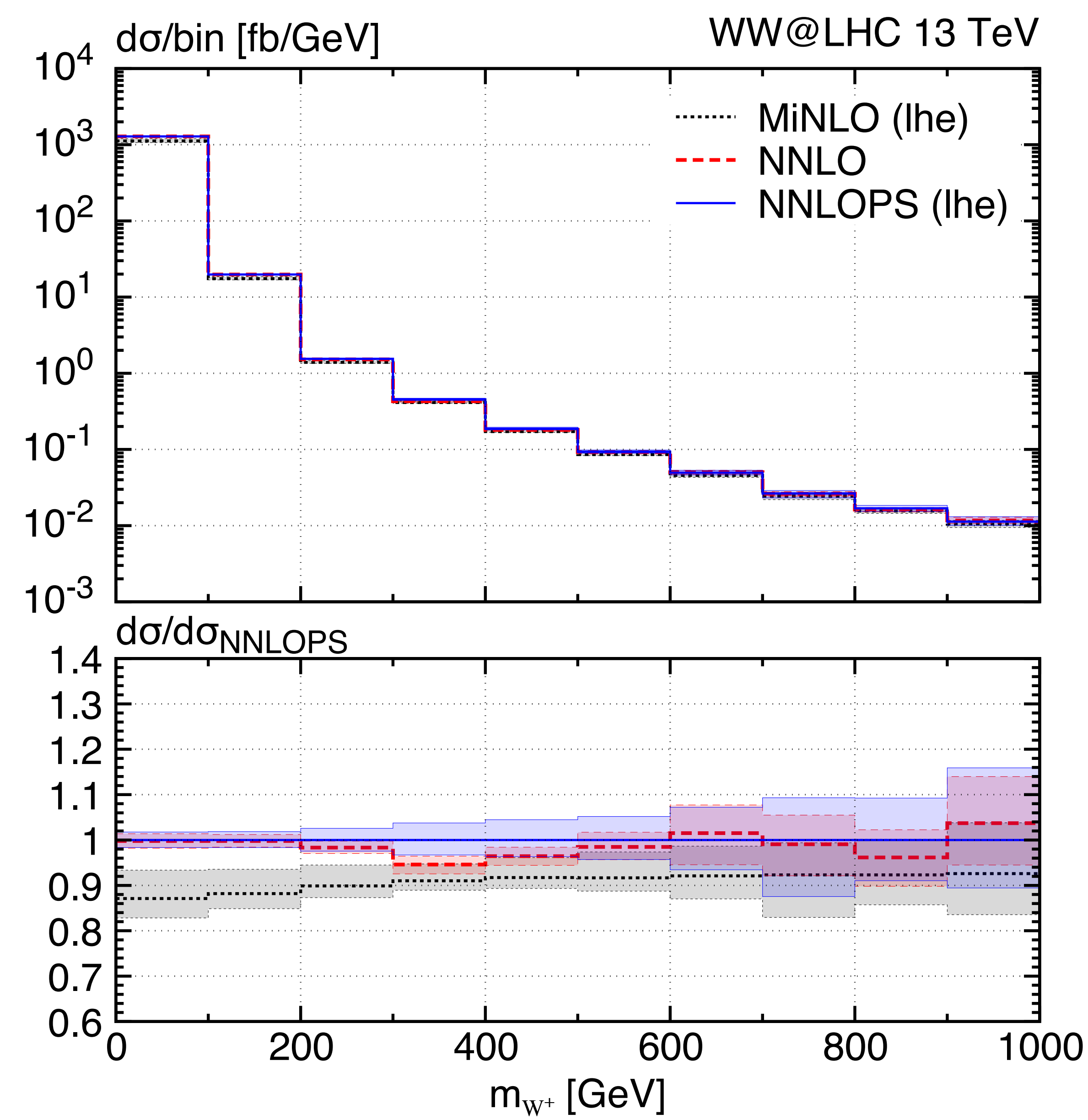
NNLOPS for WW

[Re, MW, Zanderighi '18]

Validation at LHE level:

4. NNLO distributions for invariant masses of W's reproduced ✓



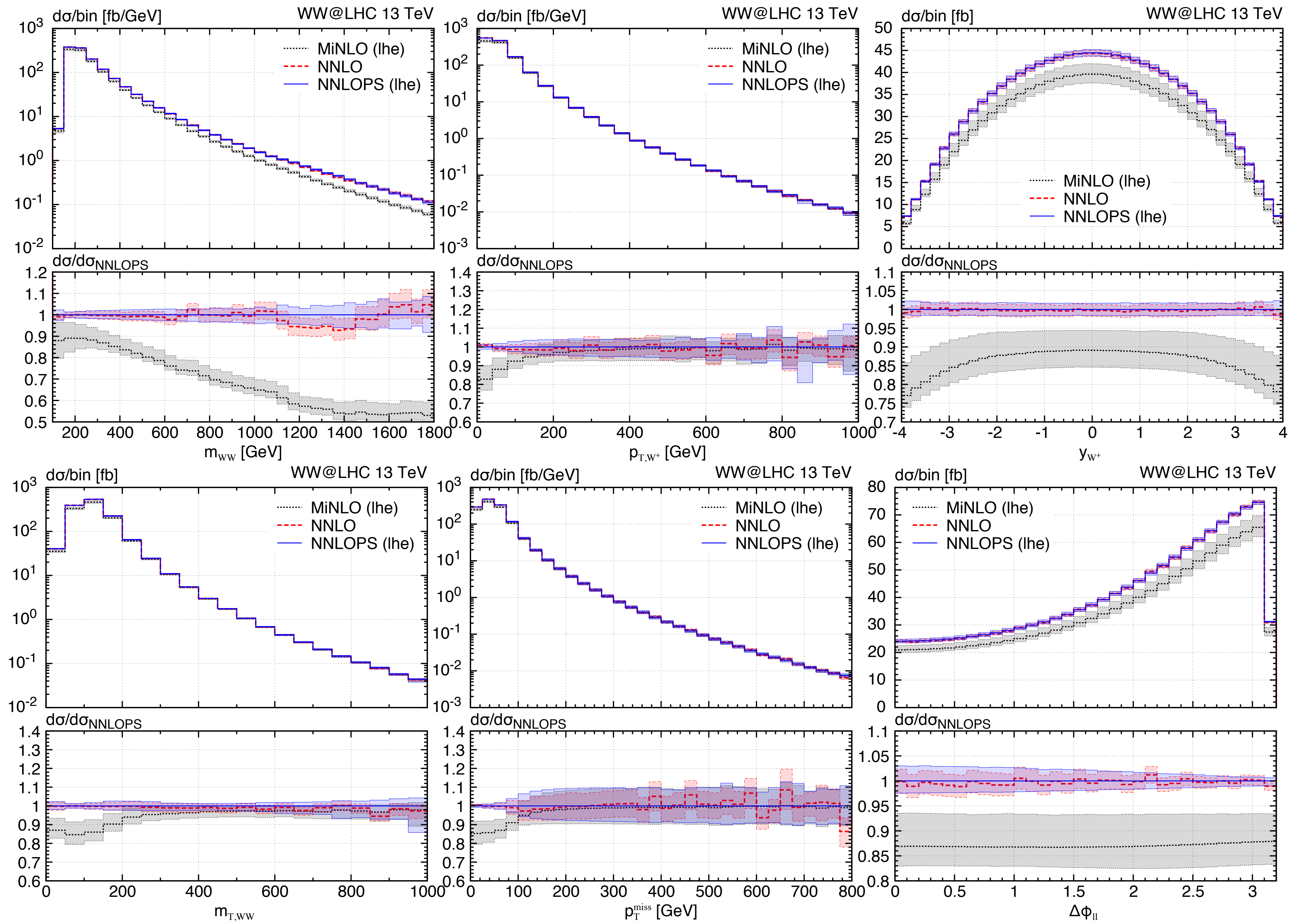


NNLOPS for WW

[Re, MW, Zanderighi '18]

Validation at LHE level:

1. NNLO distributions for other Born-level observables reproduced ✓

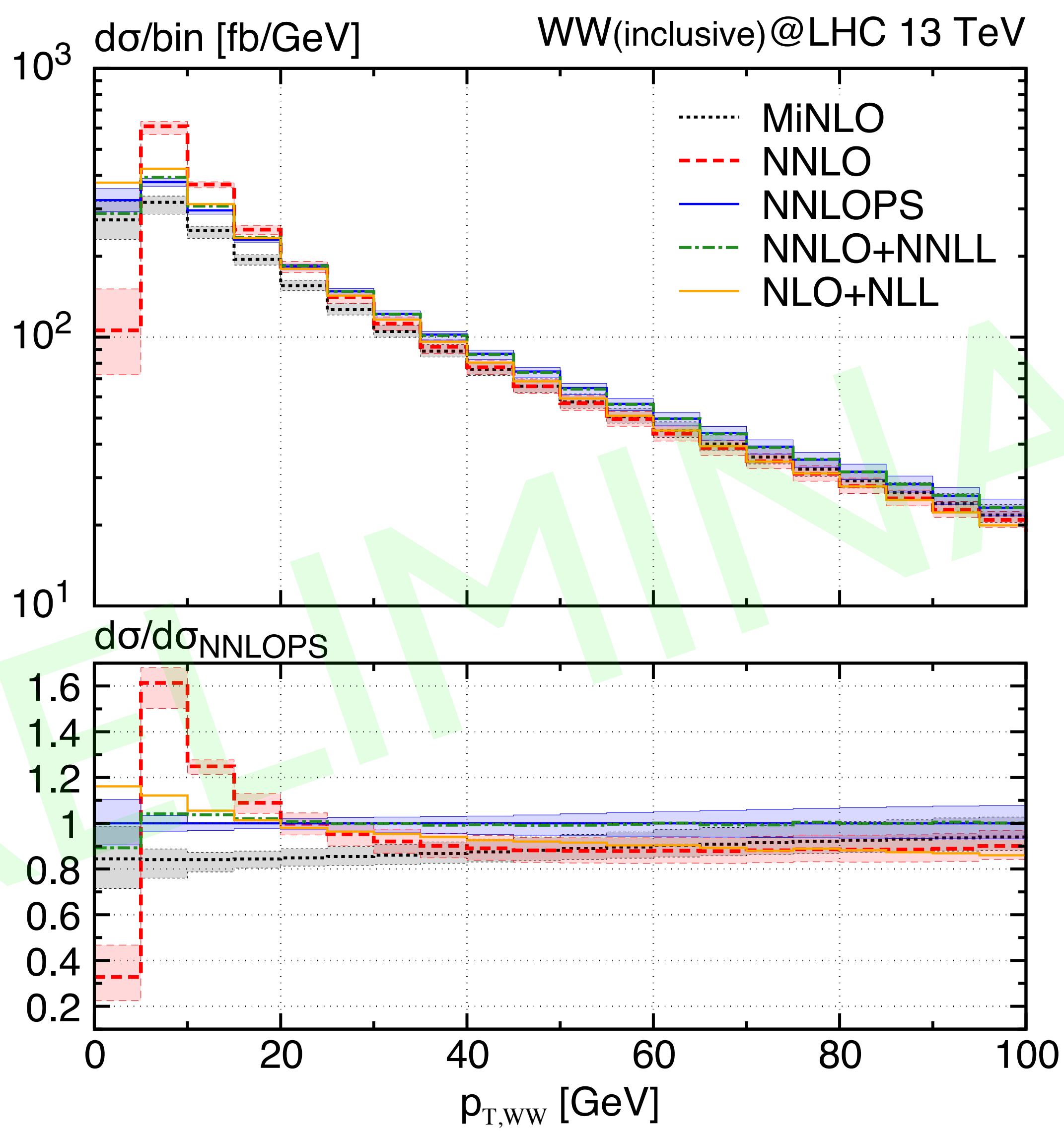


NNLOPS for WW

[Re, MW, Zanderighi '18]

Phenomenological results:

$p_{T,WW}$ (IR sensitive) compared to NNLO+NNLL



not completely fair comparison yet:
- on-shell WW for analytic resummation
- slightly different setups
→ full comparison will be done

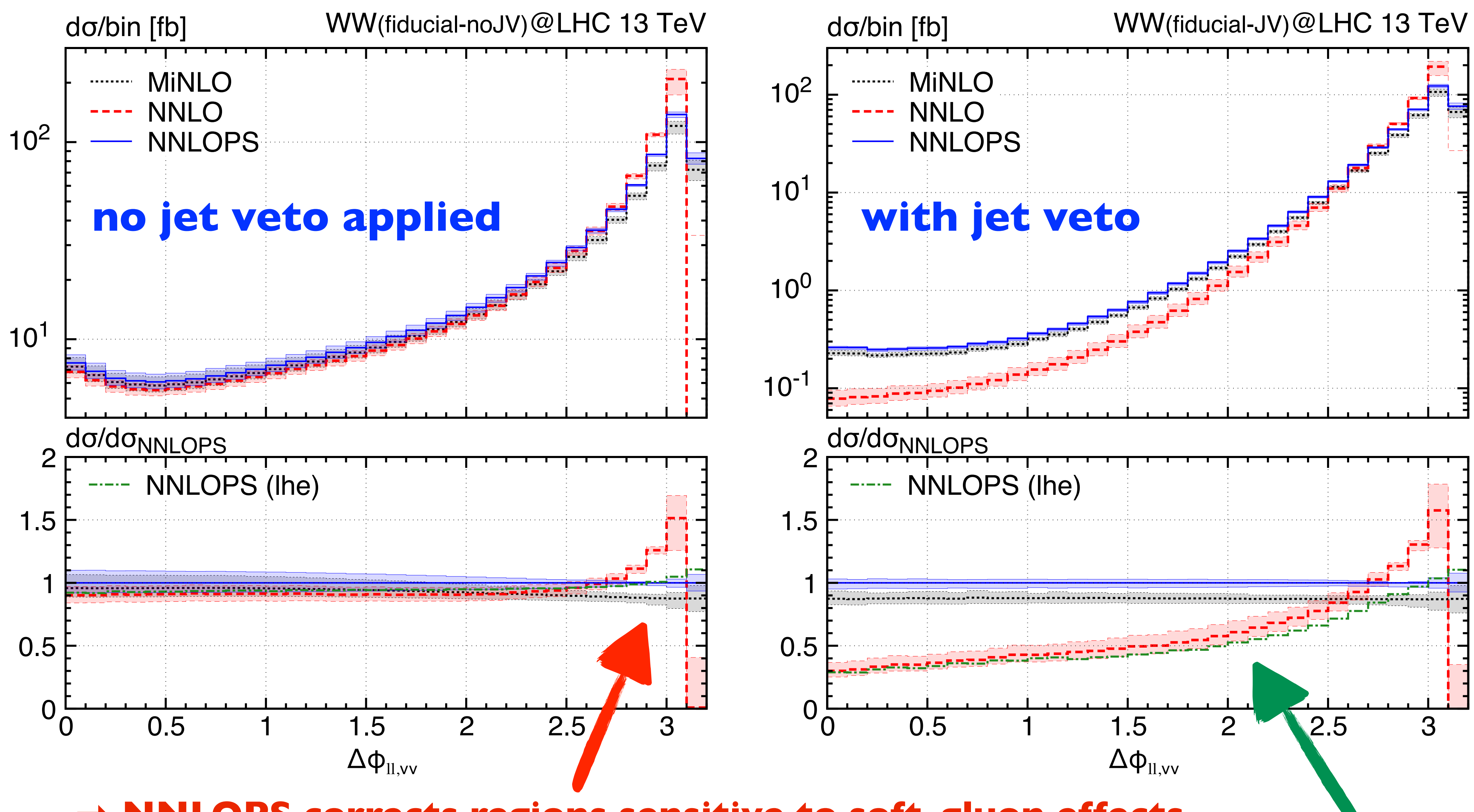
→ Resummation (analytic or shower) crucial at low p_T; NNLOPS in decent agreement with NNLL

NNLOPS for WW

[Re, MW, Zanderighi '18]

Phenomenological results:

$\Delta\Phi_{\ell\ell,\nu\nu}$ (*IR sensitive*)



→ NNLOPS corrects regions sensitive to soft-gluon effects

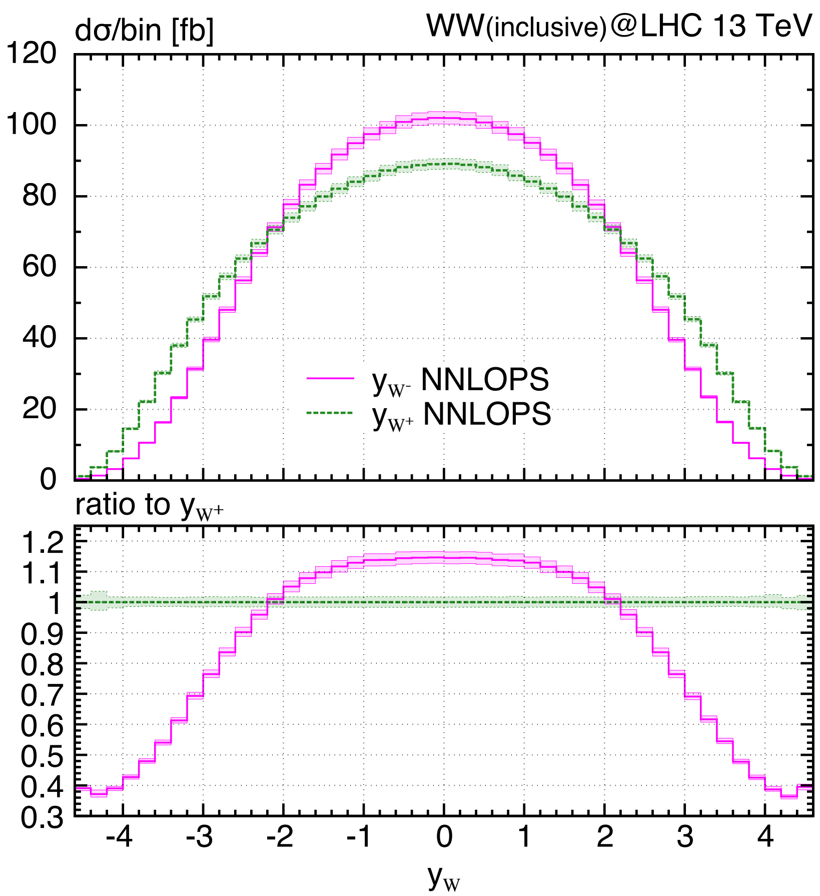
→ jet veto can turn observables sensitive soft-gluon emissions everywhere

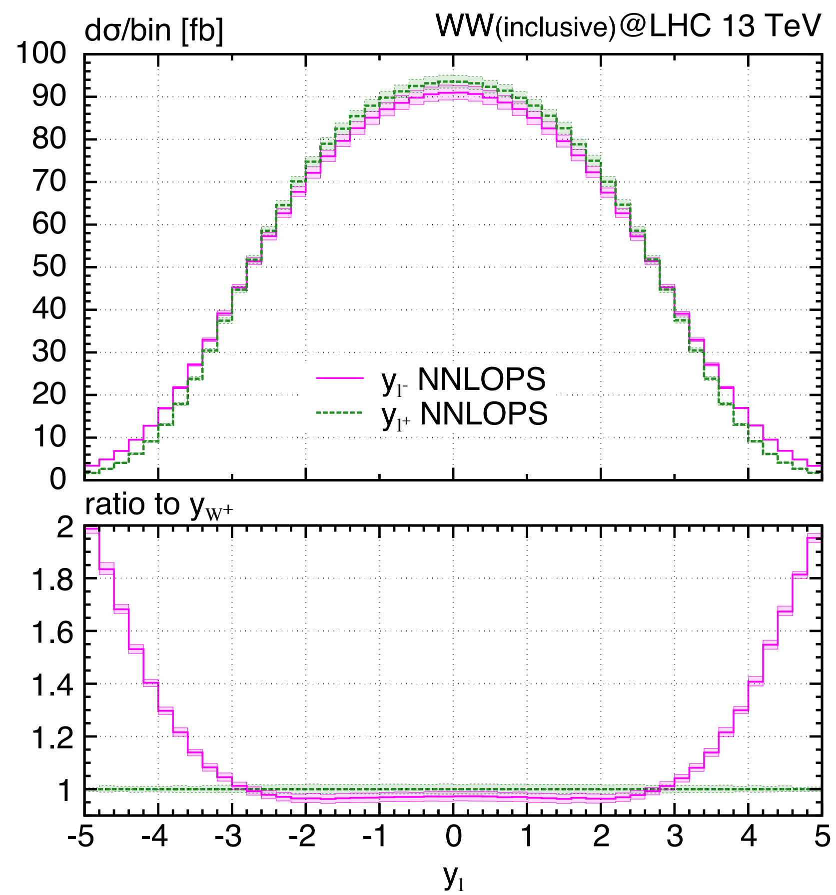
NNLOPS for WW

[Re, MW, Zanderighi '18]

Phenomenological results:

Charge asymmetry





- **W momentum cannot be reconstructed → use leptons**
- **lepton asymmetry smaller; almost vanishes in fiducial**
- **can be recovered by widening rapidity range of leptons or by considering boosted regime**
- **sensitive to W polarizations → powerful probe of new physics**

$$A_C^W = \frac{\sigma(|y_{W^+}| > |y_{W^-}|) - \sigma(|y_{W^+}| < |y_{W^-}|)}{\sigma(|y_{W^+}| > |y_{W^-}|) + \sigma(|y_{W^+}| < |y_{W^-}|)},$$

$$A_C^\ell = \frac{\sigma(|y_{\ell^+}| > |y_{\ell^-}|) - \sigma(|y_{\ell^+}| < |y_{\ell^-}|)}{\sigma(|y_{\ell^+}| > |y_{\ell^-}|) + \sigma(|y_{\ell^+}| < |y_{\ell^-}|)}.$$

NNLOPS	inclusive phase space	fiducial phase space
A_C^W	$0.1263(1)^{+2.1\%}_{-1.8\%}$	$0.0726(3)^{+2.0\%}_{-2.6\%}$
A_C^ℓ	$-[0.0270(1)^{+5.0\%}_{-6.4\%}]$	$-[0.0009(4)^{+72\%}_{-87\%}]$

MiNLO+reweighting

	X	X+jet	X+2jets	X+nj (n>2)
XJ (NLO)	—	NLO	LO	—
XJ-MiNLO	NLO	NLO	LO	PS
X@NNLO	NNLO	NLO	LO	—
X@NNLOPS	NNLO	NLO	LO	PS

1. merge *pp* → *WW* *and* *pp* → *WW+jet* *(both at NLO+PS)*

- POWHEG (F+jet): $\langle O\rangle = \int \mathrm{d}\Phi_{\mathrm{FJ}}\mathrm{d}\Phi_{\mathrm{rad}}\bar{B}(\Phi_{\mathrm{FJ}})\left[\Delta_{\mathrm{pwg}}(\Lambda)O(\Phi_{\mathrm{FJ}}) + \Delta_{\mathrm{pwg}}(p_{\mathrm{T,rad}})\frac{R(\Phi_{\mathrm{FJ}},\Phi_{\mathrm{rad}})}{B(\Phi_{\mathrm{FJ}})}O(\Phi_{\mathrm{FJJ}})\right]$
- NLO+PS (F+jet): $\bar{B}(\Phi_{\mathrm{FJ}}) = [B(\Phi_{\mathrm{FJ}}) + V(\Phi_{\mathrm{FJ}})] + \int \mathrm{d}\Phi_{\mathrm{rad}}R(\Phi_{\mathrm{FJ}},\Phi_{\mathrm{rad}})$

MiNLO+reweighting

	X	X+jet	X+2jets	X+nj (n>2)
XJ (NLO)	—	NLO	LO	—
XJ-MiNLO	NLO	NLO	LO	PS
X@NNLO	NNLO	NLO	LO	—
X@NNLOPS	NNLO	NLO	LO	PS

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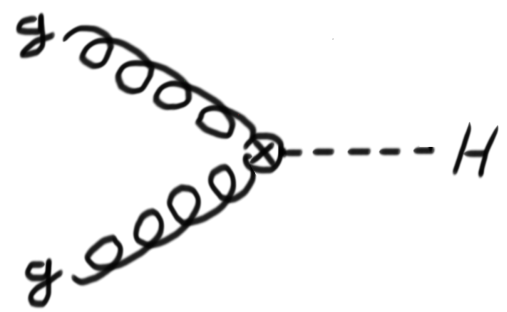
- NLO+PS (F+jet): $\bar{B}(\Phi_{\mathrm{FJ}}) = [B(\Phi_{\mathrm{FJ}}) + V(\Phi_{\mathrm{FJ}})] + \int \mathrm{d}\Phi_{\mathrm{rad}} R(\Phi_{\mathrm{FJ}}, \Phi_{\mathrm{rad}})$
- MiNLO+PS: $\bar{B}(\Phi_{\mathrm{FJ}}) = e^{-\tilde{S}(p_{\mathrm{T}})} [B(\Phi_{\mathrm{FJ}})(1 + [\tilde{S}(p_{\mathrm{T}})]^{(1)}) + V(\Phi_{\mathrm{FJ}})] + \int \mathrm{d}\Phi_{\mathrm{rad}} R(\Phi_{\mathrm{FJ}}, \Phi_{\mathrm{rad}}) e^{-\tilde{S}(p_{\mathrm{T}})}$

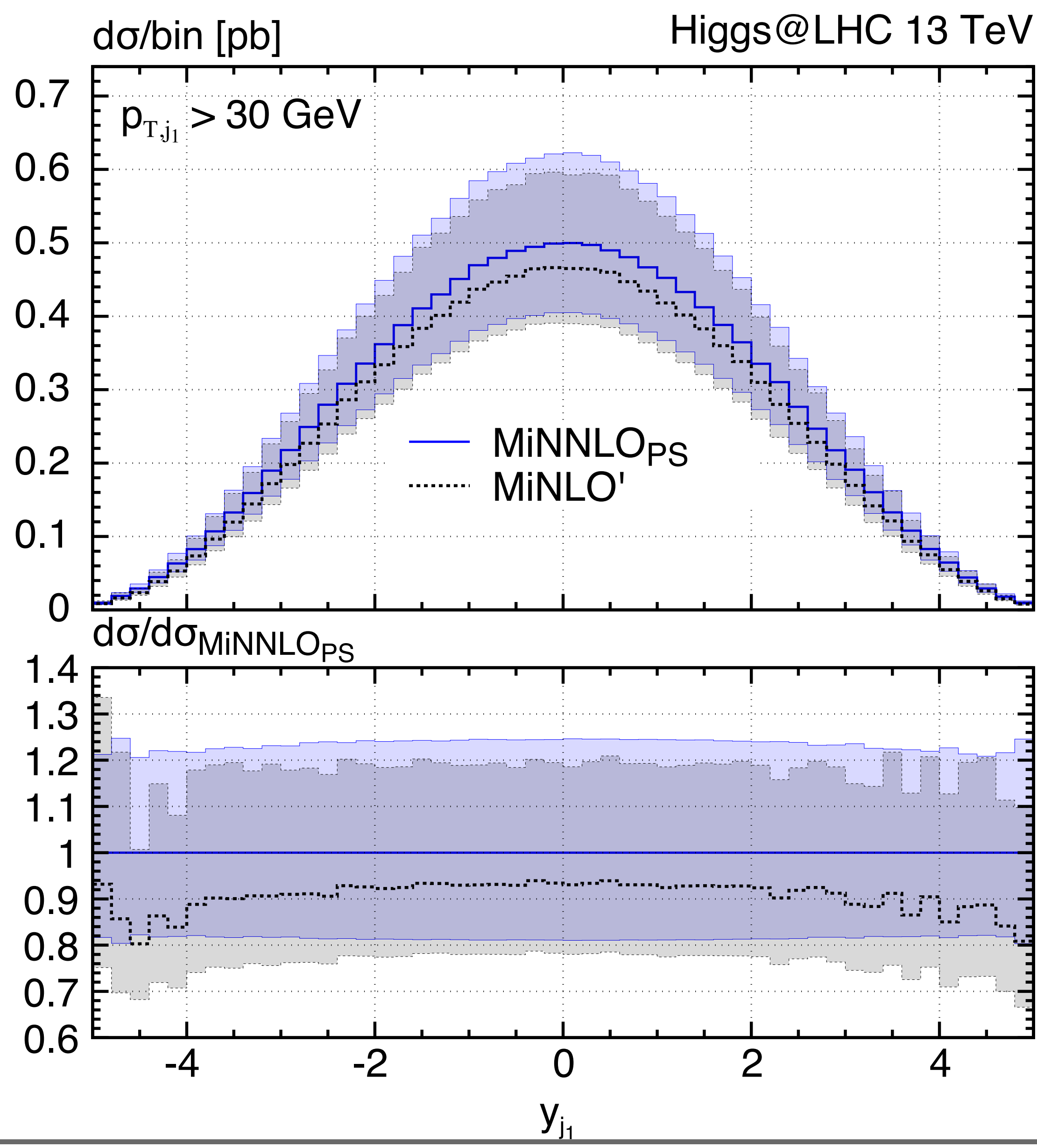
$$S(p_{\mathrm{T}}) = 2 \int_{p_{\mathrm{T}}}^{Q} \frac{\mathrm{d}q}{q} \left(A(\alpha_s(q)) \ln \frac{Q^2}{q^2} + B(\alpha_s(q)) \right),$$

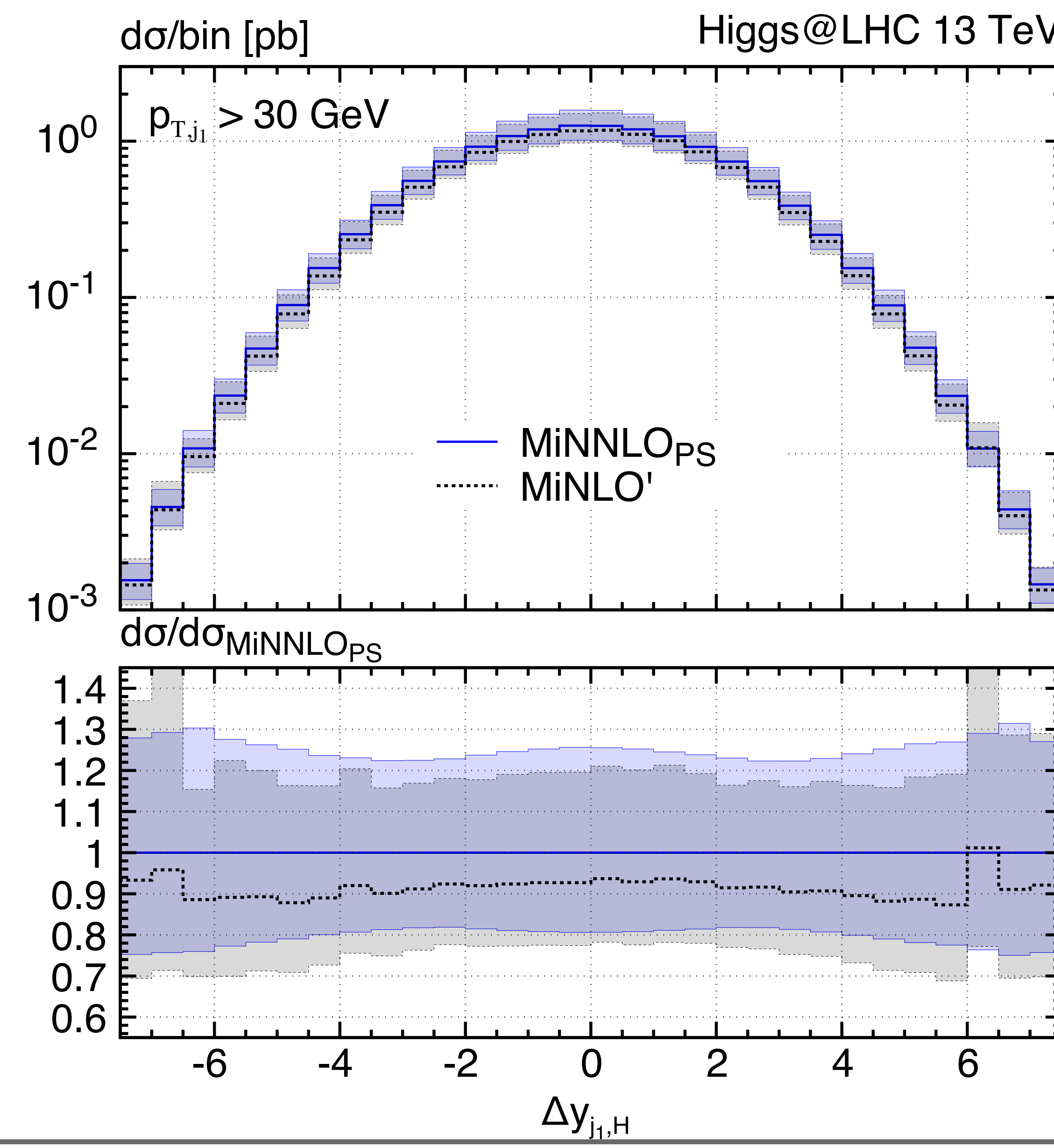
$$A(\alpha_s) = \sum_{k=1}^{2} \left(\frac{\alpha_s}{2\pi} \right)^k A^{(k)}, \quad B(\alpha_s) = \sum_{k=1}^{2} \left(\frac{\alpha_s}{2\pi} \right)^k B^{(k)},$$

MiNNLO$_{PS}$ results

[Monni, Nason, Re, MW, Zanderighi '19]







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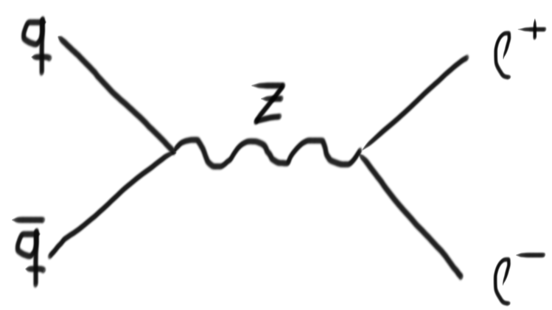

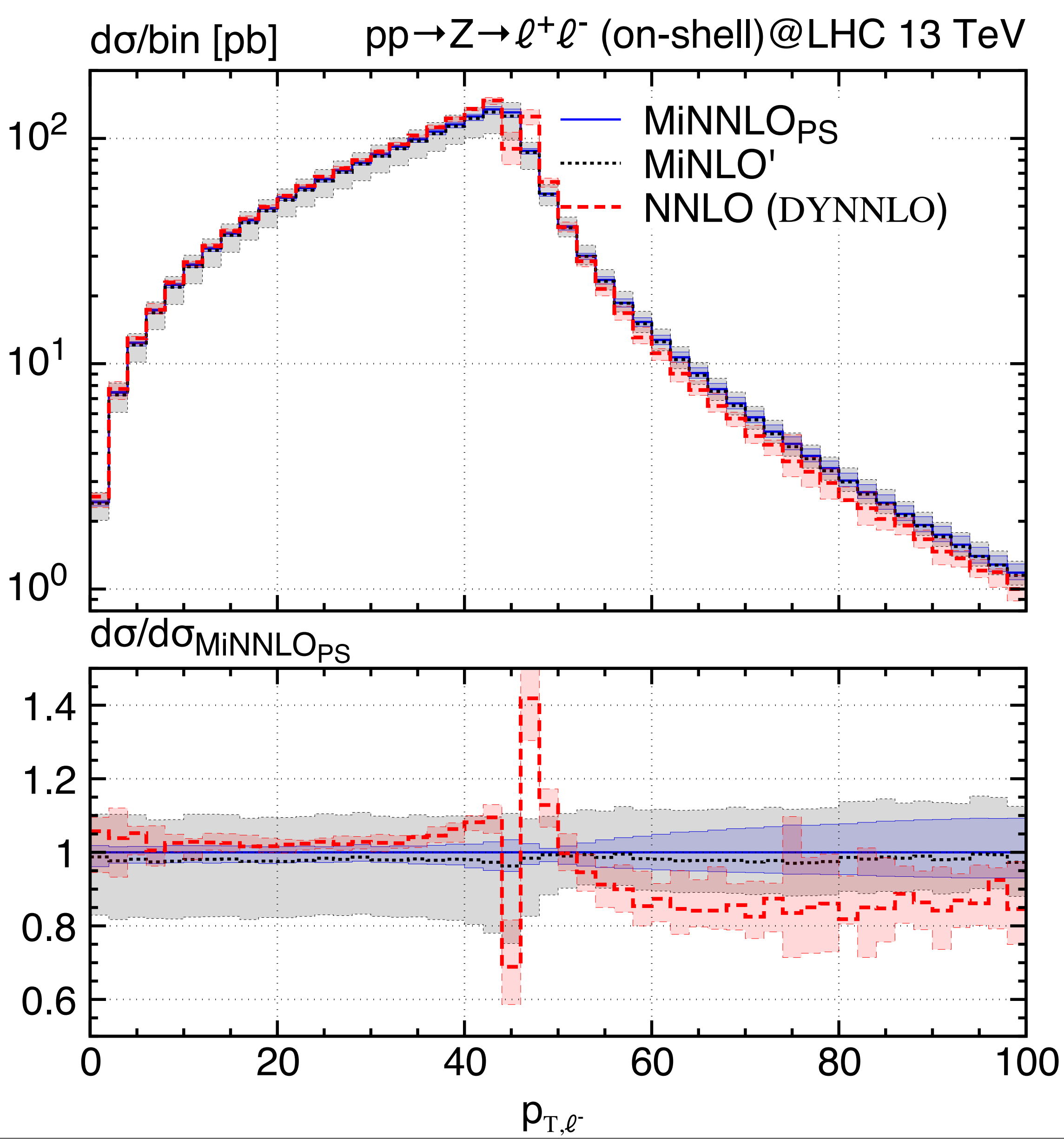

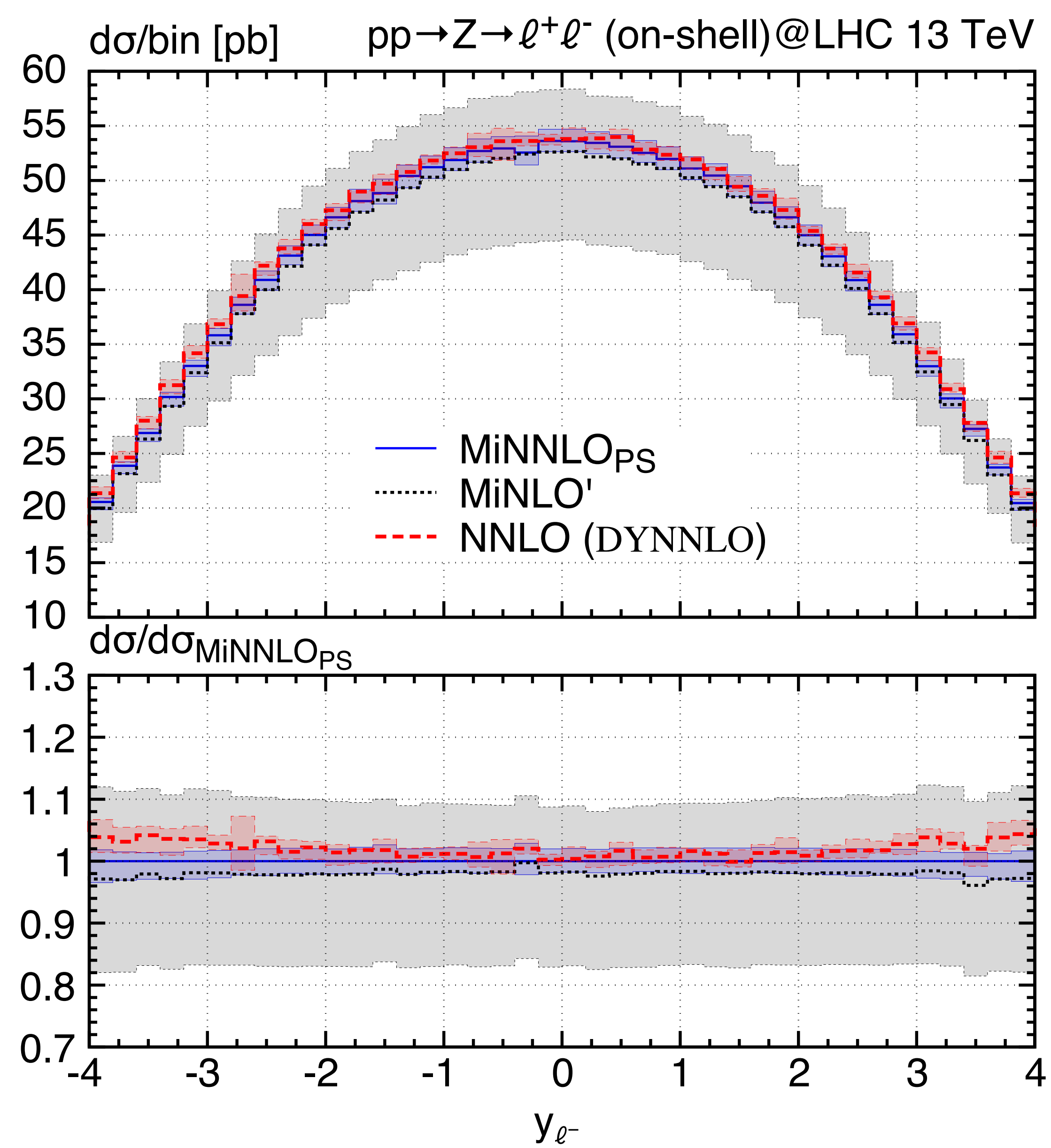

MiNNLO$_{PS}$ results

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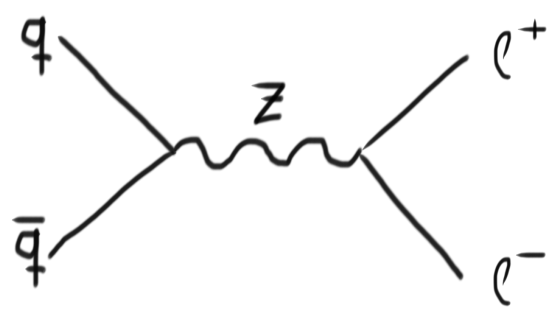

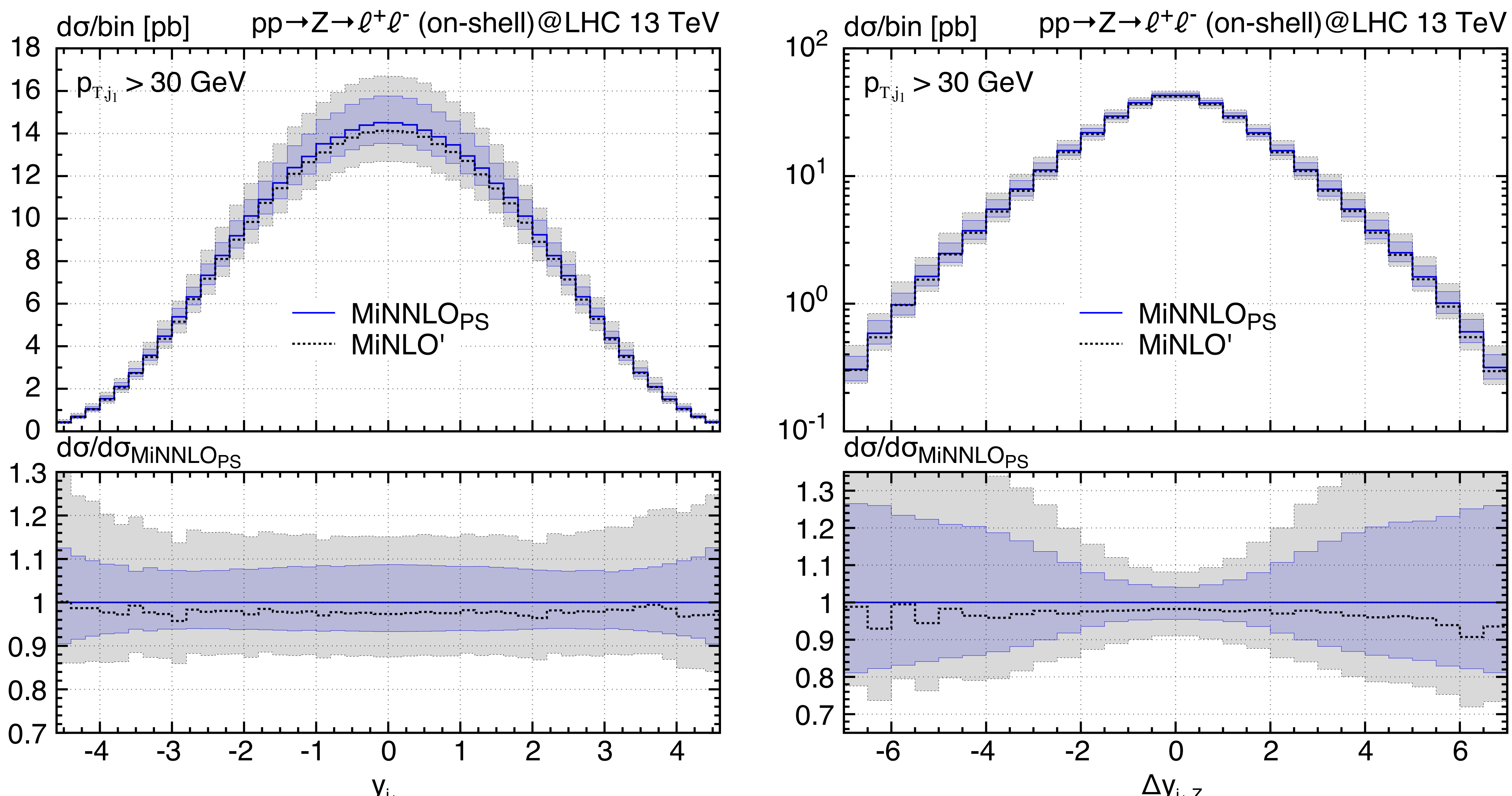

MiNNLO$_{\text{PS}}$ practical implementation

[Monni, Nason, Re, MW, Zanderighi '19]

MiNNLO$_{\text{PS}}$ master formula

$$\frac{d\sigma}{d\Phi_{\rm FJ}} = \exp[-\tilde{S}(p_{\rm T})]\left\{\frac{\alpha_s(p_{\rm T})}{2\pi}\left[\frac{d\sigma_{\rm FJ}}{d\Phi_{\rm FJ}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\rm T})}{2\pi}[\tilde{S}(p_{\rm T})]^{(1)}\right)+\left(\frac{\alpha_s(p_{\rm T})}{2\pi}\right)^2\left[\frac{d\sigma_{\rm FJ}}{d\Phi_{\rm FJ}}\right]^{(2)}+\left(\frac{\alpha_s(p_{\rm T})}{2\pi}\right)^3[D(p_{\rm T})]^{(3)}\underline{F^{\rm corr}(\Phi_{\rm FJ})}\right\}$$
$$\times\left\{\Delta_{\rm pwg}(\Lambda)+\int d\Phi_{\rm rad}\Delta_{\rm pwg}(p_{\rm T,rad})\frac{R(\Phi_{\rm FJ},\Phi_{\rm rad})}{B(\Phi_{\rm FJ})}\right\}+\mathcal{O}(\alpha_s^3)$$

$$\underline{F_\ell^{\rm corr}(\Phi_{\rm FJ})} = \frac{J_\ell(\Phi_{\rm FJ})}{\sum_{l'}\int d\Phi'_{\rm FJ}J_{l'}(\Phi'_{\rm FJ})\delta(p_{\rm T}-p_{\rm T}')\delta(\Phi_{\rm F}-\Phi'_{\rm F})},$$

$$\sum_\ell\int d\Phi'_{\rm FJ}G(\Phi'_{\rm F},p_{\rm T}')F_\ell^{\rm corr}(\Phi'_{\rm FJ}) = \int d\Phi_{\rm F}\,dp_{\rm T}G(\Phi_{\rm F},p_{\rm T})\times\sum_\ell\int d\Phi'_{\rm FJ}\delta(\Phi_{\rm F}-\Phi'_{\rm F})\delta(p_{\rm T}-p_{\rm T}')F_\ell^{\rm corr}(\Phi'_{\rm FJ}) = \int d\Phi_{\rm F}\,dp_{\rm T}G(\Phi_{\rm F},p_{\rm T})$$

$$J_\ell(\Phi_{\rm FJ}) = |M_\ell^{\rm FJ}(\Phi_{\rm FJ})|^2(f^{[a]}f^{[b]})_\ell \longrightarrow |M_\ell^{\rm FJ}(\Phi_{\rm FJ})|^2 \simeq |M^{\rm F}(\Phi_{\rm F})|^2P_\ell(\Phi_{\rm rad}) \longrightarrow J_\ell(\Phi_{\rm FJ}) = P_\ell(\Phi_{\rm rad})(f^{[a]}f^{[b]})_\ell$$

MiNNLO$_{PS}$ scale variation

[Monni, Nason, Re, MW, Zanderighi '19]

$$\frac{d\sigma^{\mathrm{sing}}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}} = \frac{d}{dp_{\mathrm{T}}}\left\{ \exp[-\tilde{S}(p_{\mathrm{T}})]\mathcal{L}(p_{\mathrm{T}}) \right\}$$

- require scale invariance ($\mu_R = K_R\, p_T\,, \mu_F = K_F\, p_T$) separately for ingredients

MiNNLOPS scale variation

[Monni, Nason, Re, MW, Zanderighi '19]

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}\Phi_{\mathrm{F}}\mathrm{d}p_{\mathrm{T}}} = \frac{\mathrm{d}}{\mathrm{d}p_{\mathrm{T}}}\left\{ \exp[-\tilde{S}(p_{\mathrm{T}})]\mathcal{L}(p_{\mathrm{T}}) \right\}$$

- require scale invariance ($\mu_R = K_R\, p_T$, $\mu_F = K_F\, p_T$) separately for ingredients
  - sudakov $\tilde{S}(p_{\mathrm{T}}) = 2\int_{p_{\mathrm{T}}}^{Q} \frac{\mathrm{d}q}{q}\left(A(\alpha_s(q))\ln\frac{Q^2}{q^2} + \tilde{B}(\alpha_s(q))\right)$
  - luminosity factor $\mathcal{L}(k_{\mathrm{T},1}) = \sum_{c,c'} \frac{\mathrm{d}|M^{\mathrm{F}}|^2_{cc'}}{\mathrm{d}\Phi_{\mathrm{F}}} \sum_{i,j} \left\{ \left(\tilde{C}^{[a]}_{ci} \otimes f^{[a]}_i\right) \tilde{H}(k_{\mathrm{T},1}) \left(\tilde{C}^{[b]}_{c'j} \otimes f^{[b]}_j\right) \right\}$

MiNNLOPS scale variation

[Monni, Nason, Re, MW, Zanderighi '19]

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$$A^{(2)}(K_{\mathrm{R}}) = A^{(2)} + (2\pi\beta_0)A^{(1)}\ln {K_{\mathrm{R}}}^2,$$
$$\tilde{B}^{(2)}(K_{\mathrm{R}}) = \tilde{B}^{(2)} + (2\pi\beta_0)B^{(1)}\ln {K_{\mathrm{R}}}^2 + (2\pi\beta_0)^2\, n_B \ln {K_{\mathrm{R}}}^2$$

  - luminosity factor $\mathcal{L}(k_{\mathrm{T},1}) = \sum_{c,c'}\frac{\mathrm{d}|M^{\mathrm{F}}|^2_{cc'}}{\mathrm{d}\Phi_{\mathrm{F}}}\sum_{i,j}\left\{ \left(\tilde{C}^{[a]}_{ci}\otimes f^{[a]}_i\right)\tilde{H}(k_{\mathrm{T},1})\left(\tilde{C}^{[b]}_{c'j}\otimes f^{[b]}_j\right) \right\}$

MiNNLOPS scale variation

[Monni, Nason, Re, MW, Zanderighi '19]

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}\Phi_{\mathrm{F}}\mathrm{d}p_{\mathrm{T}}} = \frac{\mathrm{d}}{\mathrm{d}p_{\mathrm{T}}}\left\{ \exp[-\tilde{S}(p_{\mathrm{T}})]\mathcal{L}(p_{\mathrm{T}}) \right\}$$

- require scale invariance ($\mu_R = K_R\, p_T$, $\mu_F = K_F\, p_T$) separately for ingredients
  - sudakov $\tilde{S}(p_{\mathrm{T}}) = 2\int_{p_{\mathrm{T}}}^{Q} \frac{\mathrm{d}q}{q}\left(A(\alpha_s(q))\ln\frac{Q^2}{q^2} + \tilde{B}(\alpha_s(q))\right)$

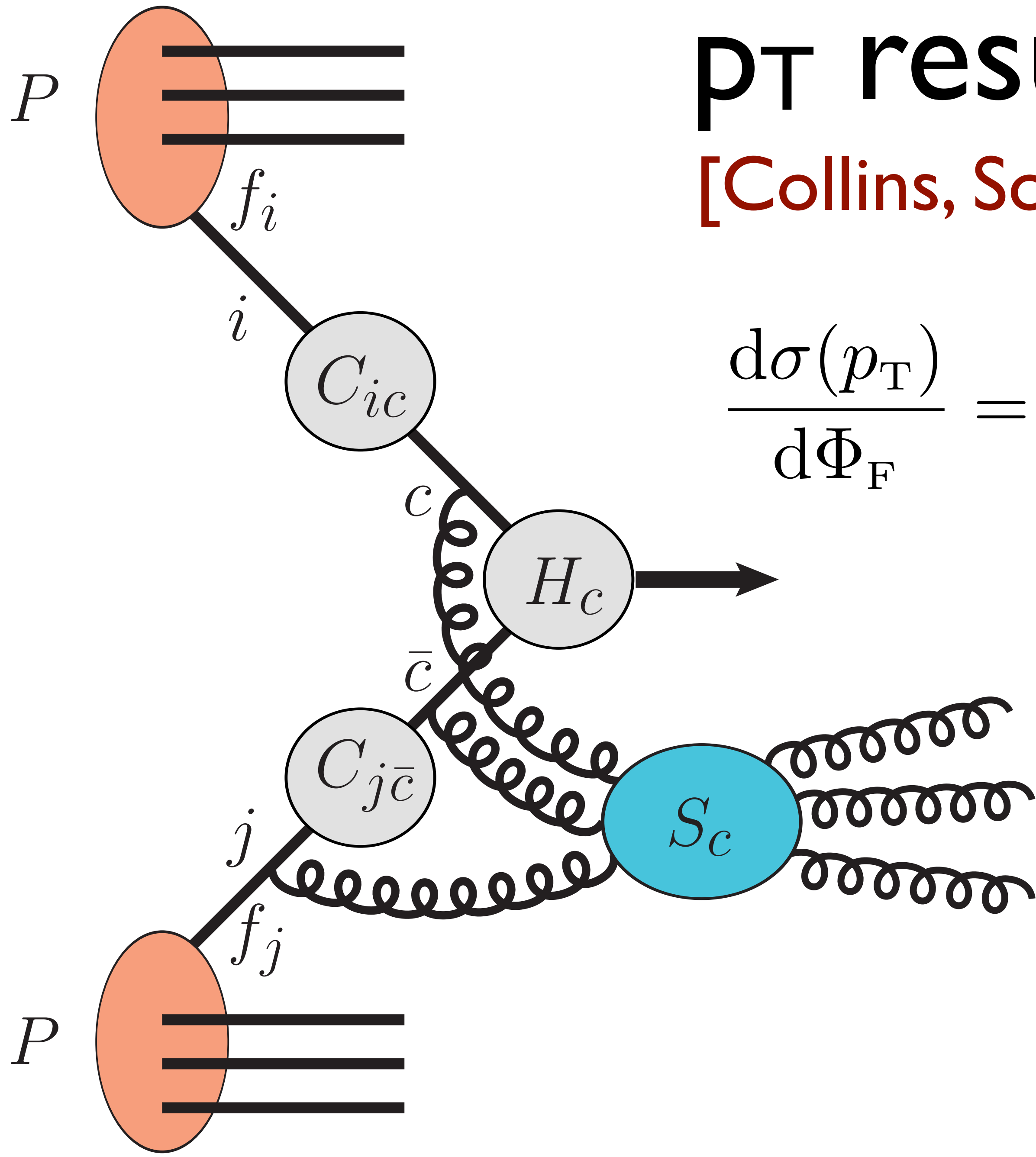
$$A^{(2)}(K_{\mathrm{R}}) = A^{(2)} + (2\pi\beta_0)A^{(1)}\ln {K_{\mathrm{R}}}^2,$$
$$\tilde{B}^{(2)}(K_{\mathrm{R}}) = \tilde{B}^{(2)} + (2\pi\beta_0)B^{(1)}\ln {K_{\mathrm{R}}}^2 + (2\pi\beta_0)^2\, n_B \ln {K_{\mathrm{R}}}^2$$

  - luminosity factor $\mathcal{L}(k_{\mathrm{T},1}) = \sum_{c,c'}\frac{\mathrm{d}|M^{\mathrm{F}}|^2_{cc'}}{\mathrm{d}\Phi_{\mathrm{F}}}\sum_{i,j}\left\{\left(\tilde{C}^{[a]}_{ci}\otimes f^{[a]}_i\right)\tilde{H}(k_{\mathrm{T},1})\left(\tilde{C}^{[b]}_{c'j}\otimes f^{[b]}_j\right)\right\}$

$$H^{(1)}(K_{\mathrm{R}}) = H^{(1)} + (2\pi\beta_0)n_B\ln {K_{\mathrm{R}}}^2$$
$$\tilde{H}^{(2)}(K_{\mathrm{R}}) = \tilde{H}^{(2)} + 4\,n_B\left(\frac{1+n_B}{2}\pi^2\beta_0^2\ln^2 {K_{\mathrm{R}}}^2 + \pi^2\beta_1\ln {K_{\mathrm{R}}}^2\right) + 2\,H^{(1)}\,(1+n_B)\,\pi\beta_0\ln {K_{\mathrm{R}}}^2$$
$$C^{(1)}(z,K_{\mathrm{F}}) = C^{(1)}(z) - \hat{P}^{(0)}(z)\ln {K_{\mathrm{F}}}^2,$$
$$\tilde{C}^{(2)}(z,K_{\mathrm{F}},K_{\mathrm{R}}) = \tilde{C}^{(2)}(z) + \pi\beta_0\hat{P}^{(0)}(z)\left(\ln^2 {K_{\mathrm{F}}}^2 - 2\ln {K_{\mathrm{F}}}^2\ln {K_{\mathrm{R}}}^2\right) - \hat{P}^{(1)}(z)\ln {K_{\mathrm{F}}}^2$$
$$+\frac{1}{2}(\hat{P}^{(0)}\otimes\hat{P}^{(0)})(z)\ln^2 {K_{\mathrm{F}}}^2 - (\hat{P}^{(0)}\otimes C^{(1)})(z)\ln {K_{\mathrm{F}}}^2 + 2\pi\beta_0 C^{(1)}(z)\ln {K_{\mathrm{R}}}^2\,,$$
$$[D(p_{\mathrm{T}})]^{(3)}(K_{\mathrm{F}},K_{\mathrm{R}})$$

p_T resummation

[Collins, Soper, Sterman '85]



$$\frac{\mathrm{d}\sigma(p_\mathrm{T})}{\mathrm{d}\Phi_\mathrm{F}} = p_\mathrm{T} \int_0^\infty db J_1(b\, p_\mathrm{T})\, e^{-S(b_0/b)} \mathcal{L}_b(Qb/b_0)$$

$$S(p_\mathrm{T}) = 2\int_{p_\mathrm{T}}^{Q} \frac{\mathrm{d}q}{q}\left(A(\alpha_s(q))\ln\frac{Q^2}{q^2} + B(\alpha_s(q))\right), \quad A(\alpha_s) = \sum_{k=1}^{2}\left(\frac{\alpha_s}{2\pi}\right)^k A^{(k)}, \qquad B(\alpha_s) = \sum_{k=1}^{2}\left(\frac{\alpha_s}{2\pi}\right)^k B^{(k)}$$

$$\mathcal{L}_b(Qb/b_0) = \sum_{c,c'} \frac{\mathrm{d}|M^\mathrm{F}|^2_{cc'}}{\mathrm{d}\Phi_\mathrm{F}} \sum_{i,j}\left\{\left(C_{ci}^{[a]} \otimes f_i^{[a]}\right)\bar{H}(Qb/b_0)\left(C_{c'j}^{[b]} \otimes f_j^{[b]}\right)\right\}$$

p_T resummation

$$\frac{d\sigma(p_T)}{d\Phi_F} = p_T \int_0^\infty db J_1(b\, p_T)\, e^{-S(b_0/b)} \mathcal{L}_b(Qb/b_0)$$

p_T resummation

$$
\begin{aligned}
\frac{\mathrm{d}\sigma(p_\mathrm{T})}{\mathrm{d}\Phi_\mathrm{F}} &= p_\mathrm{T}\int_0^\infty db J_1(b\,p_\mathrm{T})\,e^{-S(b_0/b)}\mathcal{L}_b(Qb/b_0) \\
&= e^{-S(p_\mathrm{T})}\Bigg\{\mathcal{L}_b(p_\mathrm{T})\left(1-\frac{\zeta_3}{4}S''(p_\mathrm{T})S'(p_\mathrm{T})+\frac{\zeta_3}{12}S'''(p_\mathrm{T})\right) \\
&\quad -\frac{\zeta_3}{4}\frac{\alpha_s(p_\mathrm{T})}{\pi}S''(p_\mathrm{T})\hat{P}\otimes\mathcal{L}_b(p_\mathrm{T})\Bigg\}+\mathcal{O}(\alpha_s^3(Q))\,.
\end{aligned}
$$

$\mathrm{p_T}$ resummation

$$\frac{\mathrm{d}\sigma(p_{\mathrm{T}})}{\mathrm{d}\Phi_{\mathrm{F}}} = p_{\mathrm{T}} \int_0^\infty db J_1(b\, p_{\mathrm{T}})\, e^{-S(b_0/b)} \mathcal{L}_b(Qb/b_0)$$

$$= e^{-S(p_{\mathrm{T}})} \left\{ \mathcal{L}_b(p_{\mathrm{T}}) \left(1 - \frac{\zeta_3}{4} S''(p_{\mathrm{T}}) S'(p_{\mathrm{T}}) + \frac{\zeta_3}{12} S'''(p_{\mathrm{T}}) \right) - \frac{\zeta_3}{4} \frac{\alpha_s(p_{\mathrm{T}})}{\pi} S''(p_{\mathrm{T}}) \hat{P} \otimes \mathcal{L}_b(p_{\mathrm{T}}) \right\} + \mathcal{O}(\alpha_s^3(Q))\,.$$

redefining:

$$B^{(2)} \to \tilde{B}^{(2)} = B^{(2)} + 2\zeta_3 (A^{(1)})^2 + 2\pi\beta_0 H^{(1)},$$
$$H^{(2)} \to \tilde{H}^{(2)} = H^{(2)} + 2\zeta_3 A^{(1)} B^{(1)} + \frac{8}{3} \zeta_3 A^{(1)} \pi \beta_0$$
$$C^{(2)}(z) \to \tilde{C}^{(2)}(z) = C^{(2)}(z) - 2\zeta_3 A^{(1)} \hat{P}^{(0)}(z)\,,$$

p_T resummation

$$\frac{\mathrm{d}\sigma(p_\mathrm{T})}{\mathrm{d}\Phi_\mathrm{F}} = p_\mathrm{T}\int_0^\infty db J_1(b\,p_\mathrm{T})\,e^{-S(b_0/b)}\mathcal{L}_b(Qb/b_0)$$

$$= e^{-S(p_\mathrm{T})}\left\{\mathcal{L}_b(p_\mathrm{T})\left(1-\frac{\zeta_3}{4}S''(p_\mathrm{T})S'(p_\mathrm{T})+\frac{\zeta_3}{12}S'''(p_\mathrm{T})\right)\right.$$

$$\left.-\frac{\zeta_3}{4}\frac{\alpha_s(p_\mathrm{T})}{\pi}S''(p_\mathrm{T})\hat{P}\otimes\mathcal{L}_b(p_\mathrm{T})\right\}+\mathcal{O}(\alpha_s^3(Q))\,.$$

redefining:

$$B^{(2)}\to\tilde{B}^{(2)}=B^{(2)}+2\zeta_3(A^{(1)})^2+2\pi\beta_0H^{(1)},$$
$$H^{(2)}\to\tilde{H}^{(2)}=H^{(2)}+2\zeta_3A^{(1)}B^{(1)}+\frac{8}{3}\zeta_3A^{(1)}\pi\beta_0$$
$$C^{(2)}(z)\to\tilde{C}^{(2)}(z)=C^{(2)}(z)-2\zeta_3A^{(1)}\hat{P}^{(0)}(z)\,,$$

$$\Rightarrow\quad \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_\mathrm{F}} = \exp[-\tilde{S}(p_\mathrm{T})]\mathcal{L}(p_\mathrm{T})$$

p_T resummation

$$\frac{\mathrm{d}\sigma(p_{\mathrm{T}})}{\mathrm{d}\Phi_{\mathrm{F}}} = p_{\mathrm{T}} \int_0^\infty db J_1(b\, p_{\mathrm{T}})\, e^{-S(b_0/b)} \mathcal{L}_b(Qb/b_0)$$

$$= e^{-S(p_{\mathrm{T}})} \left\{ \mathcal{L}_b(p_{\mathrm{T}}) \left(1 - \frac{\zeta_3}{4} S''(p_{\mathrm{T}}) S'(p_{\mathrm{T}}) + \frac{\zeta_3}{12} S'''(p_{\mathrm{T}}) \right) \right.$$

$$\left. - \frac{\zeta_3}{4} \frac{\alpha_s(p_{\mathrm{T}})}{\pi} S''(p_{\mathrm{T}}) \hat{P} \otimes \mathcal{L}_b(p_{\mathrm{T}}) \right\} + \mathcal{O}(\alpha_s^3(Q))\,.$$

redefining:

$$B^{(2)} \to \tilde{B}^{(2)} = B^{(2)} + 2\zeta_3 (A^{(1)})^2 + 2\pi\beta_0 H^{(1)},$$

$$H^{(2)} \to \tilde{H}^{(2)} = H^{(2)} + 2\zeta_3 A^{(1)} B^{(1)} + \frac{8}{3}\zeta_3 A^{(1)} \pi\beta_0$$

$$C^{(2)}(z) \to \tilde{C}^{(2)}(z) = C^{(2)}(z) - 2\zeta_3 A^{(1)} \hat{P}^{(0)}(z)\,,$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{\mathrm{F}} \mathrm{d}p_{\mathrm{T}}} = \frac{\mathrm{d}}{\mathrm{d}p_{\mathrm{T}}} \left\{ \exp[-\tilde{S}(p_{\mathrm{T}})] \mathcal{L}(p_{\mathrm{T}}) \right\}$$

New approach: MiNNLOPS

[Monni, Nason, Re, MW, Zanderighi '19]

* NLO (F+jet): $$\frac{d\sigma_{FJ}^{(\mathrm{NLO})}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}} = \frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(1)} + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(2)}$$

* MiNLO: $$\frac{d\sigma}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}} = \exp[-S(p_{\mathrm{T}})]\left\{\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}[S(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(2)}\right\}$$

$$S(p_{\mathrm{T}}) = 2\int_{p_{\mathrm{T}}}^{Q}\frac{dq}{q}\left(A(\alpha_s(q))\ln\frac{Q^2}{q^2} + B(\alpha_s(q))\right),$$

$$A(\alpha_s) = \sum_{k=1}^{2}\left(\frac{\alpha_s}{2\pi}\right)^k A^{(k)}, \quad B(\alpha_s) = \sum_{k=1}^{2}\left(\frac{\alpha_s}{2\pi}\right)^k B^{(k)},$$

counting:

$$\int_{\Lambda}^{Q} dp_{\mathrm{T}}\frac{1}{p_{\mathrm{T}}}\alpha_s^m(p_{\mathrm{T}})\ln^n\frac{p_{\mathrm{T}}}{Q}\exp(-S(p_{\mathrm{T}})) \approx \alpha_s^{m-\frac{n+1}{2}}(Q)$$

$$D(p_{\mathrm{T}}) \equiv -\frac{dS(p_{\mathrm{T}})}{dp_{\mathrm{T}}}\mathcal{L}(p_{\mathrm{T}}) + \frac{d\mathcal{L}(p_{\mathrm{T}})}{dp_{\mathrm{T}}}$$

* analytic all-order formula:

$$\frac{d\sigma}{d\Phi_{\mathrm{B}} dp_{\mathrm{T}}} = \frac{d}{dp_{\mathrm{T}}}\left\{\exp[-S(p_{\mathrm{T}})]\mathcal{L}(\Phi_{\mathrm{B}}, p_{\mathrm{T}})\right\} + R_f(p_{\mathrm{T}}) = \exp[-S(p_{\mathrm{T}})]\left\{D(p_{\mathrm{T}}) + \frac{R_f(p_{\mathrm{T}})}{\exp[-S(p_{\mathrm{T}})]}\right\}$$

$$= \exp[-S(p_{\mathrm{T}})]\left\{\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}[S(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(2)} + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^3[D(p_{\mathrm{T}})]^{(3)} + \text{regular terms}\right\}$$

New approach: MiNNLOPS

[Monni, Nason, Re, MW, Zanderighi '19]

- NLO (F+jet): $$\frac{d\sigma_{FJ}^{(\mathrm{NLO})}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}} = \frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(1)} + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(2)}$$

- MiNLO: $$\frac{d\sigma}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}} = \boxed{\exp[-S(p_{\mathrm{T}})]\left\{\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}[S(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(2)}\right\}}$$

$$S(p_{\mathrm{T}}) = 2\int_{p_{\mathrm{T}}}^{Q}\frac{dq}{q}\left(A(\alpha_s(q))\ln\frac{Q^2}{q^2} + B(\alpha_s(q))\right),$$

$$A(\alpha_s) = \sum_{k=1}^{2}\left(\frac{\alpha_s}{2\pi}\right)^k A^{(k)}, \quad B(\alpha_s) = \sum_{k=1}^{2}\left(\frac{\alpha_s}{2\pi}\right)^k B^{(k)},$$

counting:

$$\int_{\Lambda}^{Q} dp_{\mathrm{T}}\frac{1}{p_{\mathrm{T}}}\alpha_s^m(p_{\mathrm{T}})\ln^n\frac{p_{\mathrm{T}}}{Q}\exp(-S(p_{\mathrm{T}})) \approx \alpha_s^{m-\frac{n+1}{2}}(Q)$$

$$D(p_{\mathrm{T}}) \equiv -\frac{dS(p_{\mathrm{T}})}{dp_{\mathrm{T}}}\mathcal{L}(p_{\mathrm{T}}) + \frac{d\mathcal{L}(p_{\mathrm{T}})}{dp_{\mathrm{T}}}$$

- analytic all-order formula:

$$\frac{d\sigma}{d\Phi_{\mathrm{B}} dp_{\mathrm{T}}} = \frac{d}{dp_{\mathrm{T}}}\left\{\exp[-S(p_{\mathrm{T}})]\mathcal{L}(\Phi_{\mathrm{B}}, p_{\mathrm{T}})\right\} + R_f(p_{\mathrm{T}}) = \exp[-S(p_{\mathrm{T}})]\left\{D(p_{\mathrm{T}}) + \frac{R_f(p_{\mathrm{T}})}{\exp[-S(p_{\mathrm{T}})]}\right\}$$

$$= \boxed{\exp[-S(p_{\mathrm{T}})]\left\{\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}[S(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(2)}\right.} + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^3[D(p_{\mathrm{T}})]^{(3)} + \text{regular terms}\Big\}$$

MiNLO

New approach: MiNNLOPS

[Monni, Nason, Re, MW, Zanderighi '19]

- NLO (F+jet): $$\frac{d\sigma_{FJ}^{(\mathrm{NLO})}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}} = \frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(1)} + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(2)}$$

- MiNLO: $$\frac{d\sigma}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}} = \boxed{\exp[-S(p_{\mathrm{T}})]\left\{\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}[S(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(2)}\right\}}$$

$$S(p_{\mathrm{T}}) = 2\int_{p_{\mathrm{T}}}^{Q}\frac{dq}{q}\left(A(\alpha_s(q))\ln\frac{Q^2}{q^2} + B(\alpha_s(q))\right),$$

$$A(\alpha_s) = \sum_{k=1}^{2}\left(\frac{\alpha_s}{2\pi}\right)^k A^{(k)}, \quad B(\alpha_s) = \sum_{k=1}^{2}\left(\frac{\alpha_s}{2\pi}\right)^k B^{(k)},$$

counting:

$$\int_{\Lambda}^{Q} dp_{\mathrm{T}}\frac{1}{p_{\mathrm{T}}}\alpha_s^m(p_{\mathrm{T}})\ln^n\frac{p_{\mathrm{T}}}{Q}\exp(-S(p_{\mathrm{T}})) \approx \alpha_s^{m-\frac{n+1}{2}}(Q)$$

- analytic all-order formula:

$$D(p_{\mathrm{T}}) \equiv -\frac{dS(p_{\mathrm{T}})}{dp_{\mathrm{T}}}\mathcal{L}(p_{\mathrm{T}}) + \frac{d\mathcal{L}(p_{\mathrm{T}})}{dp_{\mathrm{T}}}$$

$$\frac{d\sigma}{d\Phi_{\mathrm{B}} dp_{\mathrm{T}}} = \frac{d}{dp_{\mathrm{T}}}\left\{\exp[-S(p_{\mathrm{T}})]\mathcal{L}(\Phi_{\mathrm{B}},p_{\mathrm{T}})\right\} + R_f(p_{\mathrm{T}}) = \exp[-S(p_{\mathrm{T}})]\left\{D(p_{\mathrm{T}}) + \frac{R_f(p_{\mathrm{T}})}{\exp[-S(p_{\mathrm{T}})]}\right\}$$

$$= \underbrace{\boxed{\exp[-S(p_{\mathrm{T}})]\left\{\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}[S(p_{\mathrm{T}})]^{(1)}\right) + \left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{d\sigma_{FJ}}{d\Phi_{\mathrm{F}} dp_{\mathrm{T}}}\right]^{(2)}\right.}}_{\text{MiNLO}} + \underbrace{\boxed{\left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^3[D(p_{\mathrm{T}})]^{(3)}}}_{\text{missing terms for NNLO accuracy}} \left. + \text{ regular terms}\right\}$$

MiNLO

missing terms for NNLO accuracy

MiNNLO$_{\text{PS}}$ practical implementation

[Monni, Nason, Re, MW, Zanderighi '19]

MiNNLO$_{\text{PS}}$ master formula

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{\mathrm{FJ}}} = \exp[-\tilde{S}(p_{\mathrm{T}})]\left\{\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\left[\frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_{\mathrm{FJ}}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}[\tilde{S}(p_{\mathrm{T}})]^{(1)}\right)+\left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^2\left[\frac{\mathrm{d}\sigma_{\mathrm{FJ}}}{\mathrm{d}\Phi_{\mathrm{FJ}}}\right]^{(2)}+\left(\frac{\alpha_s(p_{\mathrm{T}})}{2\pi}\right)^3[D(p_{\mathrm{T}})]^{(3)}F^{\mathrm{corr}}(\Phi_{\mathrm{FJ}})\right\}$$
$$\times\left\{\Delta_{\mathrm{pwg}}(\Lambda)+\int \mathrm{d}\Phi_{\mathrm{rad}}\Delta_{\mathrm{pwg}}(p_{\mathrm{T,rad}})\frac{R(\Phi_{\mathrm{FJ}},\Phi_{\mathrm{rad}})}{B(\Phi_{\mathrm{FJ}})}\right\}+\mathcal{O}(\alpha_s^3)$$

MiNNLO$_{\text{PS}}$ practical implementation

[Monni, Nason, Re, MW, Zanderighi '19]

MiNNLO$_{\text{PS}}$ master formula

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{\text{FJ}}} = \exp[-\tilde{S}(p_{\text{T}})]\left\{\frac{\alpha_s(p_{\text{T}})}{2\pi}\left[\frac{\mathrm{d}\sigma_{\text{FJ}}}{\mathrm{d}\Phi_{\text{FJ}}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\text{T}})}{2\pi}[\tilde{S}(p_{\text{T}})]^{(1)}\right)+\left(\frac{\alpha_s(p_{\text{T}})}{2\pi}\right)^2\left[\frac{\mathrm{d}\sigma_{\text{FJ}}}{\mathrm{d}\Phi_{\text{FJ}}}\right]^{(2)}+\left(\frac{\alpha_s(p_{\text{T}})}{2\pi}\right)^3\underline{[D(p_{\text{T}})]^{(3)}}F^{\text{corr}}(\Phi_{\text{FJ}})\right\}$$

$$\times\left\{\Delta_{\text{pwg}}(\Lambda)+\int \mathrm{d}\Phi_{\text{rad}}\Delta_{\text{pwg}}(p_{\text{T,rad}})\frac{R(\Phi_{\text{FJ}},\Phi_{\text{rad}})}{B(\Phi_{\text{FJ}})}\right\}+\mathcal{O}(\alpha_s^3)$$

$$\underline{[D(p_{\text{T}})]^{(3)}} = -\left[\frac{\mathrm{d}\tilde{S}(p_{\text{T}})}{\mathrm{d}p_{\text{T}}}\right]^{(1)}[\mathcal{L}(p_{\text{T}})]^{(2)}-\left[\frac{\mathrm{d}\tilde{S}(p_{\text{T}})}{\mathrm{d}p_{\text{T}}}\right]^{(2)}[\mathcal{L}(p_{\text{T}})]^{(1)}-\left[\frac{\mathrm{d}\tilde{S}(p_{\text{T}})}{\mathrm{d}p_{\text{T}}}\right]^{(3)}[\mathcal{L}(p_{\text{T}})]^{(0)}+\left[\frac{\mathrm{d}\mathcal{L}(p_{\text{T}})}{\mathrm{d}p_{\text{T}}}\right]^{(3)}$$

$$=\frac{2}{p_{\text{T}}}\left(A^{(1)}\ln\frac{Q^2}{{p_{\text{T}}}^2}+B^{(1)}\right)[\mathcal{L}(p_{\text{T}})]^{(2)}+\frac{2}{p_{\text{T}}}\left(A^{(2)}\ln\frac{Q^2}{{p_{\text{T}}}^2}+\tilde{B}^{(2)}\right)[\mathcal{L}(p_{\text{T}})]^{(1)}+\frac{2}{p_{\text{T}}}A^{(3)}\ln\frac{Q^2}{{p_{\text{T}}}^2}[\mathcal{L}(p_{\text{T}})]^{(0)}+\left[\frac{\mathrm{d}\mathcal{L}(p_{\text{T}})}{\mathrm{d}p_{\text{T}}}\right]^{(3)}$$

$$\mathcal{L}(k_{\text{T},1}) = \sum_{c,c'}\frac{\mathrm{d}|M^{\text{F}}|^2_{cc'}}{\mathrm{d}\Phi_{\text{F}}}\sum_{i,j}\left\{\left(\tilde{C}^{[a]}_{ci}\otimes f^{[a]}_i\right)\tilde{H}(k_{\text{T},1})\left(\tilde{C}^{[b]}_{c'j}\otimes f^{[b]}_j\right)+\left(G^{[a]}_{ci}\otimes f^{[a]}_i\right)\tilde{H}(k_{\text{T},1})\left(G^{[b]}_{c'j}\otimes f^{[b]}_j\right)\right\}$$

MiNNLO$_{\rm PS}$ practical implementation

[Monni, Nason, Re, MW, Zanderighi '19]

MiNNLO$_{\rm PS}$ master formula

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{\rm FJ}} = \exp[-\tilde{S}(p_{\rm T})]\left\{\frac{\alpha_s(p_{\rm T})}{2\pi}\left[\frac{\mathrm{d}\sigma_{\rm FJ}}{\mathrm{d}\Phi_{\rm FJ}}\right]^{(1)}\left(1+\frac{\alpha_s(p_{\rm T})}{2\pi}[\tilde{S}(p_{\rm T})]^{(1)}\right)+\left(\frac{\alpha_s(p_{\rm T})}{2\pi}\right)^2\left[\frac{\mathrm{d}\sigma_{\rm FJ}}{\mathrm{d}\Phi_{\rm FJ}}\right]^{(2)}+\left(\frac{\alpha_s(p_{\rm T})}{2\pi}\right)^3[D(p_{\rm T})]^{(3)}\underline{F^{\rm corr}(\Phi_{\rm FJ})}\right\}$$
$$\times\left\{\Delta_{\rm pwg}(\Lambda)+\int \mathrm{d}\Phi_{\rm rad}\Delta_{\rm pwg}(p_{\rm T,rad})\frac{R(\Phi_{\rm FJ},\Phi_{\rm rad})}{B(\Phi_{\rm FJ})}\right\}+\mathcal{O}(\alpha_s^3)$$

$$\underline{F_\ell^{\rm corr}(\Phi_{\rm FJ})} = \frac{J_\ell(\Phi_{\rm FJ})}{\sum_{l'}\int \mathrm{d}\Phi'_{\rm FJ} J_{l'}(\Phi'_{\rm FJ})\delta(p_{\rm T}-{p_{\rm T}}')\delta(\Phi_{\rm F}-\Phi'_{\rm F})},$$

$$\sum_\ell\int \mathrm{d}\Phi'_{\rm FJ}G(\Phi'_{\rm F},{p_{\rm T}}')F_\ell^{\rm corr}(\Phi'_{\rm FJ}) = \int \mathrm{d}\Phi_{\rm F}\,\mathrm{d}p_{\rm T}G(\Phi_{\rm F},p_{\rm T})\times\sum_\ell\int \mathrm{d}\Phi'_{\rm FJ}\delta(\Phi_{\rm F}-\Phi'_{\rm F})\delta(p_{\rm T}-{p_{\rm T}}')F_\ell^{\rm corr}(\Phi'_{\rm FJ}) = \int \mathrm{d}\Phi_{\rm F}\,\mathrm{d}p_{\rm T}G(\Phi_{\rm F},p_{\rm T})$$

$$J_\ell(\Phi_{\rm FJ}) = |M_\ell^{\rm FJ}(\Phi_{\rm FJ})|^2(f^{[a]}f^{[b]})_\ell \longrightarrow |M_\ell^{\rm FJ}(\Phi_{\rm FJ})|^2 \simeq |M^{\rm F}(\Phi_{\rm F})|^2 P_\ell(\Phi_{\rm rad}) \longrightarrow J_\ell(\Phi_{\rm FJ}) = P_\ell(\Phi_{\rm rad})(f^{[a]}f^{[b]})_\ell$$