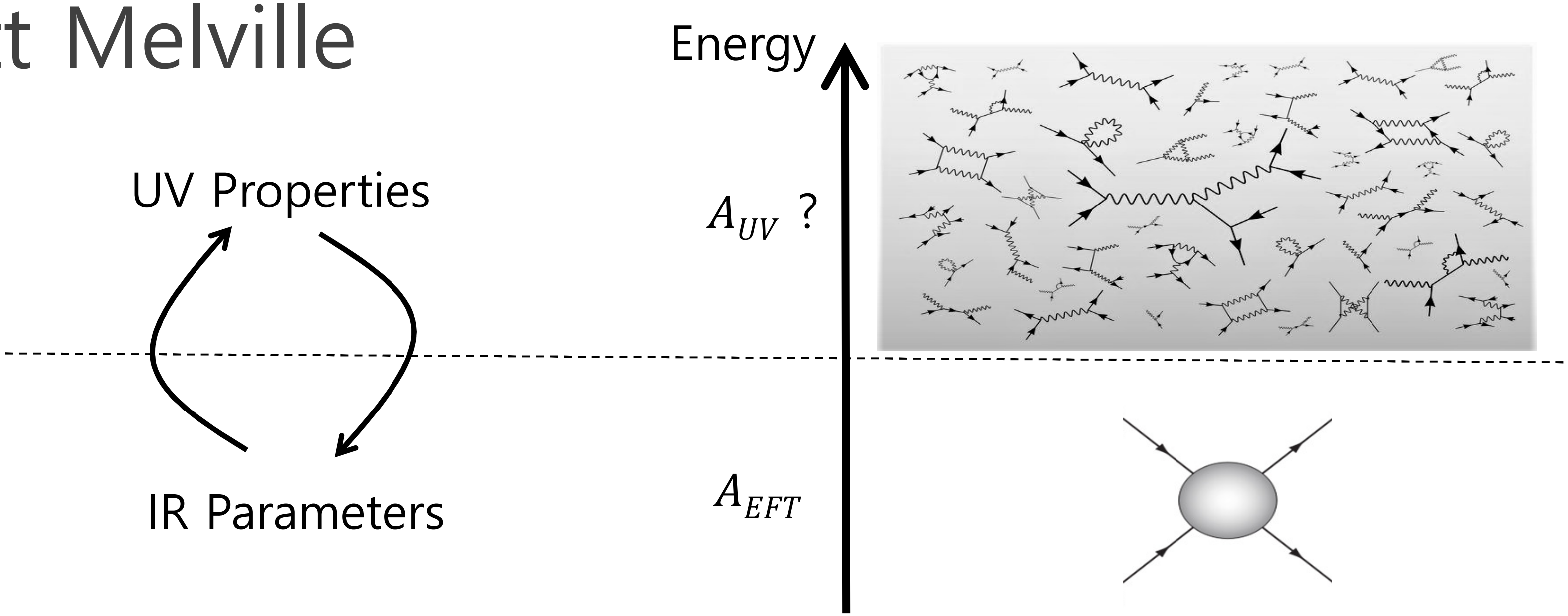
Cavendish-DAMTP Seminar 14 Feb 2019

Low Energy?
Think Positive!

Scott Melville



- Big Picture — Only some EFTs have sensible UV completions
- Positivity Constraints — … and this gives us constraining power!
- SMEFT — bounds on dimension-8 operators for VBS
- Massive Gravity — requires enhanced cutoff and weak coupling

Deriving bounds:

hep-th/0602178 Adams et al
1702.06134 SM et al
1706.02712 SM et al

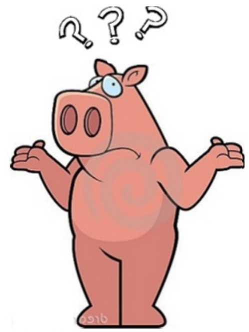
Applying bounds:

1608.02942 Cheung et al
1710.02539 Bellazzini
1710.09611 SM et al
1804.10624 SM et al
1808.00010 Zhang et al

Big Picture

UV

$$\mathcal{L}_{UV} = \quad ?$$



Λ

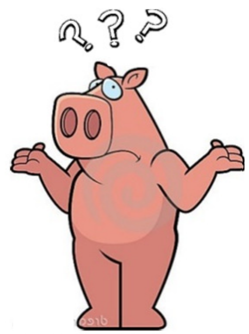
IR

$$\mathcal{L}_{EFT} = \sum^{N} \frac{c_n}{\Lambda^n} \mathcal{O}_n$$

Finite Number of
Local Operators

UV

$$\mathcal{L}_{UV} = \quad ?$$



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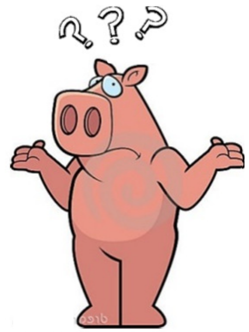

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$$\mathcal{L}_{EFT} = \sum^{N} \frac{c_n}{\Lambda^n} \mathcal{O}_n$$

Finite Number of Local Operators

with undetermined coefficients

(1) Many operators
=> Need many measurements

(2) No deeper understanding of UV

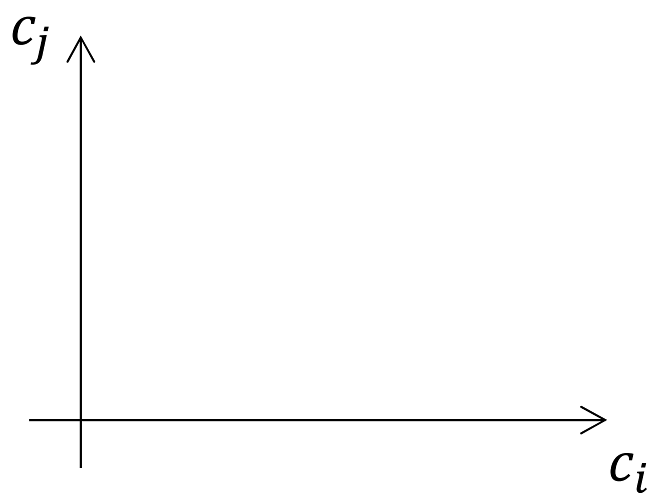
UV

$$\mathcal{L}_{UV} = \quad ?$$

Unitary, Causal, Local, …

IR

$$\mathcal{L}_{EFT} = \sum \frac{c_n}{\Lambda^n} \mathcal{O}_n$$



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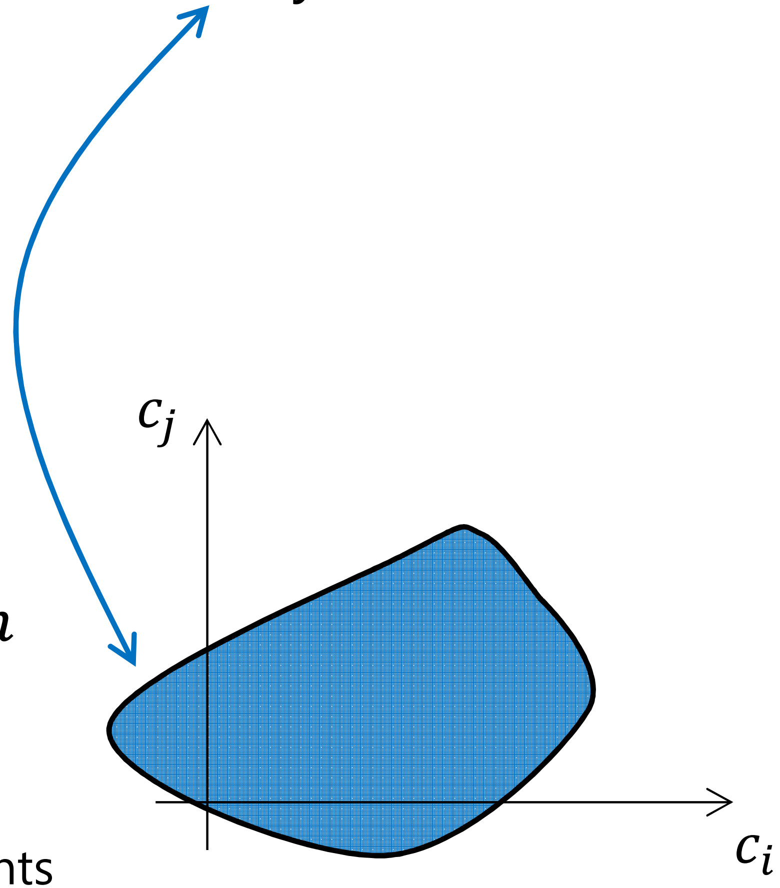

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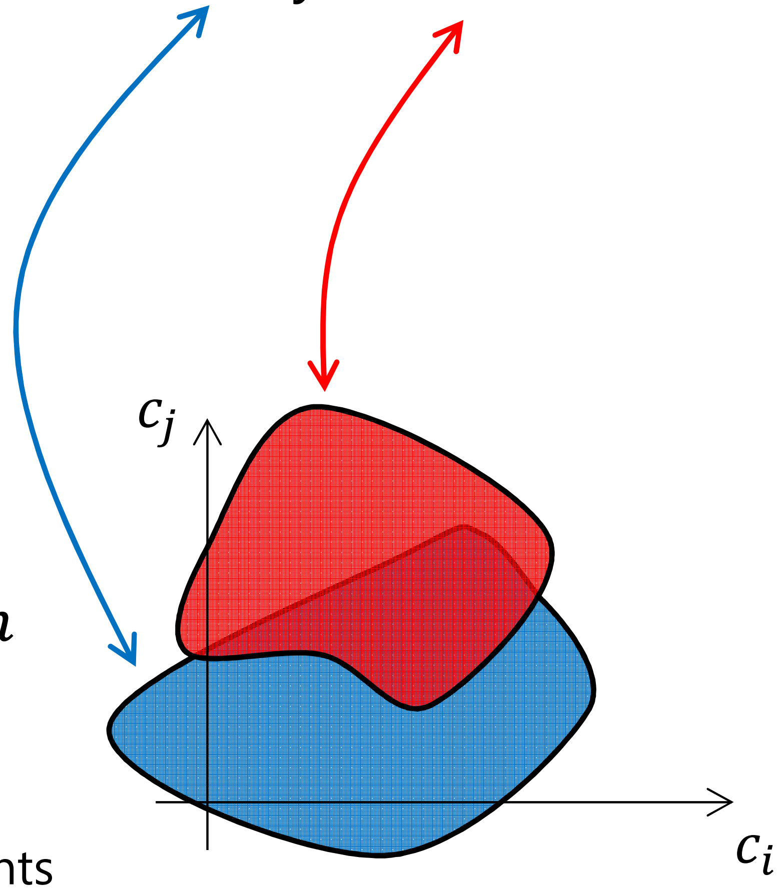

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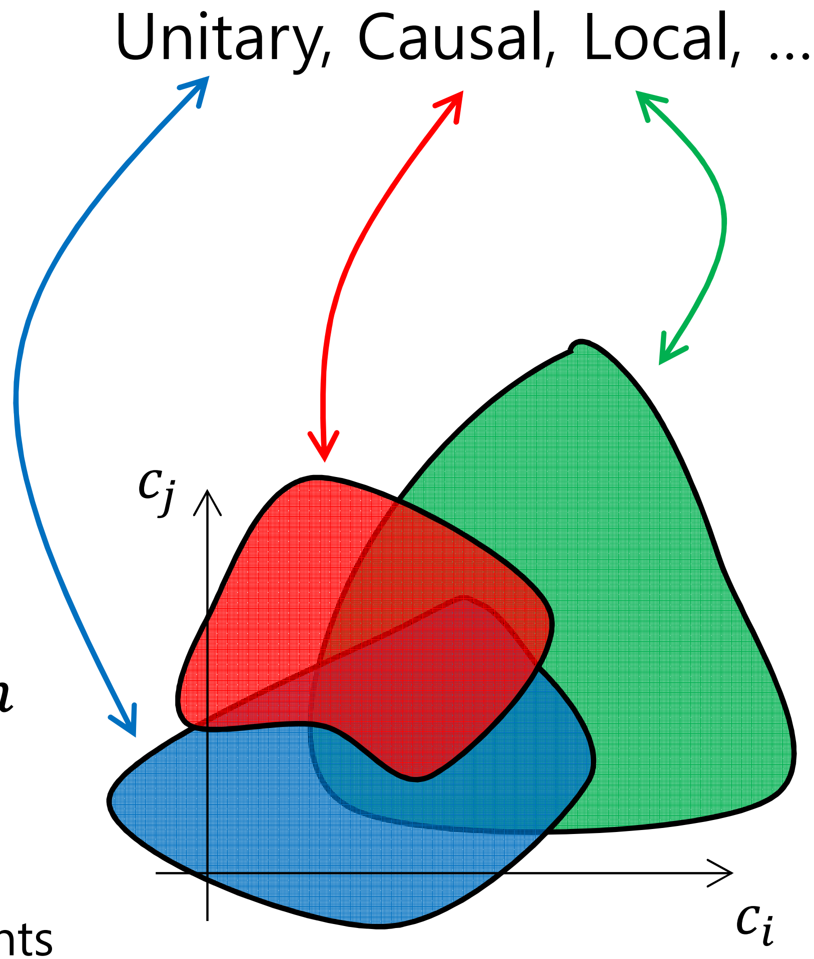

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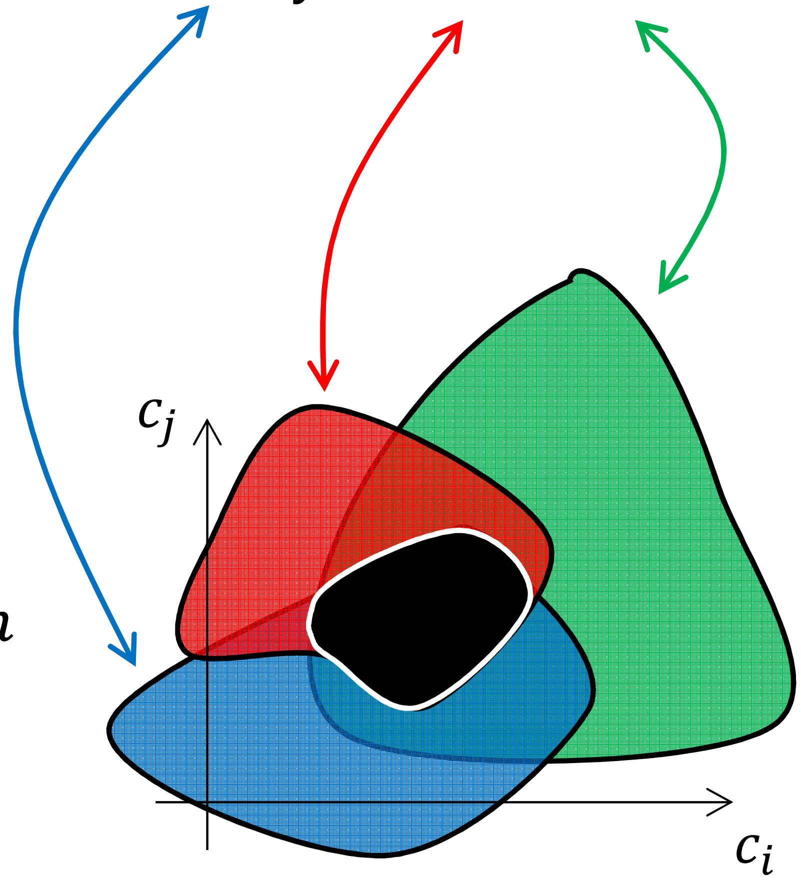


IR

$$\mathcal{L}_{EFT} = \sum \frac{c_n}{\Lambda^n} \mathcal{O}_n$$

(1) Many operators
=> Many constraints

(2) Can infer UV properties from IR measurements

Positivity Constraints

Unitarity

UV

φ $\quad$ $\color{red}{H}$

$$\frac{1}{m^2+p^2} \qquad \frac{Z}{M^2+p^2}$$

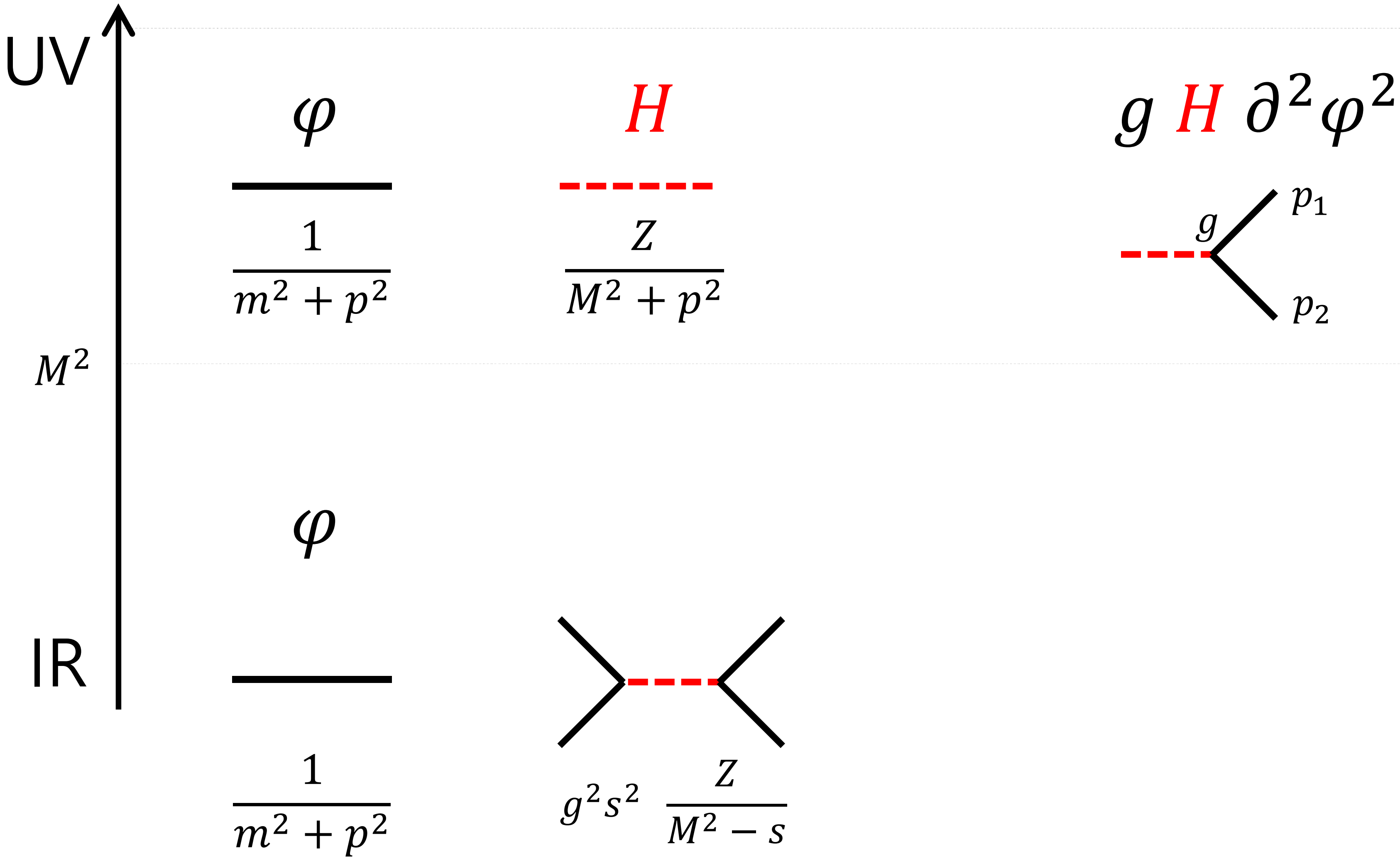
M^2

φ

IR

$$\frac{1}{m^2+p^2}$$

Unitarity



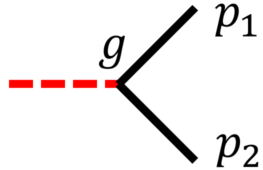
UV
φ
$\frac{1}{m^2+p^2}$
H
$\frac{Z}{M^2+p^2}$
$g\, H\, \partial^2 \varphi^2$
g
p_1
p_2
M^2
IR
φ
$\frac{1}{m^2+p^2}$
$g^2 s^2 \quad \frac{Z}{M^2-s}$

Unitarity

UV

φ $\quad$ H $\quad$ $g\, H\, \partial^2 \varphi^2$

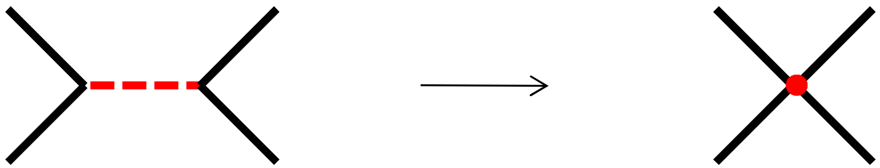
$$\frac{1}{m^2 + p^2} \qquad \frac{Z}{M^2 + p^2}$$



M^2

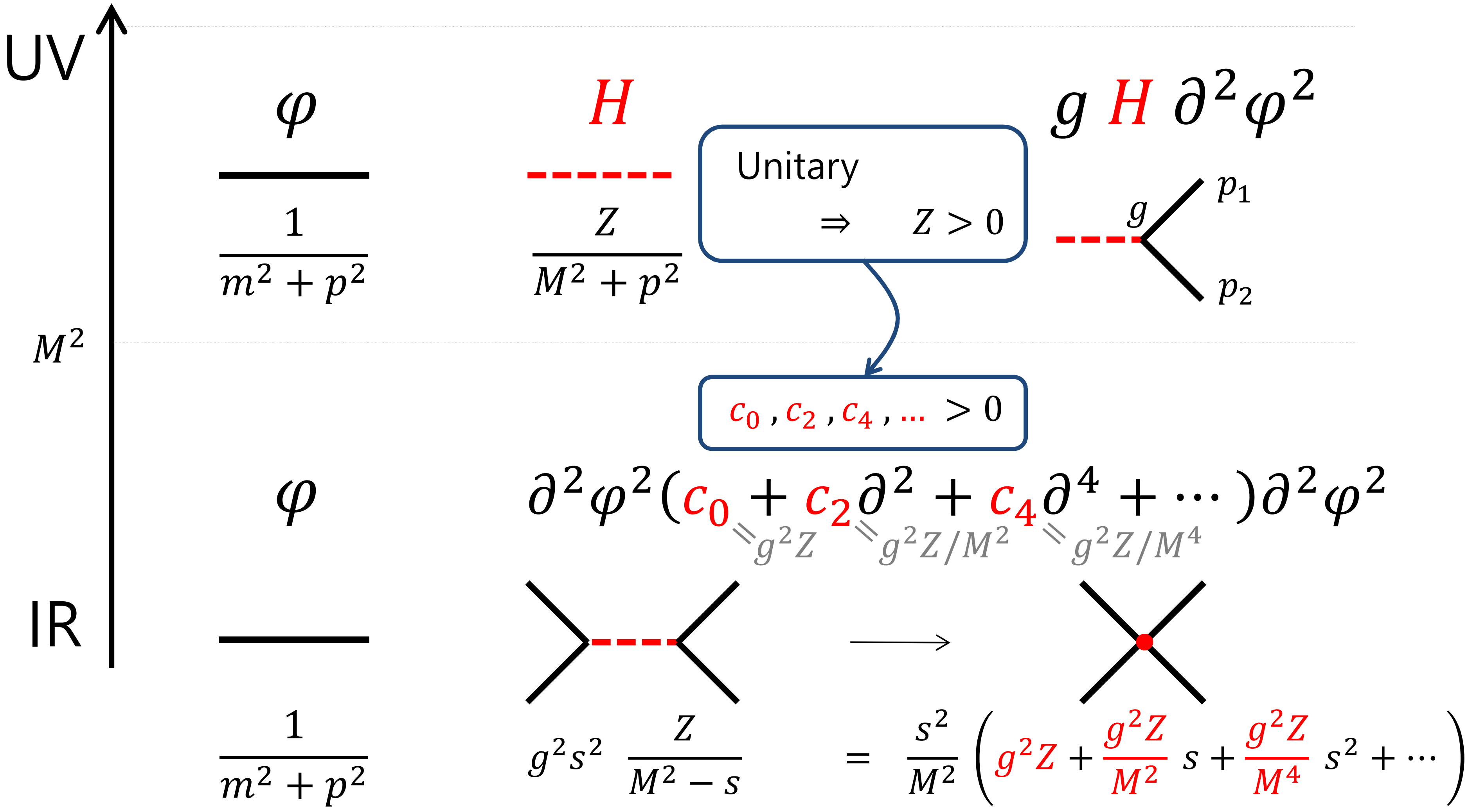
φ $\quad$ $\partial^2 \varphi^2 (c_0 + c_2 \partial^2 + c_4 \partial^4 + \cdots) \partial^2 \varphi^2$

IR

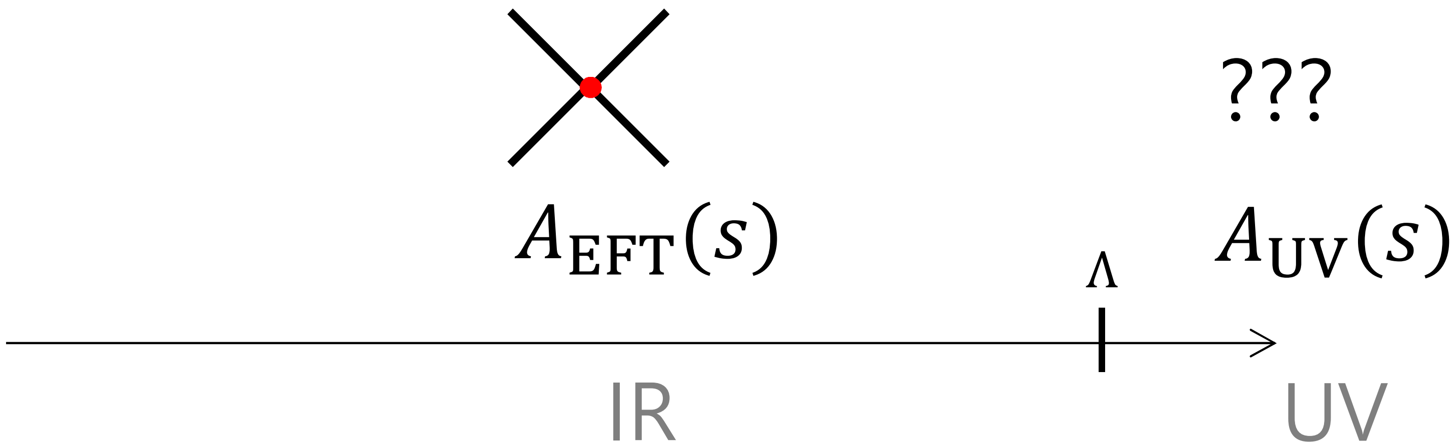


$$\frac{1}{m^2 + p^2} \qquad g^2 s^2 \frac{Z}{M^2 - s} \quad = \frac{s^2}{M^2} \left(g^2 Z + \frac{g^2 Z}{M^2} s + \frac{g^2 Z}{M^4} s^2 + \cdots \right)$$

Unitarity


UV
IR
M^2
φ
$\frac{1}{m^2+p^2}$
H
$\frac{Z}{M^2+p^2}$
Unitary $\Rightarrow$ $Z > 0$
$g\, H\, \partial^2 \varphi^2$
g
p_1
p_2
$c_0, c_2, c_4, \ldots > 0$
φ
$\partial^2\varphi^2(c_0 + c_2\partial^2 + c_4\partial^4 + \cdots)\partial^2\varphi^2$
$c_0 = g^2 Z$, $c_2 = g^2 Z/M^2$, $c_4 = g^2 Z/M^4$
$\frac{1}{m^2+p^2}$
$g^2 s^2 \frac{Z}{M^2 - s} = \frac{s^2}{M^2}\left(g^2 Z + \frac{g^2 Z}{M^2} s + \frac{g^2 Z}{M^4} s^2 + \cdots\right)$

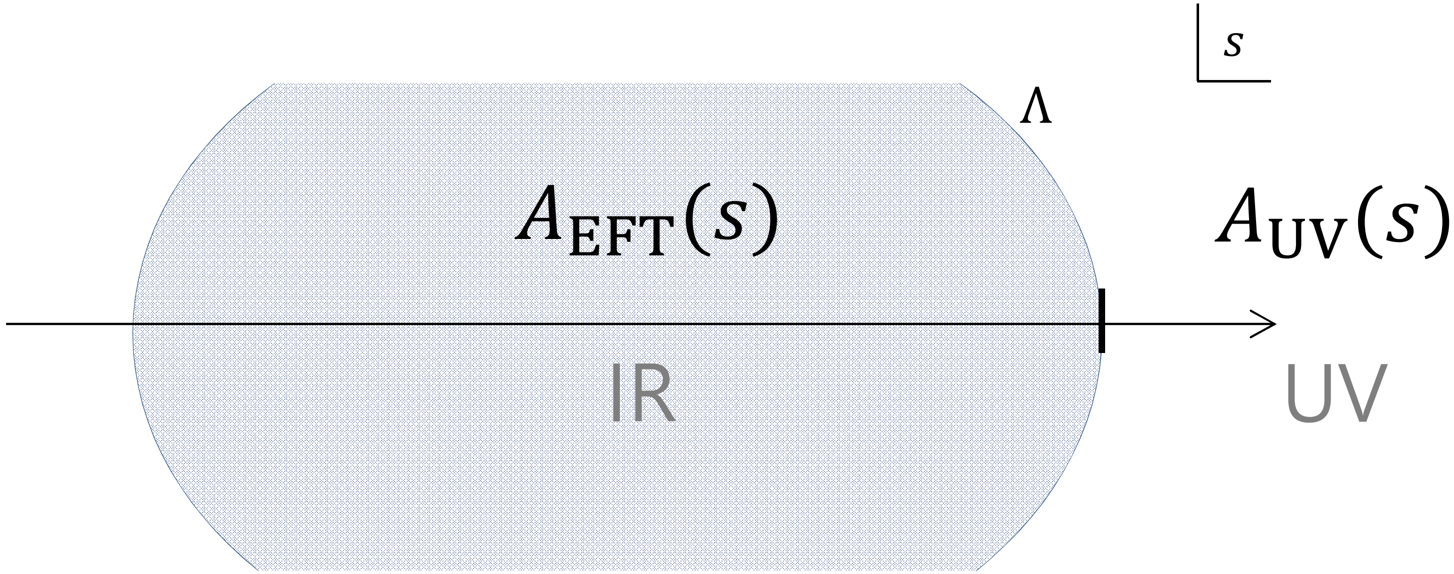
Analyticity



Causality

$\Rightarrow$ $A(s)$ is analytic (up to known poles & branch cuts)

Analyticity



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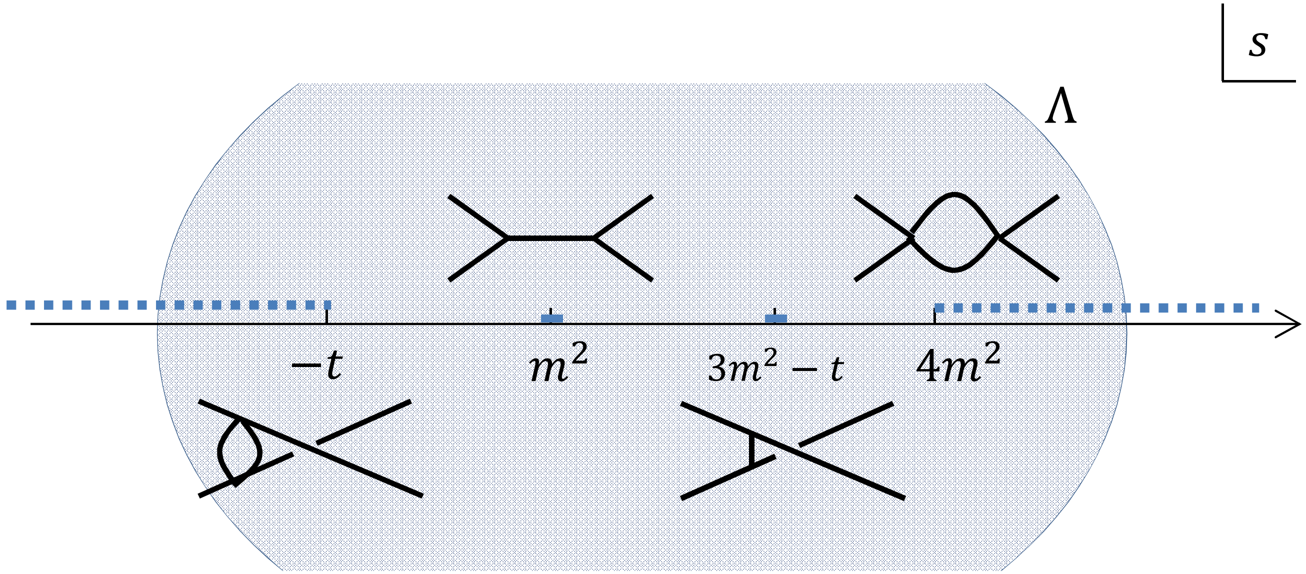
$\Rightarrow$ $A(s)$ is analytic (up to known poles & branch cuts)

At fixed t, expect singularities from s- and u- channel exchange

Analyticity



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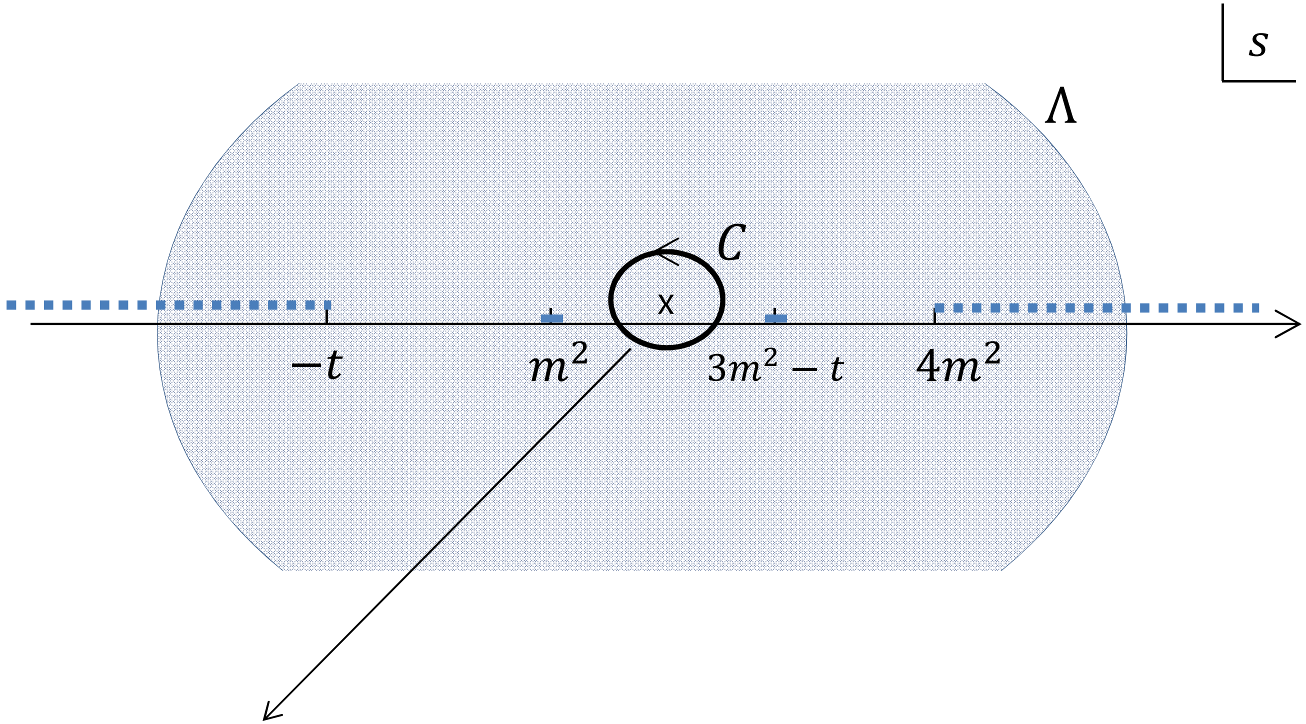
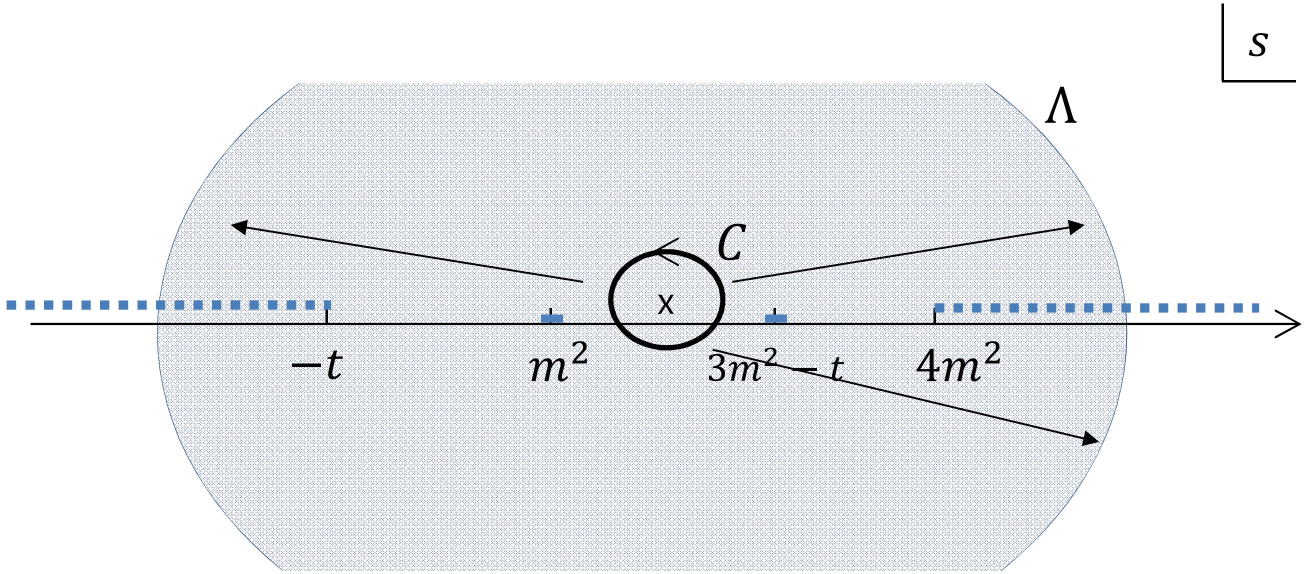


$$\partial_s^N A_{EFT}(s,t) = \oint_C \frac{d\mu}{2\pi i} \frac{A(\mu,t)}{(\mu - s)^{N+1}}$$

Analyticity



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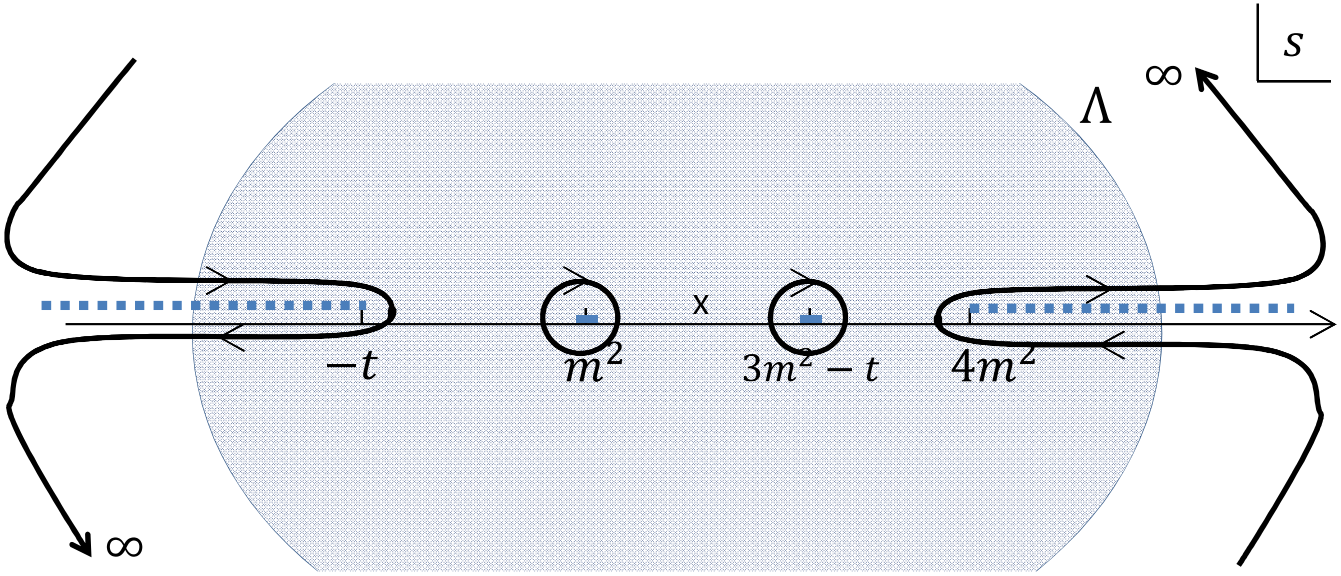

$$\partial_s^N A_{EFT}(s,t) = \text{Poles} + \left(\int_{-\infty}^{-t} + \int_{4m^2}^{\infty} \right) \frac{d\mu}{\pi} \frac{\text{Im}\, A(\mu,t)}{(\mu - s)^{N+1}} + \oint_{|\mu|=\infty} \frac{d\mu}{2\pi i} \frac{A\,(\mu,t)}{\mu^{N+1}}$$

Positivity

Analyticity
(Causality)

$$\partial_s^N \tilde{A}_{\mathrm{EFT}}(s,t) = \left(\int_{-\infty}^{-t} + \int_{4m^2}^{\infty} \right) \frac{d\mu}{\pi} \frac{\mathrm{Im}\, A(\mu,t)}{(\mu - s)^{N+1}} + \oint_{|\mu|=\infty} \frac{d\mu}{2\pi i} \frac{A\,(\mu,t)}{\mu^{N+1}}$$

Unitarity

$$\mathrm{Im}\, A(\mu,t)\Big|_{t\to 0} = \mu\, \sigma(\mu) \, > \, 0$$

Positivity

Analyticity (Causality)

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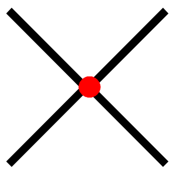
$$\boxed{\text{Analyticity connects EFT to the UV}}$$



$$A_{\text{EFT}}(s) \; = s^2(c_0 + c_2\, s + c_4\, s^2 + \cdots)$$

UV Theory $\Rightarrow$ $c_0 = g^2 Z$, $c_2 = g^2 Z/M^2$, $c_4 = g^2 Z/M^4$

Unitarity $\Rightarrow$ $Z > 0$

Positivity

Analyticity
(Causality)

$$\partial_s^N \tilde{A}_{\mathrm{EFT}}(s,t) = \left(\int_{-\infty}^{-t} + \boxed{\int_{4m^2}^{\infty} \right) \frac{d\mu}{\pi} \frac{\mathrm{Im}\, A(\mu,t)}{(\mu - s)^{N+1}}} + \oint_{|\mu|=\infty} \frac{d\mu}{2\pi i} \frac{A\,(\mu,t)}{\mu^{N+1}}$$

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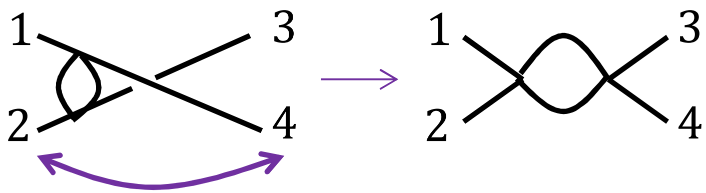
Crossing



$$\int_{4m^2}^{\infty} d\mu \;\; C_N(s;\mu,t)\, \mathrm{Im}\, A(\mu,t)$$

Positivity

Analyticity (Causality)

$$\partial_s^N \tilde{A}_{\mathrm{EFT}}(s,t) = \left(\int_{-\infty}^{-t} + \int_{4m^2}^{\infty} \right) \frac{d\mu}{\pi} \frac{\mathrm{Im}\, A(\mu,t)}{(\mu - s)^{N+1}} + \oint_{|\mu|=\infty} \frac{d\mu}{2\pi i} \frac{A(\mu,t)}{\mu^{N+1}}$$

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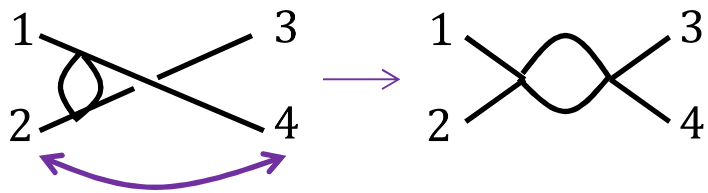

$$\int_{4m^2}^{\infty} d\mu \;\; C_N(s;\mu,t)\, \mathrm{Im}\, A(\mu,t)$$

(Locality + Gap)
Polynomially Bounded

$$\lim_{\mu\to\infty} |A(\mu,t)| < \mu^2$$

$$= 0 \text{ if } N \geq 2$$

Positivity

Analyticity
(Causality)

$$\partial_s^N \tilde{A}_{\mathrm{EFT}}(s,t) = \boxed{\left(\int_{-\infty}^{-t} \right.} + \boxed{\left. \int_{4m^2}^{\infty} \right) \frac{d\mu}{\pi} \frac{\mathrm{Im}\, A(\mu,t)}{(\mu - s)^{N+1}}} + \boxed{\oint_{|\mu|=\infty} \frac{d\mu}{2\pi i} \frac{A(\mu,t)}{\mu^{N+1}}}$$

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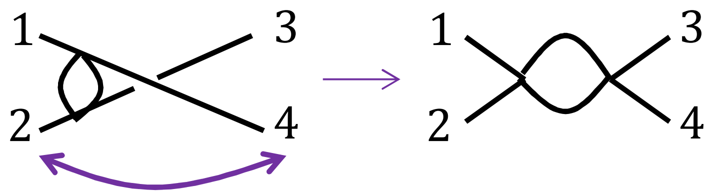

$$\int_{4m^2}^{\infty} d\mu \;\; C_N(s;\mu,t)\, \mathrm{Im}\, A(\mu,t)$$

(Locality + Gap)
Polynomially Bounded

$$\lim_{\mu\to\infty} |A(\mu,t)| < \mu^2$$

$$= 0 \text{ if } N \geq 2$$

$$\boxed{\partial_s^N \tilde{A}_{\mathrm{EFT}}(s,0) = \int d\mu \;\; C_N(s;\mu)\, \sigma(\mu) \; > \; 0}$$

Improving Positivity

$$\partial_s^N \tilde{A}_{\mathrm{EFT}}(s) = \int d\mu \;\; C_N(s;\mu)\,\sigma(\mu) \; > \; 0$$

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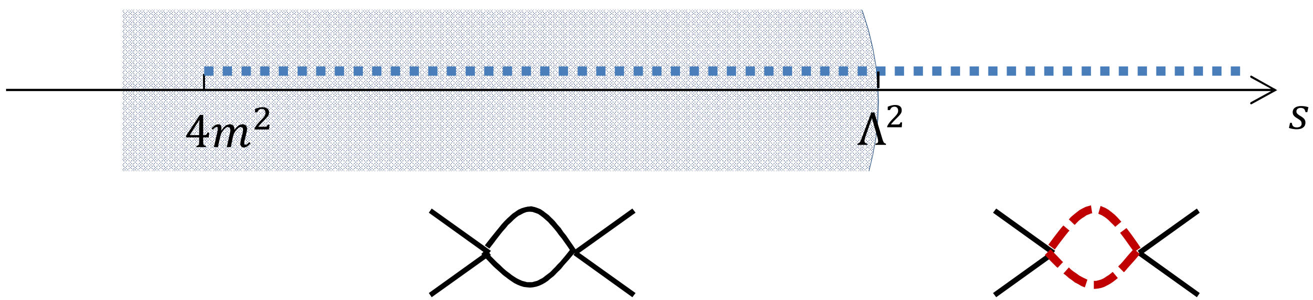

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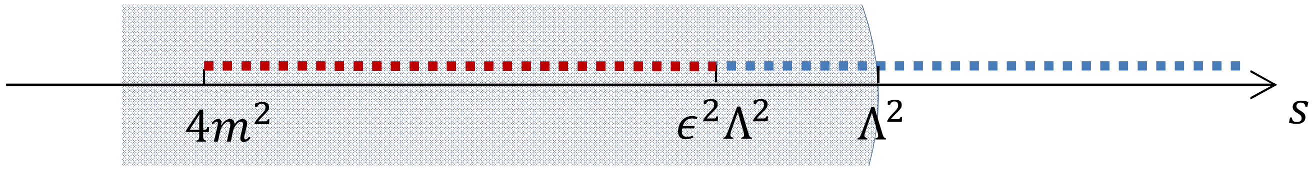

Improving Positivity

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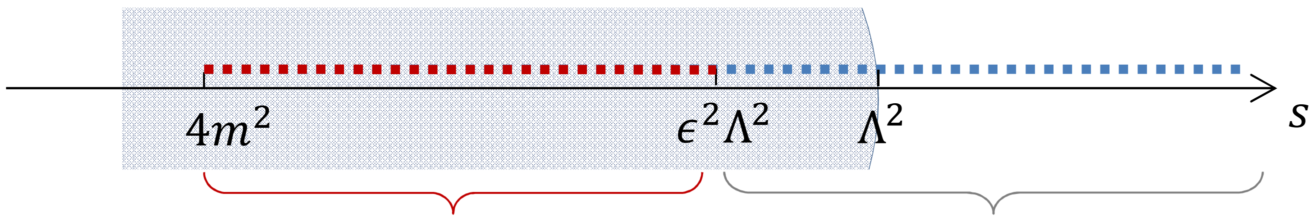
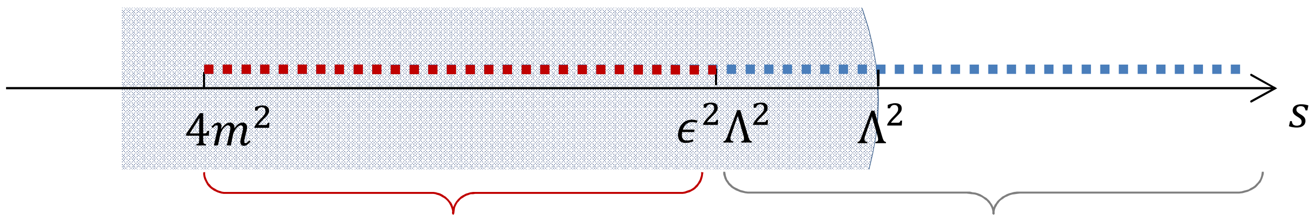


$$\int_{4m^2}^{\epsilon^2\Lambda^2} d\mu \;\; C_N \, \sigma(\mu) \qquad \int_{\epsilon^2\Lambda^2}^{\infty} d\mu \;\; C_N \, \sigma(\mu)$$

Improving Positivity

$$\partial_s^N \tilde{A}_{\mathrm{EFT}}(s) = \int d\mu \;\; C_N(s;\mu)\,\sigma(\mu) \; > \; 0$$



$$\partial_s^N \tilde{A}_{\mathrm{EFT}}(s) - \int_{4m^2}^{\epsilon^2\Lambda^2} d\mu \;\, C_N \,\sigma(\mu) \;=\; \int_{\epsilon^2\Lambda^2}^{\infty} d\mu \;\, C_N \,\sigma(\mu) > 0$$

$$\boxed{\partial_s^N \tilde{A}_{\mathrm{EFT}}(s) > \int_{4m^2}^{\epsilon^2\Lambda^2} d\mu \;\, C_N \,\sigma(\mu)}$$

Improving Positivity

$$\boxed{\partial_s^N \tilde{A}_{\mathrm{EFT}}(s) > \int_{4m^2}^{\epsilon^2\Lambda^2} d\mu \; C_N \, \sigma(\mu)}$$

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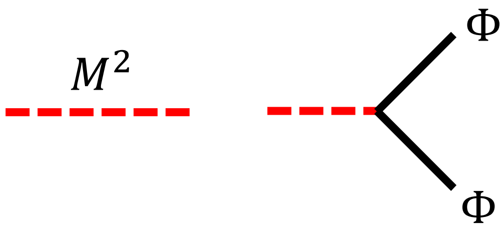


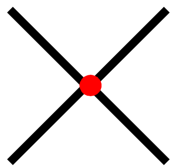
$$\mathcal{L}_{\mathrm{EFT}}\left[\frac{\Phi}{M}, \frac{\partial}{M}\right]$$

$$\frac{1}{g_*^2}\mathcal{L}_{\mathrm{EFT}}\left[\frac{g_*\Phi}{M}, \frac{\partial}{M}\right]$$

Improving Positivity

$$\boxed{\partial_s^N \tilde{A}_{\mathrm{EFT}}(s) > \int_{4m^2}^{\epsilon^2 \Lambda^2} d\mu \;\; C_N \, \sigma(\mu)}$$

$g_*^2 c_4 (\partial\Phi)^4/M^4$



$$A_{\mathrm{EFT}} = g_*^2 \frac{c_4 \, s^2}{M^4} + \cdots$$

For scalars, $\pi C_2(s;\mu) = \dfrac{\mu}{(\mu - s)^3} + \dfrac{\mu}{(\mu - 4m^2 + s)^3} \sim \dfrac{1}{\mu^2}$

$$\sigma(\mu) \sim \frac{|A(\mu)|^2}{\mu} \sim \mu^3$$

$$\frac{g_*^2 c_4}{M^4} > \frac{g_*^4 c_4^2}{16\pi^2 M^8} \int^{\epsilon^2 M^2} d\mu \;\; 2\mu \quad \Rightarrow \quad \boxed{1 > \frac{g_*^2 c_4}{16\pi^2}(\epsilon^4 + \cdots)}$$

The Scale of New Physics

Analyticity
(Causality)

$$\partial_s^N \tilde{A}_{\mathrm{EFT}}(s,t) = \int_{\mu_{\min}}^{\infty} d\mu \; C_N(s;\mu,t)\, \mathrm{Im}A(\mu,t)$$

Unitarity

$$\mathrm{Im}\, A(\mu,t)\Big|_{t\to 0} = \mu\, \sigma(\mu) \; > \; 0$$

The Scale of New Physics

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Unitarity

$$\partial_t^P \,\mathrm{Im}\, A(\mu,t) \; > \; 0$$

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Unitarity

$$\partial_t^P \mathrm{Im}\, A(\mu,t) \; > \; 0$$

$$\partial_t \partial_s^N \tilde{A}_{\mathrm{EFT}}(s,t) = \boxed{\int_{\mu_{\min}}^{\infty} d\mu \;\; C_N(s;\mu,t) \partial_t \mathrm{Im} A(\mu,t)} + \int_{\mu_{\min}}^{\infty} d\mu \, \underbrace{\partial_t C_N(s;\mu,t)}_{\text{Not positive, but known}} \, \mathrm{Im} A(\mu,t)$$

The Scale of New Physics

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Unitarity

$$\partial_t^P \mathrm{Im}\, A(\mu,t) \; > \; 0$$

$$\boxed{\partial_t \partial_s^N \tilde{A}_{\mathrm{EFT}}(s,t) + \frac{2N+1}{\mu_{\min}}\, \partial_s^N \tilde{A}_{\mathrm{EFT}}(s,t) > 0}$$

The Scale of New Physics

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$$\boxed{\partial_t \partial_s^N \tilde{A}_{\mathrm{EFT}}(s,t) + \frac{2N+1}{\mu_{\min}} \partial_s^N \tilde{A}_{\mathrm{EFT}}(s,t) > 0}$$

Can constrain *every* s and t derivative of A_{EFT}!

The Scale of New Physics

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Unitarity

$$\partial_t^P \mathrm{Im}\, A(\mu,t) > 0$$

$$\boxed{\partial_t \partial_s^N \tilde{A}_{\mathrm{EFT}}(s,t) + \frac{2N+1}{\mu_{\min}} \partial_s^N \tilde{A}_{\mathrm{EFT}}(s,t) > 0}$$

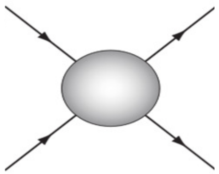
Can constrain *every* s and t derivative of A_{EFT}!

Can constrain $\mu_{\min}$, the scale of new physics!

Positivity in Practice

$$\mathcal{L} = \mathcal{L}_{\text{renorm}} + \frac{f_6 \mathcal{O}_6}{\Lambda^2} + \frac{f_8 \mathcal{O}_8}{\Lambda^4} + \frac{f_{10} \mathcal{O}_{10}}{\Lambda^6} + \cdots$$

$$A_{EFT} = c_0 + c_1 s + c_2(f_6^2, f_8)s^2 + c_3\, s^3 + \ \ldots$$

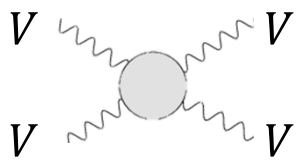


c_n Bounds
g_* Coupling

c_n Bounds
g_* Coupling
M, Mass of New States

SMEFT

$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_{i}^{18} \frac{F_i}{\Lambda^4} \mathcal{O}_i$$



$V = Z, W^{\pm}, \gamma$

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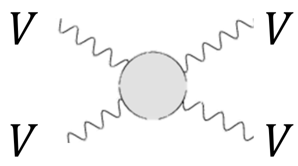

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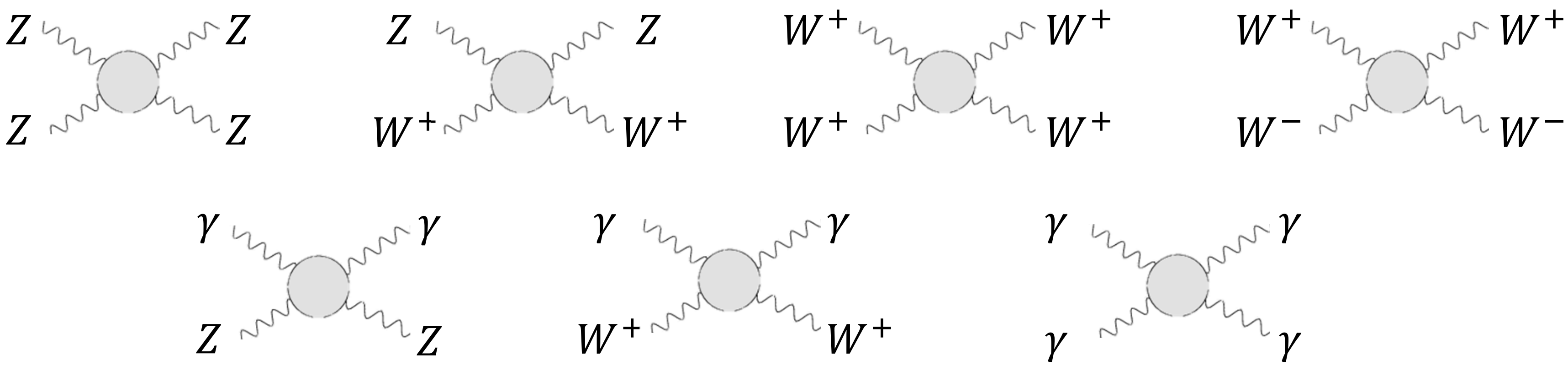


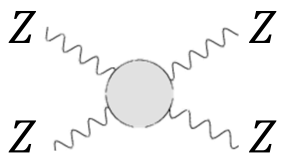
$$A^{ZZ}_{++}, A^{ZZ}_{+0}, A^{ZZ}_{+-}, A^{ZW}_{++}, A^{ZW}_{+0}, A^{ZW}_{+-}, \ldots \Rightarrow\ > 20 \text{ constraints}$$

(on 18 parameters)

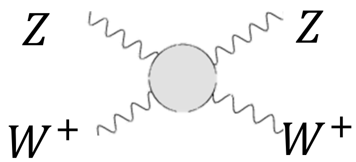
$$F_{S,0}\ \mathrm{Tr}\left[(D_\mu\Phi)^\dagger D_\nu\Phi\right]^2$$

$$F_{S,1}\ \mathrm{Tr}\left[(D_\mu\Phi)^\dagger D^\mu\Phi\right]^2$$

$$\mathrm{Tr}\left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] = M_W^2 W_\mu^- W_\nu^+\left(1+\frac{H}{v}\right)^2 + \frac{M_Z^2}{2} Z_\mu Z_\nu\left(1+\frac{H}{v}\right)^2 + \frac{1}{2}\partial_\mu H\partial_\nu H + \frac{iM_Z}{2}(Z_\mu\partial_\nu H - Z_\nu\partial_\mu H)\left(1+\frac{H}{v}\right)$$



$$\boxed{F_{S,0} + F_{S,1} > 0}$$

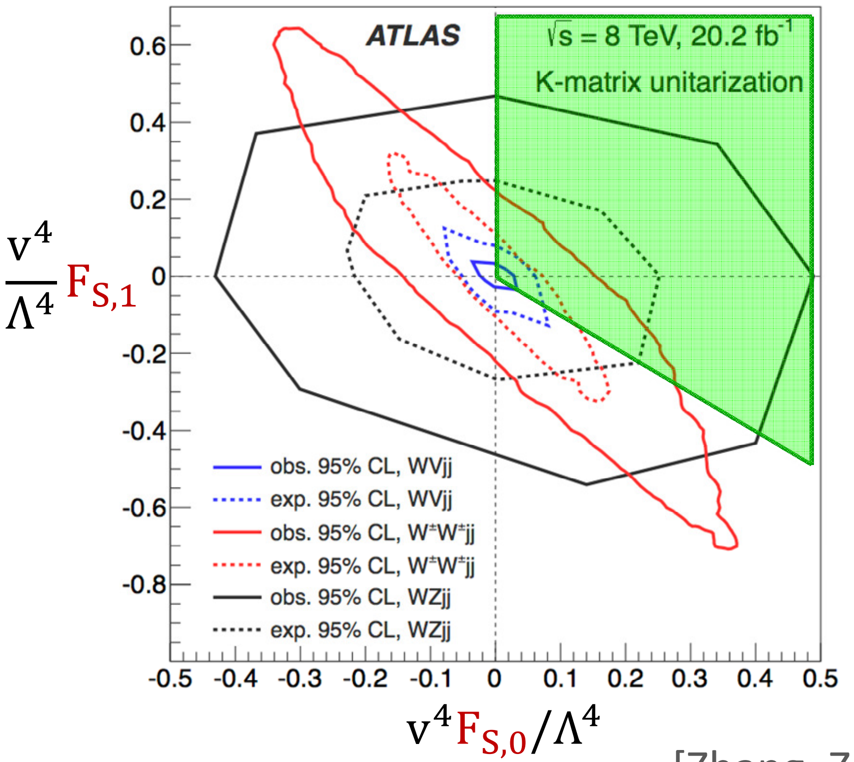


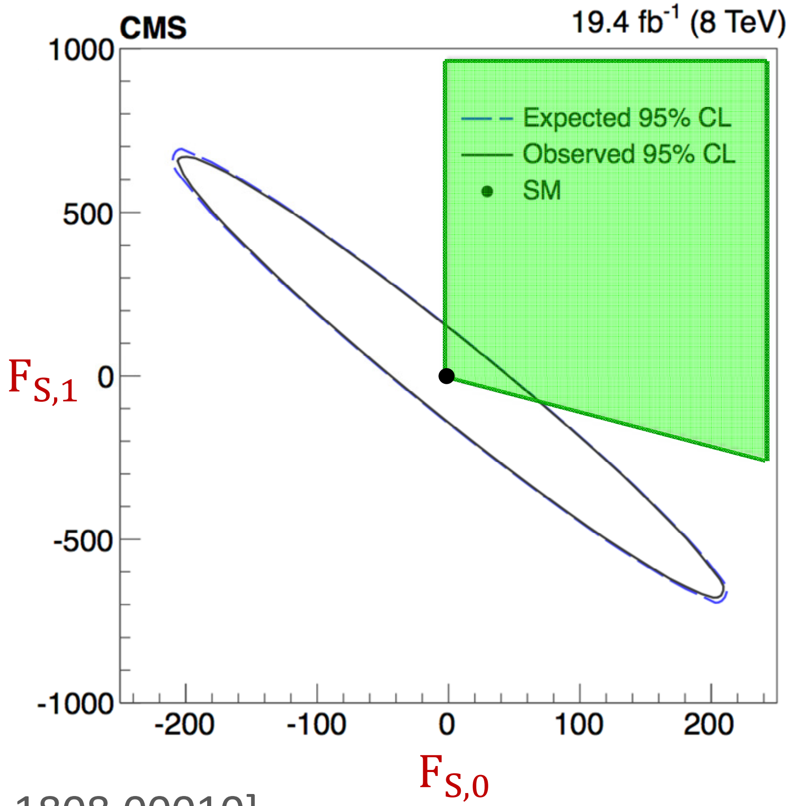
$$\boxed{F_{S,0} > 0}$$

$$F_{S,0}\,\mathrm{Tr}\left[(D_\mu\Phi)^\dagger D_\nu\Phi\right]^2$$

$$F_{S,1}\,\mathrm{Tr}\left[(D_\mu\Phi)^\dagger D^\mu\Phi\right]^2$$

$$\boxed{\begin{aligned} F_{S,0} &> 0 \\ F_{S,0} + F_{S,1} &> 0 \end{aligned}}$$





[Zhang, Zhou, 1808.00010]

Massive Gravity

Cosmological Constant (Problem)

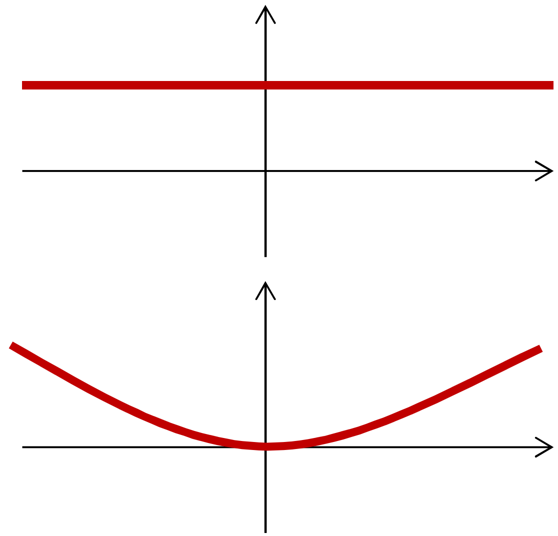
The Universe is accelerating…

$$\int d^4x \, \sqrt{-g} \, (M_P^2 R[g] + \Lambda^4)$$

or

$$\int d^4x \, \sqrt{-g} \, M_P^2 (R[g] - m^2 V(g - \bar{g}))$$

?



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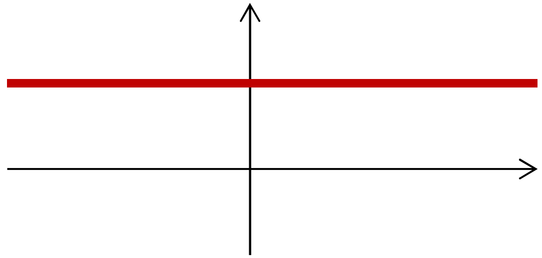

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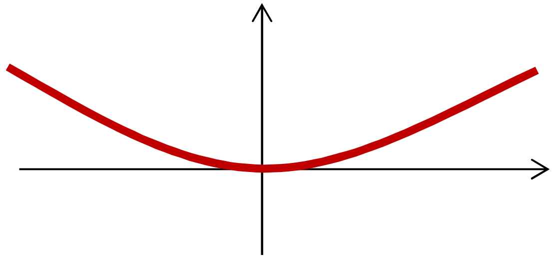

?

$$V(g-\bar{g}) = h^2 + c\, h^3 + d\, h^4 + \cdots$$

Pros

Small m^2 technically natural

Regulates IR divergences

Cons

More free parameters

Strongly coupled at $\Lambda_5 = (m^4 M_P)^{1/5}$

Massive Graviton Interactions

$$\int d^4x \sqrt{-g}\, M_P^2 (R[g] - m^2 V(g - \bar{g}))$$

$$h_{\mu\nu} \to \frac{\partial_\mu \partial_\nu \pi}{m^2 M_P} + \frac{\partial_{(\mu} A_{\nu)}}{m\, M_P} + \frac{h_{\mu\nu}}{M_P}$$

$$V(h) = [h^2] - [h]^2 + c_1[h^3] + c_2[h^2][h] + d_1[h^4] + d_2[h^2]^2$$

$$M_P^2 m^2 V(h) = m^2 h^2 + F^2 + \partial(\dots) + \frac{C_5}{\Lambda_5^5}(\partial\partial\pi)^3 + \frac{C_3}{\Lambda_3^3}(\partial\partial\pi)^2 h + \cdots + \frac{D_5 m^2}{\Lambda_5^{10}}(\partial\partial\pi)^4 + \frac{D_3}{\Lambda_3^6}(\partial\partial\pi)^3 h + \cdots$$

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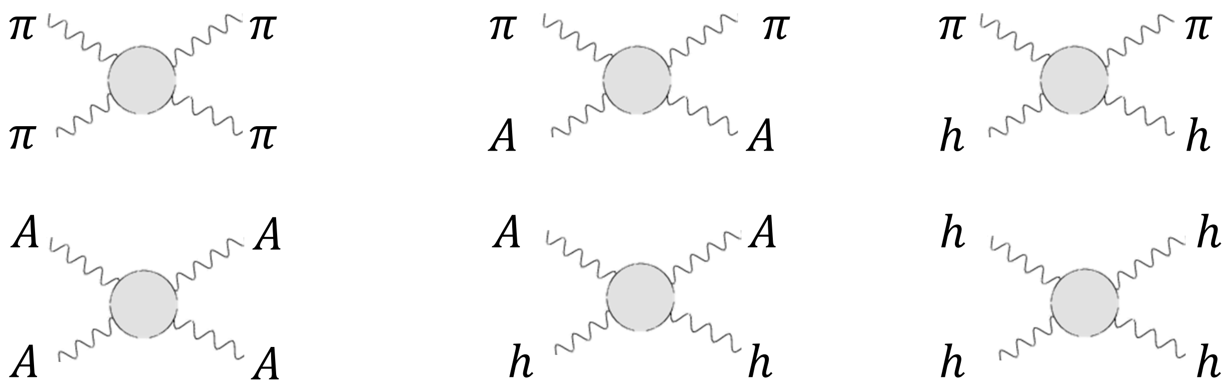
$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{GR}} + \frac{C_5}{\Lambda_5^5}\mathcal{O}_3[\pi, A, h] + \frac{D_5}{\Lambda_5^{10}}\mathcal{O}_4[\pi, A, h] + \frac{C_3}{\Lambda_3^3}\mathcal{O}'_3[\pi, A, h] + \frac{D_3}{\Lambda_3^6}\mathcal{O}'_4[\pi, A, h]$$



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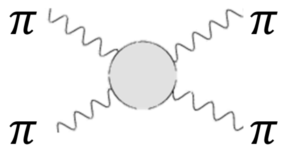
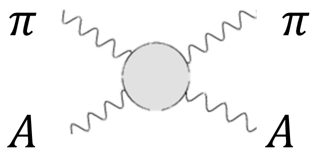
$$+ \frac{C_3}{\Lambda_3^3}\mathcal{O}'_3[\pi, A, h] + \frac{D_3}{\Lambda_3^6}\mathcal{O}'_4[\pi, A, h]$$



$$\boxed{D_5 = 0}$$



$$\boxed{C_5 = 0}$$

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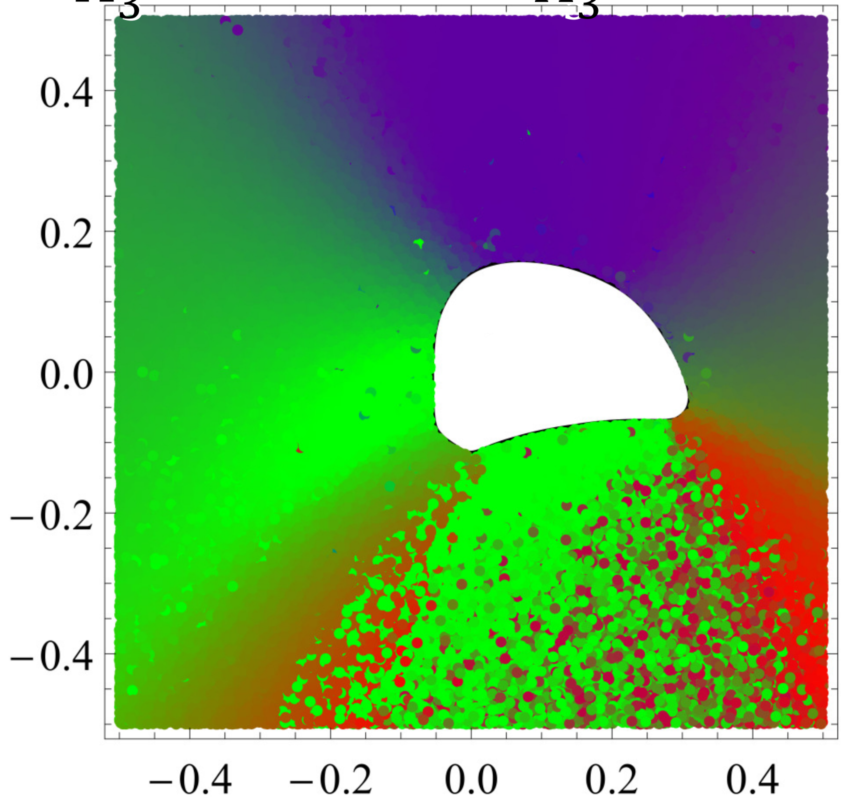
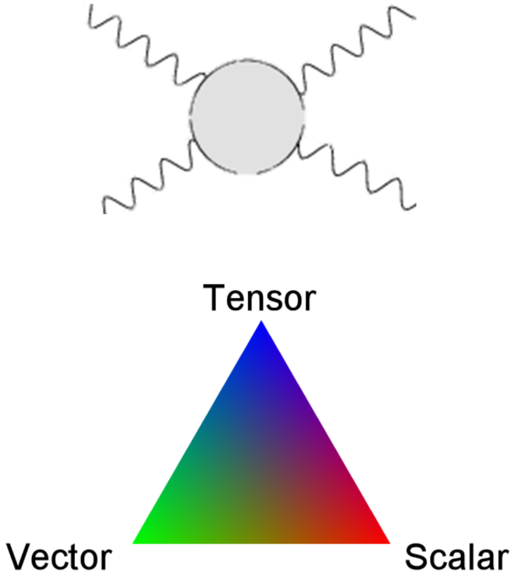
$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{GR}} + \frac{C_3}{\Lambda_3^3}\mathcal{O}'_3[\pi, A, h] + \frac{D_3}{\Lambda_3^6}\mathcal{O}'_4[\pi, A, h]$$

[Cheung, Remmen 1601.04068]





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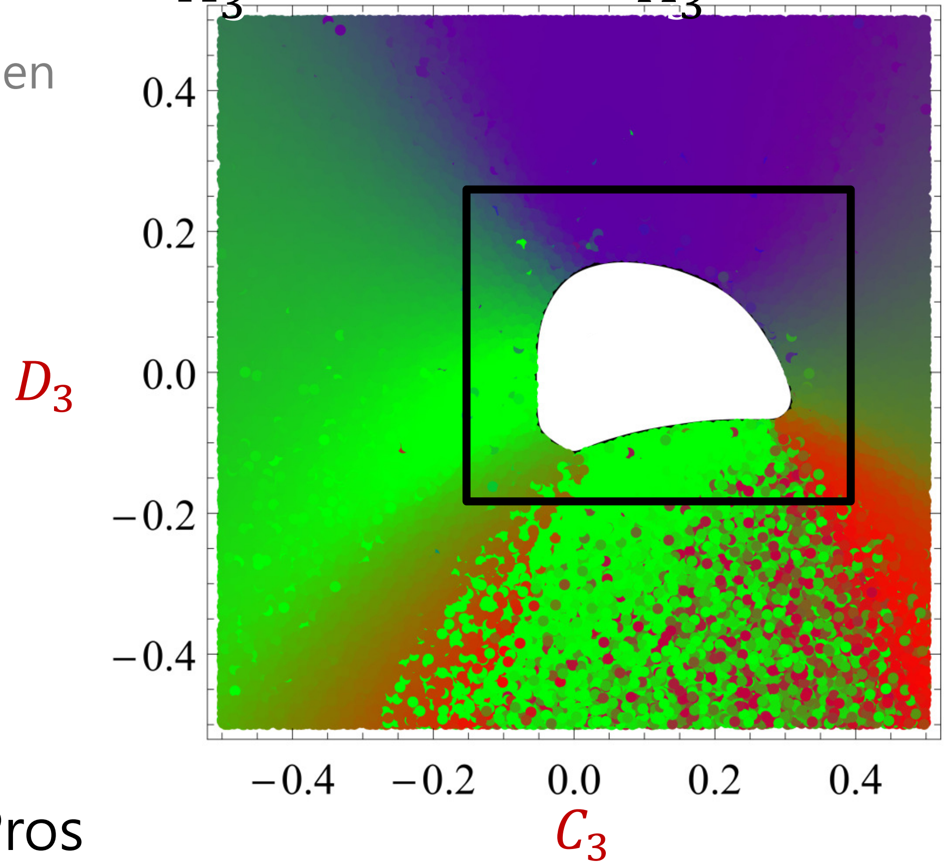

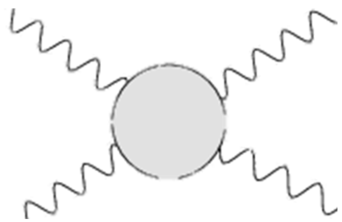

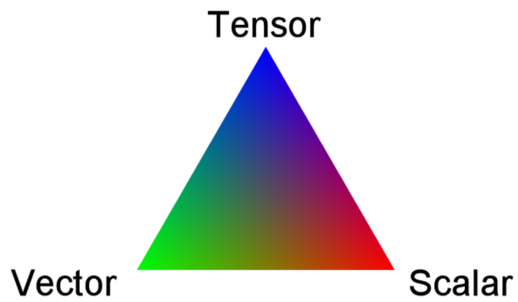

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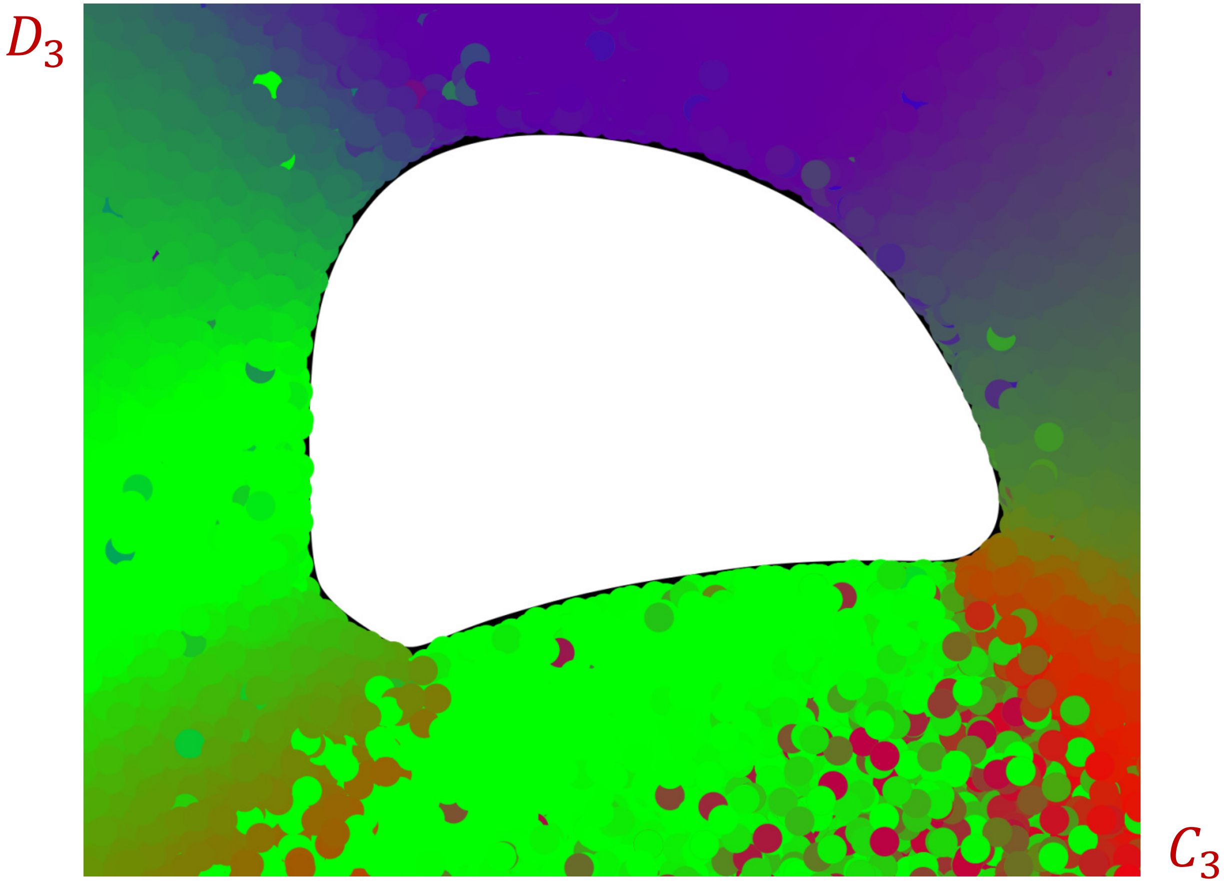

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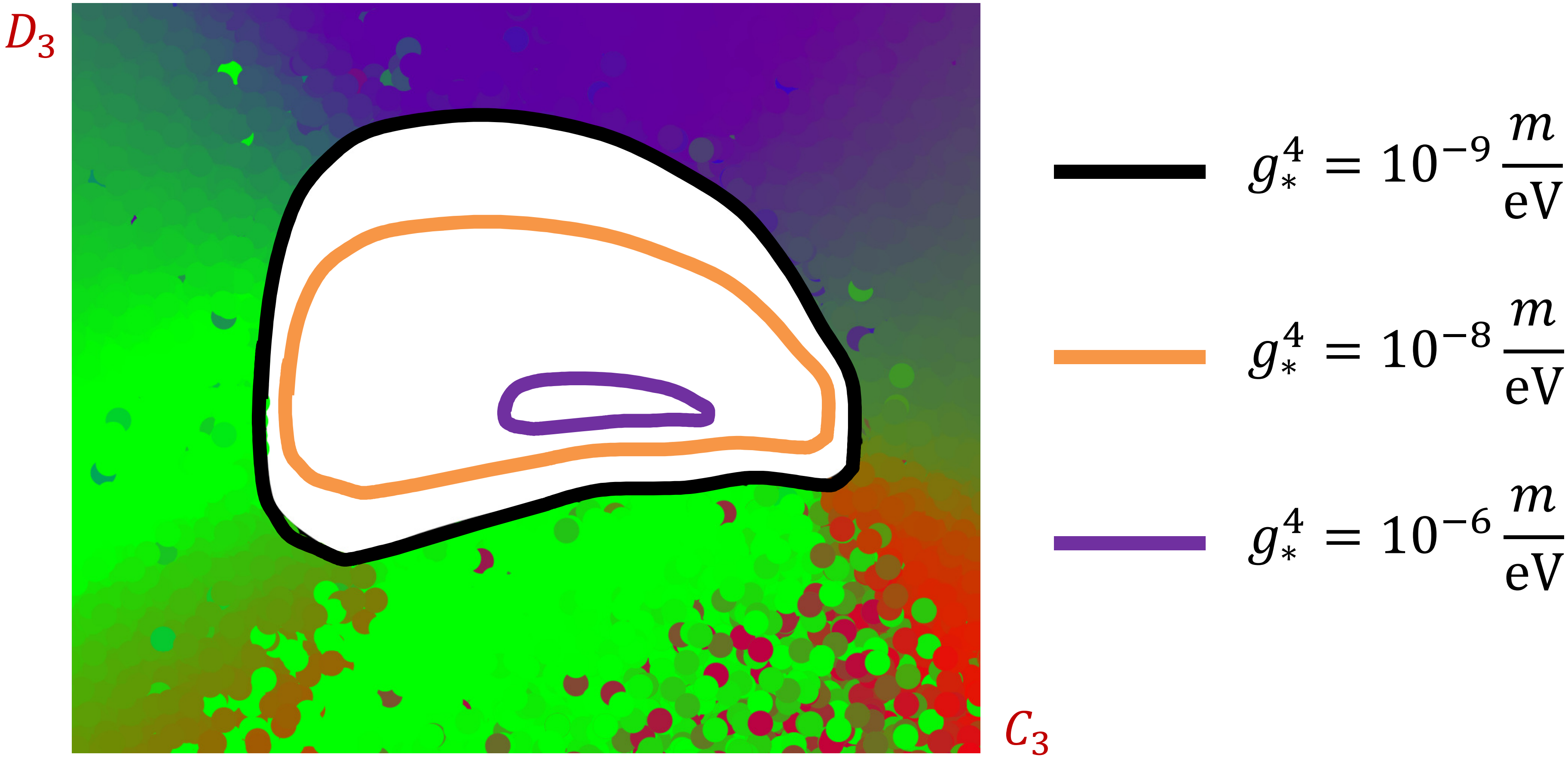
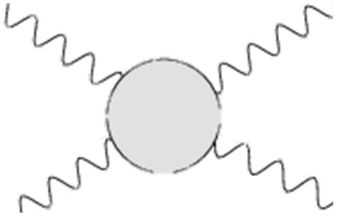
$$\to \frac{1}{g_*^2}\mathcal{L}_{\text{GR}}[g_* h] + \frac{g_* C_3}{\Lambda_3^3}\mathcal{O}'_3[\pi, A, h] + \frac{g_*^2 D_3}{\Lambda_3^6}\mathcal{O}'_4[\pi, A, h]$$



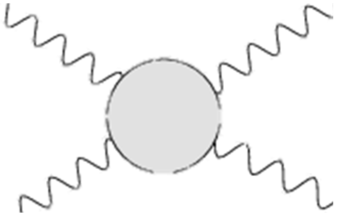
Summary

$$\frac{1}{g_*^2}\mathcal{L}_{\mathrm{EFT}}\left[\frac{g_*\Phi}{M},\frac{\partial}{M};c_n\right]$$



$$\frac{1}{g_*^2}\mathcal{L}_{\text{EFT}}\left[\frac{g_*\Phi}{M}, \frac{\partial}{M}; c_n\right]$$

Positivity Constraints



$$\partial_s^N A(s) \quad > \quad 0$$

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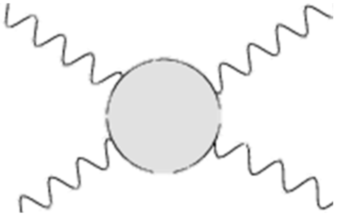

Positivity Constraints

$$\partial_s^N A(s) \quad > \int d\mu\; C_N(s,\mu)\sigma(\mu) > 0$$

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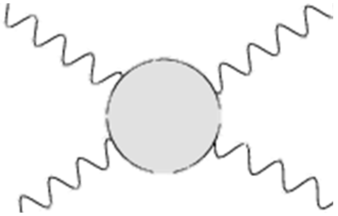

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$$\partial_s^N A(s) > \int d\mu\ C_N(s,\mu)\sigma(\mu) > 0$$

$$\partial_t\partial_s^N A(s,t) > \frac{2N+1}{M^2}\partial_s^N A(s,t)$$

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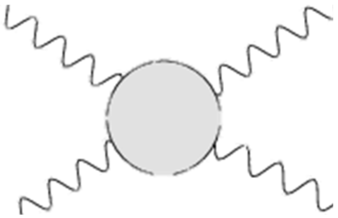

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SMEFT

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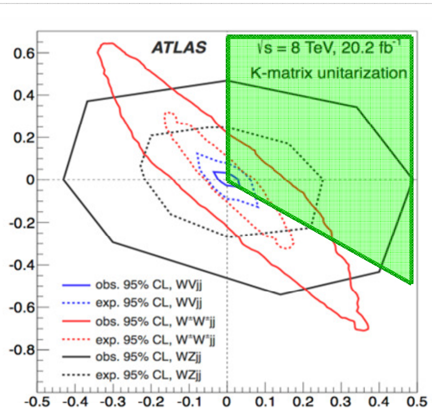

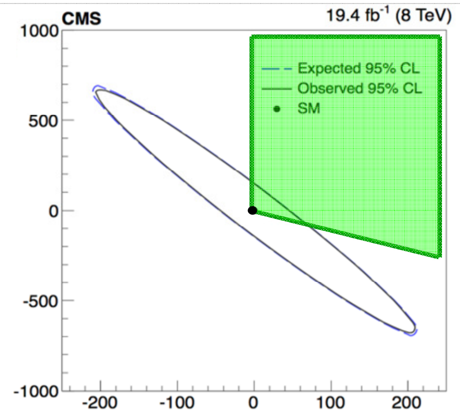

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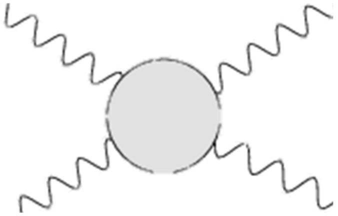

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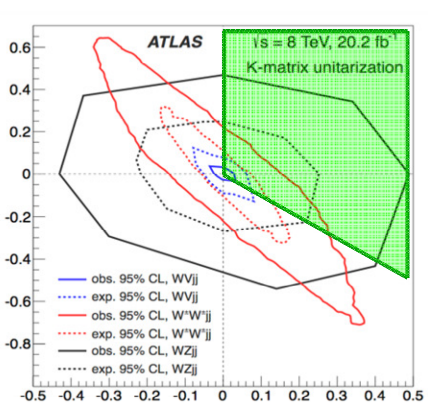
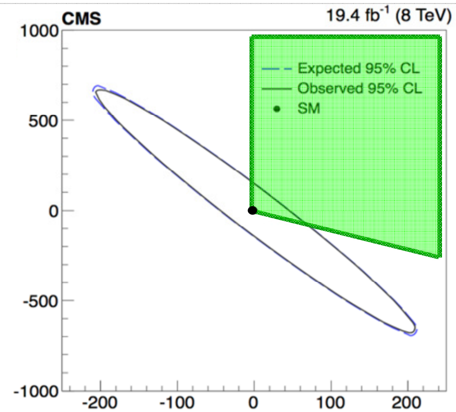
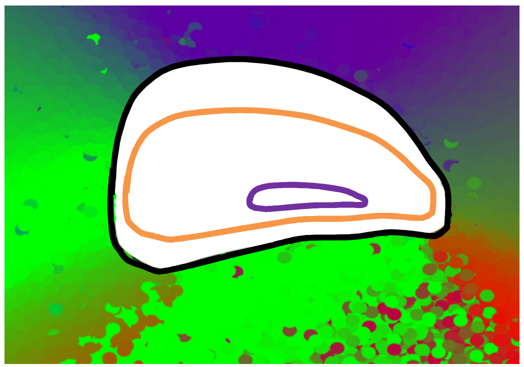




D_3



C_3

$$\boxed{\begin{aligned} C_5 &= 0 \\ D_5 &= 0 \end{aligned}}$$

Massive Gravity

Summary