High-Energy Limit of Mass-Suppressed Amplitudes

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November 30th, 2018

Beauty and Magic of QCD at High Energy

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Topics discussed

- Introduction
  - *high energy limit, Sudakov double logarithms*
  - *power corrections and renormalization group*
- QCD at high energy to $\mathcal{O}(m_q^2/Q^2)$
  - *non-Sudakov double logs in power suppressed amplitudes*
  - *soft quark emission, eikonal color charge nonconservation*
  - *factorization and resummation of non-Sudakov double logs*
  - *light quark effects in Higgs production*
  - *asymptotic formula for the quark form factors*

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  - *factorization and resummation of non-Sudakov double logs*
  - *light quark effects in Higgs production*
  - *asymptotic formula for the quark form factors*
- *Moments of beauty and magic*

Based on

T. Liu, A.A. Penin, Phys.Rev.Lett. **119** *(2017) 262001*

previous work:

T. Liu, A.A. Penin and N. Zerf, Phys.Lett. B **771** *(2017) 492*

A.A. Penin and N. Zerf, Phys.Lett. B **760** *(2016) 816*

A.A. Penin, Phys.Lett. B **745** *(2015) 69*

Sudakov limit

- Sudakov limit
  - *on-shell*
  - *exclusive*
  - *high energy*
  - *fixed angle*
- Sudakov logarithms *(leading power in m_q^2/Q^2)*
  - *each external line gets* $e^{-\frac{\alpha}{4\pi}\frac{C_R}{2}\ln^2(Q^2)}$

    Sudakov (1956); Frenkel, Taylor (1976)
  - *subleading logs exponentiate as well*

    Mueller (1979); Collins (1980); Sen (1981), ...
- What about power suppressed logs?

High energy limit beyond Sudakov approximation

- Logarithmically enhanced power corrections
  - *phenomenologically crucial e.g. bottom loop in Higgs production*
  - *intriguing from QFT point of view*

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- Power corrections and renormalization group

$$\Delta\mathcal{L} = \sum_i C_i O_i\,, \qquad \frac{d}{d\ln\mu} C_i = f_i(C_j)$$

✔	*OPE, large mass expansion*	⇨	*local composite operators*
✔	*threshold nonrelativistic limit*	⇨	*spatially nonlocal potentials*
✘	*Sudakov ultrarelativistic limit*	⇨	*light-cone Wilson lines, etc.*

High energy limit beyond Sudakov approximation

- Light quark mass effects in Higgs production
  K.Melnikov and A.Penin, JHEP **1605***, 172 (2016)*
- Next-to-eikonal soft gluon radiation
  D.Bonocore, E.Laenen, L.Magnea, L.Vernazza, C.D.White, JHEP **1612***, 121 (2016)*,
- Jettiness
  R. Boughezal, X. Liu and F. Petriello, JHEP **1703***, 160 (2017)*
- Event shapes
  I.Moult, I.W.Stewart, G.Vita, H.X.Zhu, JHEP **1808***, 013 (2018)*
- Threshold resummation $(1 - z \to 0)$
  M.Beneke et al., arXiv:1809.10631 [hep-ph]
- *many other recent studies ...*

Tools

- SCET

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- *A concrete hard problem without known solution*

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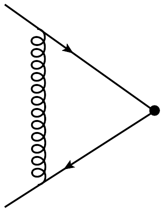
- Asymptotic expansion
  Sudakov parameters
  Ward identities

Sudakov logarithms

- Origin of double logs *(Dirac form factor)*
  - *only vertex correction contribute in a covariant gauge*
  - *gluon is soft and collinear to* p_i
  - *quarks are eikonal*



$$S = \frac{\not p_i - \not l + m_q}{(p_i - l)^2 - m_q^2} \approx -\frac{\not p_i}{2p_i l} \approx \frac{\gamma_\pm}{|\boldsymbol{l}|(\cos\theta \pm 1)}$$

- *Soft and collinear divergences*

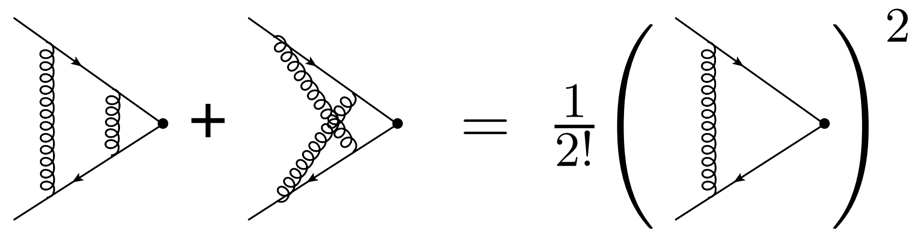
$$\int \frac{\mathrm{d}|\boldsymbol{l}|}{|\boldsymbol{l}|} \to -\frac{1}{\varepsilon} + \ln(Q^2/\mu^2) \qquad \int \frac{\mathrm{d}\cos\theta}{\cos\theta \pm 1} \to \ln(Q^2/m_q^2)$$

Sudakov logarithms

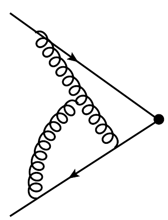
- Eikonal factorization

$$\frac{1}{p_i l_1}\frac{1}{p_i(l_1+l_2)}+\frac{1}{p_i(l_1+l_2)}\frac{1}{p_i l_2}=\frac{1}{p_i l_1}\frac{1}{p_i l_2}$$

- Exponentiation (QED)



➥ $$\text{n-loop}=\frac{(1-\text{loop})^n}{n!}$$

- *in QCD*  *cancels the non-Abelian part by Ward identity*

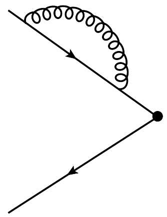
- Sudakov asymptotic result

$$F_D \sim e^{-C_F x}$$

$$(\mu = m_q, \quad x = \frac{\alpha_s}{4\pi}\ln^2(Q^2/m_q^2))$$

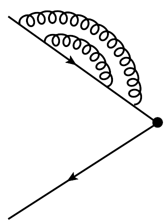
Sudakov logarithms

- Alternative picture: physical gauge



  - *forward radiation suppressed by helicity conservation*
    ➪ *only on-shell self-energies contribute*

  ➥ *universality of Sudakov DL*



  - *ordering of virtual momenta* ➪ *rainbow diagrams*

  ➥ *trivial color structure of Sudakov DL*

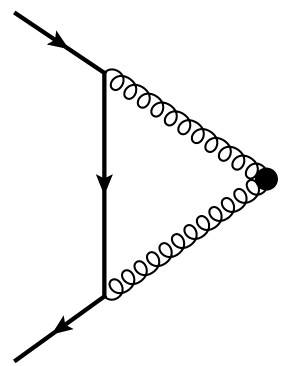
- Alternative picture: evolution equation

$$\frac{\partial}{\partial \ln Q} F = \left[-\Gamma_{cusp} + \ldots\right] F$$

  - *Cusp anomalous dimension* $\Gamma_{cusp} = -\frac{C_F \alpha_s}{\pi} \ln(m_q^2/Q^2)$

Power-suppressed amplitude

- Quark scattering by $(G^a_{\mu\nu})^2$ operator

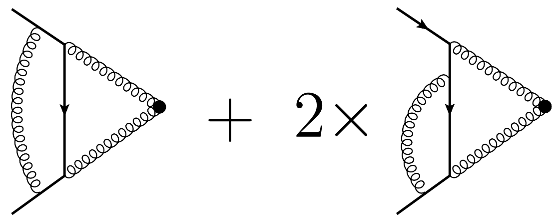


  - *helicity flip* ⇨ *mass suppression*
  - *soft quark* $S \approx m_q/l^2$ *eikonal gluons* $D \approx g_{\mu\nu}/(2p_i l)$

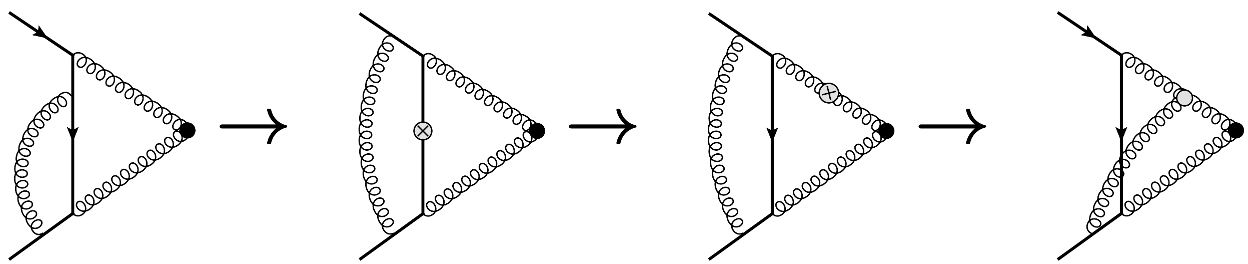
- Power suppressed double logs
  - *soft fermion exchange*
  - *eikonal color charge nonconservation*

Power-suppressed amplitude

- **Radiative corrections (QED)**



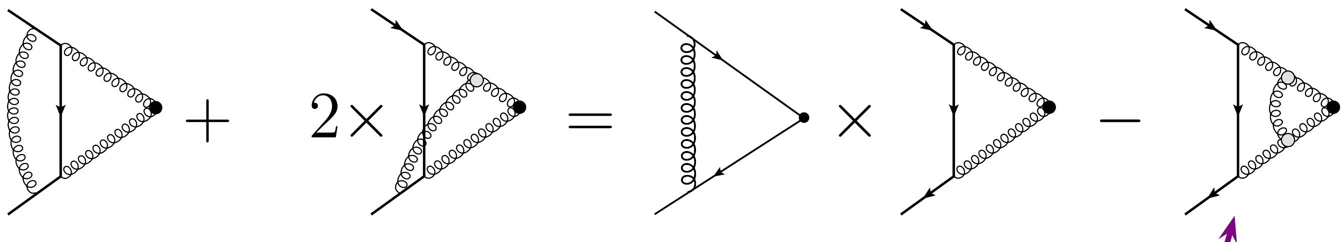
- *Ward identities*



- *crossed vertex:* $S(l) \to S(l) - S(l + l_g^+)$
- *3-photon vertex:* $2e_q p_1^\mu$, *where* e_q *is the quark charge*

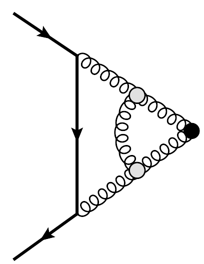
Power-suppressed amplitude

- **Sudakov logs factorization**



missing diagram to complete Sudakov factorization

- **Non-Sudakov logs**



$\propto e_q^2$

  - *multiple soft photons exponentiate*

Power-suppressed amplitude

- In QCD $$e_q^2 \to C_F - C_A$$

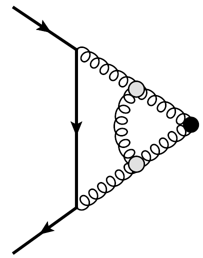
  ➥ *measures the eikonal color charge nonconservation*

- Factorization formula

$$\mathcal{G} = Z_q^2 \, g(-z) \, \mathcal{G}^{(0)}$$

  - *quark Sudakov factor* $Z_q^2 = \exp\left[-C_F\left(\frac{\alpha_s}{2\pi}\frac{\ln(m_q^2/Q^2)}{\varepsilon} + x\right)\right]$
  - *non-Sudakov double-logarithmic function* $g(-z)$*,*
    where $z = (C_A - C_F)x, \; x = \frac{\alpha_s}{4\pi}\ln^2(Q^2/m_q^2), \; g(0) = 1$

Evaluation of $g(z)$

$g(-z) \sim$ 

- *New variables*
  - *Sudakov parameters* $l = up_1 + vp_2 + l_\perp$
  - *hypercube coordinates*

    $\eta = \ln v / \ln(m_q^2/Q^2),\ \xi = \ln u / \ln(m_q^2/Q^2),\ 0 < \eta,\ \xi < 1$
  - *strict ordering* $\eta_i < \eta_j,\ \ldots$, *onshell condition* $\eta_i + \xi_i < 1$
- *Integral representation*

$$g(z) = 2\int_0^1 \mathrm{d}\xi \int_0^{1-\xi} \mathrm{d}\eta \ \ e^{2z\eta\xi}$$

soft gluon loop exponent →

Result

- Exact formula

$$g(z) = {}_2F_2\,(1,1;3/2,2;z/2)$$

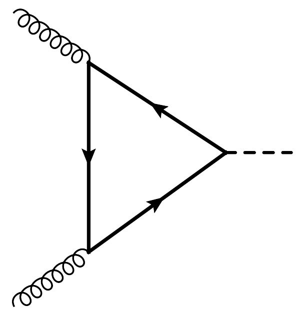
- Asymptotic behavior at $z \to +\infty$

$$g(-z) \sim \frac{\ln(2z)+\gamma_E}{z}, \quad g(z) \sim \left(\frac{2\pi e^z}{z^3}\right)^{1/2},$$

- The amplitude $\mathcal{G} \propto g(-z)$
  - *in QCD* $z > 0$ ⇨ *power suppressed*
  - *in QED* $z < 0$ ⇨ *exponentially enhanced*

Bottom loop in $gg \to H$

- Leading contribution

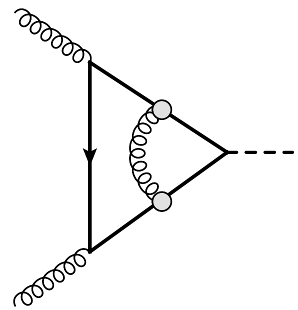


$$\propto \ln^2(m_H^2/m_b^2)\frac{m_b^2}{m_H^2}$$

  - *effective expansion parameter* $\alpha_s \ln^2(m_H^2/m_b^2) \sim 40\alpha_s$
  - ⮑ *resummation is mandatory (main source of uncertainty)*
  - *factorization structure similar to* $\mathcal{G}$

Moment of beauty

- Non-Sudakov logs



  - *inverted color flow:* $C_F \leftrightarrow C_A$, $z \to -z$

- Factorization formula

$$\mathcal{M}^{b}_{gg\to H} = Z_g^2\, g(z)\, \mathcal{M}^{b\,(0)}_{gg\to H}$$

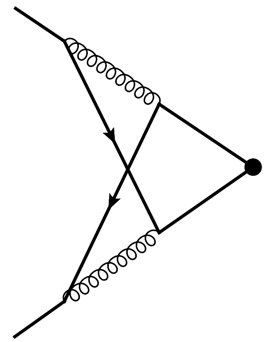
  - *gluon Sudakov factor* $Z_g^2 = \exp\left[-\frac{C_A}{\varepsilon^2}\frac{\alpha_s}{2\pi}\frac{\mu^{2\varepsilon}}{m_H^{2\varepsilon}}\right]$
  - $g(z)$ *exponentially enhanced in QCD*

Dirac form factor beyond the leading power

- High-energy expansion

$$F_D = Z_q^2 \left(1 + \frac{m_q^2}{Q^2} F_D^{(1)} + \ldots\right),$$

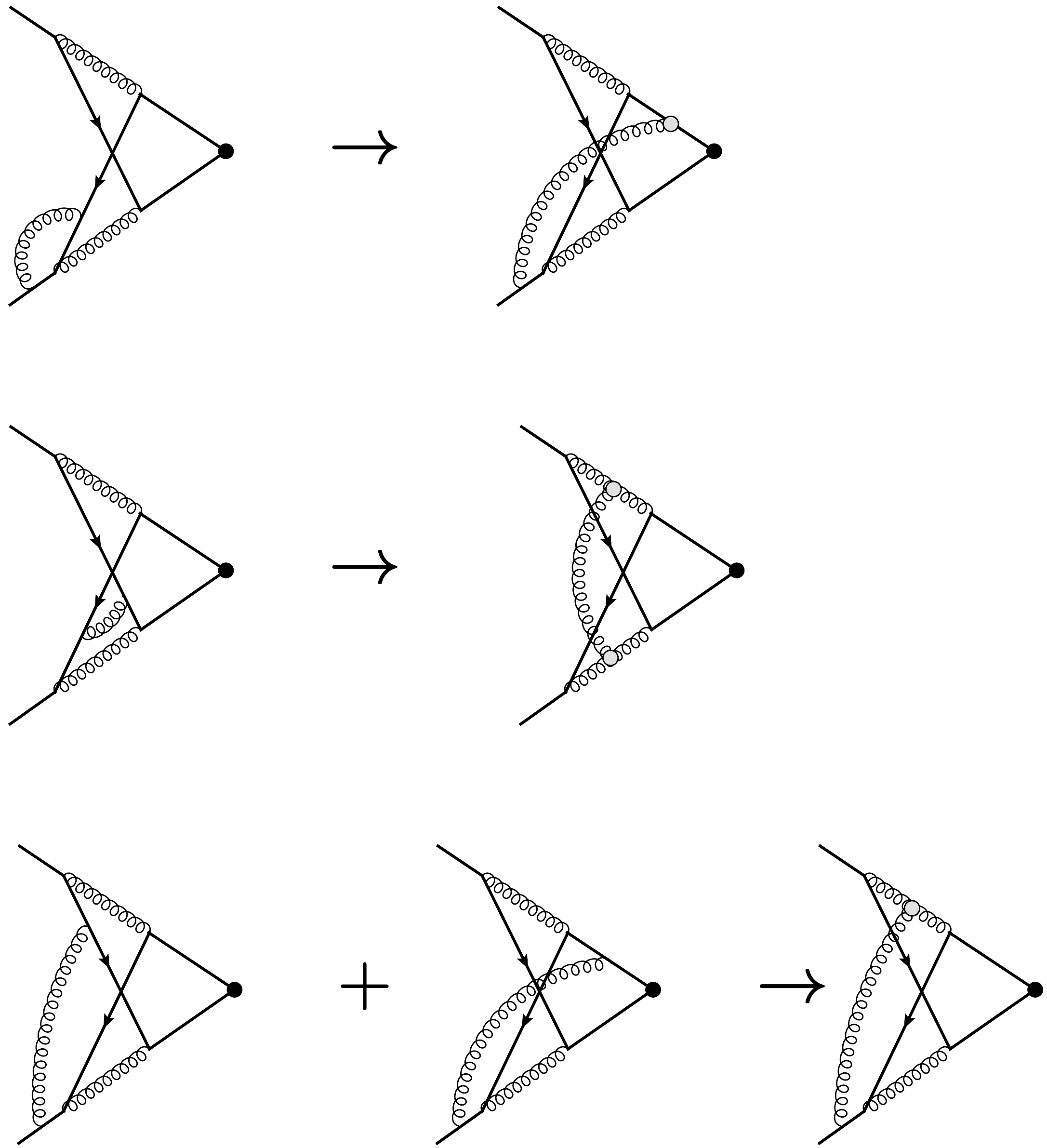
- Double logarithms in $F_D^{(1)}$



  - *induced by soft quark pair exchange*

A.A. Penin, Phys.Lett. B **745** (2015) 69

Diagram transformation



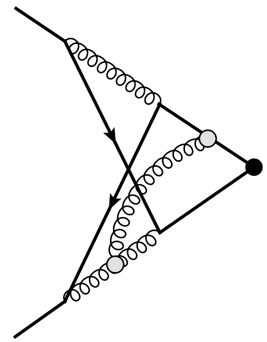
Dirac form factor beyond the leading power

- All-order result

$$F_1^{(1)} = \left[\frac{C_F(C_A-2C_F)}{6}x^2\right] f(-z)$$

two-loop contribution

- non-Sudakov double-logarithmic function

$$f(-z) \sim$$  $$+ \quad \textit{symmetric}$$

Result for $f(z)$

- Integral representation

$$f(z) = 12\int_0^1 \mathrm{d}\eta_1 \int_{\eta_1}^1 \mathrm{d}\eta_2 \int_0^{1-\eta_2} \mathrm{d}\xi_2 \int_{\xi_2}^{1-\eta_1} \mathrm{d}\xi_1 \, e^{2z\eta_1(\xi_1-\xi_2)} e^{2z\xi_2(\eta_2-\eta_1)}$$

- Taylor series $f(z) = \sum_n c_n z^n$
  - *high-order behavior* $c_n \sim \frac{\ln n}{n! 2^n n^{5/2}}$

- Asymptotic behavior at $z \to +\infty$

$$f(-z) \sim C_- \left(\frac{\ln z}{z}\right)^2, \quad f(z) \sim C_+ \ln z \left(\frac{e^z}{z^5}\right)^{1/2}$$

$$C_- = 3.6\ldots, C_+ = 14.8\ldots$$

➥ *exponentially enhanced in QED, power suppressed in QCD*

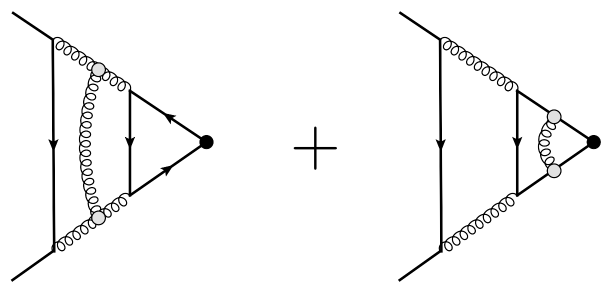
Moment of magic

- Scalar form factor

$$F_S^{(1)} = \left[-\frac{C_F T_F}{3} x^2\right] f_S(-z)$$

- non-Sudakov double-logarithmic function

$$f_S(-z) \sim$$



- *Amazing universality* $\quad f_S(z) = f(z)$

➥ *the same function for axial and pseudoscalar form factors*

Higgs phenomenology

- Higgs production in gluon fusion
  - *no jets, veto on soft emission* $\sim m_b$

$$\delta\sigma_{gg\to H} = -3\rho\ln^2\rho\left(1+\frac{z}{6}+\frac{z^2}{45}+\frac{z^3}{420}+\ldots\right)\sigma_{gg\to H}$$

$$z = \frac{\alpha_s}{4\pi}(C_A - C_F)\ln^2\rho \approx 1.2 \qquad 3\rho\ln^2\rho \approx 20\% \qquad \rho = m_b^2/m_H^2$$

  - *three-loop correction approximately* -0.6%

Summary

- The first LL resummation beyond leading power in QCD

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  - *relations between different amplitudes and gauge groups*

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  - *universality* ⇨ *two functions* $g(z)$ *and* $f(z)$
  - *relations between different amplitudes and gauge groups*
- Bottom mass effect in $gg \to H$
  - -0.6% *to exclusive cross section*