

# Lattice investigation of charmed and bottom hadrons

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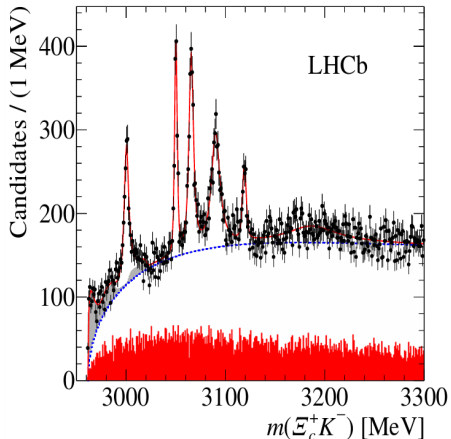
Regensburg, Germany

14<sup>th</sup> November 2019

Collaborators: Nilmani Mathur, HSC and ILGTI.

Based on PRL 119(2017), 042001; PRL 121(2018), 202002;  
PRD 99(2019), 031501; PRD 99 (2019), 034507.

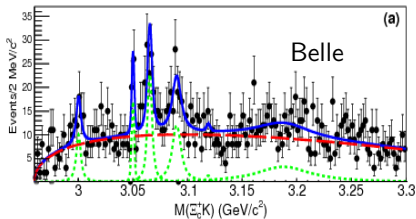
# LHCb discovery of excited $\Omega_c^0$ baryons: 2017



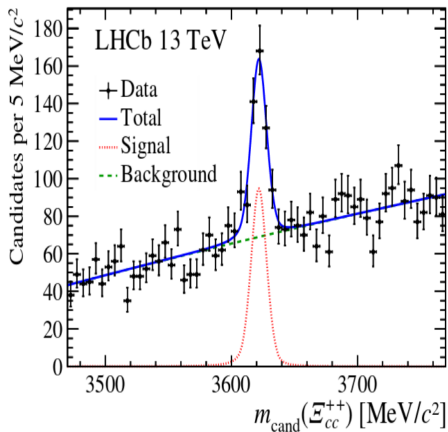
Aaij *et al.* (LHCb) 1703.04639

Confirmation by Belle : Yelton *et al.* (Belle) 1711.07927

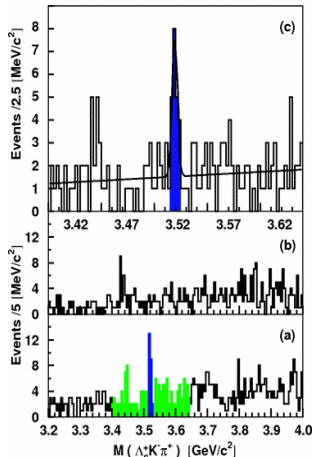
Resonance	Energy	Width	Q.no.
$\Omega_c^0$	2695(2)	-	$1/2^+$
$\Omega_c^0(2770)$	2766(2)	-	$3/2^+$
$\Omega_c^0(3000)$	3000(1)	4.5(1)	?
$\Omega_c^0(3050)$	3050(1)	1(-)	?
$\Omega_c^0(3066)$	3066(1)	3.5(-)	?
$\Omega_c^0(3090)$	3090(1)	8.7(1)	?
$\Omega_c^0(3119)$	3119(1)	1(1)	?



# A year of heavy baryons (2017) : $\Xi_{cc}$



Aaij *et al.* (LHCb) 1707.01621



Mattson *et al.* (SELEX) hep-ex/0208014

Other papers from LHCb on properties of  $\Xi_{cc}$  : 1905.02421; 1807.01919; 1806.02744

# Doubly heavy tetraquarks : possibly stable system

PRL **119**, 202001 (2017)

PHYSICAL REVIEW LETTERS

week ending  
17 NOVEMBER 2017

## Discovery of the Doubly Charmed $\Xi_{cc}$ Baryon Implies a Stable $bb\bar{u}\bar{d}$ Tetraquark

Marek Karliner<sup>1,\*</sup> and Jonathan L. Rosner<sup>2,†</sup><sup>1</sup>*School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel*<sup>2</sup>*Enrico Fermi Institute and Department of Physics, University of Chicago, 5620 South Ellis Avenue, Chicago, Illinois 60637, USA*

(Received 28 July 2017; published 15 November 2017)

Heavy Quark Symmetry implies for a sufficiently heavy “heavy” quark, the doubly heavy tetraquarks should be stable to strong decays.

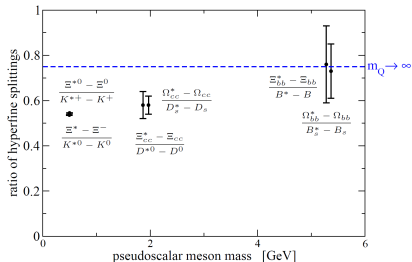
Carlson *et al.* PRD37 744 (1988); Eichten and Quigg 1707.09575

For a heavy-light meson-like system of doubly charm baryons

$$\frac{M(\Xi_{QQ}^*) - M(\Xi_{QQ})}{M(V_Q) - M(PS_Q)} \rightarrow \frac{3}{4}$$

Brambilla *et al* hep-ph/0506065Data : Brown *et al* 1409.0497

Figure : Lewis CHARM-2018



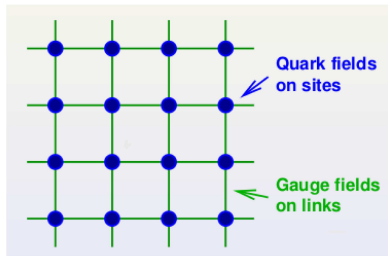
# Heavy hadrons

- Clean signature for many observables, even though rarity in events.
- A large number of discoveries in heavy hadron sector over the past decade.
  - usual quarkonium mesons like  $h_c(1P)$ ,  $h_b(1P)$ ,  $h_b(2P)$ ,  $\psi(1^3D_2)$ .
  - heavy baryons like  $\Xi_{cc}$  and highly excited states of  $\Omega_c$  (LHCb).
  - new beauty baryons like  $\Sigma_b^\pm(6097)$ ,  $\Lambda_b(6146)$  and  $\Lambda_b(6152)$  by LHCb.
  - tetraquarks (LHCb, Belle) and pentaquarks (LHCb).
  - many other states with less theoretical understanding (XYZ).
- Charmed-bottom hadrons : largely unexplored unlike others.  
 Discovered :  $B_c(1S, 0^-)$  at 6275(1) MeV  
 $B_c(2S, 0^-)$  at 6842(6) MeV (ATLAS)  
 $B_c(2S, 0^-)$  at 6872(6) MeV and  $B_c(2S, 1^-)$  (LHCb) at 6841(6) MeV\*

# Lattice QCD : theoretical prospects

LQCD : A non-perturbative, gauge invariant regulator for the QCD path integrals.

- Quarks lives on sites
- Gauge fields lives on links
- Lattice spacing : UV cut off
- Lattice size : IR cut off



Discretization  $\Rightarrow$  Finite number of degrees of freedom

$\Rightarrow$  Infinite dimensional path integrals  $\rightarrow$  finite dimensional integrals.

Employ Monte Carlo importance sampling methods on Euclidean metric for numerical studies.

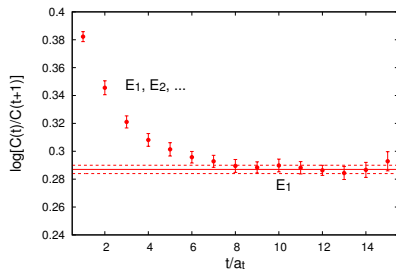
# QCD spectrum from Lattice QCD

- Aim : to extract the physical states of QCD.
- Euclidean two point current-current correlation functions

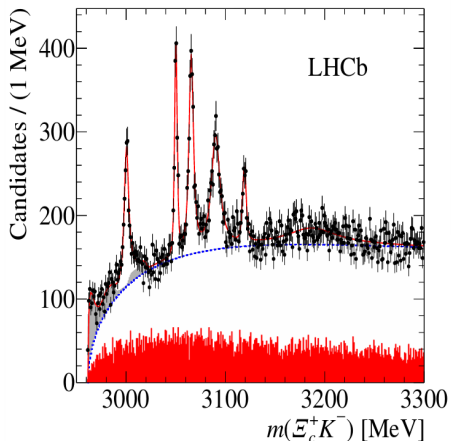
$$C_{ji}(t_f - t_i) = \langle 0 | \Phi_j(t_f) \bar{\Phi}_i(t_i) | 0 \rangle = \sum_n \frac{Z_i^{n*} Z_j^n}{2m_n} e^{-m_n(t_f - t_i)}$$

where  $\Phi_j(t_f)$  and  $\bar{\Phi}_i(t_i)$  are the desired interpolating operators and  $Z_j^n = \langle 0 | \Phi_j | n \rangle$ .

- Effective mass defined as  $\log\left[\frac{C(t)}{C(t+1)}\right]$



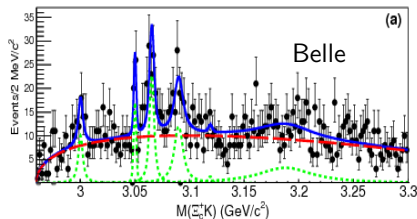
- The ground states : from the exponential fall off at large times.  
Non-linear fitting techniques.

LHCb discovery of excited  $\Omega_c^0$  baryons: 2017

Aaij *et al.* (LHCb) 1703.04639

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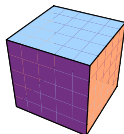




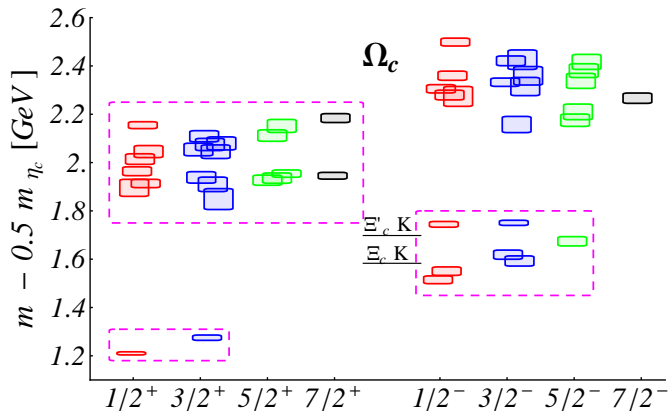
# Excited baryon spectroscopy

- Aim : Extraction of highly excited states.  
Local operators  $\rightarrow$  low lying states.  
Extended operators  $\rightarrow$  Radial and orbital excitations.
- Continuum operators with well defined quantum nos.  
Reduce/subduce into the irreps of the reduced symmetry.

 $O(3)$ 

 $O_i^{J^{PC}, M}$ 
 $O_h$ 

 $O_{i, \Lambda}^{[J^{PC}]}$ 

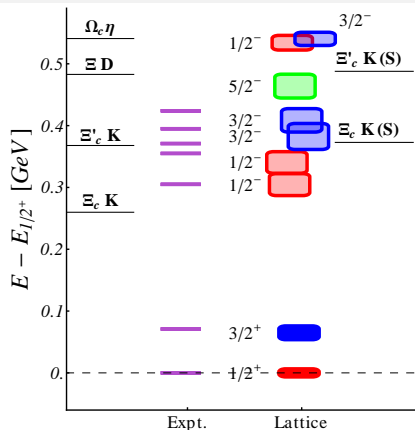
- Variational analysis of correlation matrices,  $C_{ji}$ .
- Rigorous spin identification procedure using operator state overlaps,  $Z$ .

Results :  $\Omega_c$  spectrum (L1)

MP & Mathur 1704.00259, Charm 2013 (1311.4806), Charm 2015 (1508.07168).

Magenta ellipses : States with strong non-relativistic content.  
The low lying spectrum same as non-relativistic expectations.

# Experiment vs. lattice predictions (L1)



Energy	Expt.	Lattice
$\Delta E_{\Omega_c^0(3119)}$	422(1)	464(20)
$\Delta E_{\Omega_c^0(3090)}$	395(1)	409(19)
$\Delta E_{\Omega_c^0(3066)}$	371(1)	383(21)
$\Delta E_{\Omega_c^0(3050)}$	355(1)	341(18)
$\Delta E_{\Omega_c^0(3000)}$	305(1)	304(17)
$\Delta E_{\Omega_c^0(2770)}$	70.7(1)	65(11)
$E_{\Omega_c^0} - \frac{1}{2} E_{\eta_c}$	1203(2)	1209(7)

Spin  $1/2, 3/2, 5/2$

$\Omega_{ccc}$  : HSC 1307.7022

$\Xi_{cc}$  and  $\Omega_{cc}$  : HSC 1502.01845

On anisotropic  $N_f = 2 + 1$  lattices  
 $L \sim 1.9 \text{ fm}$ ,  $a_t m_c = 0.114$   
 and  $m_\pi = 391 \text{ MeV}$

Edwards *et al.* 0803.3960

Here  $\Delta E = E - E_{\Omega_c^0}$ .  
 The new states correspond to the excited  $p$ -wave excitations.

## Heavy quarks on the lattice

Fermion action:  $\bar{\psi}(\gamma \cdot \mathbf{D} + \mathbf{m})\psi$

Only dimensionless quantities defined on the lattice

Lattice fermion action:  $\bar{\psi}(\mathbf{x})(\gamma \cdot \mathbf{D}_L + \mathbf{am})\psi(\mathbf{x})$

Discretization  $\rightarrow \mathbf{D}_L$ ; clover, staggered, overlap, domain-wall, etc.

Discretization errors on observables  $\mathcal{O}(\mathbf{am})$ .

Charm quarks:  $m_c \sim 1.275$  GeV

$$am_c = 0.5 \Rightarrow a \sim 0.075 \text{ fm}$$

$$am_c = 0.3 \Rightarrow a \sim 0.046 \text{ fm}$$

Bottom quarks:  $m_b \sim 4.66$  GeV

$$am_b = 0.5 \Rightarrow a \sim 0.021 \text{ fm}$$

$$am_b = 0.3 \Rightarrow a \sim 0.013 \text{ fm}$$

## Another lattice calculation: L2

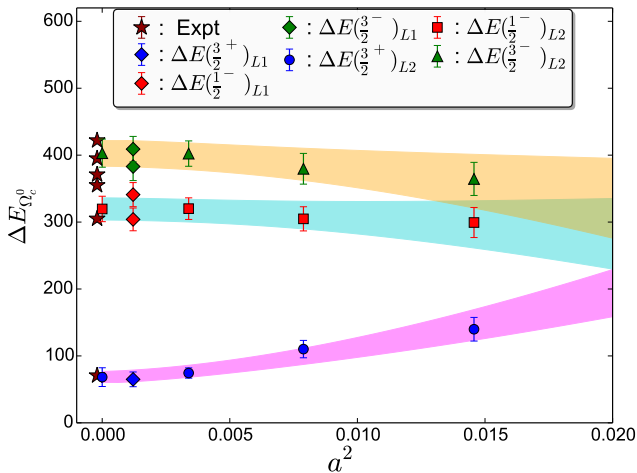
- State-of-the-art ensembles :  $N_f = 2+1+1$  HISQ (MILC)  
 $a = 0.1207(11), 0.0888(8)$  and  $0.0582(5)$  *fm*.  
 MILC Collaboration 1212.4768
- Chiral fermion action for quark masses from light to charm.  
 Exact chiral symmetry at finite lattice spacing. No  $\mathcal{O}(am)$  errors.
- Heavy quark mass tuning Dowdall *et al.* 1110.6887  
 $\Delta E_{hfs}^{1S, \bar{c}c} = 115(2)(3)$  MeV (*Lattice*) &  $\Delta E_{hfs}^{1S, \bar{c}c} = 114$  MeV (*Expt*).
- We work with energy splittings

$$\Delta M_H = [M_H^L - n_c \overline{1S}_c / 2] a^{-1}.$$

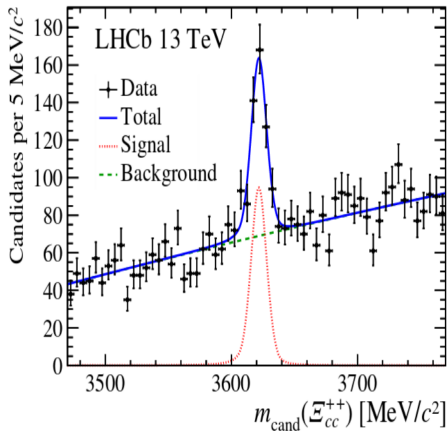
- Continuum extrapolation fit forms  $Q^f = A + a^2 B$ ,  $L^f = A + a^2 \log(a) B$ .
- We extract the mass of hadrons from

$$M_H^c = \Delta M_H^c + n_c (\overline{1S}_c)_{phys} / 2.$$

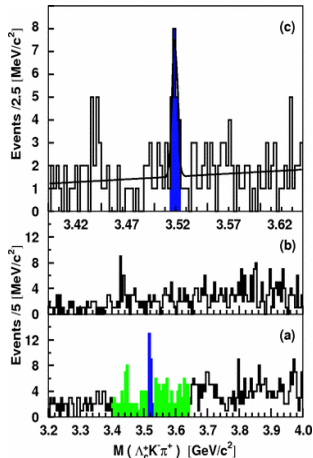
# Comparison between two lattice determinations



# A year of heavy baryons (2017) : $\Xi_{cc}$

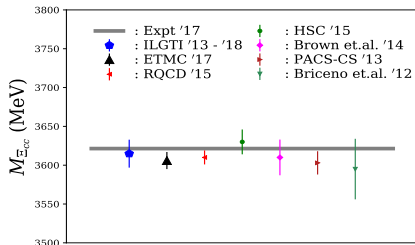
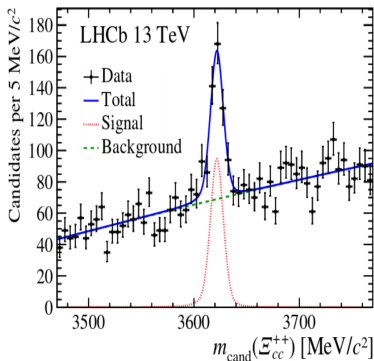


Aaij *et al.* (LHCb) 1707.01621



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$\Xi_{cc}$  from lattice QCDAaij *et al.* (LHCb) 1707.01621Alexandrou *et al.* (ETMC) 1704.02647Briceño *et al.* 1207.3536Namekawa *et al.* (PACS) 1301.4743MP *et al.* (HSC) 1502.01845 (L1)Brown *et al.* 1409.0497Pérez-Rubio *et al.* (RQCD) 1503.08440

ILGTI 1807.00174, 1312.3050 (L2)

 $\Xi_{cc}$  isospin splitting (LQCD), 2.16(11)(17) MeV : BMW 1406.4088SELEX measurement (3519 MeV) : Mattson *et al.* hep-ex/0208014



# Doubly charmed baryons from lattice QCD

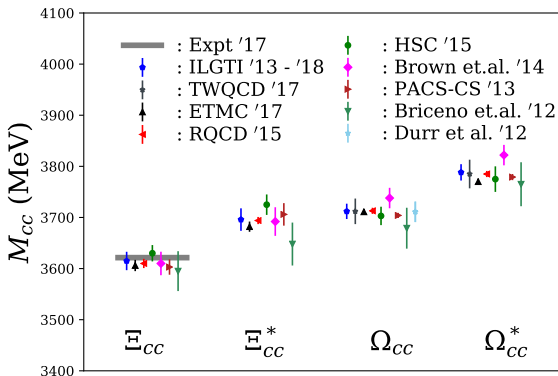


Figure from arXiv:1905.09651 [hep-lat]

$\Omega_{CC}$ : probably the next doubly charm baryon to be observed in LHCb.

$\Xi_{CC}^*$ : radiative decay might be predominant over strong decays.

So might be difficult for LHCb to discover.

## Singly charmed baryons from lattice QCD

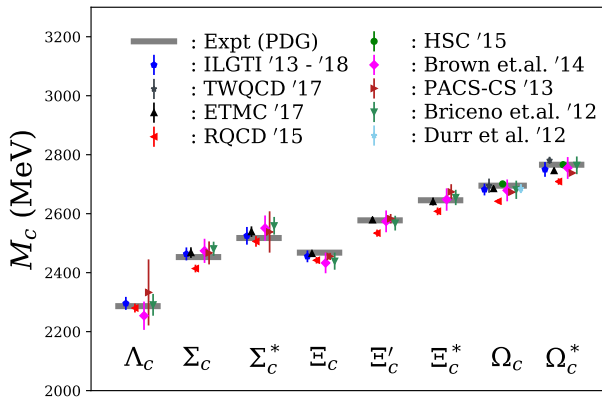


Figure from arXiv:1905.09651 [hep-lat]

# Charmed-bottom hadrons from lattice QCD: L2

- State-of-the-art ensembles :  $N_f = 2+1+1$  HISQ (MILC)  
 $24^3 \times 64$ ,  $32^3 \times 96$  and  $48^3 \times 144$   
 $a = 0.1207(11)$ ,  $0.0888(8)$  and  $0.0582(5)$  fm.

MILC Collaboration 1212.4768

- Chiral fermion action for quark masses from light to charm.  
 Exact chiral symmetry at finite lattice spacing. No  $\mathcal{O}(am)$  errors.
- Bottom quarks realized using NRQCD formulation.  
 Includes terms in the Hamiltonian through  $\mathcal{O}(\alpha\nu_b^4)$ .

HPQCD Collaboration 1110.6887, Hammant *et al* 1303.3234.

- Heavy quark mass tuning Dowdall *et al.* 1110.6887

$$\Delta E_{hfs}^{1S, \bar{c}c} = 115(2)(3) \text{ MeV and } \Delta E_{hfs}^{1S, \bar{b}b} = 63(3)(5) \text{ MeV} \quad \text{Lattice}$$

$$\Delta E_{hfs}^{1S, \bar{c}c} = 114 \text{ MeV and } \Delta E_{hfs}^{1S, \bar{b}b} = 62(1) \text{ MeV} \quad \text{Expt.}$$

# Charmed-bottom hadrons from lattice QCD: L2

- We work with energy splittings and dimensionless ratios

$$\Delta M_H = [M_H^L - n_b \overline{1S}_b/2 - n_c \overline{1S}_c/2] a^{-1} \quad \text{and}$$

$$R_H = \frac{M_H^L - n_b \overline{1S}_b/2}{M_{B_c(0^-)}^L - n_b \overline{1S}_b/2}.$$

- Chiral extrapolation ( $\Xi_{bc}$ ) using  $A + m_\pi^2 B$  and other fancy chiral extrapolation forms as in Brown *et al* 1409.0497.
- Continuum extrapolation fit forms  $Q^f = A + a^2 B$ ,  $C^f = A + a^3 B$ ,  $L^f = A + a^2 \log(a) B$ .
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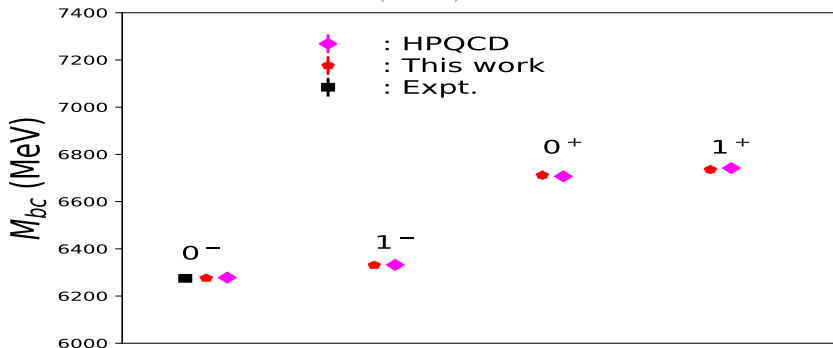
$$M_H^c = \Delta M_H^c + n_b (\overline{1S}_b)_{phys}/2 + n_c (\overline{1S}_c)_{phys}/2 \quad \text{and}$$

$$M_H^c = R_H^c \times (M_{B_c(0^-)} - n_b \overline{1S}_b/2)_{phys} + n_b (\overline{1S}_b)_{phys}/2.$$

# $B_c$ meson results (L2)

Hadrons	Lattice	HPQCD	Wurtz <i>et al</i>	Experiment
$B_c(0^-)$	6276(3)(6)	6278(9)	-	6274.9(8)
$B_c^*(1^-)$	6331(4)(6)	6332(9)	6332.5(3)	?
$B_c(0^+)$	6712(18)(7)	6707(17)	6711(2)	?
$B_c(1^+)$	6736(17)(7)	6742(16)	6752(2)	?

Mathur, MP, Mondal 1806.04151; Dowdall *et al.* (HPQCD) 1207.5149; Wurtz *et al.* 1505.04410



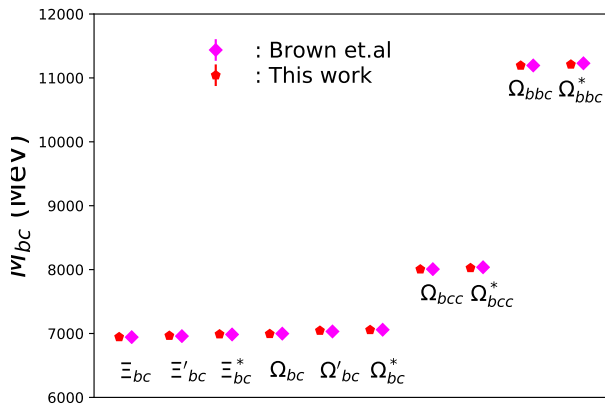
## bc baryons: L2 Vs other lattice results

Hadrons	Lattice	Brown <i>et al</i>	Experiment
$\Xi_{cb}(cbu)(1/2^+)$	6945(22)(14)	6943(33)(28)	?
$\Xi'_{cb}(cbu)(1/2^+)$	6966(23)(14)	6959(36)(28)	?
$\Xi_{cb}^*(cbu)(3/2^+)$	6989(24)(14)	6985(36)(28)	?
$\Omega_{cb}(cbs)(1/2^+)$	6994(15)(13)	6998(27)(20)	?
$\Omega'_{cb}(cbs)(1/2^+)$	7045(16)(13)	7032(28)(20)	?
$\Omega_{cb}^*(cbs)(3/2^+)$	7056(17)(13)	7059(28)(21)	?
$\Omega_{ccb}(1/2^+)$	8005(6)(11)	8007(9)(20)	?
$\Omega_{ccb}^*(3/2^+)$	8026(7)(11)	8037(9)(20)	?
$\Omega_{cbb}(1/2^+)$	11194(5)(12)	11195(8)(20)	?
$\Omega_{cbb}^*(3/2^+)$	11211(6)(12)	11229(8)(20)	?

Brown *et al* 1409.0497

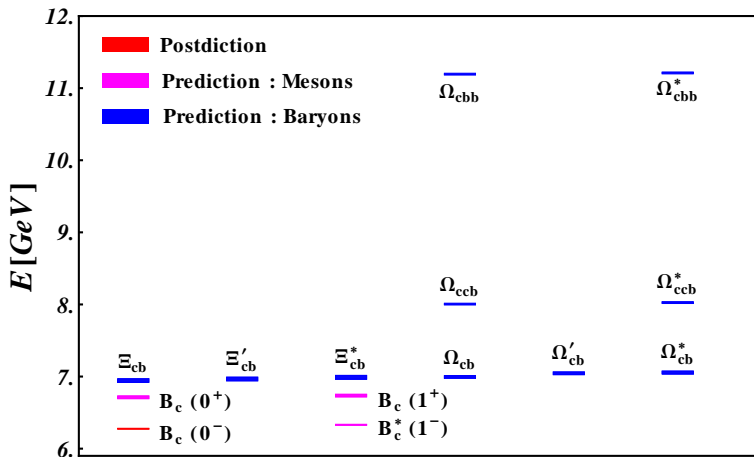
Mathur, MP, Mondal 1806.04151

# $bc$ baryons: L2 Vs other lattice results

Brown *et al* 1409.0497

Mathur, MP, Mondal 1806.04151

# Summary in predictions for $bc$ hadrons (L2)





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Marek Karliner<sup>1,\*</sup> and Jonathan L. Rosner<sup>2,†</sup><sup>1</sup>*School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel*<sup>2</sup>*Enrico Fermi Institute and Department of Physics, University of Chicago, 5620 South Ellis Avenue, Chicago, Illinois 60637, USA*

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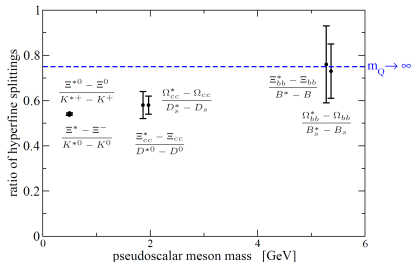
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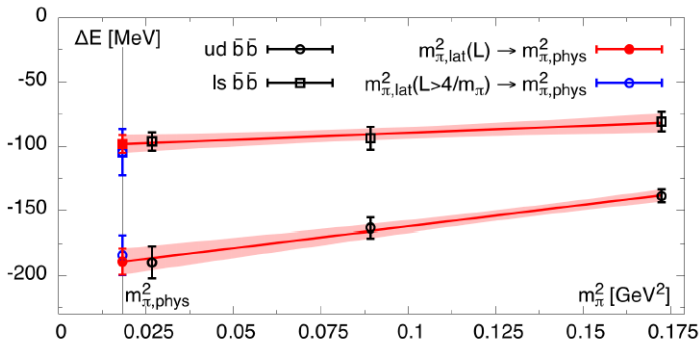
For a heavy-light meson-like system of doubly charm baryons

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Brambilla *et al* hep-ph/0506065Data : Brown *et al* 1409.0497

Figure : Lewis CHARM-2018



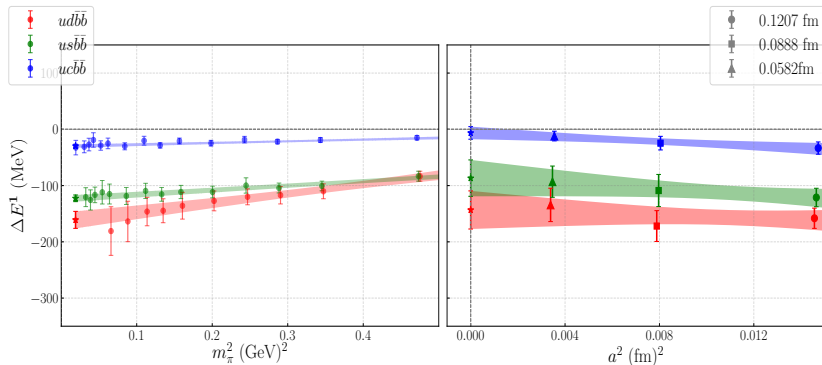
$\bar{b}\bar{b}ud$  and  $\bar{b}\bar{b}ls$  tetraquarks on latticeFrancis *et al.* 1607.05214

Study of pseudovector doubly bottom tetraquarks on  $N_f = 2 + 1$  PACS-CS lattices  
 Correlation matrices out of tetraquark as well as meson-meson interpolators  
 Binding energies :  $\Delta E_{\bar{b}\bar{b}ud} = 189(10)$  MeV and  $\Delta E_{\bar{b}\bar{b}us} = 98(7)$  MeV

Other existing lattice calculations :

Bicudo *et al.* 1510.03441, Leskovec *et al.* 1904.04197

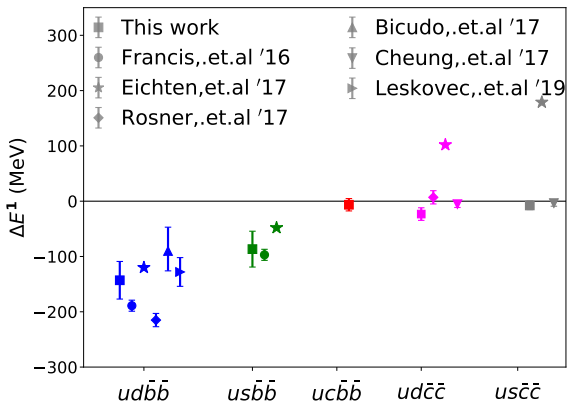
# Doubly heavy tetraquarks from L2



Junnarkar, Mathur & MP (ILGTI) 1810.12285

- Extensive study of quark mass dependence on binding energies of doubly heavy tetraquarks.
- Correlation matrices out of tetraquark as well as meson-meson interpolators.
- Binding energies from difference in the lowest non-interacting levels.

# Summary doubly heavy tetraquarks ( $J^P = 1^+$ )



1810.12285 (ILGTI, This work), 1607.05214 (Francis),  
 1707.09575 (Eichten and Quigg) 1707.07666 (Karlina and Rosner),  
 1510.03441 (Bicudo), 1709.01417 (HSC, Cheung), 1904.04197 (Leskovec).

# Summary

- Discussed exploratory investigation of excited  $\Omega_c$  baryon spectrum. Highlighted predictions for the quantum numbers of LHCb discovered excited  $\Omega_c$  baryons.
- Summarized lattice QCD predictions for singly and doubly charm baryons. Compared lattice results with experimental values.
- Presented precision lattice QCD predictions/postdictions for masses of charmed-bottom hadron ground states. Predictions compared with other existing lattice estimates and postdictions compared with the respective experimental values and other lattice results.
- Beyond conventional hadrons : Promising platforms to study. Doubly heavy tetra-quarks: possibly deeply bound states. Discussed our study on  $m_q$  dependence of these binding energies. Ongoing investigations for  $bc\bar{u}\bar{d}$  tetraquarks.

Thank you...

# Spin identification : $J > \frac{3}{2}$ (L1)

- For example, a continuum operator  $O = [ccc \otimes (\frac{3}{2}^+)_S^1 \otimes D_{L=2,S}^{[2]}]^{J=\frac{5}{2}}$ .  
Projects on to  $\frac{5}{2}^+$ .

- In the continuum,  $\langle 0 | O | \frac{5}{2}^+ \rangle = Z$ .

- On lattice,  $O$  gets subduced over two lattice irreps  $H_g$  and  $G_{2g}$ .

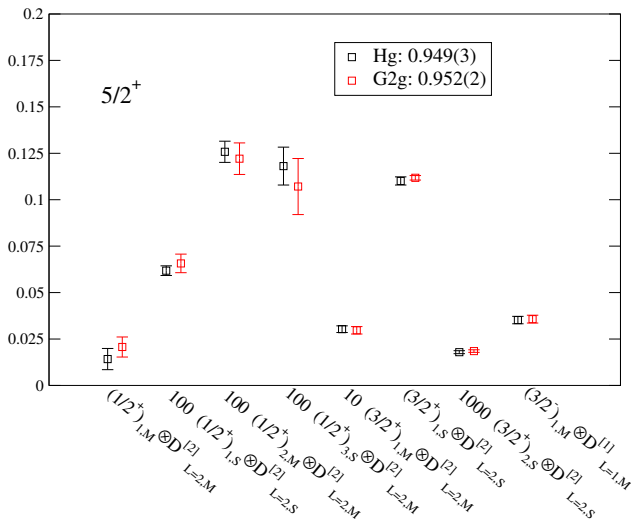
- Then

$$\langle 0 | O_{H_g} | \frac{5}{2}^+ \rangle = Z_1 \alpha \quad \& \quad \langle 0 | O_{G_{2g}} | \frac{5}{2}^+ \rangle = Z_2 \beta$$

where  $\alpha$  and  $\beta$  are the Clebsch-Gordan coefficients.

- If “close” to the continuum, then  $Z \sim Z_1 \sim Z_2$ .

# Overlap factors ( $Z$ ) across multiple irreps : $5/2^+$ (L1)

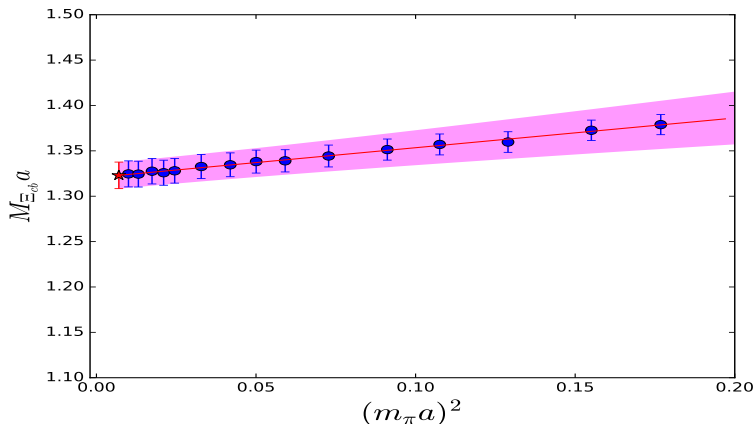




## Errors in predictions for $bc$ hadrons (L2)

- Wall sources to reduce statistical errors. All well below percent level.
- Discretization errors : Largest found to be in  $\Xi_{bc}$ ,  $\sim 6$  MeV.
- Scale setting : Independently calibrate the lattice using  $\Omega$  baryon.  
This work :  $a = 0.1192(14)$ ,  $0.0877(10)$  and  $0.0582(5)$  fm.  
MILC( $r_1$ ) :  $a = 0.1207(11)$ ,  $0.0888(8)$  and  $0.0582(5)$  fm.  
Largest errors to be  $\sim 6$  MeV.
- Chiral extrapolation : Robust with different extrapolation forms.
- Uncertainties in NRQCD : missing higher order terms,  $\mathcal{O}(\alpha^2 v^4)$  and  $\mathcal{O}(\alpha v^6)$ .  
 $\sim 4$  MeV for mesons  
 $\sim 5, 5$  and  $6$  MeV for  $bcq$ ,  $bcc$  and  $bbc$  baryons respectively.
- Other sources : Quark mass mis-tuning, unphysical sea quark mass effects, electromagnetism, isospin breaking effects and absence of dynamical bottom quarks collectively to be within few MeV.

# $\Xi_{bc}$ baryon chiral extrapolation (L2)



On the coarse lattice. Chiral extrapolation form  $A + m_{\pi}^2 B$ .

Mathur, MP, Mondal 1806.04151