Dielectric haloscopes: a new way to search for axion DM

Alex Millar on behalf of the new MADMAX Collaboration

A. Caldwell et al arXiv:1611.05865
A. Millar et al arXiv:1612.07057
A. Ioannisian et al arXiv:1707.00701
A. Millar et al arXiv:1707.04266

SFB 1258
Neutrinos
Dark Matter
Messengers

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Partially based on a talk by Georg Raffelt

Dark Matter

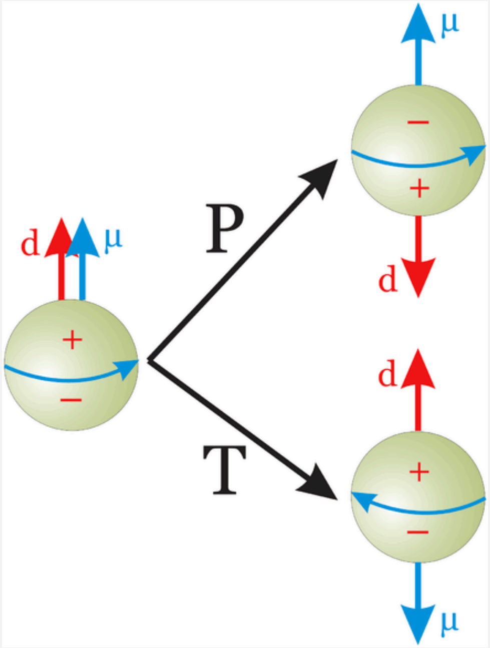
- Strong experimental evidence that General Relativity cannot describe the cosmos when the known baryonic matter is used.
- Modifications to gravity both have significant theoretical problems, and fail to explain the full suite of cosmological observations.
- Need a new massive particle to explain observations (or at least something that behaves like one)

Strong CP problem

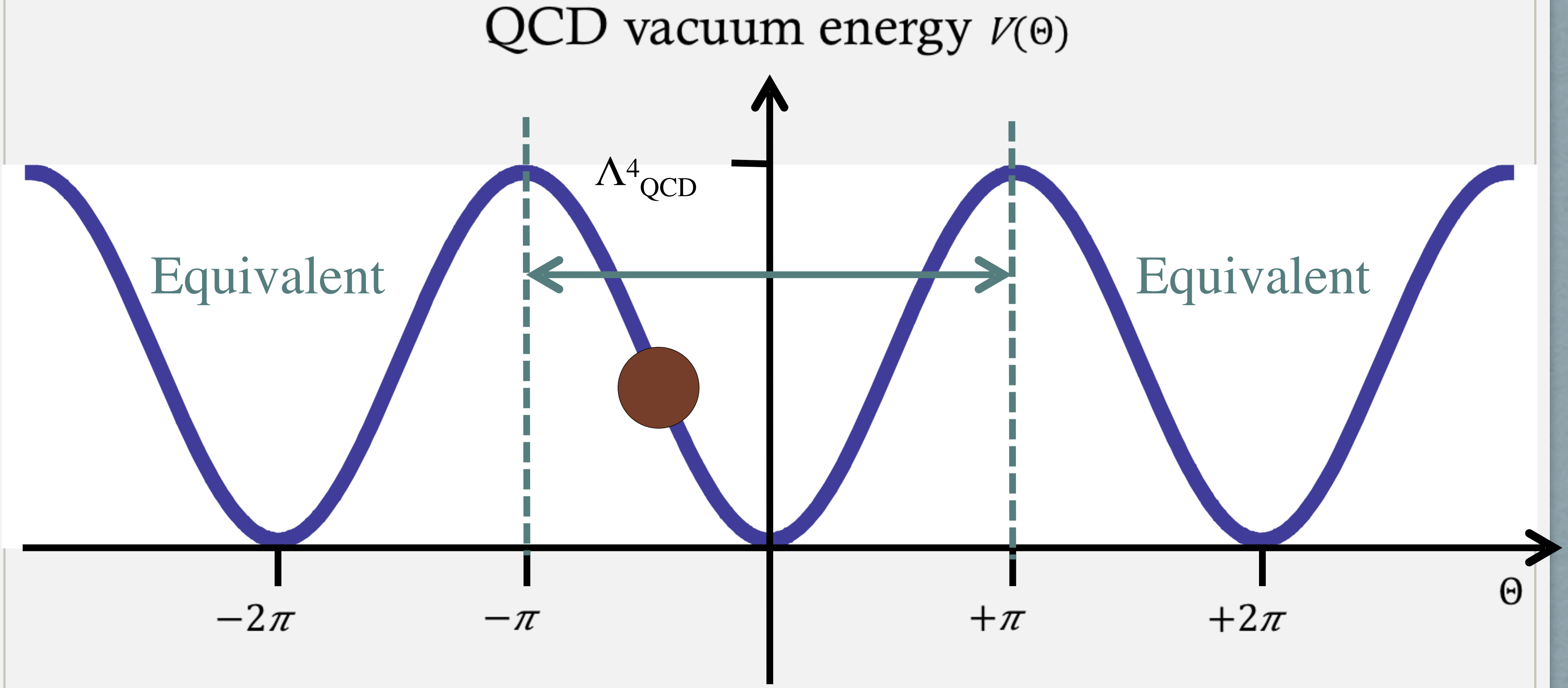
- QCD potentially has a CP violating term

$$\theta \frac{g^2}{32\pi^2} G\widetilde{G}$$

- Measurement of neutron EDM (which requires T violation) gives $\theta<10^{-10}$
- No reason for θ to be so ridiculously small (even anthropics can't explain it)



Strong CP problem



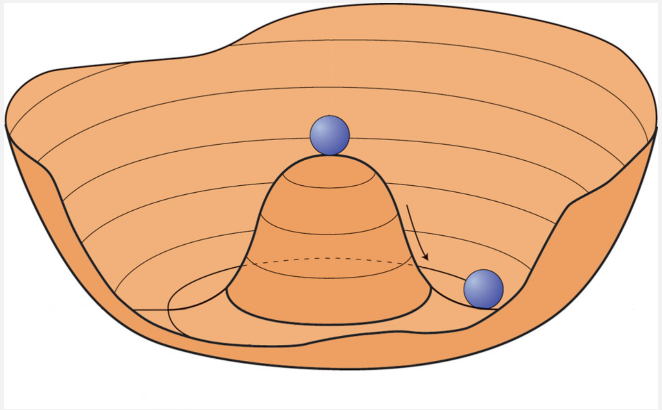
- θ=0 minimizes the vacuum energy, but θ is not a dynamical term

What are Axions?

- Solution to the Strong CP problem: make θ a dynamical field so it can minimise the energy and send θ to zero

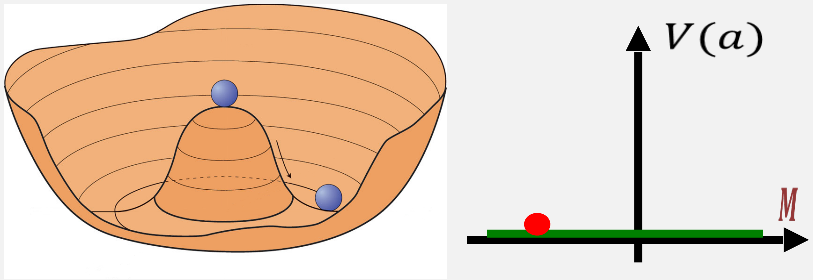
$$\begin{aligned}\mathcal{L}_{\text{stand mod + axion}} &= \ldots + \frac{1}{2}\partial_\mu a \partial^\mu a \\ &+ \frac{g^2}{32\pi^2}\frac{a(x)}{f_a} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}\end{aligned}$$

- Need a new anomalous U(1) chiral symmetry (Peccei-Quinn), which is broken at high temperature ~f_a (around 10^{12} GeV)

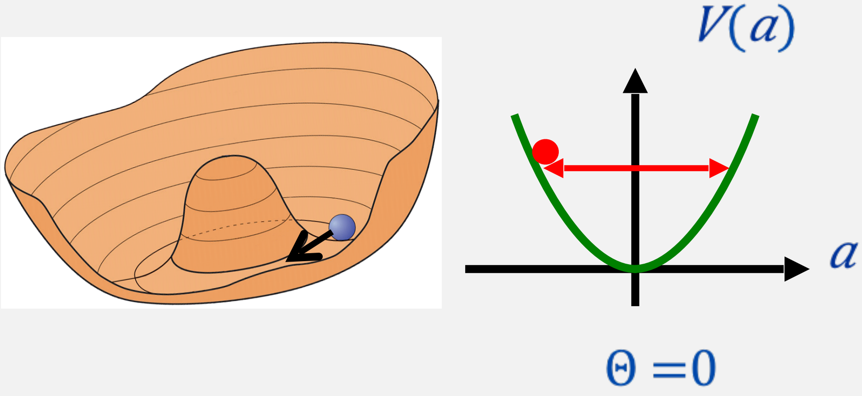


What are Axions?

- The axion is the angular degree of freedom which is unbroken at intermediate temperatures

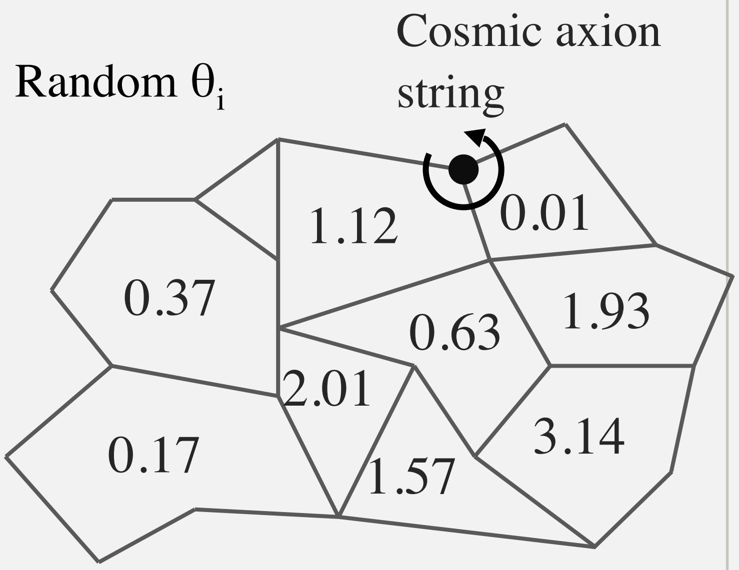


- At the QCD scale the potential tilts as the axion acquires a mass – axion rolls down to a CP conserving minimum



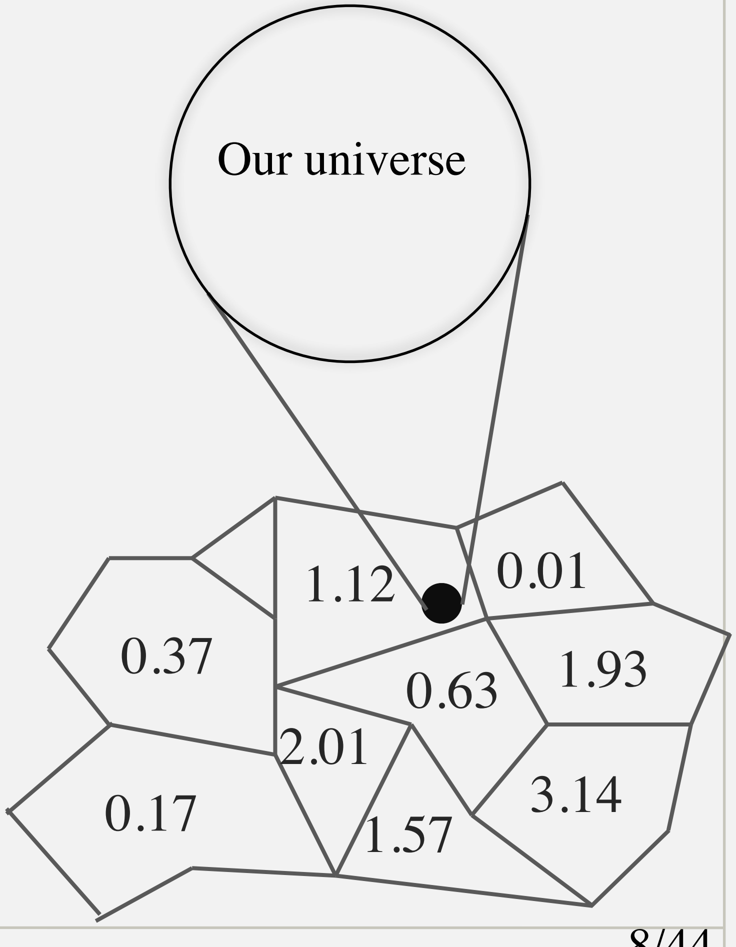
Axion DM: scenario 1

- Scenario 1: PQ broken after inflation
- θ_i has random values in every casual region, with the dark matter density determined by the average
- Topological defects such as strings and domain walls exist in the early universe



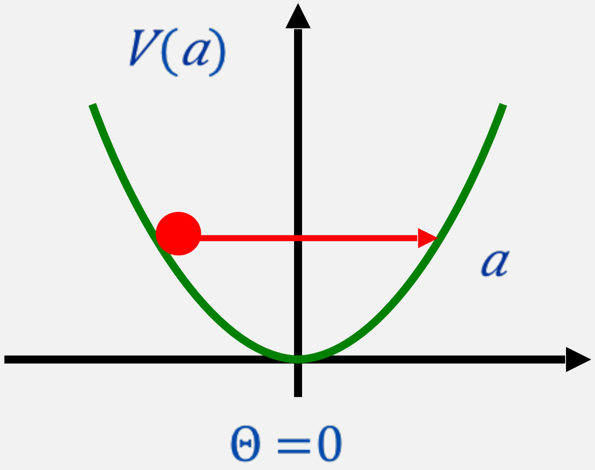
Axion DM: scenario 2

- Scenario 2: PQ broken before inflation
- θ_i has a single random value which determines the dark matter density
- No topological defects

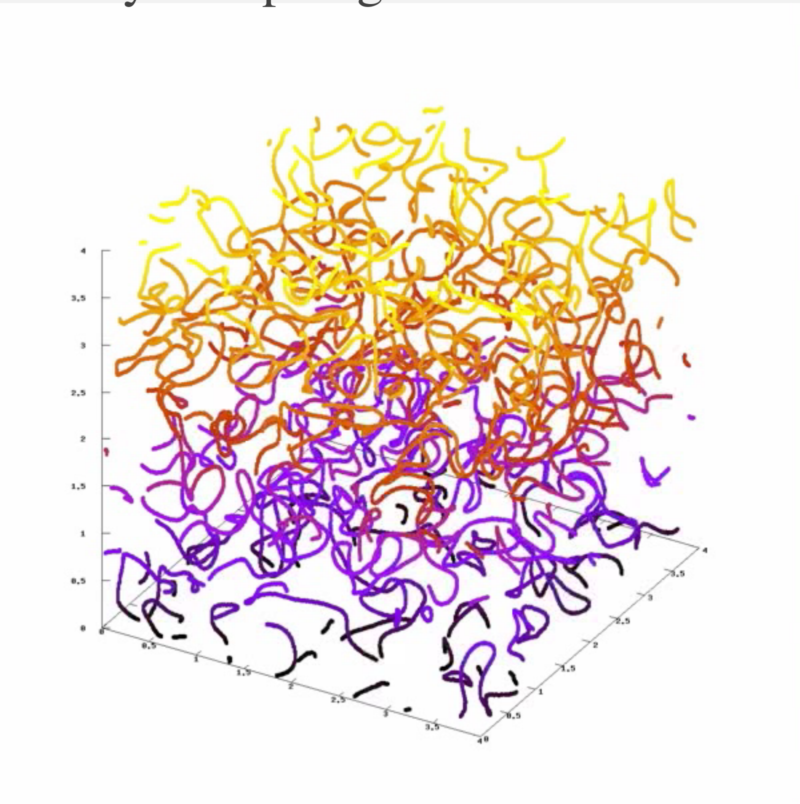


Axion production mechanisms

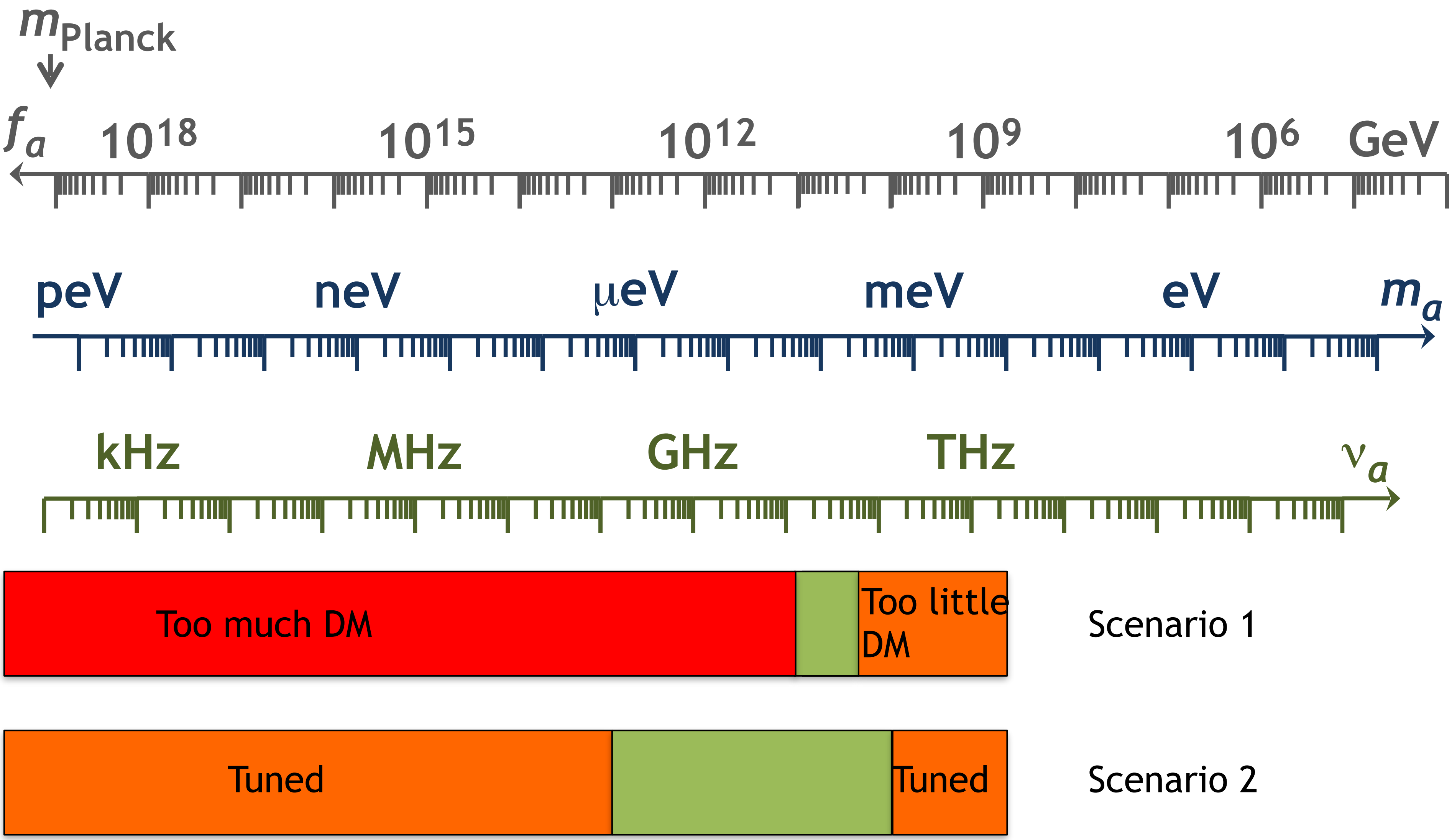
Vacuum Misalignment



Decay of topological defects



Animation credit: Javier Redondo



Too much DM | Too little DM — Scenario 1

Tuned | Tuned — Scenario 2

Axion DM is a classical field

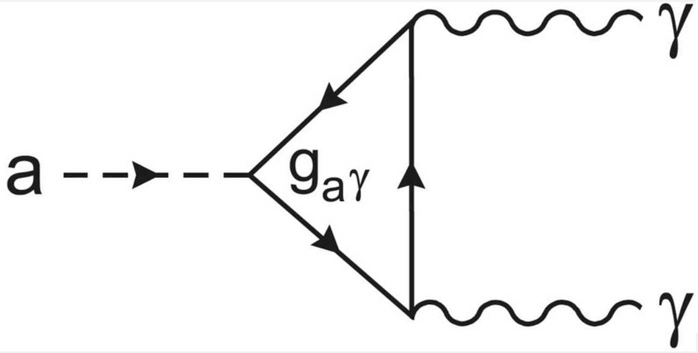
- Two classical limits of QFT: point particles and classical fields
- Wimps are an example of the first: heavy (~100 GeV) and low in number – direct detection looks for scatterings
- Axions are light (~10^{15} times lighter) and highly degenerate
- Totally different phenomenology

Axion-electrodynamics

- Axions and ALPs interact with photons through an anomaly term

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - J^{\mu}A_{\mu} + \frac{1}{2}\partial_{\mu}a\partial^{\mu}a - \frac{1}{2}m_a^2a^2 - \boxed{\frac{g_{a\gamma}}{4}F_{\mu\nu}\widetilde{F}^{\mu\nu}a,}$$

- This coupling is tiny, but still important



$$m_a = 5.70(7)\,\mu\text{eV}\,\frac{10^{12}\text{GeV}}{f_a}\,,$$
$$g_{a\gamma} = \frac{\alpha}{2\pi f_a}C_{a\gamma} = 2.04(3)\times 10^{-16}\ \text{GeV}^{-1}\,\frac{m_a}{\mu\text{eV}}\,C_{a\gamma}\,,$$
$$C_{a\gamma} = \frac{E}{N} - 1.92(4)\,,$$

Axion induced E-field

- Maxwell's inhomogenous equations get new terms: axion acts as a current

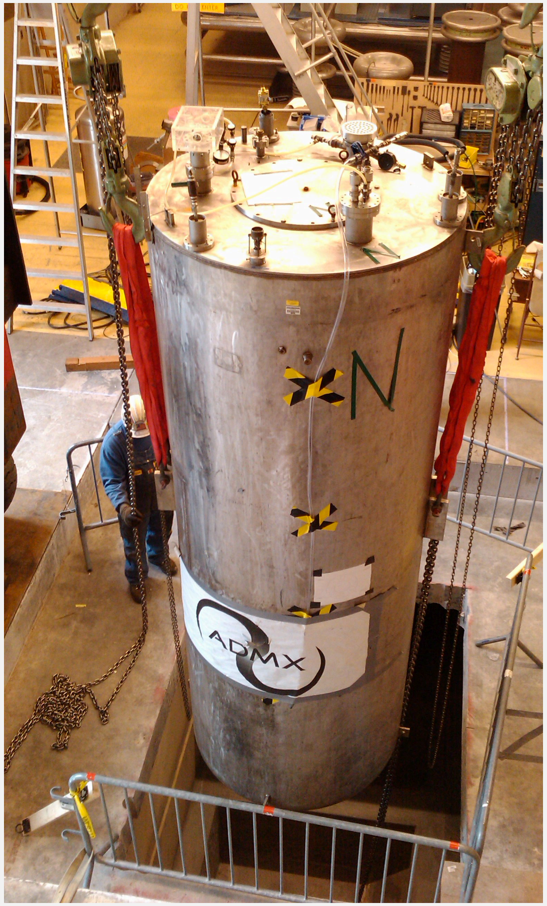
$$\begin{aligned} \epsilon\boldsymbol{\nabla}\cdot\mathbf{E} &= \rho - g_{a\gamma}\mathbf{B}_{\mathrm{e}}\cdot\boldsymbol{\nabla}a\,, \\ \boldsymbol{\nabla}\times\mathbf{H} - \dot{\mathbf{E}} &= \mathbf{J} + g_{a\gamma}\mathbf{B}_{\mathrm{e}}\dot{a}\,, \\ \ddot{a} - \boldsymbol{\nabla}^2 a + m_a^2 a &= g_{a\gamma}\mathbf{E}\cdot\mathbf{B}_{\mathrm{e}}\,, \end{aligned}$$

- The upshot is that in an external B-field the axion sources an E-field

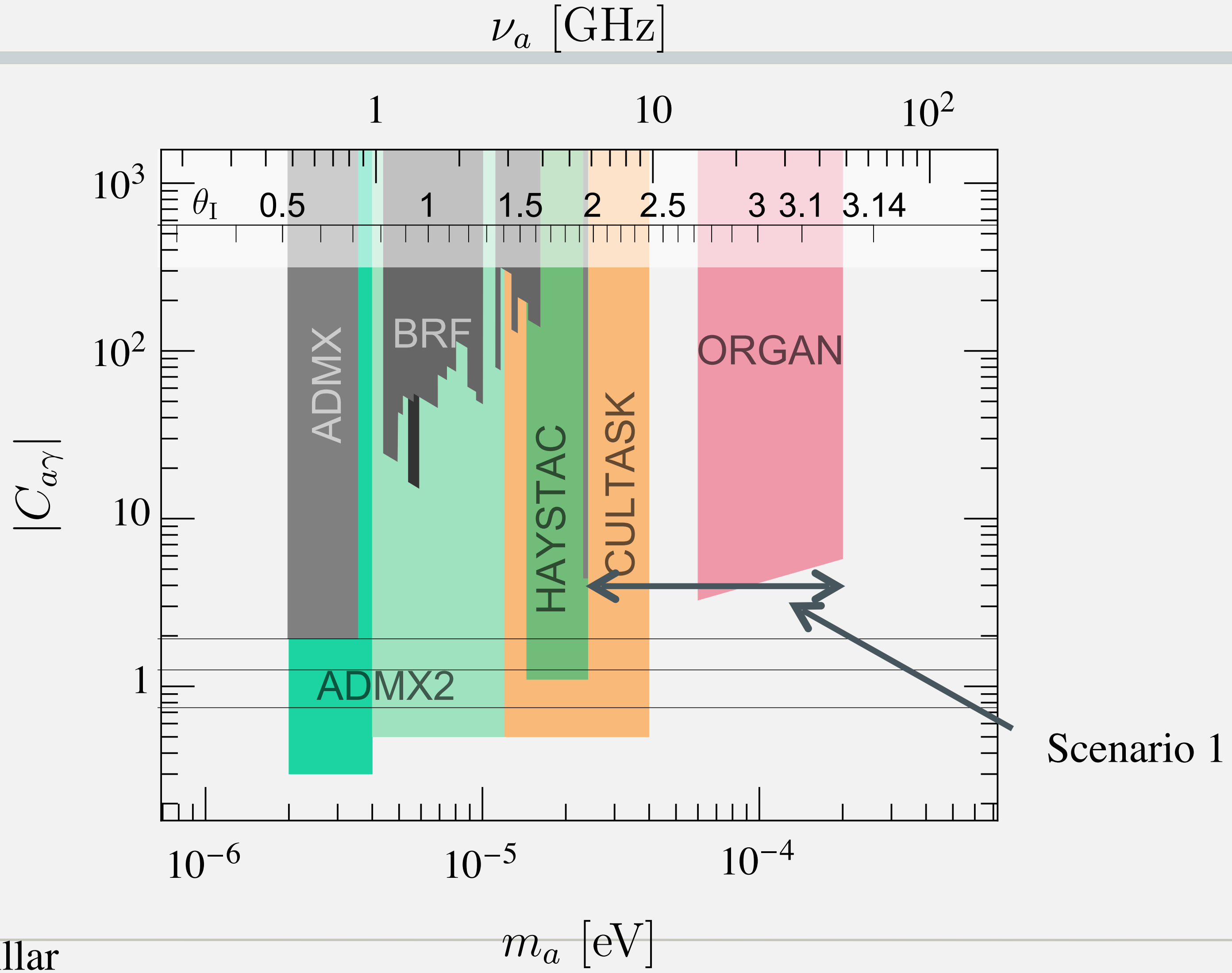
$$\mathbf{E}_a = -\frac{g_{a\gamma}\mathbf{B}_e a_0}{\epsilon} e^{-i m_a t} = 1.3\times 10^{-12}\ \mathrm{V/m}\ \frac{B_{\mathrm{e}}}{10\ \mathrm{T}}\ \frac{C_{a\gamma} f_{\mathrm{DM}}^{1/2}}{\epsilon}\,.$$

Cavity Haloscopes

- Build a cavity at the same scale as the axion's Compton wavelength – resonant enhancement
- Hugely increases signal, but only in a very narrow range
- Power and bandwidth inversely related
- Requires large volume – hard to do for large axions masses (small wavelengths)



Cavity Haloscopes

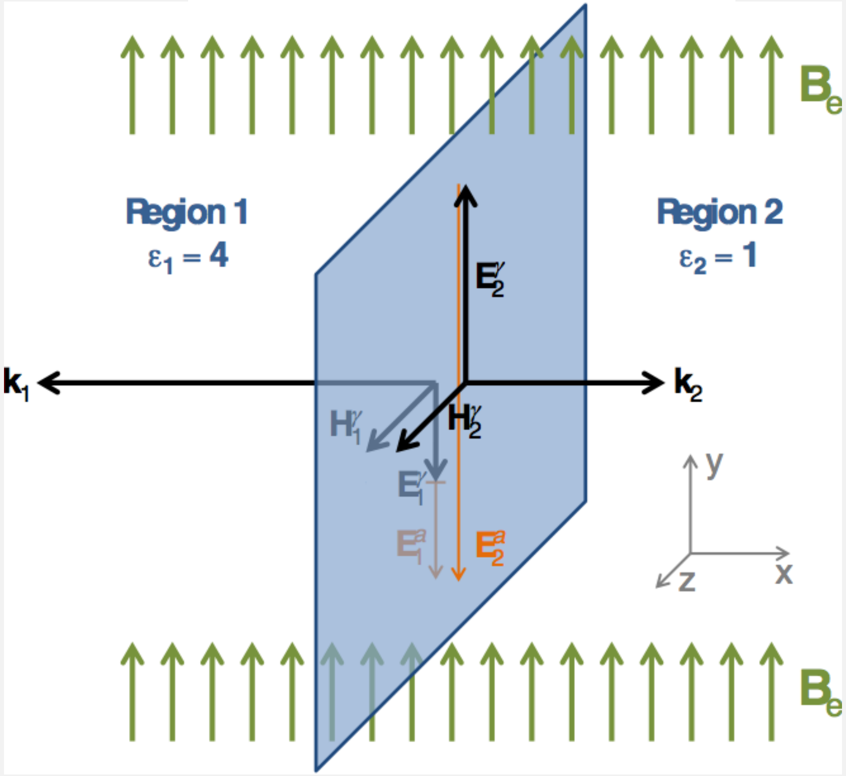


Single interface

(fun with boundary conditions)

- E_a depends on the medium, so changing media causes a discontinuity.
- EM won't tolerate discontinuities in the parallel E and H fields
- Regular EM waves are emitted to compensate

$$\mathbf{E}_a(t) = -\frac{g_{a\gamma}\mathbf{B}_\mathrm{e}}{\epsilon}\, a(t)$$



Single interface

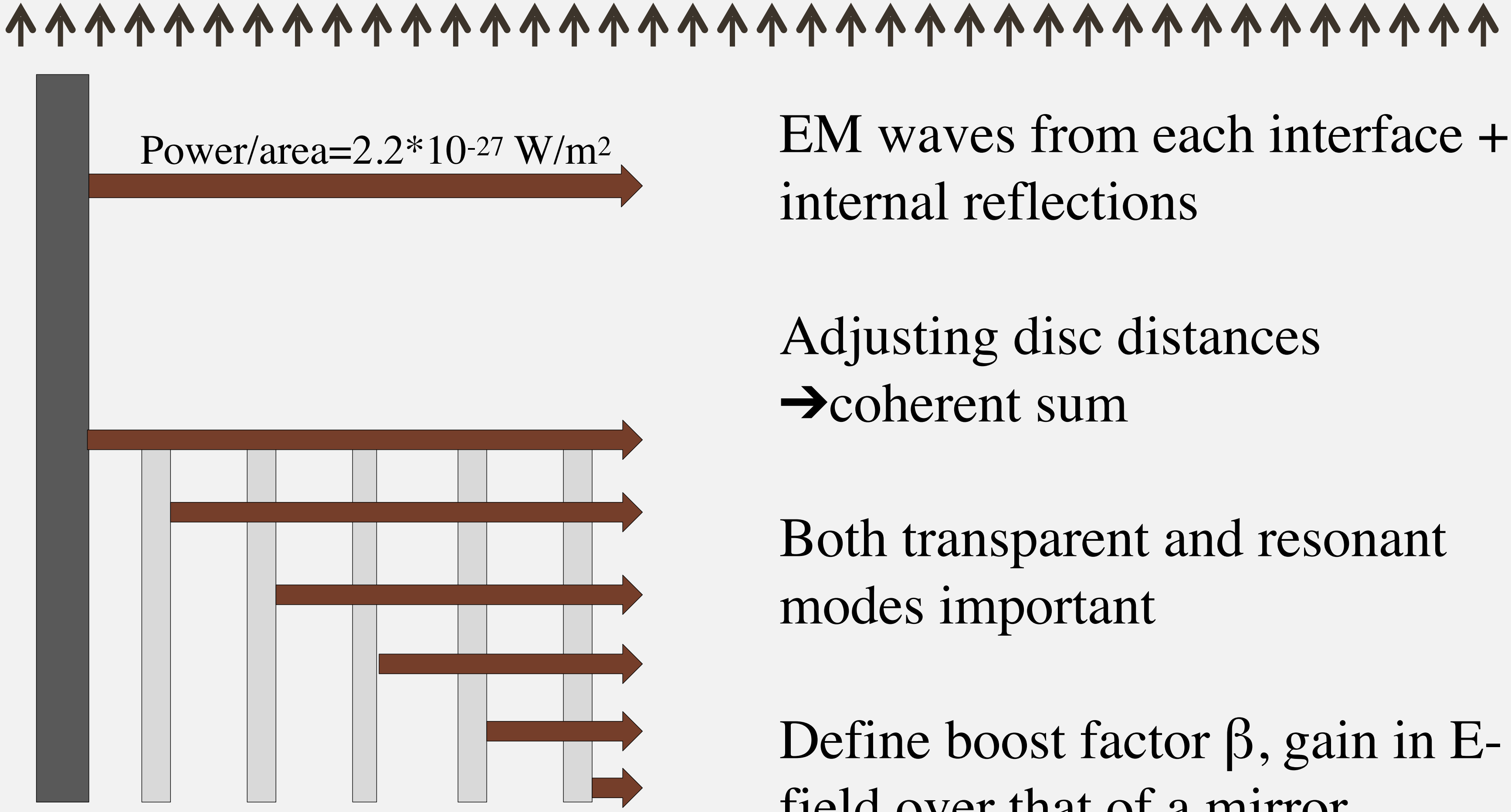
(fun with boundary conditions)

- The ideal single interface is a mirror, which provides

$$\frac{P^\gamma}{A} = (\mathbf{E}_2^\gamma \times \mathbf{H}_2^\gamma)_x = \frac{E_0^2}{2} = 2.2 \times 10^{-27} \frac{W}{\mathrm{m}^2} \left(\frac{|\mathbf{B}_e|}{10\,\mathrm{T}}\right)^2 C_{a\gamma}^2 f_{\mathrm{DM}},$$

- 4-5 orders of magnitude too small for the QCD axion to be detected with modern technology
- Need more power!

Multiple layers: dielectric haloscope



EM waves from each interface + internal reflections

Adjusting disc distances ➔coherent sum

Both transparent and resonant modes important

Define boost factor β, gain in E-field over that of a mirror

Transfer matrix formalism

- Encode every interface and distance as a matrix
- Add in a new source term at each interface to account for the axions

$$\begin{pmatrix} R \\ L \end{pmatrix}_m = \mathsf{T} \begin{pmatrix} R \\ L \end{pmatrix}_0 + E_0 \mathsf{M} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\mathsf{G}_r = \frac{1}{2n_{r+1}} \begin{pmatrix} n_{r+1}+n_r & n_{r+1}-n_r \\ n_{r+1}-n_r & n_{r+1}+n_r \end{pmatrix}$$

$$\mathsf{P}_r = \begin{pmatrix} e^{+i\delta_r} & 0 \\ 0 & e^{-i\delta_r} \end{pmatrix},$$

$$\mathsf{S}_r = \frac{A_{r+1} - A_r}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$\mathsf{M} = \sum_{s=1}^{m} \mathsf{T}_s^m \mathsf{S}_{s-1}$$

$$\mathsf{T}_b^a = \mathsf{G}_{a-1}\mathsf{P}_{a-1}\mathsf{G}_{a-2}\mathsf{P}_{a-2} \ldots \mathsf{G}_{b+1}\mathsf{P}_{b+1}\mathsf{G}_b\mathsf{P}_b$$

Overlap integral formalism

- Resonant cavities use Sikivie's overlap integral formalism… Can we generalise?

$$P_{\rm cav} = \kappa \mathcal{G} V \frac{Q}{m_a} \rho_a g_{a\gamma}^2 B_{\rm e}^2, \;\; \mathcal{G} = \frac{\left(\int dV \, \mathbf{E}_{\rm cav} \cdot \mathbf{B}_{\rm e}\right)^2}{V B_{\rm e}^2 \int dV \, \mathbf{E}_{\rm cav}^2}$$

- Yes! Actually all the axion experiments can be handled with overlap integrals…
- The overlap integral can be (most easily) proved by a QFT calculation

Quantum calculation

- Need to calculate the probability of a single axion converting to a photon
- Lowest order QFT→Fermi’s golden rule

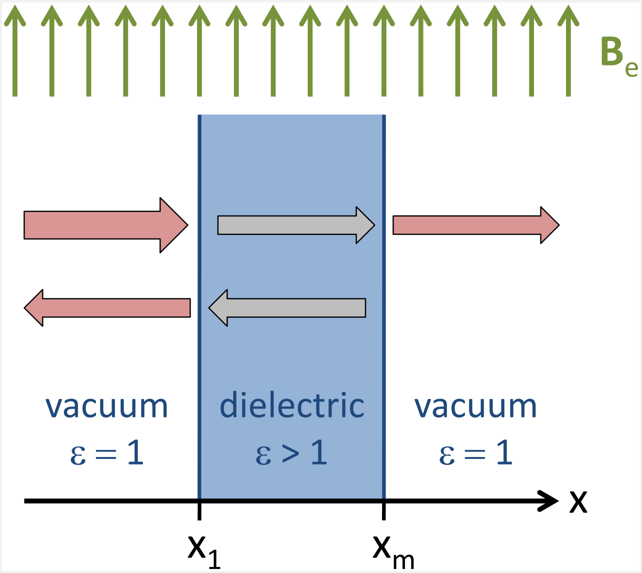
$$\Gamma_{a\to\gamma} = 2\pi \sum_{\mathbf{k}} |\mathcal{M}|^2 \, \delta(\omega_a - \omega_{\mathbf{k}}) \, .$$

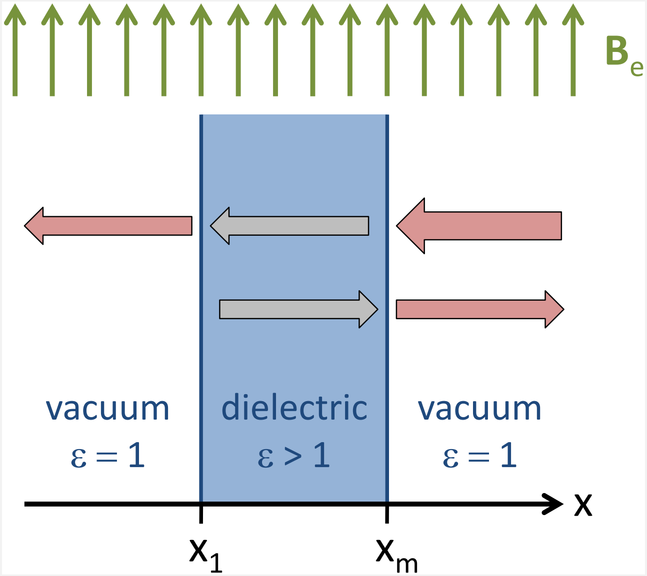
- Matrix element is given by the overlap of the axion and photon wave functions

$$\mathcal{M} = \frac{g_{a\gamma}}{2\omega V} \int d^3\mathbf{r} \, e^{i\mathbf{p}\cdot\mathbf{r}} \, \mathbf{B}_{\rm e}(\mathbf{r}) \cdot \mathbf{E}^*_{\mathbf{k}}(\mathbf{r})$$

Overlap integral formalism

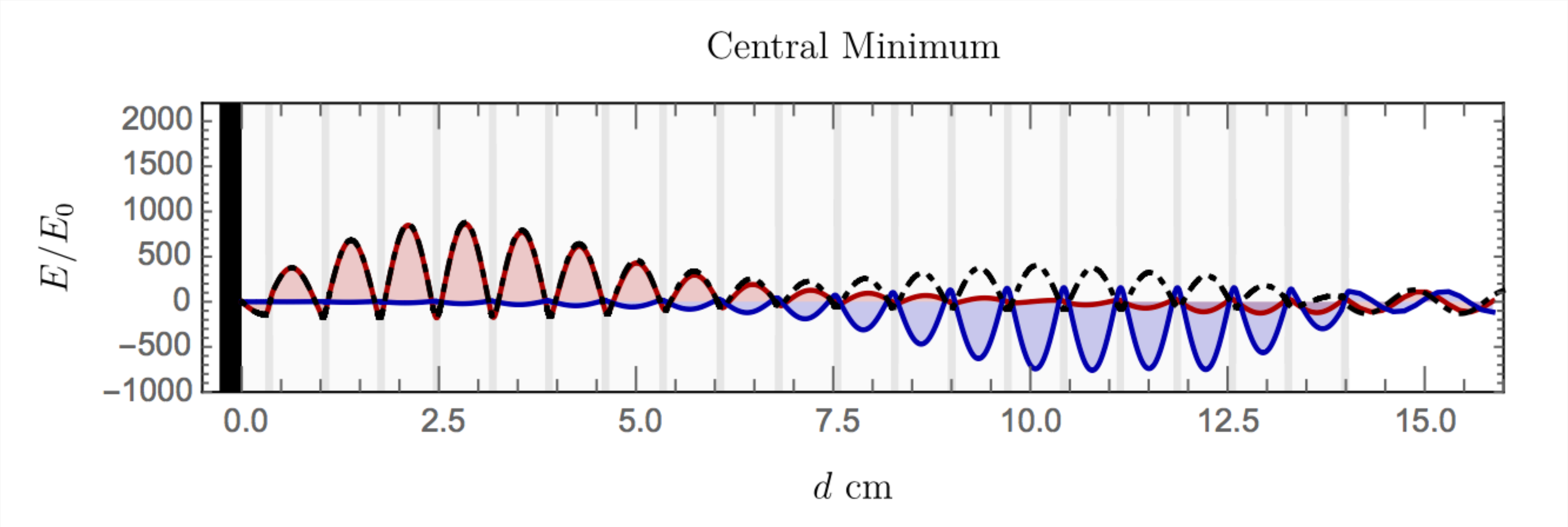
- The main trick is choosing the right free-photon wave functions: Garibian wave functions,





Overlap integral formalism

- The E-field only encodes boundary conditions: in general it isn't excited

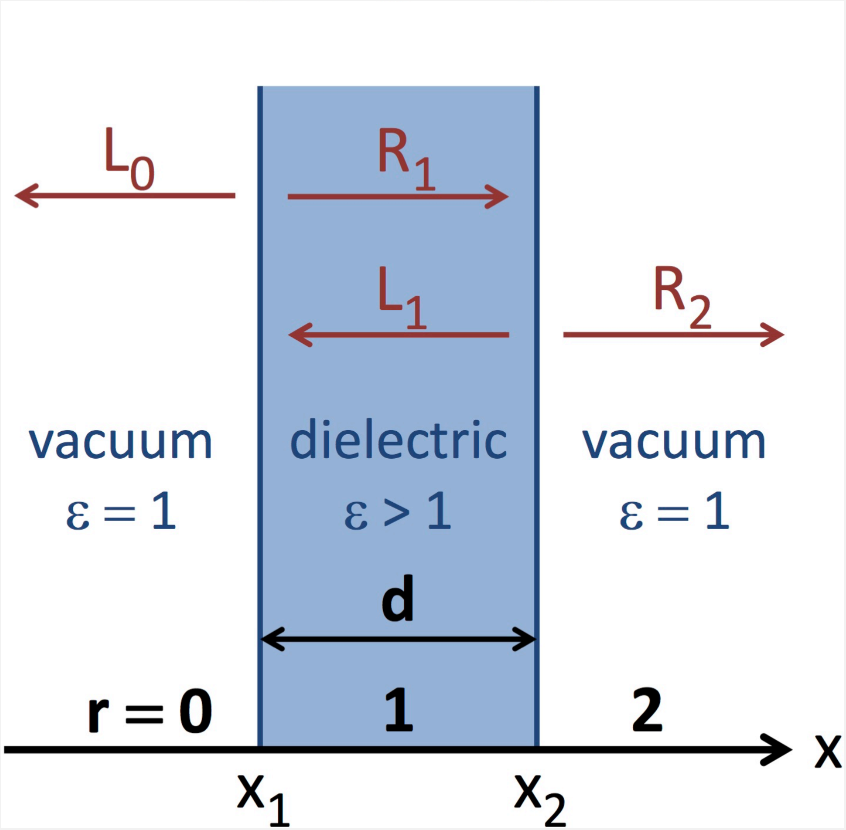


Theoretical formalisms

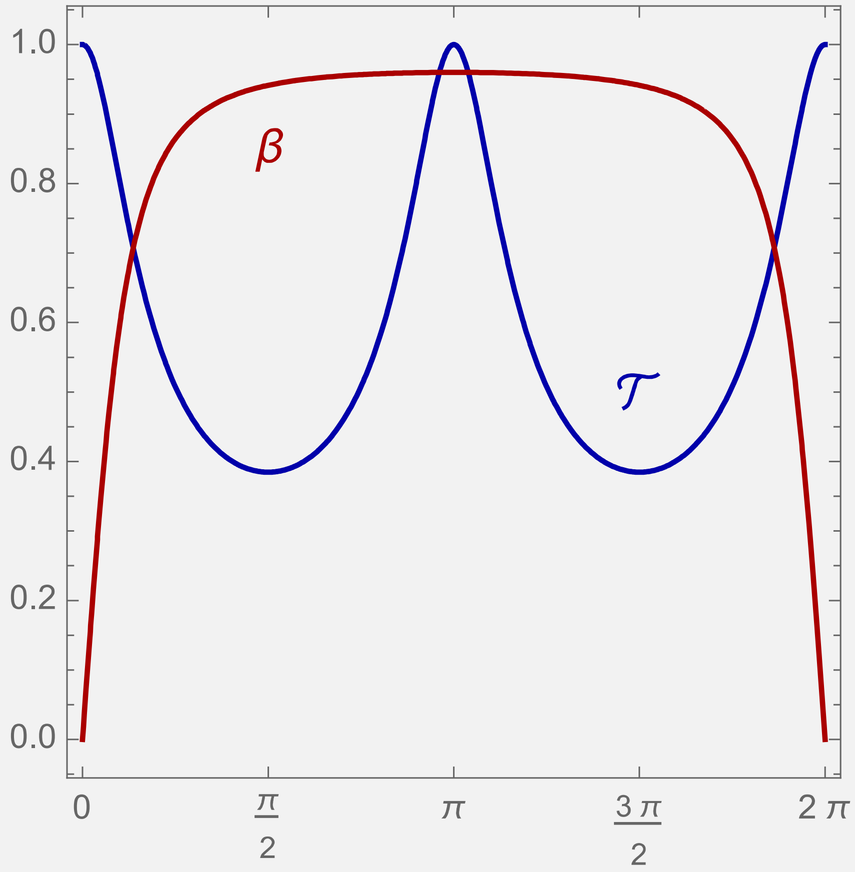
- **Transfer matrices (classical calculation)**
- All the action is at the interfaces
- Combination of axion and photon field satisfies axion-Maxwell equations: axion-photon wave function
- Solving for the classical E-field everywhere
- **Overlap integral (quantum field calculation)**
- All space is involved
- Axion and photons wave functions treated separately: photon wave function satisfies regular Maxwell equations
- Calculating transition probability

Dielectric disk

- What about two interfaces?
- Each interface emits a wave – reflections and interference
- Frequency dependent emission



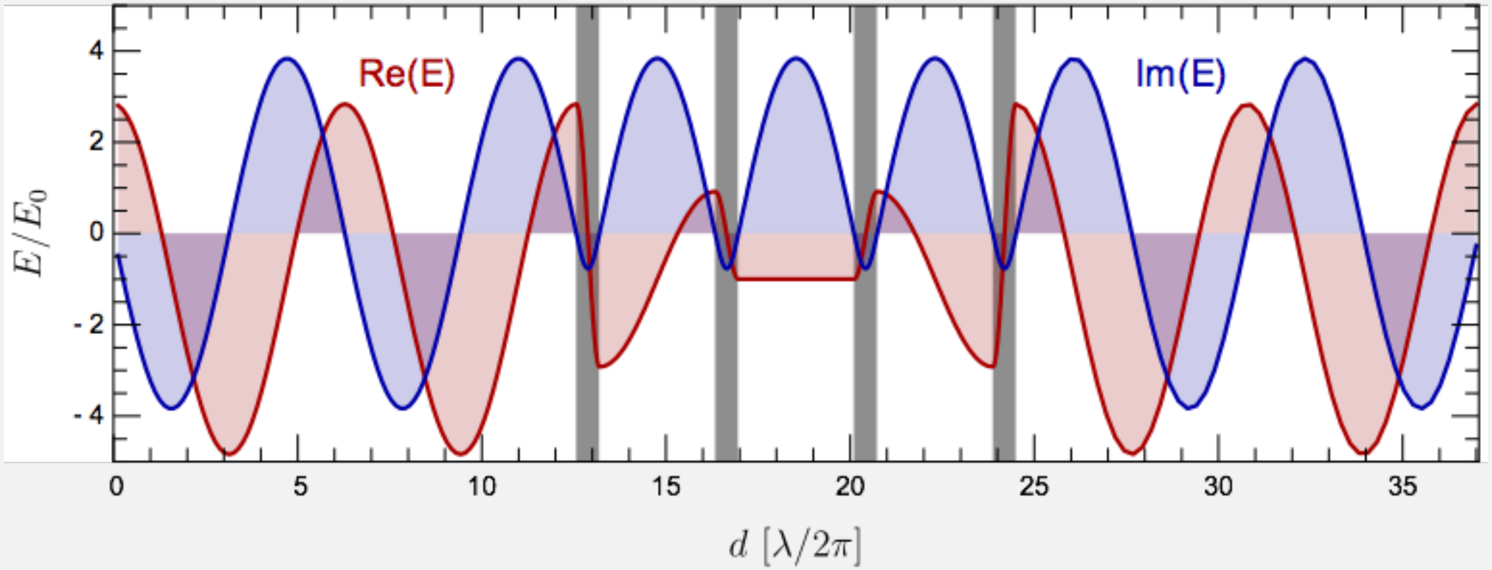
Dielectric disk



$$\delta = n\omega d$$

More disks: transparent mode

- Limiting case… just use transparent disks[2] ($\lambda/2$)



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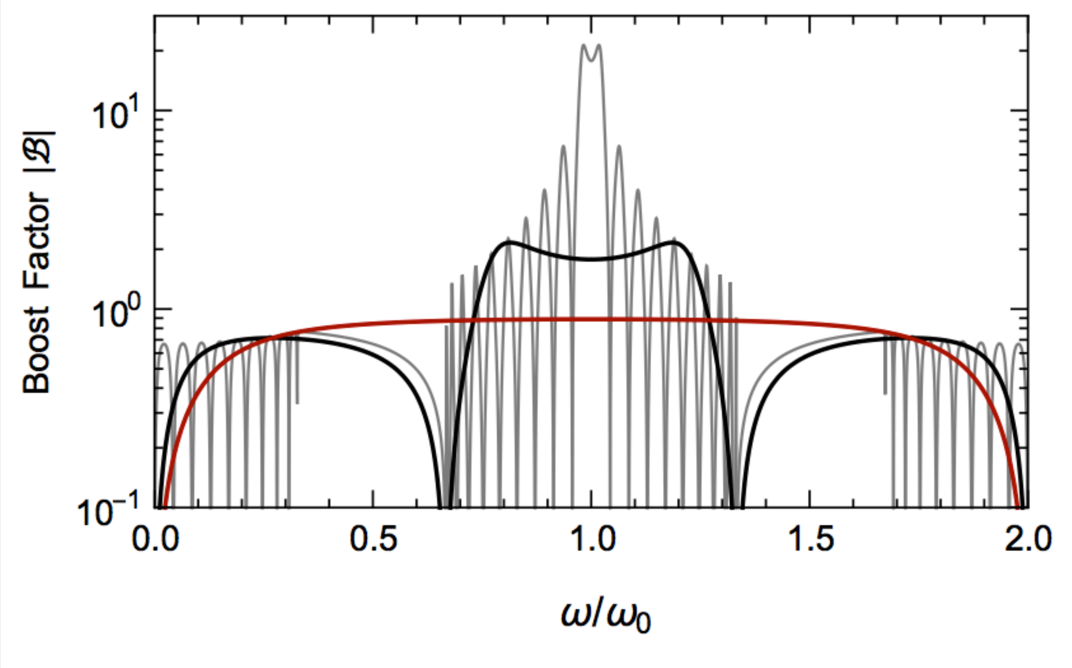
[2] J. Jaeckel and J. Redondo, Resonant to broadband searches for cold dark matter consisting of weakly interacting slim particles, Phys. Rev. D **88** (2013) 115002 [arXiv:1308.1103].

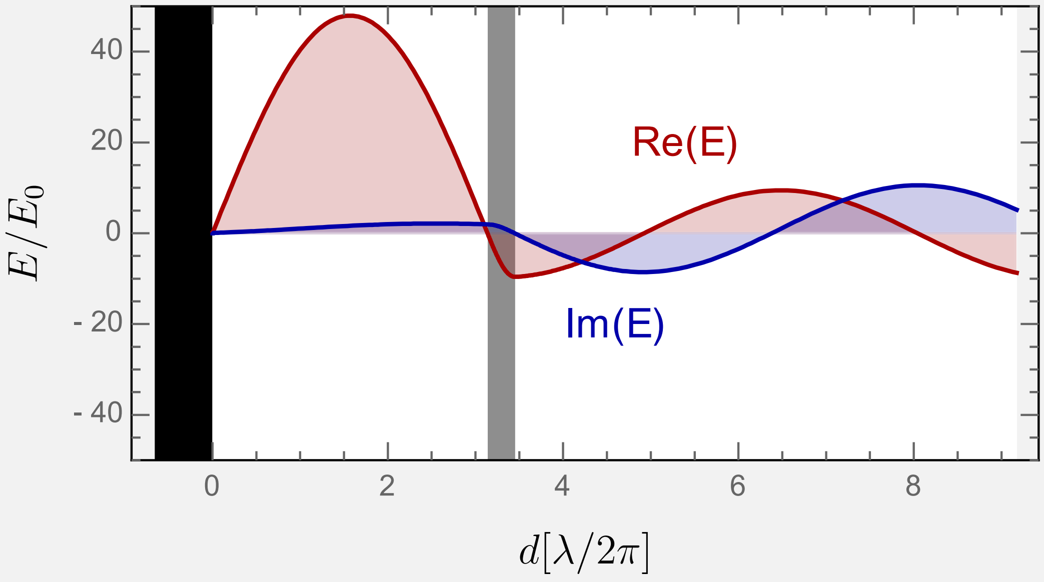
More disks: transparent mode

- N disks give β~2N+1 (with a mirror included)
- Width decreases with N: P/Δν=N
- Can’t choose width



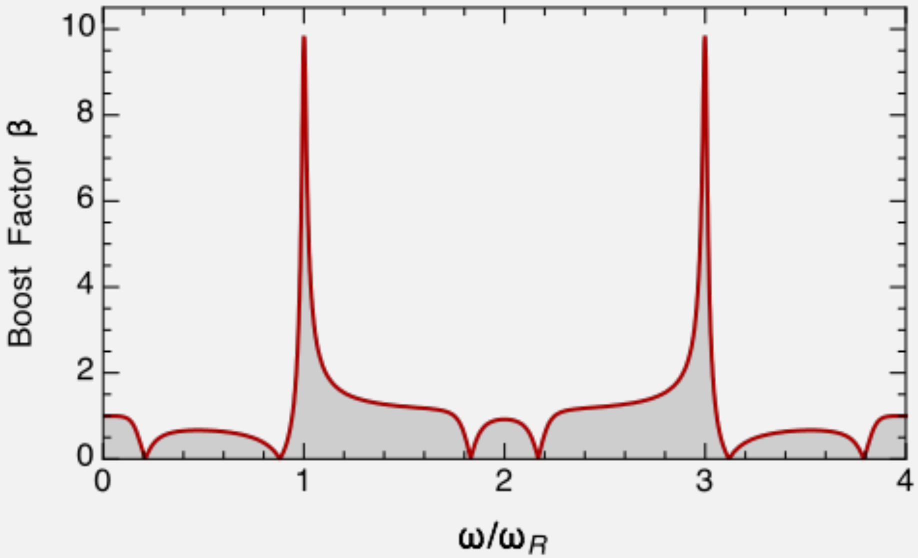
Simple resonator

- What about using the reflections?
- Use λ/4 disks to maximise reflectivity
- Increases density of final states



Simple resonator

- β~2n
- Width decreases with n^2: P/Δν=1
- Much narrower then transparent mode
- Can't get high n materials

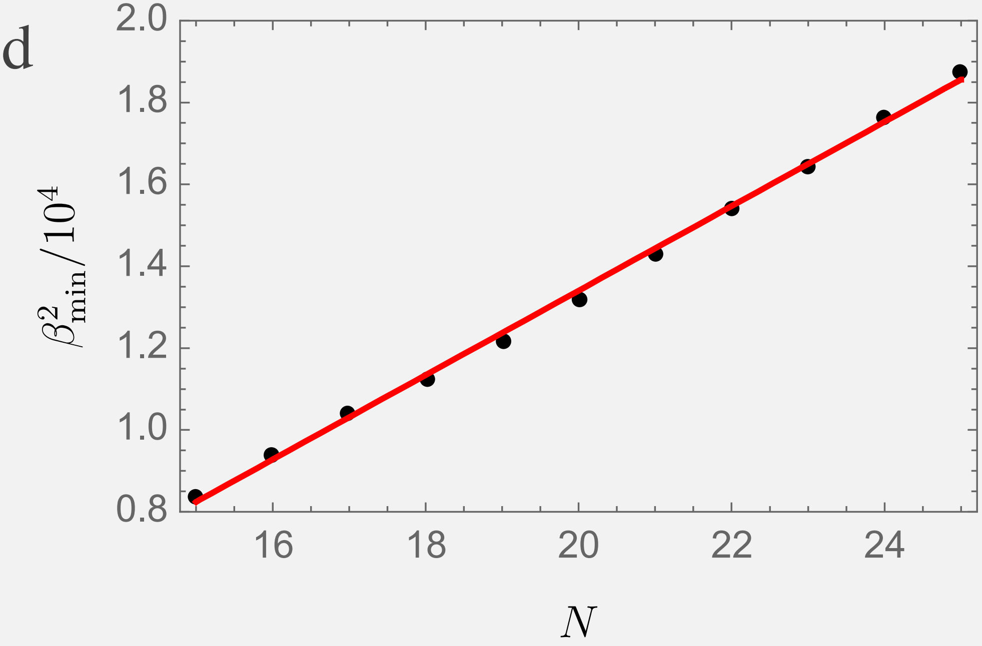


Boost factor and bandwidth

- β and bandwidth related
- "Area Law"

$$\int \beta^2 d\nu \propto N$$

- As this area is roughly conserved, can trade bandwidth for β
- Can extrapolate to many disk solutions



Practical solutions

- Need: non-specific disk thicknesses

$$\Delta t = \left(\frac{\mathrm{S}}{\mathrm{N}}\right)^2 \left(\frac{T_{\mathrm{sys}}}{P_\gamma}\right)^2 \Delta\nu_a$$

- Scanning rate~$P^2\Delta\nu$, but the Area Law implies $P\Delta\nu$~const
- Naively expect that a narrow resonance wins by a factor P

Practical solutions

- Don't forget the readjustment time!
- No point making the measurements much quicker than the down time
- Ideal optimum is readjustment time = measurement time

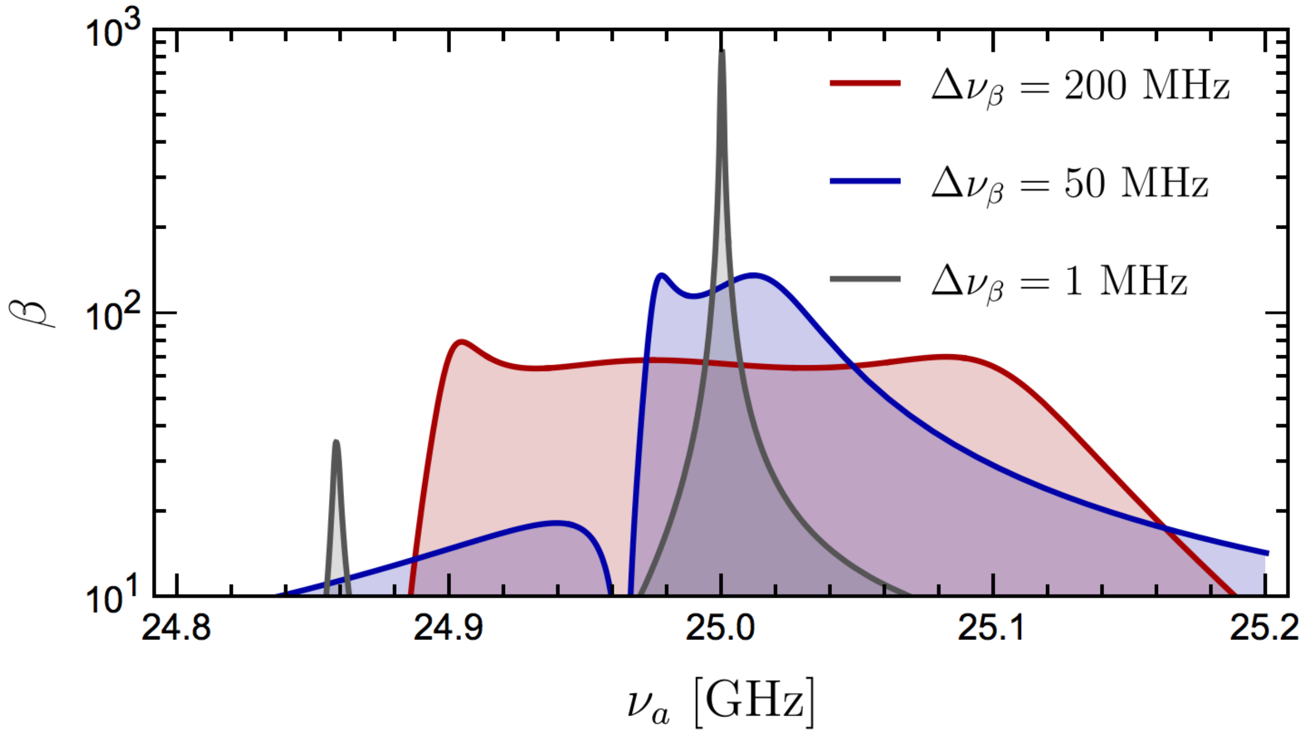
$$t_{\mathrm{o}} = \frac{2t_{\mathrm{R}}}{2p+1}\frac{\beta_2^2}{K}\nu_2$$

- Instead, gain by adding bandwidth
- Goal: rectangular, broadband response for scanning, narrow resonance for confirming/rejecting discovery

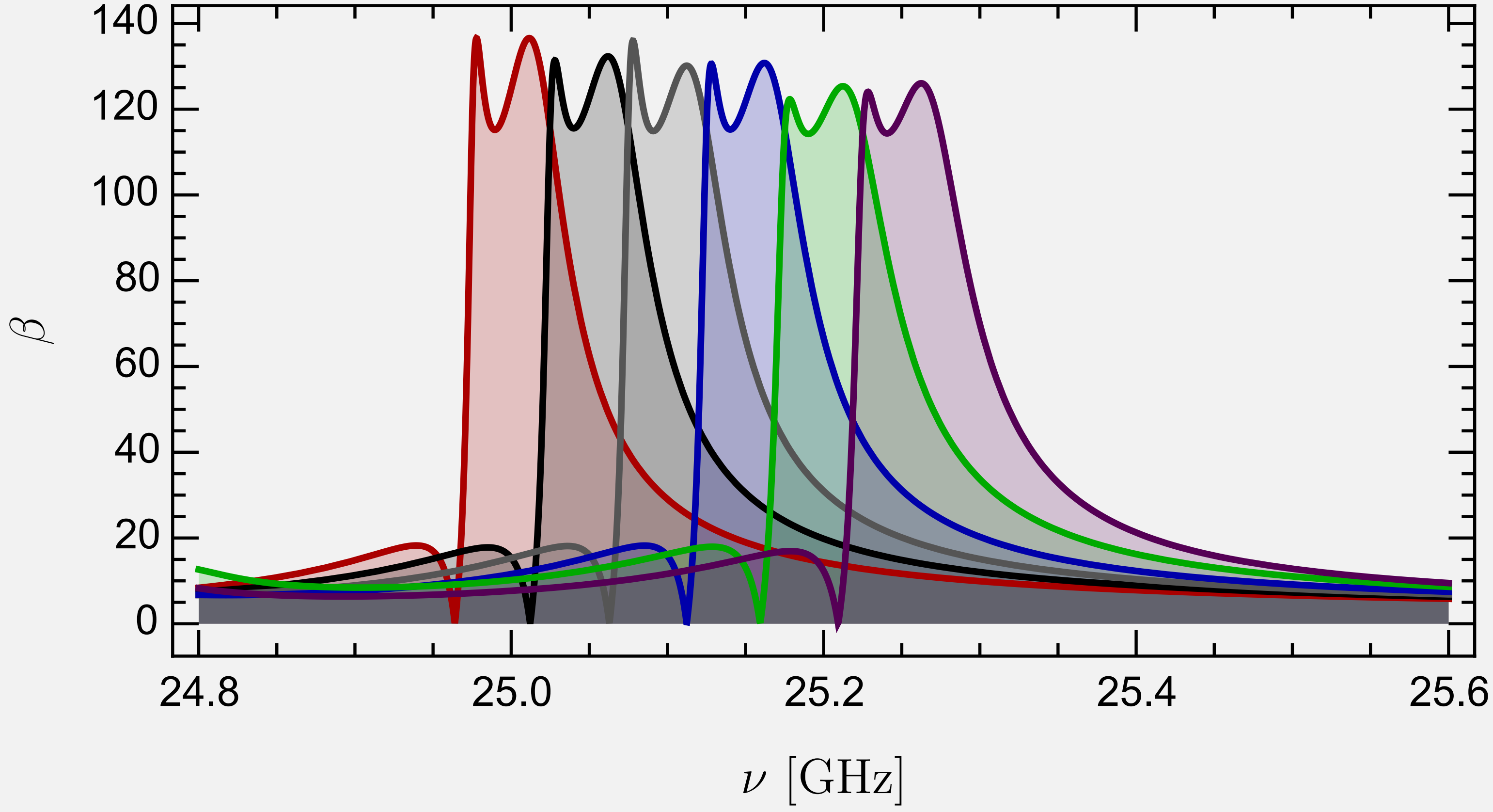
Example solutions

- Non-trivial task to optimise solutions
- Fix disk thicknesses (1 mm)
- N disks means N dimensional parameter space
- 20 disks and a mirror: less computationally intensive, complicated behaviour
- 80 disks and a mirror: more realistic for an experiment, longer to solve

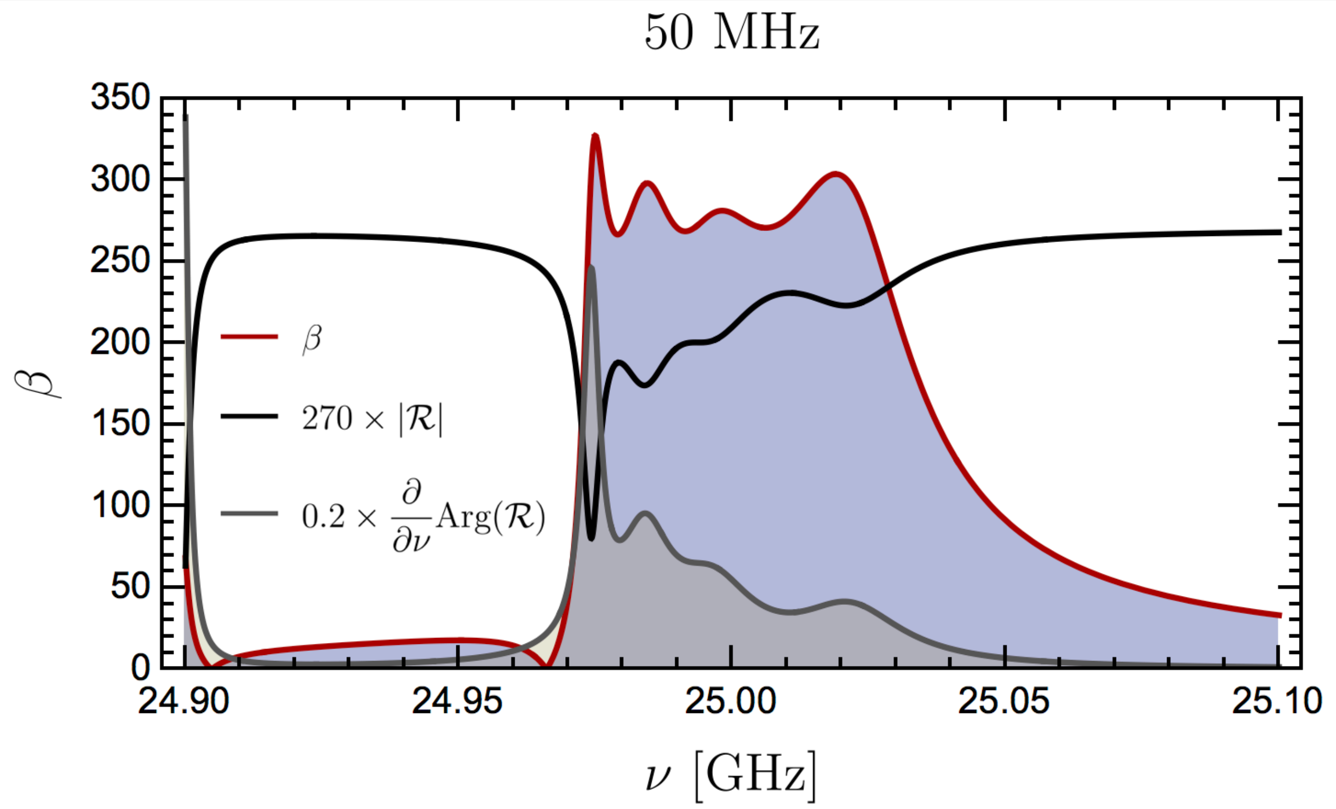
Example solutions: 20 disks



Example scan



Example solutions: 80 disks

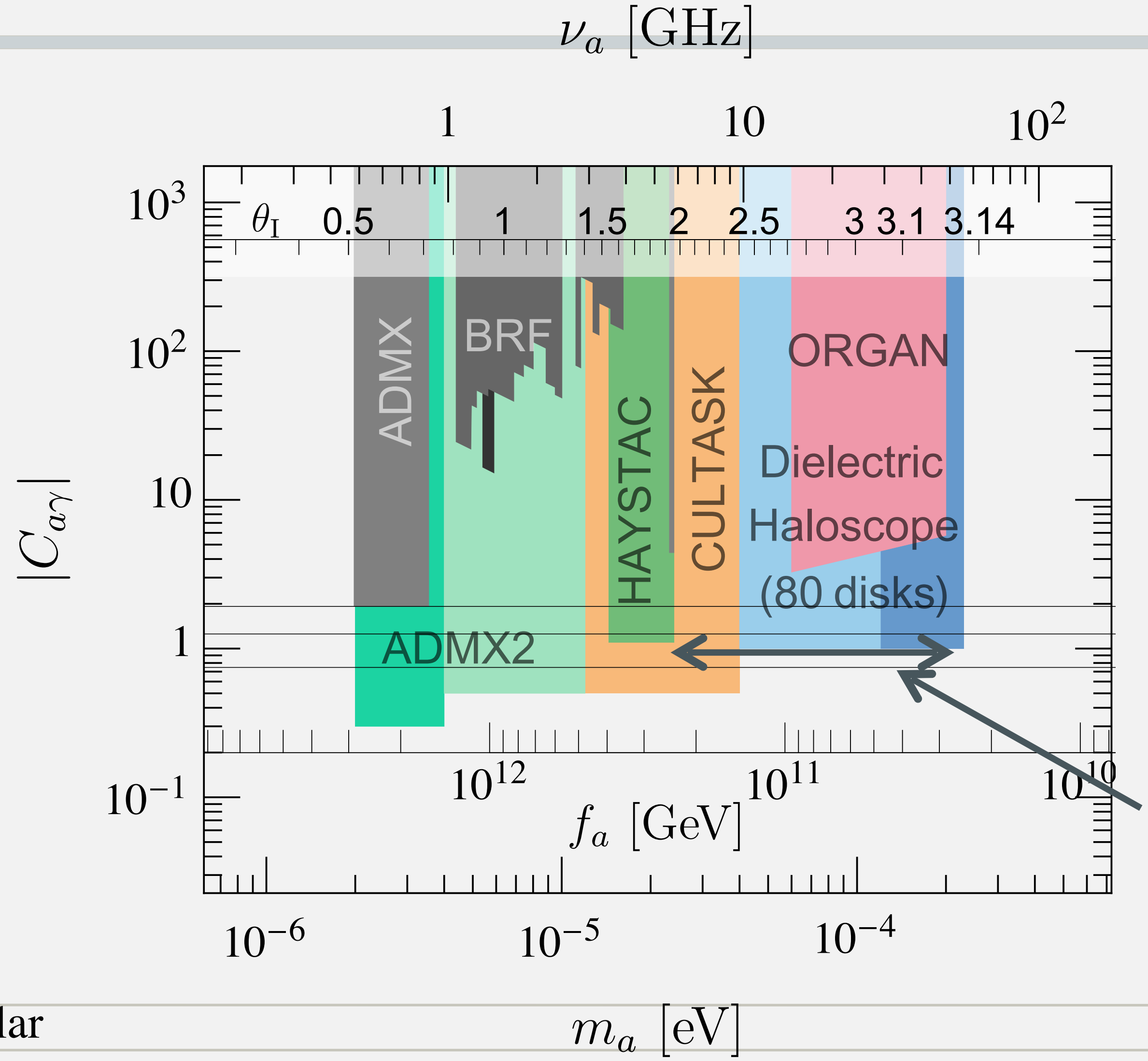


Discovery potential

- Can use the 80 disk solutions and the area law to extrapolate the general behaviour of many disk setups
- This allows us to predict the discovery potential of an experiment

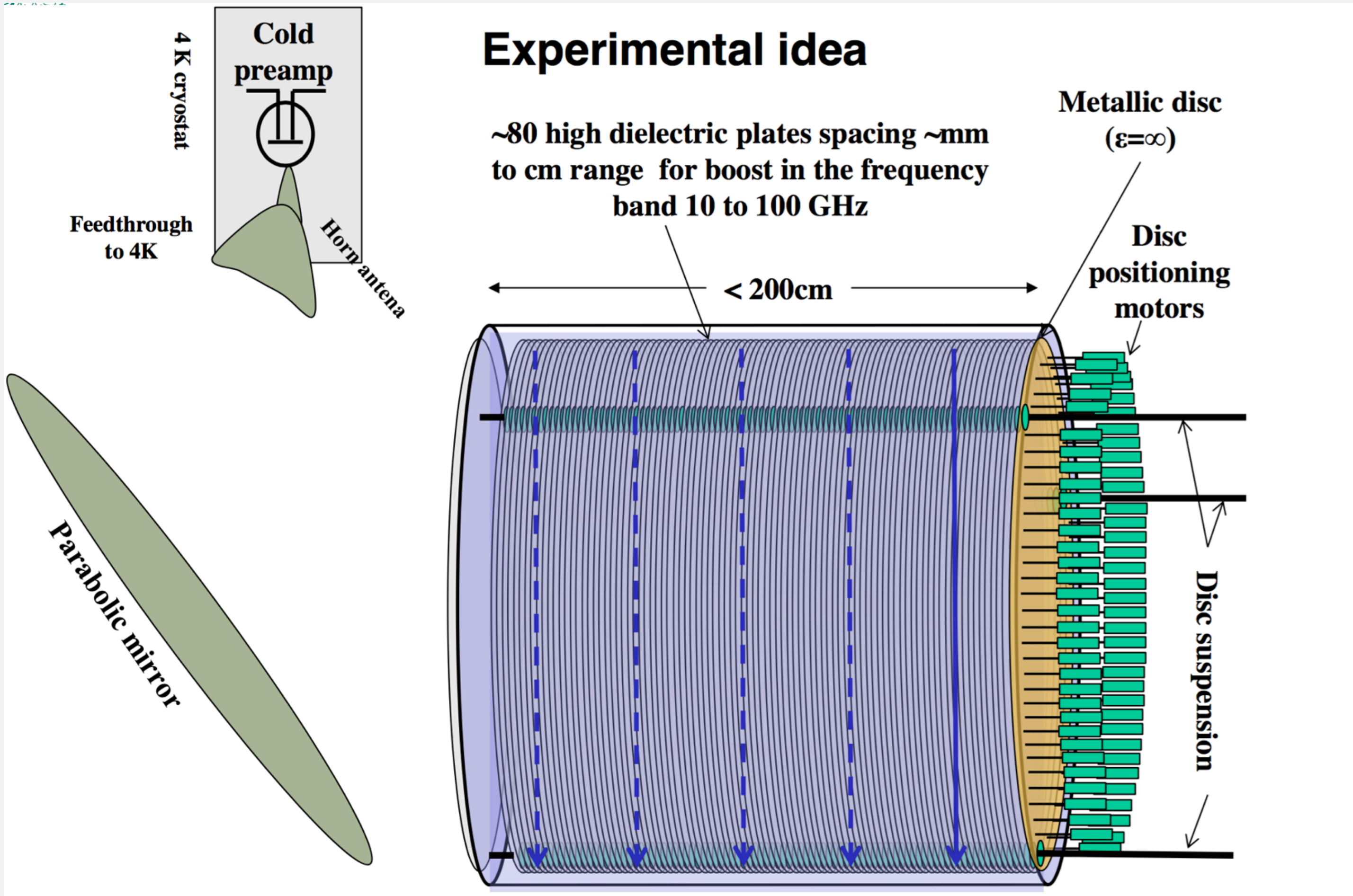
$$\frac{\Delta t}{1.3\,\mathrm{days}} \sim \left(\frac{\mathrm{S/N}}{5}\right)^2 \left(\frac{400}{\beta}\right)^4 \left(\frac{\mathrm{m}^2}{A}\right)^2 \left(\frac{m_a}{100\ \mu\mathrm{eV}}\right) \left(\frac{T_{\mathrm{sys}}}{8\,\mathrm{K}}\right)^2 \left(\frac{10\,\mathrm{T}}{B_{\mathrm{e}}}\right)^4 \left(\frac{0.8}{\eta}\right)^2 C_{a\gamma}^4$$

Discovery potential



Scenario 1

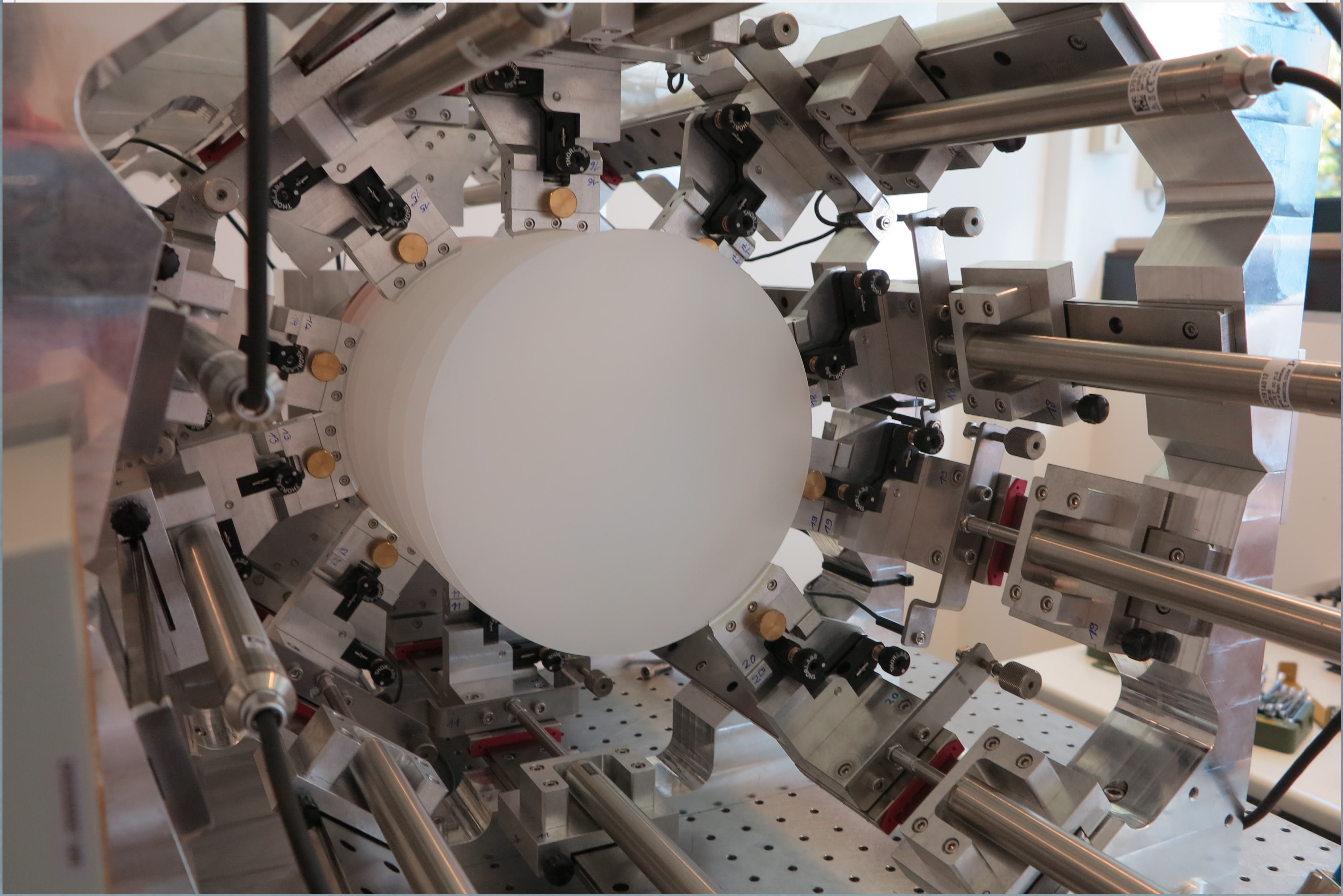
Towards an experiment: MADMAX



Ongoing experimental efforts

- Newly formed into an official collaboration, chose experimental site (DESY's HERA north hall)
- Two ongoing magnet design studies
- Trying to understand 3D effects, relation between boost factor and reflectivity
- Algorithms for correct disk placement, mechanical considerations for holding large disks
- Looking at disk tiling

20 Disk Prototype



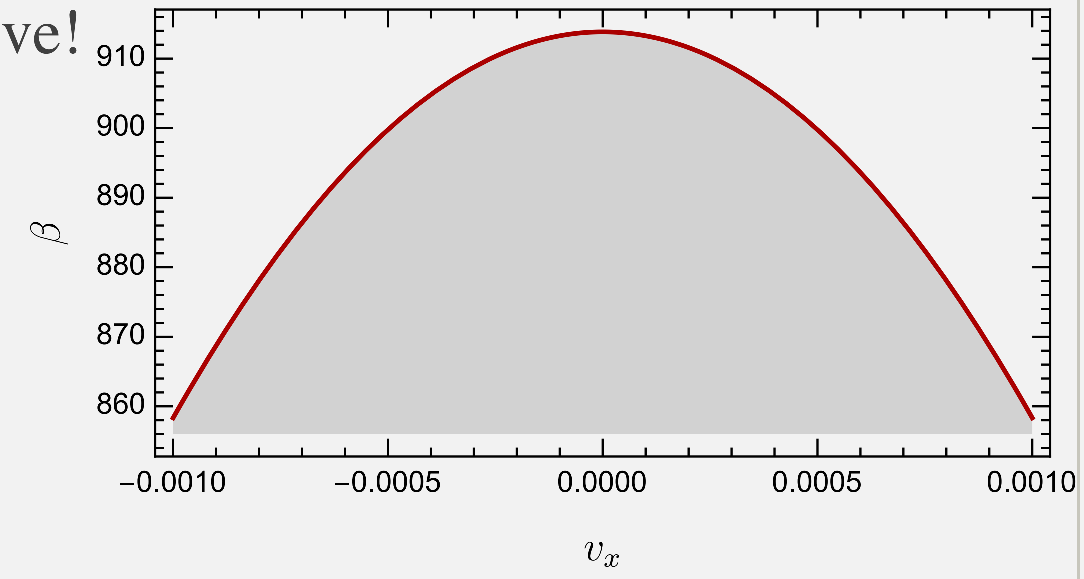
MADMAX Collaboration

Conclusions

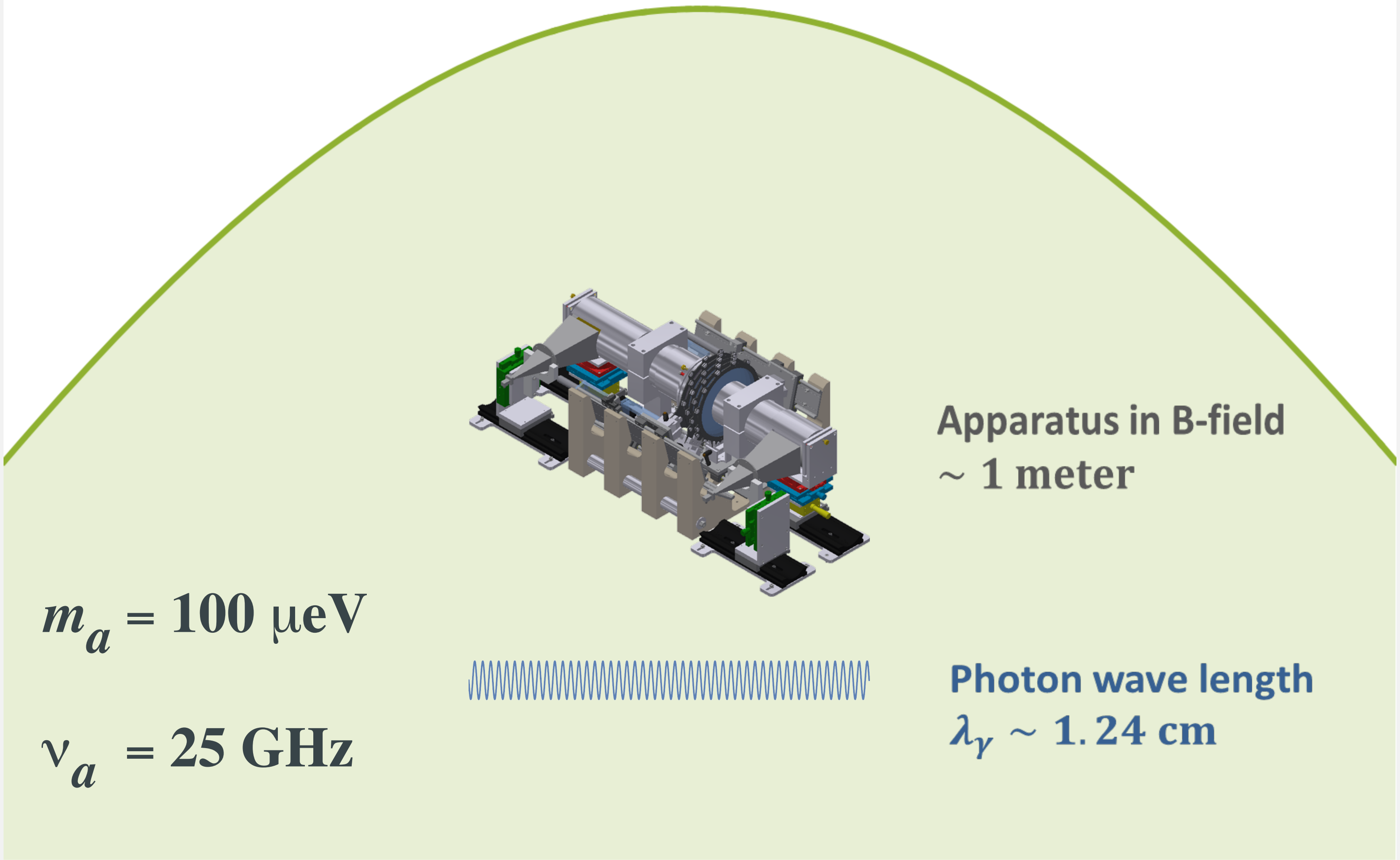
- Axions are a highly well motivated dark matter candidate with a unique phenomenology
- Dielectric haloscopes are an exciting new method for searching for axion dark matter
- They have the potential to detect axions produced in the predictive and mostly unexplored post inflationary scenario
- Didn’t have time for: dielectric haloscopes have excellent potential for directional sensitivity

Directional sensitivity

- If the device is larger than ~20% of the axion de Broglie wavelength, the change of the axion's phase becomes important
- Directionally sensitive!



Length Scales



$m_a = 100\ \mu\text{eV}$

$\nu_a = 25\ \text{GHz}$

Apparatus in B-field
~ 1 meter

Photon wave length
$\lambda_\gamma \sim 1.24$ cm

Magnets

- Need 10 T dipole magnets… of 1m aperture
- Currently two ongoing design studies
- Never been done before… challenging but exciting