Axion Superradiance in Rotating Neutron Stars

Jamie McDonald

based on **1904.08341** with Francesca Day

(Also work in progress with L Ventura (U. Aveiro) and R. Battye *et. al* at U. Manchester)

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Outline

- Introduction to superradiance in various forms

- Briefly review axion superradiance in black holes

- Axion like particles and their coupling to electromagnetism

- Axion superradiance in neutron stars

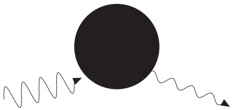
- Axion DM conversion in neutron star magnetospheres
- Timing delay of photons and superradiant BHs

- Summary

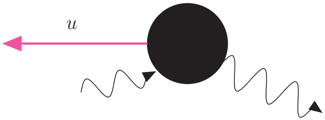
Superradiance - a working picture

Reviews: Beckenstein Schiffer 1998, Brito et al 2015

Take an object which can absorb radiation:



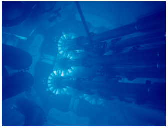
Then move it very fast: $u > c_s$:

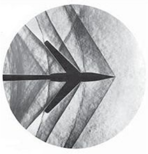


Low frequency waves $c_s \leq u$ are **amplified**

$\Longrightarrow$ **Superradiant Scattering**

E.g. Cherenkov Radiation, sonic booms

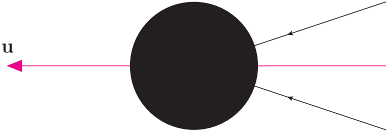




$$\omega - \mathbf{k} \cdot \mathbf{u} < 0 \quad \Leftrightarrow \quad \textbf{superradiance}$$

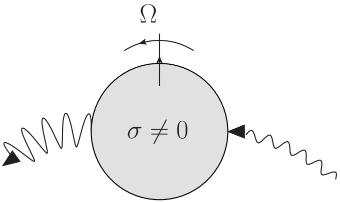
For **linear** trajectories must break Lorentz invariance with sound speed $c_s < 1$:

$$c_s/u < \cos\theta, \qquad \cos\theta = \hat{\mathbf{k}} \cdot \hat{\mathbf{u}}$$



Works in other geometries - **rotational superradiance** Zel'dovich 1971.

Conducting Rotating Sphere:



$m\Omega > \omega$

Maxwell's eqs. $\quad \Box A^\mu = j^\mu, \qquad j^\mu = \sigma F^{\mu\nu} u_\nu \quad$ Ohm's law

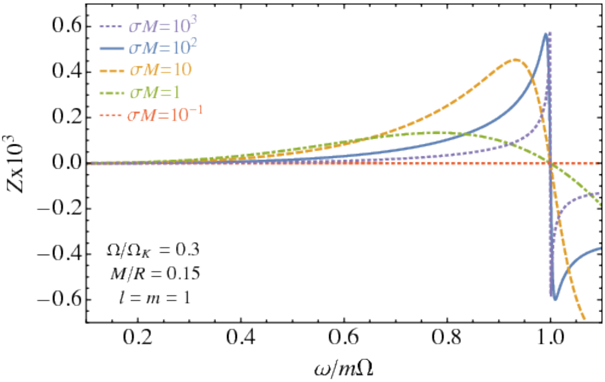
$$A^\mu \sim \frac{A}{r} e^{-i\omega t} \left[0,\ 0,\ 1/\sin\theta\partial_\varphi,\ -\sin\theta\partial_\theta\right] Y_{\ell m}$$

Radial "Schrödinger" equation:

$$\left[\frac{d}{dr^2} + \frac{\ell(\ell+1)}{r^2} + \underbrace{i\sigma(m\Omega - \omega)}_{\textbf{amplifiction}}\right] A_{\ell m}(r) = \omega^2 A_{\ell m}(r)$$

$$\lim_{r\to\infty} A \simeq |A_{\text{out}}| e^{ir\omega} + |A_{\text{in}}| e^{-ir\omega}$$

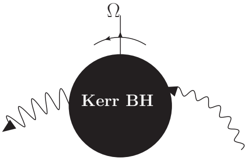
Amplification Factor : $Z = 1 - \frac{|A_{\text{in}}|}{|A_{\text{out}}|}$



Cardoso et al 2017

Modes with $\omega < m\Omega$ superradiantly scattered.

Even works with black holes: event horizon $\simeq$ absorptive surface



$m\Omega > \omega$

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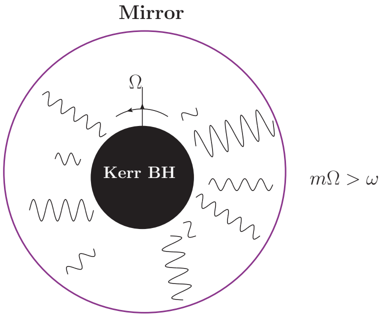
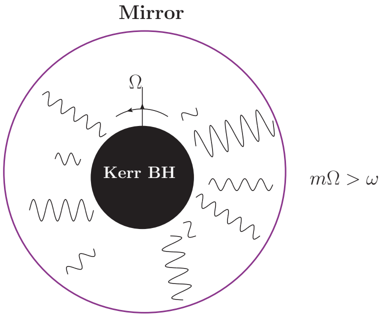


But what if we could **trap** the superradiantly scattered radiation?

Even works with black holes:



But what if we could **trap** the superradiantly scattered radiation?

Superradiant Instability - black hole bomb! Press, Teukolsky, Saul A. 1972.

growth driven by angular momentum extraction.

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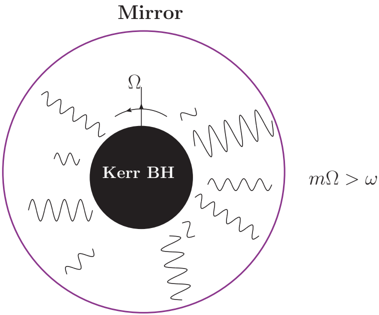


But what if we could **trap** the superradiantly scattered radiation?

Superradiant Instability - black hole bomb! Press, Teukolsky, Saul A. 1972.

growth driven by angular momentum extraction.

But there is a natural trapping mechanism - **Gravity!**

Axion : $g^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \mu^2\phi = 0$

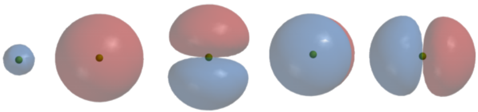
$$ds^2 = -dt^2\left(1 - \frac{2GM}{r}\right) + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Hydrogen-like solutions - e.g.Detweiler 1980

$$\phi = \psi(r) Y_{\ell m}(\theta,\varphi) e^{-i\omega t} \qquad -\frac{d\psi}{dr^2} + V(r)\psi = \omega^2\psi$$

$$V(r) \simeq \frac{\ell(\ell+1)}{r^2} - \frac{2GM\mu^2}{r} + \mu^2,$$ "Gravitational atom"

Gives rise to eigenvalue problem for spinning black holes

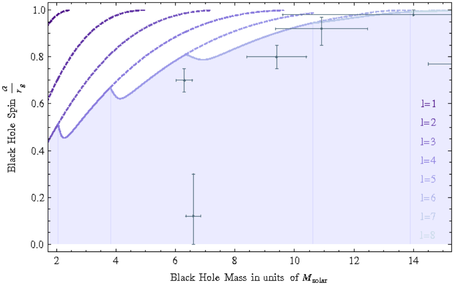


Combing the **rotation** and **confining** nature of a Kerr BH leads to an instability with discrete eigenvalues

$$\phi \sim e^{-i\omega t} \qquad \omega = \omega_R + i\omega_I \qquad a = J/M$$

$$\omega_R = \mu - \frac{\mu}{2}\left(\frac{GM\mu}{\ell + m + 1}\right)^2, \qquad \omega_I \sim (GM\mu)^{4\ell+5}\left(\frac{am}{GM} - 4\mu GM\right)$$

Axion feeds on BH angular momentum, spinning it down:



$\mu \simeq 10^{-11}$eV

Arvanitaki,Dubovsky (2011)

Much interest in going beyond vanilla BH superradiance

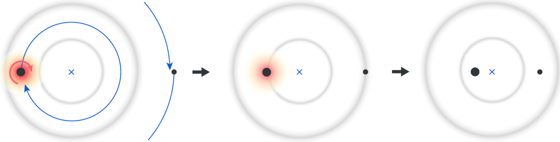
$$\mathcal{L} \supset \sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\mu^2}{2}\phi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g_{a\gamma\gamma}}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} - A_\mu j^\mu\right],$$

ALP-EM instabilities of BHs

"Axionic instabilities and new black hole solutions" - Cardoso et al (2019)
"Blasts of Light from Axions" - Cardoso et al (2019)
"Stimulated axion decay in superradiant clouds around PBHs" - Rosa, Kephart (2018)

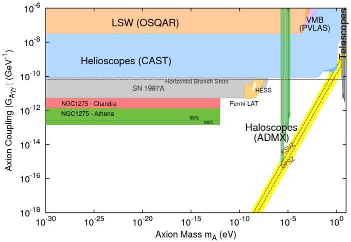
Superradiance in BH binaries

"Probing Ultralight Bosons with Binary Black Holes" " Baumann et al 2018



Superradiance in Neutron Stars

$$\mathcal{L}_{a\gamma\gamma} = -\frac{g_{a\gamma\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$



Magnetised conducting plasma: fluctuations A^μ and ϕ (JM, F. Day)

$$\Box\phi + \mu^2\phi = -\frac{g_{a\gamma\gamma}}{2}F_{\mu\nu}\tilde{F}_B^{\mu\nu},$$

$$\partial_\mu F^{\mu\nu}[A] - \sigma F^{\nu\mu}[A]u_\mu = -g_{a\gamma\gamma}(\partial_\mu\phi)\tilde{F}_B^{\mu\nu},$$

Background magnetic field:

$$\tilde{F}_B^{\mu\nu} = \begin{pmatrix} 0 & -\mathbf{B} \\ \mathbf{B} & 0 \end{pmatrix} \qquad j^\mu = \sigma F^{\mu\nu}u_\nu \quad \text{Ohm's law}$$

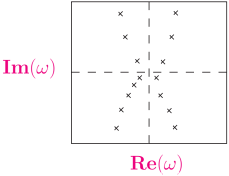
Mixing equations for axion and photon

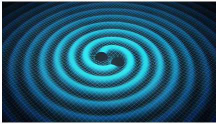
$$\Box\phi + \mu^2\phi = -g_{a\gamma\gamma}\left[\nabla A^0 + \dot{\mathbf{A}}\right]\cdot\mathbf{B},$$

$$\Box A^0 = -g_{a\gamma\gamma}\nabla\phi\cdot\mathbf{B} - \sigma\mathbf{u}\cdot\left[\nabla A^0 + \dot{\mathbf{A}}\right],$$

$$\Box\mathbf{A} = g_{a\gamma\gamma}\dot{\phi}\mathbf{B} - \sigma\left[\nabla A^0 + \dot{\mathbf{A}}\right] + \sigma\mathbf{u}\times\left[\nabla\times\mathbf{A}\right],$$

Stationary backgrounds, $A, \phi \sim e^{-i\omega t}$ + Dirichlet BCs:



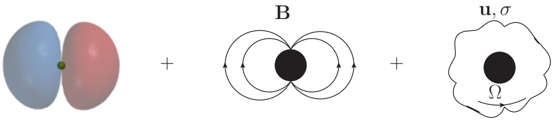


Analogous to BH QNMs

Difficult Problem!

(i) two mixing fields (4 d.o.f.)
(ii) all (ℓ, m) are coupled
(iii) 3-dimensional

Quantum mechanical perturbation theory



Axion Boundstate Mangetic Mixing Plamsa

$$[H(\sigma_*) + V(\sigma, g_{a\gamma\gamma})] \begin{pmatrix} |\phi\rangle \\ |A^0\rangle \\ |\mathbf{A}\rangle \end{pmatrix} = \omega^2 \begin{pmatrix} |\phi\rangle \\ |A^0\rangle \\ |\mathbf{A}\rangle \end{pmatrix}, \qquad \hat{p} \leftrightarrow -i\nabla$$

Mixing : $$V_{a\gamma\gamma} = i g_{a\gamma\gamma} \begin{pmatrix} 0 & \mathbf{B}(\hat{x})\cdot\hat{p} & -\omega\mathbf{B}(\hat{x}) \\ \mathbf{B}(\hat{x})\cdot\hat{p} & 0 & 0 \\ \omega\mathbf{B}(\hat{x}) & 0 & 0 \end{pmatrix}$$

Plasma : $$V_{\mathrm{pl}} = i\sigma_M(\hat{x}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mathbf{u}(\hat{x})\cdot\hat{p} & -\omega\mathbf{u}(\hat{x}) \\ 0 & \hat{p} & -\omega - \mathbf{u}(\hat{x})\times\hat{p} \end{pmatrix}$$

Quantum mechanical perturbation theory

Perturbing an eigenspectrum:

$$\omega = \omega_0 + \omega_1 + \omega_2 + \omega_3 + \cdots, \qquad \omega^{(n)} = \mathcal{O}\left(V^n\right)$$

Unperturbed axion states $|\phi_{\ell mn}\rangle$, complete basis of *continuous* photon states

$$\sum_{\ell m} \int \frac{d\omega}{2\pi} \left|A_{\ell m}, \omega\right\rangle \left\langle A_{\ell m}, \omega\right| = \mathbb{I}, \qquad -\nabla^2 \left|A_{\ell m}, \omega\right\rangle = \omega^2 \left|A_{\ell m}, \omega\right\rangle$$

$$\omega_1 = \langle \phi_{\ell mn} | V | \phi_{\ell mn} \rangle = 0$$

$$\omega_2 = \sum_{\ell' m'} \int \frac{d\omega}{2\pi} \frac{\langle \phi_{\ell mn} | V | A_{\ell' m'}, \omega \rangle \langle A_{\ell' m'}, \omega | V | \phi_{\ell mn} \rangle}{\omega_0^2 - \omega^2}$$

etc...

Quantum mechanical perturbation theory

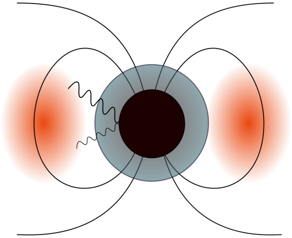
$$\omega_3 = \sum_{\ell m} \int \frac{d\omega}{2\pi}\frac{d\omega'}{2\pi} \frac{\langle\phi_{\ell mn}|\, V\, |A_{\ell' m'}\rangle \langle A_{\ell' m'}|\, V\, |A_{\ell'' m''}\rangle \langle A_{\ell' m'}|\, V\, |\phi_{\ell mn}\rangle}{(\omega_0^2 - \omega^2)(\omega_0^2 - \omega'^2)}$$

Take residues:

$$\mathrm{Im}\,\omega_3 = \Gamma, \qquad \Gamma \sim \langle\phi|\, V_B\, |A\rangle \langle A|\, V_{\rm pl}\, |A\rangle \langle A|\, V_B\, |\phi\rangle$$

Instability (JM, F. Day)

$$\phi(\ell, m) \sim e^{\Gamma t} \qquad \Gamma \sim g_{a\gamma\gamma}^2 B^2 \sigma (m\Omega - \omega)(R\omega)^{2\ell+3}(2GM\mu)^{2\ell+5},$$



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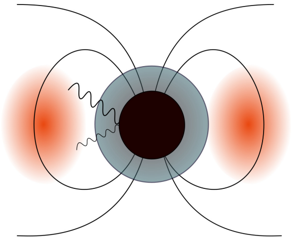
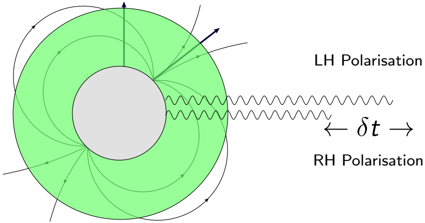


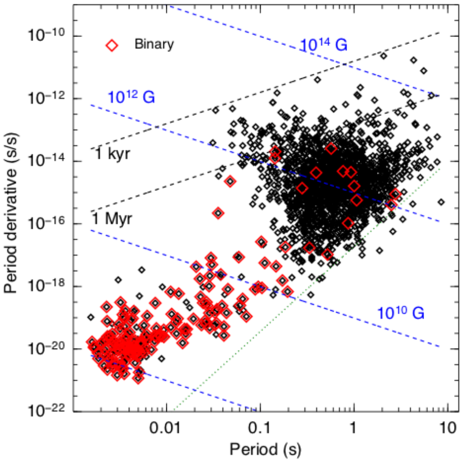
Step 1 Perturb system with initial axion boundstate
Step 2 Excites photon modes via $\mathbf{B}_{\mathrm{NS}}$-field mixing
Step 3 star = conducting (σ), rotating (Ω)
$\Longrightarrow$ Photon modes superradiantly scatter off NS magnetosphere
Step 4 Photons dump extracted rotational energy into axion sector

Observational Implications

- Axion constraints from pulsar spin down measurements Cardoso et al 2017
- $\mathcal{L}_{\phi\gamma\gamma} = g_{\gamma\phi}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$ violates **parity** - polarisation dependent time delay δt of radio signals from photons travelling through axion cloud. Mohanty, Nayak 1993



Pulsar timescales and spin down ($P, \dot{P}$)



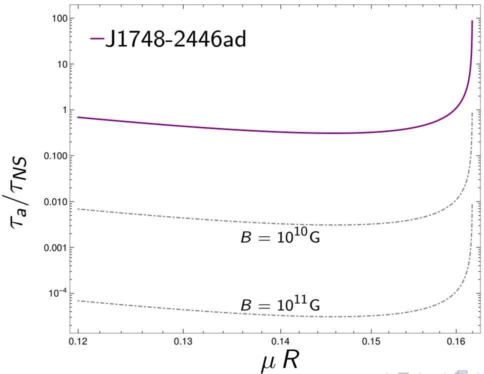
The characteristic NS time scale:

$$\tau_{NS} = \Omega/\dot{\Omega}$$

$$E_{\rm rot} = \frac{1}{2} I\Omega^2 : \qquad \frac{dE_{\rm rot}}{dt} = -\frac{2}{\tau_{NS}} E_{\rm rot},$$

spin down time can be very long: $10^5 - 10^{11}$yr Harding 2013

Compare instability timescale $\tau_a = 1/\Gamma$ to τ_{NS}



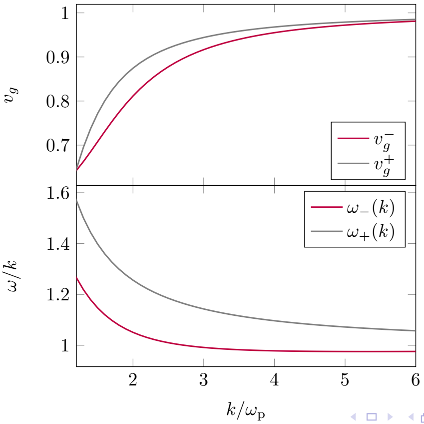
More on axions, EM, neutron stars and black holes - in progress

Axion backgrounds as dispersive (polarisation dependent) media

$$\mathcal{L}_{\phi\gamma\gamma} = -\frac{g_{a\gamma\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} = g_{a\gamma\gamma}\phi \mathbf{E}\cdot\mathbf{B} \qquad \text{violates } \textbf{parity}$$

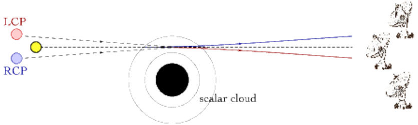
$\implies$ **LH** and **RH** polarisations different dispersion in axion background

Dispersion and Birefringence

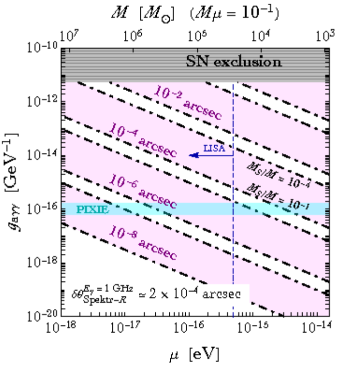


Polarisation-dependent bending around BH (or pulsar?)

Plascencia, Urbano (2017)

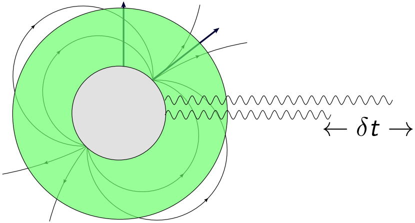


Refractive index $n = c/v_p \implies \mathcal{O}(g_{a\gamma\gamma})$ effect: $\delta\phi \lesssim 10^{-5}$arcsec.



Aside - $\mathcal{O}(g_{a\gamma\gamma})$ bending recently (Tues.) questioned by Blas, Caputo, Ivanov, Sberna 1910.06128

Polarisation-dependent time delay from axion clouds



$$\Box\phi + \mu^2\phi = g_{a\gamma\gamma}\,(\mathbf{E}.\mathbf{B})_{\rm NS}$$

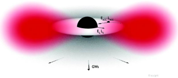
$$\delta t_\gamma = \int d\ell \frac{1}{v_g}$$ Mohanty, Nayak (1993)

PSR B1937+21 : $\quad \delta t \lesssim 10^{-6}s \implies g_{a\gamma\gamma} \lesssim 10^{-11}\mathrm{GeV}$

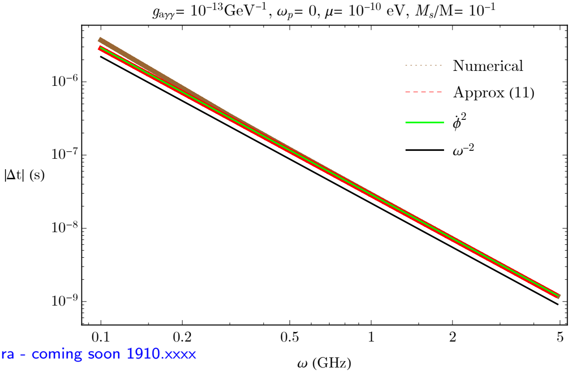
Dispersive time delays from Superradiant BH

$$t_a = -\frac{g_{a\gamma\gamma}^2}{8k_\gamma^2}\int d\ell\,\dot{\phi}^2$$

$$\Delta t_a(\omega_{\rm p}) = \pm\frac{g_{a\gamma\gamma}\omega_{\rm p}{}^2}{2k_\gamma^3}\int d\ell\,\dot{\phi}$$



$$\Delta t = \pm\frac{g_{a\gamma\gamma}^3}{8k_\gamma^3}\int d\ell\,\dot{\phi}^3$$



JM with L Ventura - coming soon 1910.xxxx

Primordial black holes

Many black holes along line of sight

-Estimate number of binary collisions which have produced remnant BH with high spin $a > a_c$:

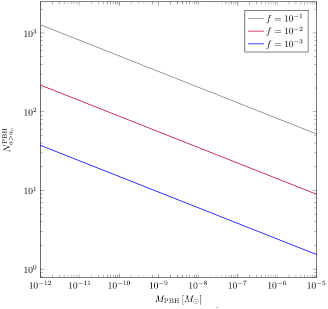
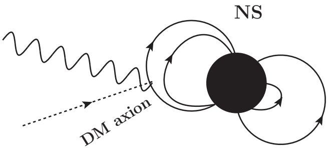


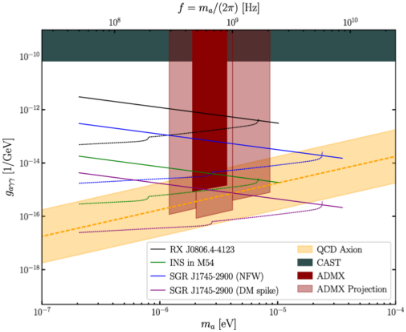
Figure: Number of PBHs with $a > a_c$ along line of sight.

Enhanced effect $N_c \times \delta t$.

Dark matter axion conversion in NS magnetospheres:
$m_a \sim 10^{-6}\mathrm{eV}$
Axions fall towards NS from galactic halo and mix (resonantly!) with photons in neutron star magnetosphere (where $\omega_{\rm pl} = m_a$) producing GHz radio signatures (Hook 2018, Pshirkov 2007)

$$\omega_\gamma \simeq m_a$$





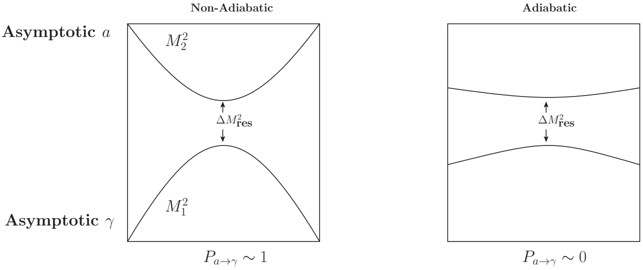
1. Axion-photon mixing in NS magnetosphere: (Hook et al 2018)

$$-\frac{d^2}{d^2z}\begin{pmatrix} a \\ E_{\parallel} \end{pmatrix} + \begin{pmatrix} m_a^2 & -i\omega g_{a\gamma\gamma}B \\ i\omega g_{a\gamma\gamma}B & m_\gamma^2(z) \end{pmatrix}\begin{pmatrix} a \\ E_{\parallel} \end{pmatrix} = \omega^2 \begin{pmatrix} a \\ E_{\parallel} \end{pmatrix}$$

Resonant conversion at $m_\gamma(z) = m_a$.

$$\text{Landau-Zener}: \quad P_{a\to\gamma} = 1 - e^{-2\pi\gamma_{\text{res}}} \qquad \gamma_{\text{res}} \sim \left.\frac{(\Delta M^2)^2}{\omega dm_\gamma^2/dz}\right|_{\text{res}}$$

Conversion depends on the **mass gap**, and how **rapidly** the background varies.

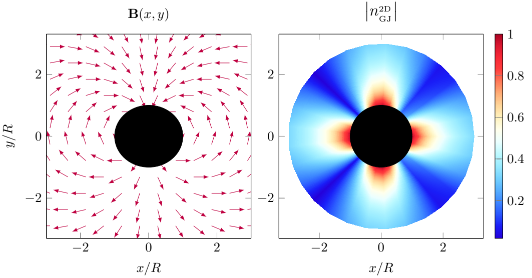


Importance of 3D effects?

$$\mathbf{3D}: -\nabla^2 \mathbf{E} + \nabla(\nabla \cdot \mathbf{E}) + \sigma \cdot \dot{\mathbf{E}} - \omega^2 \mathbf{E} = \omega^2 g_{a\gamma\gamma} a \mathbf{B}_0$$

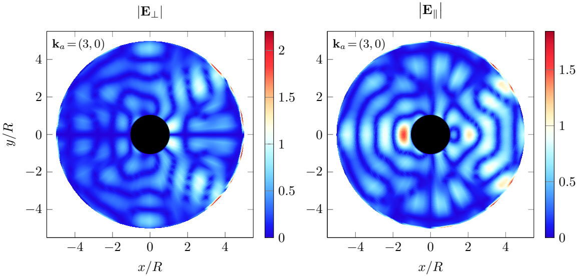
Conductivity σ arises from **B** and plasma density $\omega_{\rm p} \propto n_e$:

Warm up in 2D



JM with R. Battye *et al*

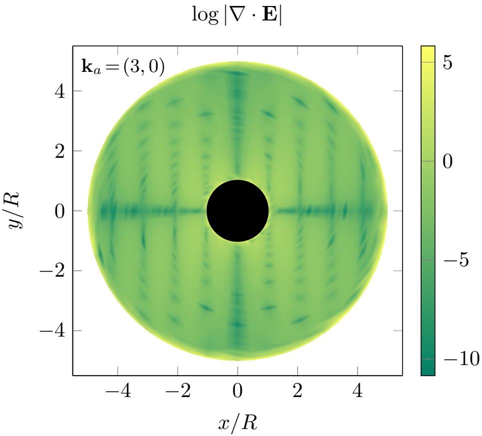
$$\mathbf{3D}: -\nabla^2 \mathbf{E} + \nabla(\nabla \cdot \mathbf{E}) + \sigma \cdot \dot{\mathbf{E}} - \omega^2 \mathbf{E} = \omega^2 g_{a\gamma\gamma} a \mathbf{B}_0\,,$$



Component normal to magnetic field $\mathbf{E}_\perp$ also active

Photon also acquires longitudinal polarisation:

$$\textbf{3D}: -\nabla^2 \mathbf{E} + \nabla(\nabla \cdot \mathbf{E}) + \sigma \cdot \dot{\mathbf{E}} - \omega^2 \mathbf{E} = \omega^2 g_{a\gamma\gamma} a \mathbf{B}_0 \,,$$



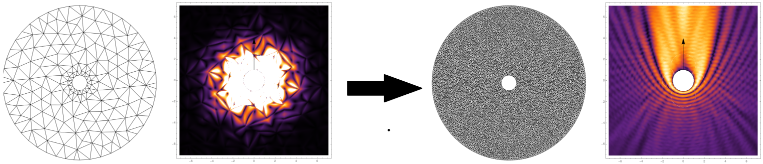
Gauss Law $\quad \nabla \cdot \mathbf{E} = \nabla \cdot \left(\frac{\sigma}{\omega} \cdot \mathbf{E}\right) - g_{a\gamma\gamma} \mathbf{B}_0 \cdot \nabla a$

3D Results

- So far only qualitative statements about polarisations JM, Battye et al
- 3D haloscopes examined by Redondo, Ringwald, Knirck 1906.02677
- Need quantitative analysis but large hierarchy of scales present numerical challenges:

$$\lambda_\gamma = 1/m_a = \text{cm} - \text{meter} \ll R_{\rm NS} = 10\text{km}.$$

-High level of integration cells needed to resolve structure



- Solution?Coarse graining procedure to derive transport equations with wave-front structure integrated out. Track only field amplitudes?

Summary

(i) Astrophysical environments remain interesting settings in which to probe axion physics.

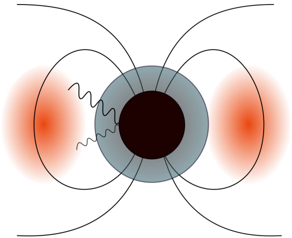
(ii) Neutron stars as well as BHs can exhibit superradiance provided there is source of dissipation.

(iii) Superradiant axion clouds can lead to time-delays of $10^{-6}s$ at $g_{a\gamma\gamma} = 10^{-13}\mathrm{GeV}$. Plasma can also enhance time-delay.

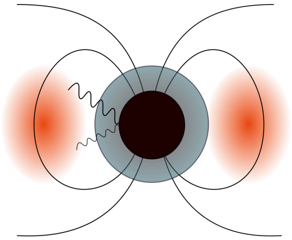
(iv) Study 3D effects for axion-photon conversion in plasmas to better understand DM conversion in neutron star magnetospheres.

Thanks for listening!

Quenching of the Instability



Quenching of the Instability



can radiation leak out?

Preliminary - work in progress!

Consider a boundstate coupled to a continuum

$$\begin{pmatrix} \ddot{\phi} \\ \ddot{A} \end{pmatrix} - \frac{d^2}{dx^2}\begin{pmatrix} \phi \\ A \end{pmatrix} + \begin{pmatrix} V(x) & B \\ B & \alpha\partial_t \end{pmatrix}\begin{pmatrix} \phi \\ A \end{pmatrix} = 0$$

Harmonic Oscillator:

$$V(x) = \frac{1}{4}\kappa^2 x^2, \qquad \phi_0(x) = e^{-x^2\kappa^2/4}$$

Question. what happens when friction is switched off $\alpha = 0$

$$M = \begin{pmatrix} 0 & B \\ B & 0 \end{pmatrix}$$ **hermitian**: surely no dissipation if $\alpha = 0$?

Perhaps just a real spectral shift?

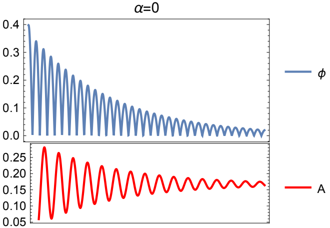
$$\omega_n^2 \to \omega_n^2 + \sum_\omega \frac{\langle\phi_n|\, V\, |A\rangle \langle A|\, V\, |\phi_n\rangle}{\omega_n^2 - \omega^2}$$

One might argue eigenvalues still real since M hermitian

$|\langle A|\, V\, |\phi_n\rangle|^2$ real? No dissipation at 2nd order in PT?

Preliminary - work in progress!

But that's not what happens:



When $\alpha = 0$ boundstate decays via magnetic mixing and EM radiation (see also Cardoso 2018/19)

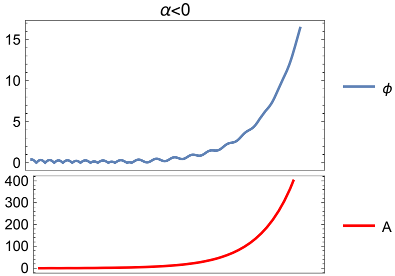
$$\sum_{\omega} \frac{|\langle\phi_n| V |A\rangle|^2}{\omega_n^2 - \omega^2} : \qquad \sum_{\omega} \to \int d\omega \Leftrightarrow 2\pi i \times (\text{residues})$$

$$\Gamma \sim |\langle\phi_n| V |A\rangle|^2 \qquad \text{Fermi's Golden Rule}$$

for a continuous spectrum modes escape, energy lost at infinity - outgoing Neumann boundary conditions.

Preliminary - work in progress!

Good news! instability still persists for $\alpha < 0$



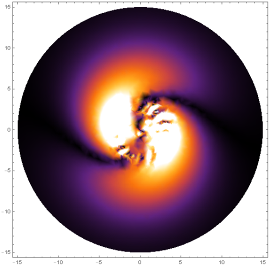
Threshold condition $\alpha \leq -|\alpha_c|$ for energy extraction via friction to overcome loss to leakage.

Preliminary - work in progress!

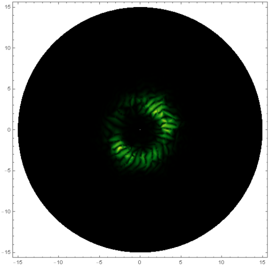
2D Simulations JM 2019 in progress

$$\Box_{t,x,y}\begin{pmatrix}\phi\\ A\end{pmatrix}+\begin{pmatrix}\mu^2-\frac{r_s\mu^2}{r} & -g_{a\gamma}B\partial_t\\ g_{a\gamma}B\partial_t & \sigma(\partial_t+\mathbf{u}\cdot\nabla)\end{pmatrix}\begin{pmatrix}\phi\\ A\end{pmatrix}=0$$

Axion



Photon

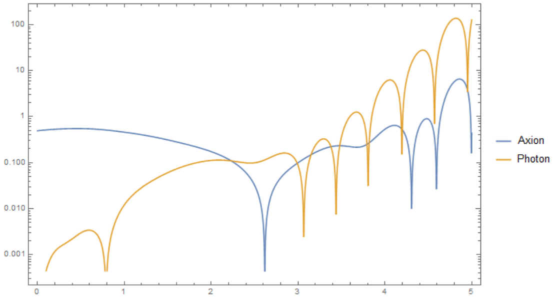


"Ohm's law" term: $\sigma(\partial_t+\mathbf{u}\cdot\nabla)\to\sigma(m\Omega-\omega)$

Preliminary - work in progress!

2D Simulations

$$\Box_{t,x,y}\begin{pmatrix}\phi\\ A\end{pmatrix}+\begin{pmatrix}\mu^2-\frac{r_s\mu^2}{r} & -g_{a\gamma}B\partial_t\\ g_{a\gamma}B\partial_t & \sigma(\partial_t+\mathbf{u}\cdot\nabla)\end{pmatrix}\begin{pmatrix}\phi\\ A\end{pmatrix}=0$$

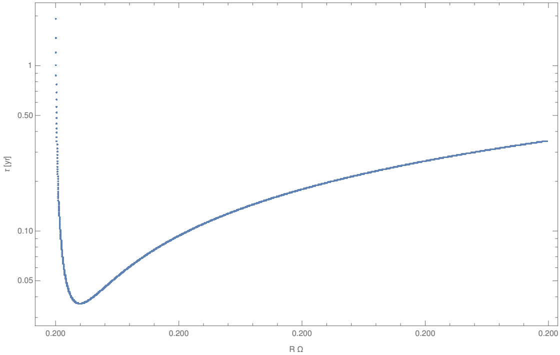


t

$u^\mu=(1,0,0,\Omega),\quad m\Omega>\omega\quad \sigma\geq\sigma_c\qquad\Leftrightarrow$ **unstable!**

Little more work to finish 3D simulations + analytics

Instability time-scale from stellar interior B $= 10^{14}\,G$, $\sigma \gg \mu$



Very high conductivity in interior $\rightarrow$ narrow resonance at $\mu \sim m\Omega$.
(Mixing of axion with a specific photon mode)

Thanks for listening!