# **Precise QCD predictions for Higgs boson pair production**

#### Javier Mazzitelli



Cambridge, June 2018

#### Outline

#### • Introduction:

Motivation, main production and decay modes

Status and prospects for the LHC

• QCD corrections in the large M<sub>t</sub> limit:

NNLO cross section

Threshold resummation at NNLL

• NNLO predicitons with finite M<sub>t</sub> effects:

Technical ingredients, NNLO approximations

Total cross sections and differential distributions

Conclusions

## **Multi-Higgs production**

- Higgs couplings to fermions and gauge bosons so far compatible with SM
- What happens for the Higgs self-couplings? Are they relevant? How can we measure them?





• Self-couplings determined by the Higgs potential

$$V(H)=\frac{1}{2}M_H^2H^2+\lambda vH^3+\frac{1}{4}\lambda'H^4$$
 In the SM:  $\lambda=\lambda'=M_H^2/(2v^2)$ 

# 

Produce an **off-shell** Higgs boson that decays into: Trilinear coupling  $H^* \to HH$ 

Quartic coupling  $H^* \to H H H$ 

Experimentally very challenging!



At the LHC:



### **Double Higgs production mechanisms**



[1] Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira 12;
 [2] Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro 14;
 [3] Ling, Zhang, Ma, Guo, Li, Li 14;
 [4] Li, Wang 16;
 [5] Li, Li, Wang 17;

#### **Di-Higgs decay channels**



**Relevant channels:** in general at least one  $H \rightarrow bb$  to have large BR

bbbb: highest BR, high QCD and tt contamination
bbWW: high BG, large irreducible tt background
bbtt: relatively low background and low BR
bbyy: high purity, very low BR

### **LHC results**

1

BSM scenarios can substantially enhance the HH cross section or produce a resonance

Both resonant and non-resonant searches have been performed at ATLAS and CMS

 $\sigma/\sigma_{\rm CM} = 05\%$  C.L. (evp)

	0/0	$S_{\rm M}$ JU/0 C.L.	(cxp)
Results from non-resonant		ATLAS	CMS
searches, upper limits:	bbbb	<29 (38)	<342 (308)
• Run-I			
ATLAS combined: 70 x SM CMS bbvv: 74 x SM	bbWW		<79 (89)
	bbττ		<28 (25)
• Run-II			
Reaching O(10) xSM sensitivity	bbyy	<117 (161)	<19 (17)
• SM sensitivity: full HL-LHC statistics	WWγy	<747 (386)	
	P. Meridiani, EPS17	3 fh <sup>-1</sup> 12 fh	-1 36 fb <sup>-1</sup>

## **Prospects for the LHC and beyond**

- Assuming a SM rate, HH production should be observed at the HL-LHC
- Expected uncertainty on the signal yield: O(50%) using bbyy and  $bb\tau\tau$
- Combination with other decay channels (specially 4b) will reduce this uncertainty

[ATL-PHYS-PUB-2014-019, ATL-PHYS-PUB-2015-046, CMS PAS FTR-15-002]

Higgs pair production should be observed at the HL-LHC... but we also want to measure  $\lambda$ 



Assuming a SM-like scenario

- Determination of  $\lambda$  will require full HL-LHC integrated luminosity and the combination of the different channels
- $\bullet$  Even then, uncertainties on  $\lambda$  will be large
- Complementary information from loop effects in single Higgs and EW precision observables
- Precision determination of  $\lambda$ : one motivation for a future collider

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## HH production via gluon fusion



[1] Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke 16; [2] Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, JM 18; [3] Heinrich, Jones, Kerner, Luisoni, Vryonidou 17; [4] Jones, Kuttimalai 17; [5] Shao, Li, Li, Wang 13; [6] de Florian, JM 15; [7] Ferrera, Pires 16; [8] de Florian, Fabre, JM 17;

## NLO with full top mass dependence

Calculation of QCD corrections is really difficult: exact NLO only became available in 2016
 Borowka et al. arXiv:1604.06447



- NLO corrections are very large (~66% for total cross section at 14TeV)
- Beyond that: heavy top quark mass limit (HTL, also called HEFT)



 Typically, corrections computed in the HEFT and normalized by exact LO differentially in M<sub>hh</sub> (16% overestimation at NLO – further improvements also possible, but more about this later)

## **QCD corrections in the HEFT**

E.g.: virtual corrections



• The effective vertices have the same structure!

 $\mathcal{L}_{ggH} \propto G_{\mu\nu} G^{\mu\nu} H/v$  $\mathcal{L}_{ggHH} \propto G_{\mu\nu} G^{\mu\nu} (H/v)^2$ 

 Profit from the single Higgs production results!

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• Profit from the single Higgs production results!

• We can split the calculation

$$Q^{2}\frac{d\hat{\sigma}}{dQ^{2}} = \hat{\sigma}^{a} + \hat{\sigma}^{b}$$
Single-Higgs like

**Starts at NLO** 

New topologies with two effective vertices

## **QCD corrections in the HEFT**

• At NNLO we have



• Similar idea for real-virtual and double-real corrections

We obtained analytical results for the NNLO total cross section, differential only in the HH invariant mass

• Extended to include BSM effects via EFT dimension 6 operators (backup slides)





## **Threshold Resummation**

All-order summation of threshold enhanced contributions



 $z = \frac{Q^2}{\hat{s}}$ 



• Originated by **soft gluon** emissions

• Threshold enhanced contributions: 
$$(\ln N)^k$$



Resummed contributions should account for a large part of the uncalculated missing higher orders



#### **Threshold Resummation**



**Universal structure:** only process dependence encoded in FO virtual corrections D. de Florian, JM, arXiv:1209.0673

Resummed contribution is matched to the fixed order result

$$\sigma^{NNLL} = \sigma^{res} - \sigma^{res}|_{\mathcal{O}(\alpha_S^4)} + \sigma^{NNLO}$$

### **NNLO+NNLL numerical results**



Here NNLL means

- Shape: small differences between FO and resummed distributions
- Uncertainty reduction from NNLO to NNLL
- Resummed contributions increase of the cross section

• NNLL/NNLO ratio vs. HH invariant mass



- NNLL always larger than NNLO, ratio is almost linear in Q
- Larger collider energies Smaller resummation effects (further from threshold)





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## **HEFT vs full theory**

- HEFT: large Mt limit  $\rightarrow$  Worse than for single Higgs (larger invariant mass)
- $\bullet$  Born improved overestimates the NLO total XS by a 15%
- Poor description of the tail of some distributions (associated with hard radiation)

NLO distributions [S. Borowka et al., arXiv:1608.04798]



- To obtain accurate NNLO results, we need to combine the HEFT NNLO with the full NLO
- $\bullet$  Moreover, we need to include finite  $M_t$  effects in the NNLO corrections

#### HH at NNLO with $M_t$ effects

Higgs boson pair production at NNLO with top quark mass effects M. Grazzini, G. Heinrich, S. Jones, S. Kallweit, M. Kerner, J. Lindert, JM [arXiv:1803.02463]

• Fully differential predictions for Higgs boson pair production via gluon fusion

- Combination of full NLO with large- $M_t$  NNLO
- NNLO piece improved with different reweighting techniques to account for finite-M<sub>t</sub> effects
- Estimation of remaining  $M_t$  uncertainty at NNLO
- Most advanced perturbative prediction available to date





Analytical results for NNLO two-loop corrections in the HEFT [de Florian, JM, '13]

NNLO subtraction formalism: q<sub>T</sub>-subtraction [Catani, Grazzini, '07]

Implementation based on public code MATRIX [Kallweit, Grazzini, Wiesemann, '17]

We worked with three different **approximations** for the pure NNLO piece:

NLO-improved approximation – NNLO<sub>NLO-i</sub>

- Born-projected approximation NNLO<sub>B-proj</sub>
- Full-theory approximation NNLO<sub>FTapprox</sub>

### **qT** subtraction



Our implementation is based on the public code MATRIX [Kallweit, Grazzini, Wiesemann]

## **qT** subtraction

[Catani, Grazzini, '12] [Catani, Cieri, de Florian, Ferrera, Grazzini, '14]



#### NLO-improved approximation – $NNLO_{NLO-i}$

Done originally in Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk and Zirke, arXiv:1608.04798 [hep-ph]



- Observable level reweighting, technically simple
- Finite Mt effects in the NNLO piece enter via the full NLO
- Has to be repeated for each observable and binning (bin size dependent!)
- $\bullet$  We compute the total cross section based on the  $M_{hh}$  distribution

#### **Born-projected approximation – NNLO**<sub>B-proj</sub>



Projection to Born kinematics needed

#### We make use of the qT-recoil procedure:

Catani, de Florian, Ferrera and Grazzini, arXiv:1507.06937 [hep-ph]

- Momenta of the Higgs bosons remain unchanged
- The new initial state partons momenta absorb the qT due to the additional radiation
- Initial state momenta remain massless, and their transverse component goes to zero when qT goes to zero (and then qT-cancellation is not spoiled)

Finite Mt effects entering only via the Born amplitude: no information about real radiation

### **Full-theory approximation - NNLO**<sub>FTapprox</sub>

- Double real corrections can be computed in the full theory (one-loop amplitudes)
- Idea: construct an approximation in which they are treated in an exact way

We perform a subprocess-wise reweighting: for each n-loop squared amplitude

$$\mathcal{A}_{\rm HEFT}^{(n)}(ij \to HH + X)$$

we apply the reweighting

$$\mathcal{R}(ij \to HH + X) = \frac{\mathcal{A}_{\text{Full}}^{\text{Born}}(ij \to HH + X)}{\mathcal{A}_{\text{HEFT}}^{(0)}(ij \to HH + X)}$$

- Same partonic subprocess used for reweighting: no need for a projection
- Amplitudes that are tree-level in the HEFT are treated exactly
- At NLO this agrees with the FTapprox in Maltoni, Vryonidou and Zaro, arXiv:1408.6542 [hep-ph]
- Great performance at NLO (4% difference with full NLO) + full  $M_t$  dependence in double reals

[Discussion numerical stability in backup slides]

Our best NNLO prediction

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#### **Numerical results**

Setup of the calculation:

- M<sub>h</sub> = 125GeV M<sub>t</sub> = 173GeV
- PDF4LHC15 sets at each corresponding order
- Central scale value  $\mu_0 = M_{hh}/2$
- Scale uncertainties: 7-point variation
- Results for 13, 14, 27 and 100TeV
- No bottom quark contributions (effect below 1% at LO)
- No top quark width effects (2% at LO for the total cross section)
- $\bullet$  On-shell scheme for  $M_{t},$  no estimation of scheme uncertainties

$\sqrt{s}$	$13 { m TeV}$	$14 { m TeV}$	$27 { m TeV}$	$100 { m TeV}$
NLO [fb]	$27.78^{+13.8\%}_{-12.8\%}$	$32.88^{+13.5\%}_{-12.5\%}$	$127.7^{+11.5\%}_{-10.4\%}$	$1147^{+10.7\%}_{-9.9\%}$
$\rm NLO_{FTapprox}$ [fb]	$28.91  {}^{+15.0\%}_{-13.4\%}$	$34.25^{+14.7\%}_{-13.2\%}$	$134.1^{+12.7\%}_{-11.1\%}$	$1220{}^{+11.9\%}_{-10.6\%}$
$NNLO_{NLO-i}$ [fb]	$32.69^{+5.3\%}_{-7.7\%}$	$38.66^{+5.3\%}_{-7.7\%}$	$149.3^{+4.8\%}_{-6.7\%}$	$1337^{+4.1\%}_{-5.4\%}$
$NNLO_{B-proj}$ [fb]	$33.42^{+1.5\%}_{-4.8\%}$	$39.58^{+1.4\%}_{-4.7\%}$	$154.2^{+0.7\%}_{-3.8\%}$	$1406{}^{+0.5\%}_{-2.8\%}$
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$M_t$ unc. NNLO <sub>FTapprox</sub>	$\pm 2.6\%$	$\pm 2.7\%$	$\pm 3.4\%$	$\pm 4.6\%$
$NNLO_{FTapprox}/NLO$	1.118	1.116	1.096	1.067

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Increase with respect to NLO at 14TeV:						
B-proj: 20% NLO-i: 18% FTapprox: 12%						

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- Size of perturbative corrections decreases with the energy for the FTapprox
- This doesn't happen for the other two approximations
- Not fully surprising: similar behavior for NLO K-factor

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Strong reduction of the scale					
uncertainties at NNLO		Even stron	ger reduction		
<ul> <li>About a factor of 3 for at 14TeV</li> </ul>	<ul> <li>About a factor of 3 for the FTapprox</li> <li>at 14TeV</li> </ul>			00TeV	

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• At NLO the FTapprox overestimates full NLO by 4% - 11% for the pure NLO contribution

• Assuming a ±11% uncertainty for the pure NNLO piece +1.2% uncertainty at NNLO

• Multiply by a factor of 2 to be more conservative

(14TeV)

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$M_t$ unc. NNLO <sub>B-proj</sub>	$\pm 14\%$	$\pm 15\%$	$\pm 20\%$	$\pm 36\%$

• At NLO the FTapprox overestimates full NLO by 4% - 11% for the pure NLO contribution

- Assuming a  $\pm 11\%$  uncertainty for the pure NNLO piece  $\implies \pm 1.2\%$  uncertainty at NNLO
- Multiply by a factor of 2 to be more conservative

We can repeat the procedure for the Born-projected approximation

Compatible results even without the factor of 2

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$NNLO_{NLO-i}$ [fb]	$32.69^{+5.3\%}_{-7.7\%}$	$38.66  {}^{+5.3\%}_{-7.7\%}$	$149.3^{+4.8\%}_{-6.7\%}$	$1337^{+4.1\%}_{-5.4\%}$
$NNLO_{B-proj}$ [fb]	$33.42^{+1.5\%}_{-4.8\%}$	$39.58{}^{+1.4\%}_{-4.7\%}$	$154.2^{+0.7\%}_{-3.8\%}$	$1406^{+0.5\%}_{-2.8\%}$
$NNLO_{FTapprox}$ [fb]	$31.05{}^{+2.2\%}_{-5.0\%}$	$36.69{}^{+2.1\%}_{-4.9\%}$	$139.9^{+1.3\%}_{-3.9\%}$	$1224_{-3.2\%}^{+0.9\%}$
$M_t$ unc. NNLO <sub>FTapprox</sub>	$\pm 2.3\%$	$\pm 2.4\%$	$\pm 2.7\%$	$\pm 3.1\%$
$M_t$ unc. NNLO <sub>B-proj</sub>	$\pm 14\%$	$\pm 15\%$	$\pm 20\%$	$\pm 36\%$

• But the difference between FTapprox and NLO-i increases with the collider energy faster than this uncertainty estimate

• To be more conservative, take half the difference between FTapprox and NLO-i

$\sqrt{s}$	$13 { m TeV}$	$14 { m TeV}$	$27 { m ~TeV}$	$100 { m TeV}$
NLO [fb]	$27.78^{+13.8\%}_{-12.8\%}$	$32.88^{+13.5\%}_{-12.5\%}$	$127.7^{+11.5\%}_{-10.4\%}$	$1147^{+10.7\%}_{-9.9\%}$
$\rm NLO_{FTapprox}$ [fb]	$28.91^{+15.0\%}_{-13.4\%}$	$34.25^{+14.7\%}_{-13.2\%}$	$134.1^{+12.7\%}_{-11.1\%}$	$1220{}^{+11.9\%}_{-10.6\%}$
$NNLO_{NLO-i}$ [fb]	$32.69^{+5.3\%}_{-7.7\%}$	$38.66^{+5.3\%}_{-7.7\%}$	$149.3^{+4.8\%}_{-6.7\%}$	$1337^{+4.1\%}_{-5.4\%}$
$NNLO_{B-proj}$ [fb]	$33.42^{+1.5\%}_{-4.8\%}$	$39.58{}^{+1.4\%}_{-4.7\%}$	$154.2^{+0.7\%}_{-3.8\%}$	$1406^{+0.5\%}_{-2.8\%}$
$NNLO_{FTapprox}$ [fb]	$31.05{}^{+2.2\%}_{-5.0\%}$	$36.69{}^{+2.1\%}_{-4.9\%}$	$139.9^{+1.3\%}_{-3.9\%}$	$1224_{-3.2\%}^{+0.9\%}$
$M_t$ unc. NNLO <sub>FTapprox</sub>	$\pm 2.3\%$	$\pm 2.4\%$	$\pm 2.7\%$	$\pm 3.1\%$
$M_t$ unc. NNLO <sub>B-proj</sub>	$\pm 14\%$	$\pm 15\%$	$\pm 20\%$	$\pm 36\%$
$M_t$ unc. NNLO <sub>FTapprox</sub>	$\pm 2.6\%$	$\pm 2.7\%$	$\pm 3.4\%$	$\pm 4.6\%$

• But the difference between FTapprox and NLO-i increases with the collider energy faster than this uncertainty estimate

• To be more conservative, take half the difference between FTapprox and NLO-i

Small difference for LHC, more conservative for larger energies

#### Differential distributions - $M_{\rm hh}$

 $\sqrt{s} = 14 \text{ TeV}$ 



- B-proj and NLO-i have similar behaviors
- FTapprox presents larger corrections at threshold, minimum corrections at Mhh ~ 400GeV, slow increase towards the tail
- Scale uncertainties are substantially reduced in the whole range
- Overlap with the NLO band

## Differential distributions - $M_{\rm hh}$

![](_page_42_Figure_1.jpeg)

- Previous features enhanced at 100TeV
- Slower decrease in the tail of the distribution
- Larger separation between the different NNLO predicitons, smaller corrections for the FTapprox
- FTapprox different behavior at threshold even stronger: due to contributions from events with hard radiation

#### **Differential distributions - p<sub>T,hh</sub>**

![](_page_43_Figure_1.jpeg)

#### **Differential distributions - p<sub>T,hh</sub>**

![](_page_44_Figure_1.jpeg)

• Different behaviors are more pronounced at 100TeV

• Larger separation between FTapprox and NLO-i (almost full agreement in the tail)

- FTapprox agrees with B-proj for low  $p_{\tau,\text{hh}}$ 

## **Differential distributions - p**T,j1

![](_page_45_Figure_1.jpeg)

- Huge unphysical corrections in the tail for the B-proj approximation
- More pronounced differences between FTapprox and NLO-i compared to  $p_{\mbox{\tiny T,hh}}$
- FTapprox predicts a softer spectrum, corrections contained in the NLO uncertainty band

### Differential distributions - $p_{T,h1}$ and $p_{T,h2}$

![](_page_46_Figure_1.jpeg)

### **M**<sub>t</sub> uncertainties for distributions

Based on the performance of the FTapprox at NLO and on the separation between the NNLO approximations, we can roughly estimate the size of the Mt uncertainties for distributions

![](_page_47_Figure_2.jpeg)

#### $\mathbf{M}_t$ uncertainties for distributions

Based on the performance of the FTapprox at NLO and on the separation between the NNLO approximations, we can roughly estimate the size of the Mt uncertainties for distributions

![](_page_48_Figure_2.jpeg)

![](_page_49_Figure_0.jpeg)

### **BSM EFT**

![](_page_50_Figure_1.jpeg)

[1] Gröber, Mühlleitner, Spira, Streicher 15; [2] de Florian, Fabre, JM 17; [3] Buchalla, Capozi, Celis, Heinrich, Scyboz (To appear)

#### Conclusions

- HH production is the main way of measuring Higgs self-coupling
- Current limit: ~O(10) x SM cross section
- Should be observed in the HL-LHC
- Precision measurement of  $\lambda \rightarrow$  future collider

• Lot of recent progress in the theoretical predictions:

NLO: full Mt dependence

Beyond: Large-Mt limit

- NNLO in the large-Mt limit
- Threshold resummation at **NNLL**

Further reduction of scale uncertainties

Suggests to use  $\mu_0 = M_{hh}/2$  for fixed order predictions

#### Conclusions

- We **combined** the full NLO with the NNLO corrections computed in the HEFT
- Fully differential results, using q<sub>T</sub>-subtraction
- NNLO piece improved via different **reweightings** to account for **finite Mt effects**
- Our best prediction includes the **full double-real loop-induced** amplitudes
- Increase with respect to NLO from 12% at 13TeV to 7% at 100TeV
- Remaining Mt uncertainty: few percent level
- Most advanced perturbative prediction for HH available to date

**Outlook:** NNLO<sub>FTapprox</sub> for non-SM self-couplings, inclusion of Higgs decays, estimation of Mt renormalization scheme uncertainties, BSM EFT at NLO with full Mt dependence

#### Thanks!

#### **Backup slides**

## **Numerical stability**

• Loop-induced double real amplitudes can became unstable close to *dipole singularities* 

Small 
$$\ lpha = rac{p_i \cdot p_j}{\hat{s}}$$
 , i and j emitters

- Quadruple precision rescue non viable (~10 minutes per PS point for  $gg \rightarrow HHgg$ )
- $\bullet$  Using a too large cut on  $\alpha$  spoils the qT-cancellation

![](_page_54_Figure_5.jpeg)

#### **Numerical stability**

Solution: we introduced a new parameter,  $\alpha_{L-i,cut}$  , below which we approximate the loop-induced amplitudes by the Born reweighted HEFT

• We avoid evaluating the double real loop induced amplitudes in the unstable regions

• We can use a lower overall dipole cut — we don't spoil the qT-cancellation

$$\alpha_{\text{L-i,cut}} = 10^{-3} \text{ to } 10^{-5}$$

$$\alpha_{\rm cut} = 10^{-10}$$

**Results independent in this range** 

## **Numerical stability**

![](_page_56_Figure_1.jpeg)

• Extrapolation to  $r_{cut} \rightarrow 0$  via linear least  $\chi^2$  fit (vs quadratic in default MATRIX)

• Upper bound of the interval varied to get the best fit and uncertainty estimation

#### **NNLO including dim 6 operators**

We extended the computation to include BSM effects via EFT dimension 6 operators

$$\mathcal{L}_{\text{eff}} = -M_t \,\bar{t}t \left( c_t \frac{h}{v} + c_{tt} \frac{h^2}{2v^2} \right) - c_3 \frac{1}{6} \left( \frac{3M_h^3}{v} \right) h^3 + \frac{\alpha_s}{\pi} G^{a\,\mu\nu} G^a_{\mu\nu} \left( c_g \frac{h}{v} + c_{gg} \frac{h^2}{2v^2} \right)$$

• All relevant dimension 6 operators that vanish when h=0

• The SM corresponds to  $c_t=c_3=1$  and  $c_{tt}=c_g=c_{gg}=0$ 

![](_page_57_Figure_6.jpeg)

## **NNLO including dim 6 operators**

![](_page_58_Figure_1.jpeg)

• Same couplings we have already in the SM HTL  $\rightarrow$  easy to obtain the NNLO corrections

![](_page_58_Figure_3.jpeg)

![](_page_58_Figure_4.jpeg)

#### **NNLO including dim 6 operators**

Does the size of the corrections change when we move away from the SM?

• We vary one coupling at a time:

 $\begin{array}{rcl} c_3 &=& 1+10\;\xi\,,\\ c_t &=& 1+0.35\;\xi\,,\\ c_{tt} &=& 1.5\;\xi\,,\\ c_g &=& 0.15\;\xi\,,\\ c_{gg} &=& 0.15\;\xi\,. \end{array}$ 

• Large corrections, but small dependence on the couplings

$$\begin{split} \Delta K^{c_{gg}} &= \frac{\max |K(c_{gg}) - K_{SM}|}{K_{SM}} \approx 15.8\% & \text{ at } c_{gg} = 0.15 \,. \\ & \Delta K^{c_3} \approx 7.2\% & \text{ at } c_3 = 4.20 \,, \\ & \Delta K^{c_{tt}} \approx 5.7\% & \text{ at } c_{tt} = 0.66 \,, \\ & \Delta K^{c_g} \approx 3.4\% & \text{ at } c_g = -0.15 \,, \\ & \Delta K^{c_t} \approx 0.5\% & \text{ at } c_t = 0.65 \,. \end{split}$$

![](_page_59_Figure_6.jpeg)

### Degeneracy and $M_{\rm hh}$ distribution

Different combinations of couplings can give a total XS similar to SM

![](_page_60_Figure_2.jpeg)

## Degeneracy and $M_{\rm hh}$ distribution

The invariant mass distribution can help disentangling the degeneracy

S. Borowka et al. [arXiv:1608.04798] de Florian, Fabre, JM [arXiv:1704.05700]

![](_page_61_Figure_3.jpeg)

Threshold is particularly sensitive due to triangle-box cancellations in the SM

#### Sensibility to self-coupling for the different production mechanisms

![](_page_62_Figure_1.jpeg)

From arXiv:1212.5581 [hep-ph]

#### Scale variation for fixed order and resummed total XS

![](_page_63_Figure_1.jpeg)

#### Differential distributions - $y_{\rm hh}$

![](_page_64_Figure_1.jpeg)

- Not very different behaviors between the different approximations (besides normalization)
- Largest shape difference in the central region for NLO-i

## Differential distributions – $\Delta\phi_{\rm hh}$

![](_page_65_Figure_1.jpeg)

- Trivial at LO: back-to-back. NNLO effectively NLO
- Large corrections above 50%, sizable scale uncertainties
- B-proj approximations predicts larger corrections in the region dominated by hard radiation
- $\bullet$  Good general agreement between FTapprox and NLO-i, larger differences close to  $\pi$

## $\mathbf{M}_t$ uncertainties for distributions

Based on the performance of the FTapprox at NLO and on the separation between the NNLO approximations, we can roughly estimate the size of the Mt uncertainties for distributions

![](_page_66_Figure_2.jpeg)

## $\mathbf{M}_t$ uncertainties for distributions

Based on the performance of the FTapprox at NLO and on the separation between the NNLO approximations, we can roughly estimate the size of the Mt uncertainties for distributions

![](_page_67_Figure_2.jpeg)