Numerical stability in hard and IR regions in OpenLoops2

arXiv:1906.?????
arXiv:1908.?????

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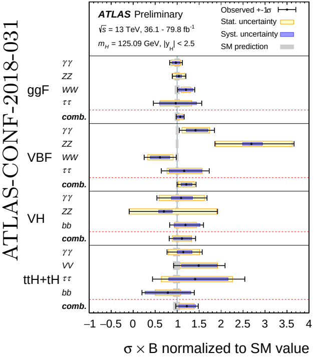
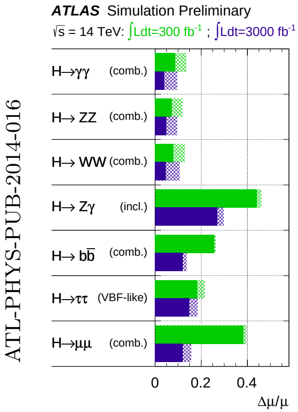
University of Cambridge



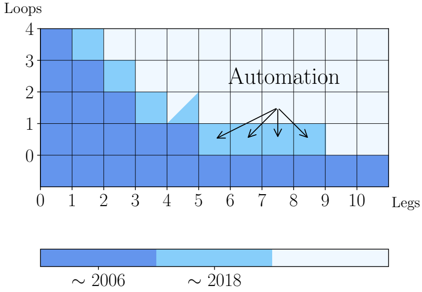
- 7 years since Higgs discovery
- Standard Model is very well consistent with LHC results
- No excesses observed with respect to the Standard Model
- Limited precision $\sim 20\%$
  Expect NP at $\mathcal{O}\left(\delta_{\mathrm{EW}}\right) \sim 5\%$?

Experiment future



Theory future

- Precision in SM predictions
  - Complete NLO for high multiplicities
  - NNLO QCD for $2 \to 2$, $2 \to 3$, . . .
- Precision in BSM predictions
  - UV complete models
  - SMEFT
- Automation



Computation of one-loop amplitudes (QCD & EW) is quite advanced

FeynArts / FormCalc	[Hahn et al]
GoSam	[Chiesa,Greiner,Heinrich,Jahn,Jones,Kerner,Luisoni Mastrolia, Ossola, Peraro, Schlenk, Scyboz, Tramontano]
NLOX	[Honeywell, Quackenbush, Reina, Reuschle]
MadGraph5 aMC@NLO	[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro]
NGluon	[Badger, Biedermann, Uwer, Yundin]
OpenLoops	[Cascioli, Lindert, Maierhöfer, Pozzorini]
BlackHat	[Bern, Dixon, Cordero, Höche, Ita, Kosower, Maitre, Ozeren]
HELAC-NLO	[Bevilacqua,Czakon,Garzelli,van Hameren,Kardos,Papadopoulos,Pittau,Worek]
Recola	[Actis, Denner, Hofer, Lang, Scharf, Uccirati]
...	

Used in many projects:

- Event generators
  Complete NLO corrections available in:
  - SHERPA arXiv:1712.07975
  - MADGRAPH5_AMC@NLO arXiv:1804.10017
- In two-loop computations
  For the 1-loop part + real-virtual contributors e.g.: in MATRIX arXiv:1711.06631

We need *high-performance* one-loop providers that are *reliable* in *all phase-space.*

Content of this Talk

The OpenLoops2 algorithm

Numerical stability in the on-the-fly reduction

Performance and stability benchmarks

The OpenLoops2 algorithm

- Fully automated framework for tree and one-loop amplitudes

$$\sum_{\text{colour/spin}} |\mathcal{M}_0|^2, \qquad \sum_{\text{colour/spin}} 2\mathrm{Re}\left[\mathcal{M}_0^*\mathcal{M}_1\right], \qquad \sum_{\text{colour/spin}} |\mathcal{M}_1|^2, \ldots$$

I **Process generator:**
Generation of (symbolic) amplitudes using FeynArts [Hahn]
Transformation into recursive representation (OpenLoops algorithm)

II **Process computation:**

$$\mathcal{M}_1 = \sum_i \mathcal{N}_{i,\mu_1\ldots} T_i^{\mu_1\cdots} + \mathcal{M}_{\mathrm{CT}} + \mathcal{M}_{\mathrm{R}_2}$$

- OpenLoops [Cascioli, Lindert, Maierhöfer, Pozzorini]:
  Computes coefficients and uses third-party tools for the reduction: Collier[Denner, Dittmaier, Hofer], CutTools[Ossola, Papadopoulos, Pittau], OneLoop[van Hameren].
- OpenLoops2 [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller]:
  Combines computation of coefficients and reduction for tree-loop, requires scalar integral providers (Collier, OneLoop)

Ingredients for complete one-loop computations:

- Complete SM renormalization
  - N_f flavour scheme for α_s, $\alpha(0), \alpha(M_\mathrm{Z}), G_\mathrm{F}$
  - On-shell renormalization
  - Complex-mass scheme [Denner, Dittmaier]

$$M_V^2 \rightarrow \mu_V^2 = M_i^2 - \mathrm{i}\Gamma_V M_V$$

- Complete set of R_2 [Pittau et al].
- Generation of full tower of orders

$$|\mathcal{M}_0|^2 \qquad \mathcal{O}\left(\alpha_\mathrm{s}^{n-1}\alpha^{m+1}\right) \qquad \mathcal{O}\left(\alpha_\mathrm{s}^{n}\alpha^{m}\right) \qquad \cdots$$

$$\mathrm{Re}\,[\mathcal{M}_0^*\mathcal{M}_1] \quad \mathcal{O}\left(\alpha_\mathrm{s}^{n-1}\alpha^{m+2}\right) \quad \mathcal{O}\left(\alpha_\mathrm{s}^{n}\alpha^{m+1}\right) \quad \mathcal{O}\left(\alpha_\mathrm{s}^{n+1}\alpha^{m}\right) \quad \cdots$$

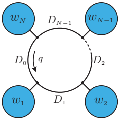
OpenLoops2 publicly available (temporary link)

`https://openloops.hepforge.org`

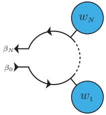
Computation of One-Loop diagrams in OpenLoops

Cut colour-stripped loop-amplitudes can be written as

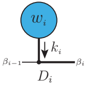
$$A_1^{(d)} = \int \mathrm{d}^D q \, \frac{\mathrm{Tr}\left[\mathcal{N}(q,h)\right]}{D_0 D_1 \dots D_{N-1}} =$$



$\Rightarrow$



Numerator factorises into segments:

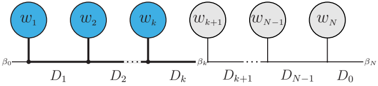


$$\left[\mathcal{N}(q,h)\right]_{\beta_0}^{\beta_N} = \left[S_1(q,h_1)\right]_{\beta_0}^{\beta_1} \left[S_2(q,h_2)\right]_{\beta_1}^{\beta_2} \dots \left[S_N(q,h_N)\right]_{\beta_{N-1}}^{\beta_N}$$

OpenLoops 1-loop recursion (dressing steps):

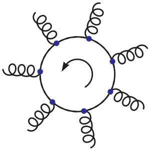
$$\mathcal{N}_k(q,\hat{h}_n) = \mathcal{N}_{k-1}(q,\hat{h}_{k-1}) \cdot S_k(q,h_k)$$

$$=$$



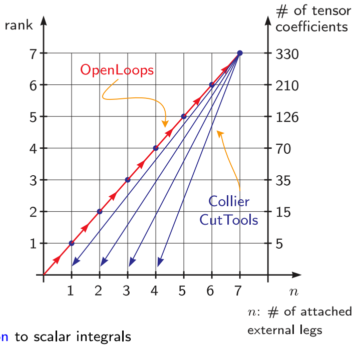
OpenLoops amplitude construction and reduction

$$\mathcal{N}(q,h) = \sum_{r=0}^{R} \mathcal{N}_{\mu_1\ldots\mu_r}(h)\; q^{\mu_1}\ldots q^{\mu_r}$$

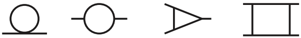


Example of a 7g diagram

complexity grows exponentially with tensor rank



Numerical tensor integral reduction to scalar integrals



Drawback/bottlenecks

1. Large structure growth prior to reduction (due to high rank)
2. Evaluation for each helicity configuration h

On-the-fly operations in OpenLoops2

arXiv:1710.11452 [Buccioni, Pozzorini, Zoller '17]

1 **On-the-fly helicity summation**
Uses tree amplitude information in tree-loop interference

$$\mathcal{N}_k(q) \to \left(\sum_{\text{col}} \mathcal{M}_0^* C^{(d)} \right) \mathcal{N}_k(q)$$

and sums over external helicities after dressing steps.
⇒ No evaluation of one-loop helicity amplitudes

2 **On-the-fly reduction**
Reduce open loops by integrand and integral reduction identities
⇒ Avoids high rank objects at any stage in computation

Advantages

- Significant gain in CPU performance
- New algorithm which combines construction and reduction
- New systematic way to address numerical instabilities

On-the-fly reduction

Use reduction identities valid at integrand level [del Aguila, Pittau '05].

$$q^\mu q^\nu = A^{\mu\nu}_{-1} + A^{\mu\nu}_0 D_0 + \left(B^{\mu\nu}_{-1,\lambda} + \sum_{i=0}^{3} B^{\mu\nu}_{i,\lambda} D_i \right) q^\lambda, \qquad (\spadesuit)$$

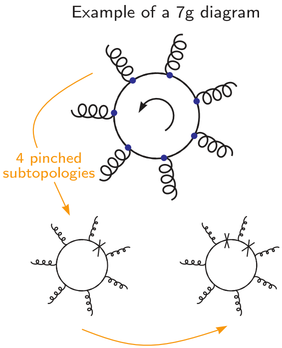
$$D_i = (q + p_i)^2 - m_i^2$$

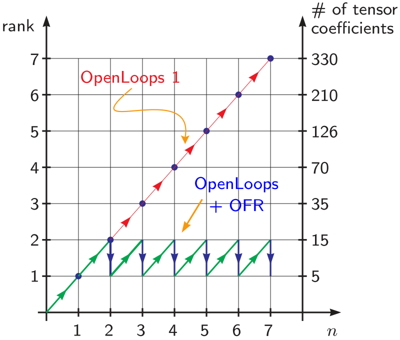
- Reduction identity follows from loop-momentum decomposition:

$$q^\mu = \sum_{i=1}^{4} c_i l_i^\mu, \quad l_i = l_i(p_1, p_2)$$

- Integrand identity ♠ requires another *independent* momentum p_3
- $A_i^{\mu\nu}, B_i^{\mu\nu}$ are constants depending on p_1, p_2, p_3
- p_1, p_2, p_3 can be chosen freely to cancel propagators D_i

OpenLoops2 amplitude construction and reduction





On-the-fly reduction step

$$\frac{\mathcal{V}^{\mu\nu} q_\mu q_\nu}{D_0 \dots D_{N-1}} = \frac{\mathcal{V}^{\mu}_{-1} q_\mu + \mathcal{V}_{-1}}{D_0 \dots D_{N-1}} + \sum_{i=0}^{3} \frac{\mathcal{V}^{\mu}_{i} q_\mu + \mathcal{V}_{i}}{D_0 \dots \not{D}_i \dots D_{N-1}}$$

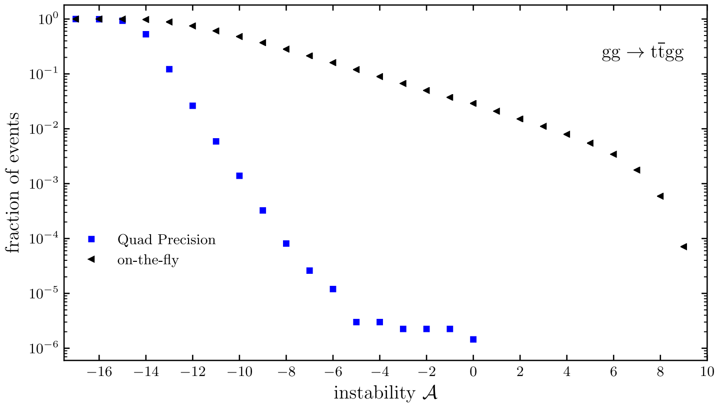
Final integral reduction

- Reduce rank-2 2-point, rank-3 triangles and rank-1 boxes with integral level identities [del Aguila, Pittau '05].
- Reduce rank-0 and rank-1 N-point for N$\geq$ 5 with OPP reduction [Ossola, Papadopoulos, Pittau '07].
- Use COLLIER[Denner, Dittmaier, Hofer '16] or ONELOOP[van Hameren '10] for evaluation of scalar master integrals

Numerical stability in the on-the-fly reduction

Native implementation of the on-the-fly reduction

Case study $gg \to t\bar{t}gg$



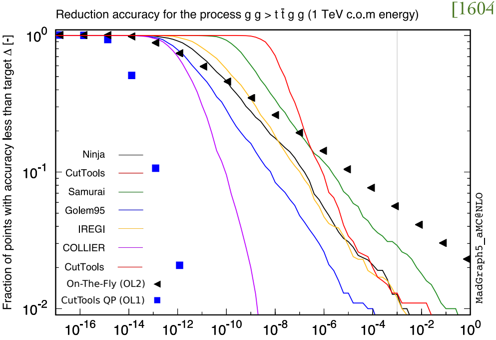
- Sample of 10^6 hard events: $p_\mathrm{T} > 50\,\mathrm{GeV}$, $\Delta R_{ij} >= 0.5$
- $\mathcal{M}_{qp}$ CutTools
- $\mathcal{A} = \dfrac{|\mathcal{M}_\mathrm{qp} - \mathcal{M}_\mathrm{dp}|}{|\mathcal{M}_\mathrm{qp} + \mathcal{M}_\mathrm{dp}|}$

Comparison to other reduction methods

Case study $gg \to t\bar{t}gg$

New Loop Reductions in MadLoop

[1604.01363]



Valentin Hirschi, SLAC
25.08.2016

- Large number of reduction methods available
- All algorithm suffer from (severe) numerical instabilities
- Necessity for a high-precision rescue system

Approach to cure numerical instabilities

I Consider stability distributions for **large samples**

- Sample corresponding to size of **real-life applications**
- Estimate stability, e.g. ~~by rescaling of dimension full parameter?~~
- **Better:** Derive stability from high-precision computation

II Construct **stability correlator**

- Capture *all* points in the tails of stability distributions

III **Cure instability**

- Avoid reduction steps with critical correlation
- Perform expansions
- ...

Sources of numerical instability in OFR

The on-the-fly reduction suffers from *Gram determinant* instabilities

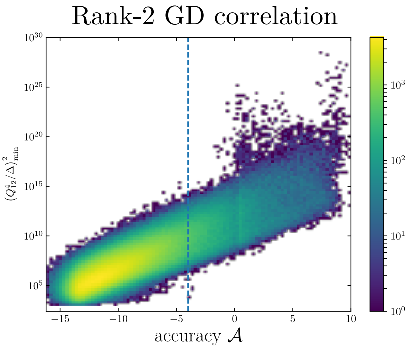
$$q^\mu q^\nu = A^{\mu\nu} + B^{\mu\nu}_\lambda q^\lambda$$

$$A^{\mu\nu}_i = \frac{1}{\Delta_{12}} a^{\mu\nu}_i,$$

$$B^{\mu\nu}_{i,\lambda} = \frac{1}{\Delta_{12}^2}\frac{1}{\sqrt{\Delta_{123}}} b^{(1),\mu\nu}_{i,\lambda} + \frac{1}{\Delta_{12}} b^{(2),\mu\nu}_{i,\lambda},$$

Instabilities for

- $\Delta_{12} = (p_1 \cdot p_2)^2 - p_1^2 p_2^2 \to 0$
- $\sqrt{\Delta}_{123} \sim p_3 \cdot l_{3/4}(p_1, p_2) \to 0$



- *Severe* numerical instabilities as $\Delta_{12} \to 0$
- Instabilities propagate $A_i/B_i \times A_i/B_i \times ... \sim \Delta_{12}^{-k}$ and amplify
- Moderate values of $1 \gg \frac{\Delta_{12}}{Q^2} \gg 0$ lead to numerical instabilities

On-the-fly solution to (single) rank-2 GD instabilities

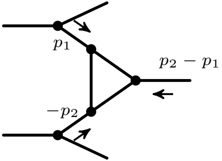
[Buccioni, Pozzorini, Zoller '17]

I Use freedom of on-the-fly reduction, choose i_1, i_2 such that $\sim \Delta_{i_1 i_2}$ maximal. Corresponds to propagator permutation

$$\frac{\mathcal{V}^{\mu\nu} q_\mu q_\nu}{D_0 D_1 D_2 D_3 \dots} \to \frac{\mathcal{V}^{\mu\nu} q_\mu q_\nu}{D_0 D_{i_1} D_{i_2} D_{i_3} \dots}, \quad i_1, i_2, i_3 \in [1,2,3]$$

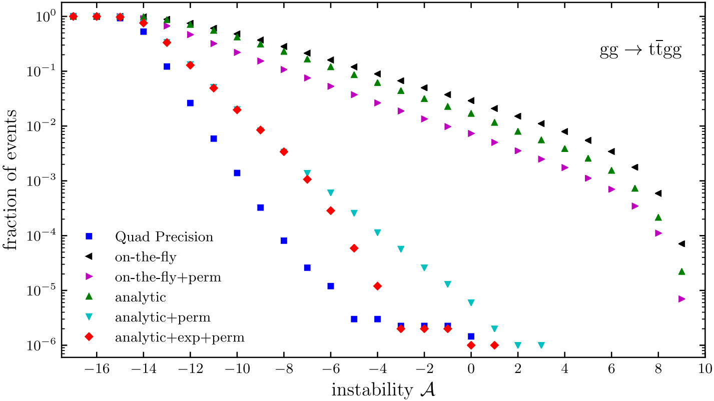
$\Rightarrow$ Avoids small rank-2 GD up to triangle reduction ✓

II Only t-channel topology introduces numerical instabilities (hard region)



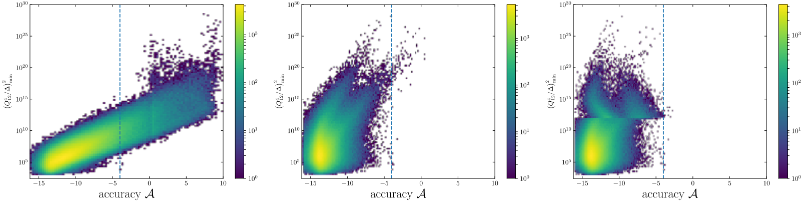
- $p_i^2 < 0$
- $(p_1 - p_2)^2 = 0$

- Analytic solution instead of on-the-fly reduction
- Perform expansions in Δ_{12}



(D_1, D_2, D_3)-permutation, no expansion

(D_1, D_2, D_3)-permutation + expansions to $\mathcal{O}(\delta^2)$



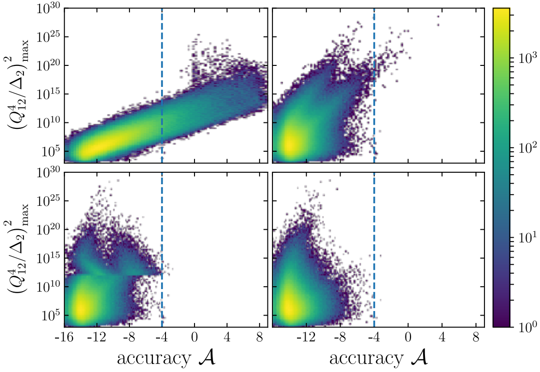
Curing residual instabilities in the OFR

Goal: Cure residual instabilities in the tail

- Full control in hard region crucial for stability in IR regions
- Get rid of stability system which requires 2 fold evaluation

Obstacle: Clean QP Benchmarks

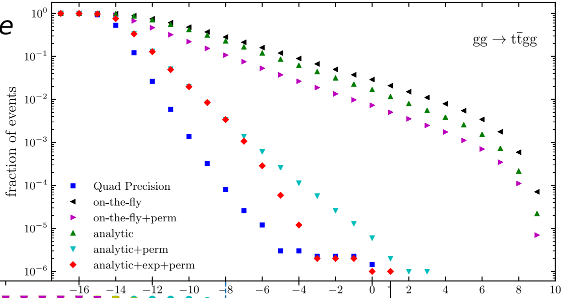
- On-the-fly reduction can operate in full QP ✓
  (no double precision contamination)
- Truncation spoils rescaling tests
  Any-order expansions ✓



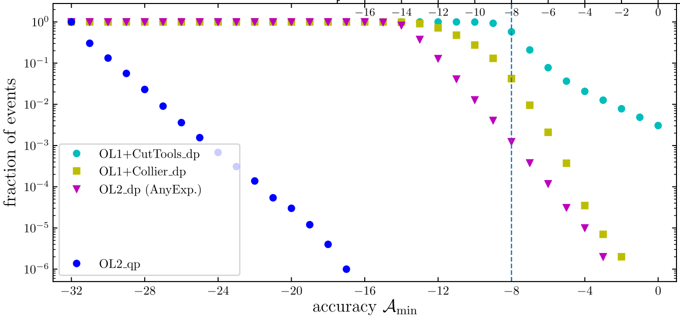
- Expansion to *arbitrary order*
- **Analytic cancellation** of Δ_{12} poles
- **Multi-precision target accuracy** (DP or QP)
- Efficient implementation for tensor integrals, allowing to reach ~ 1000 terms.
- All QCD cases implemented in library TRRED

Conclusion rank-2 GD instabilities

Before



After



The local error estimation and propagation

- Each step in the OL2 algorithm has an (inherited) error

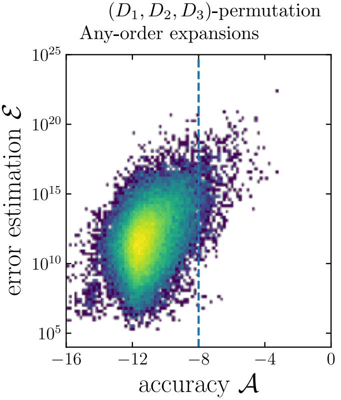
Local error sources

I Reduction steps
Estimated via reduction coefficients

II Scalar integrals
Estimated using COLLIER
(via mod. Cayley determinant)

III Reduction basis
Estimated via rank 3 Gram determinant

⇒ Propagate and combine to construct **global error estimation $\mathcal{E}$**



On-the-fly trick to alleviate instabilities

Step I: Avoid error propagation

- Error propagation only in on-the-fly reduction
- New reduction: choose i_1, i_2, i_3 such that $\sim \Delta_{i_1 i_2}$ maximal and $\sim \Delta_{i_1 i_2 i_3}$ maximal $\simeq$ propagator permutations + look ahead

$$\frac{\mathcal{V}^{\mu\nu} q_\mu q_\nu}{D_0 D_1 D_2 D_3 D_4 \ldots} \to \frac{\mathcal{V}^{\mu\nu} q_\mu q_\nu}{D_0 D_{i_1} D_{i_2} D_{i_3} D_{i_4} \ldots}, \quad i_1, i_2, i_3, i_4 \in [1, 2, 3, 4]$$

Step II: Cure remaining instabilities

- Expansion? ×
- Perform cancellation numerically in targeted way.

Hybrid precision mode

- Promote open loops $\mathcal{N}$ with $\mathcal{E} > \mathcal{E}^{\text{thres}}$ to quad precision

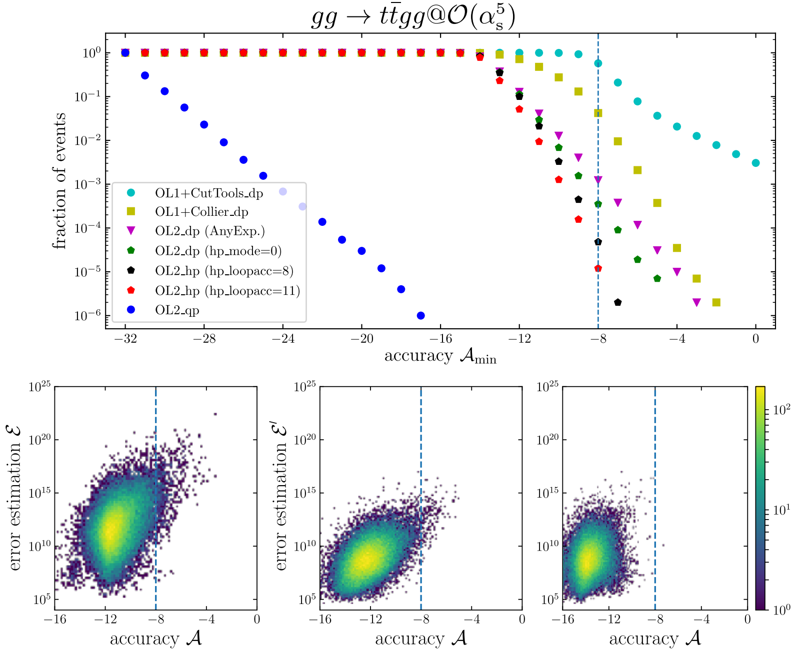
$$\mathcal{N} \to \mathcal{N} = \begin{pmatrix} \mathcal{N}_{\text{DP}} \\ \mathcal{N}_{\text{QP}} \end{pmatrix}, \quad \text{Upgrade}(\mathcal{N}) = \begin{pmatrix} 0 \\ \mathcal{N}_{\text{QP}} + \mathcal{N}_{\text{DP}} \end{pmatrix}$$

- Consecutive operations f performed in respective precision

$$\mathcal{N}_{\text{out}} = f(\mathcal{N}_{\text{in}}) \to \mathcal{N}_{\text{out}} = \begin{pmatrix} f_{\text{DP}}(\mathcal{N}_{\text{DP}}) \\ f_{\text{QP}}(\mathcal{N}_{\text{QP}}) \end{pmatrix}$$

- Cancellation takes places in QP in the last step

$$\text{Result} = \left(\sum_i \mathcal{N}^i_{\text{DP}} \text{SI}^i_{\text{DP}} \right) + \underbrace{\left(\sum_i \mathcal{N}^i{}_{\text{QP}} \text{SI}^i{}_{\text{QP}} \right)}_{\text{numerical cancellations in here}}$$

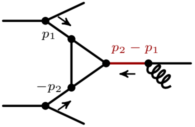

$gg \to t\bar{t}gg@\mathcal{O}(\alpha_s^5)$
fraction of events
accuracy $\mathcal{A}_{\min}$
OL1+CutTools_dp
OL1+Collier_dp
OL2_dp (AnyExp.)
OL2_dp (hp_mode=0)
OL2_hp (hp_loopacc=8)
OL2_hp (hp_loopacc=11)
OL2_qp
error estimation $\mathcal{E}$
error estimation $\mathcal{E}'$
accuracy $\mathcal{A}$

Frequent appearance of **double small rank 2 GD** instabilities

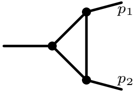
$$\Delta_{ij} \approx 0, \quad \Delta_{kl} \approx 0$$

Unstable triangle reductions

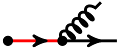
- IR t-channel $(p_2 - p_1)^2 \approx 0$



- IR triangles $\Delta_{12} \approx 0$



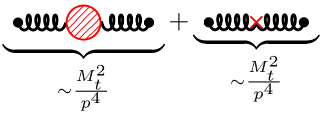
IR kinematics/computation invariants



$$\sim \frac{1}{(p+k)^2 - m^2} = \frac{1}{2p \cdot k}$$

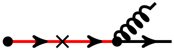
Cancellations: virtual CT R2

- *Gluon self-energy*



$$\overset{!}{\propto} \frac{1}{p^2}$$

- *Quark 2-point CT dressing*



$$\sim (\not{p} + \not{k})^3$$

New triggers/features encoded in `hp_mode=2`.

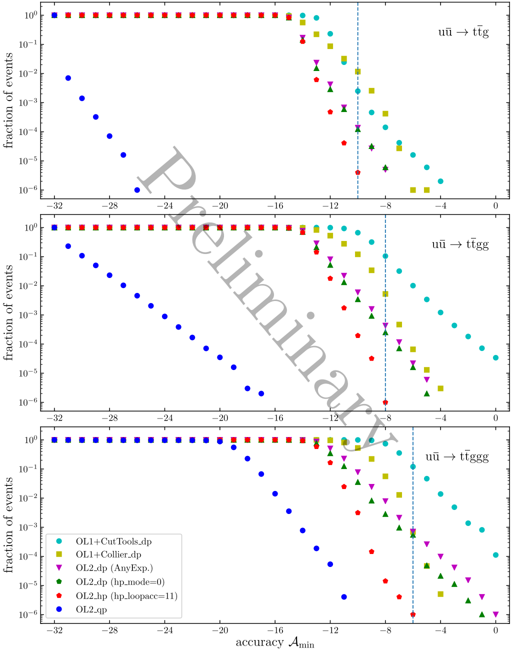
Performance and stability benchmarks

processes @ NLO QCD	OL2[ms], hp_loopacc=8	OL2, hp_mode=0	OL2, hp_loopacc=11	OL1
$gg \to t\bar{t}$	0.96	1.0	1.01	1.32
$gg \to t\bar{t}g$	21.4	1.0	1.04	1.56
$gg \to t\bar{t}gg$	600	1.0	1.15	2.17
$gg \to t\bar{t}b\bar{b}$	95	1.0	1.18	2.05
$u\bar{u} \to t\bar{t}$	0.23	1.0	1.0	0.93
$u\bar{u} \to t\bar{t}g$	3.1	1.0	1.06	1.19
$u\bar{u} \to t\bar{t}gg$	73	1.0	1.16	1.45
$u\bar{u} \to t\bar{t}ggg$	2085	0.99	1.26	1.88
$d\bar{u} \to W^+g$	0.33	1.0	1.03	0.79
$d\bar{u} \to W^+gg$	5.6	1.0	1.05	0.92
$d\bar{u} \to W^+ggg$	134	0.99	1.16	1.28
$d\bar{u} \to W^+gggg$	3760	0.98	1.14	1.41
$u\bar{u} \to W^+W^-$	0.19	1.0	1.0	1.19
$u\bar{u} \to W^+W^-g$	6.7	1.0	1.16	1.24
$u\bar{u} \to W^+W^-gg$	154	0.99	1.19	1.63
$u\bar{u} \to W^+W^-ggg$	3660	0.98	1.17	2.18

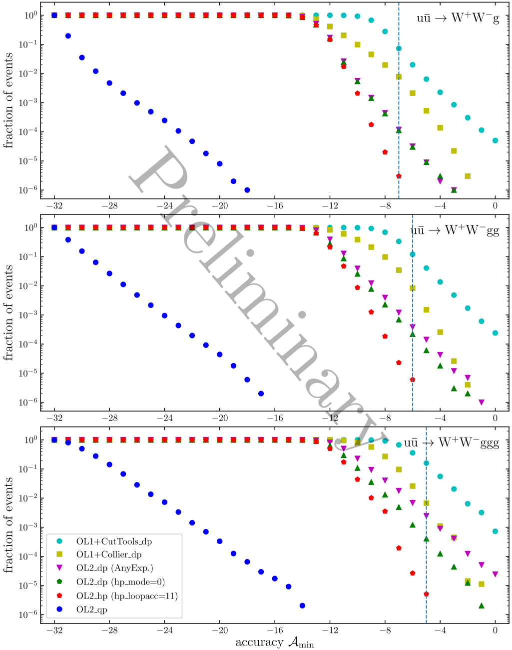
processes @ NLO EW	OL2[ms], hp_loopacc=8	OL2, hp_mode=0	OL2, hp_loopacc=11	OL1
$d\bar{u} \to s\bar{c}h$	1.8	1.0	1.06	1.78
$u\bar{u} \to t\bar{t}h$	12.7	0.99	1.17	1.0
$d\bar{d} \to e^-\bar{\nu}_e\mu^+\nu_\mu$	12.4	0.98	1.5	1.45
$u\bar{u} \to W^+W^+dd$	335	0.99	1.11	1.01

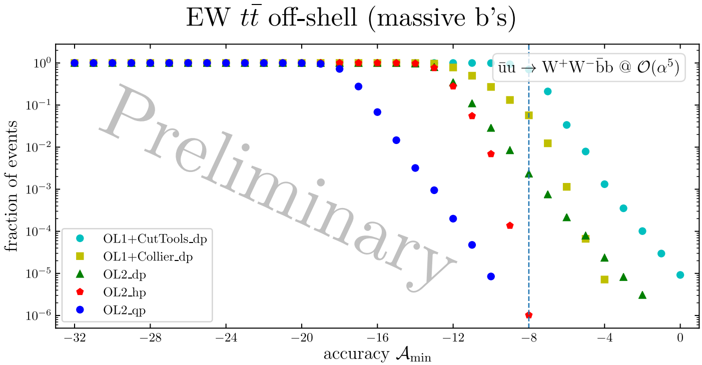
Table: Intel i7-4790K @ 4.00GHz. OL2[ms] is default hybrid precision mode (hp_loopacc=8) given in milliseconds. Other columns display ratio t/t_{OL2}. OL1 with COLLIER run, but no stability system.

$t\bar{t} + ng@\mathcal{O}(\alpha_s^{3+n})$

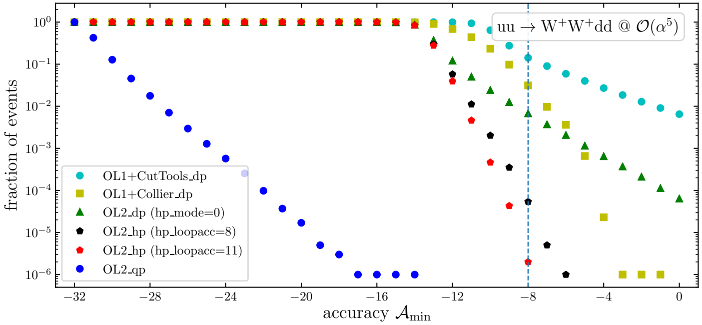


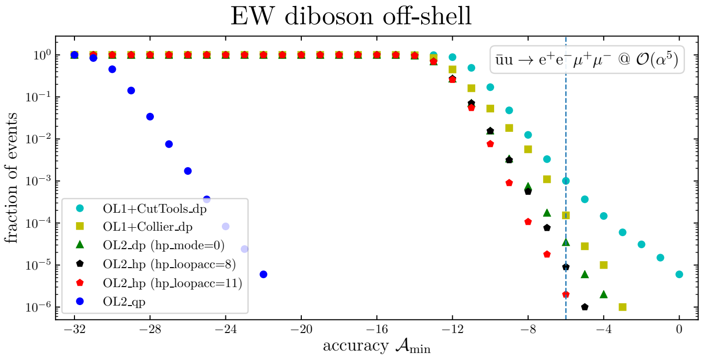
$W^+W^- + ng@\mathcal{O}(\alpha^2\alpha_s^{1+n})$




EW $t\bar{t}$ off-shell (massive b's)
$\bar{u}u \to W^+W^-\bar{b}b$ @ $\mathcal{O}(\alpha^5)$
Preliminary
fraction of events
accuracy $\mathcal{A}_{min}$
OL1+CutTools_dp
OL1+Collier_dp
OL2_dp
OL2_hp
OL2_qp

EW vector-boson scattering




EW diboson off-shell
ūu → e+e−μ+μ− @ O(α5)
fraction of events
accuracy A_min
OL1+CutTools_dp
OL1+Collier_dp
OL2_dp (hp_mode=0)
OL2_hp (hp_loopacc=8)
OL2_hp (hp_loopacc=11)
OL2_qp

IR phase-space point generation and performance

The soft and collinear events generated from hard underlying events

$$\xi_{\text{soft}} = E_{\text{soft}}/Q$$
$$\xi_{\text{coll}} = \arccos\left(\frac{\boldsymbol{p}_i \cdot \boldsymbol{p}_j}{|\boldsymbol{p}_i||\boldsymbol{p}_j|}\right)^2$$

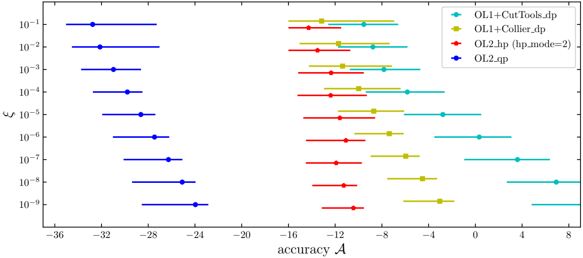
- E_{soft}: energy of a soft particle
- Q: the center of mass energy
- $\boldsymbol{p}_i, \boldsymbol{p}_j$: spacial momenta of collinear particles

The performance ranges between (*preliminary*)

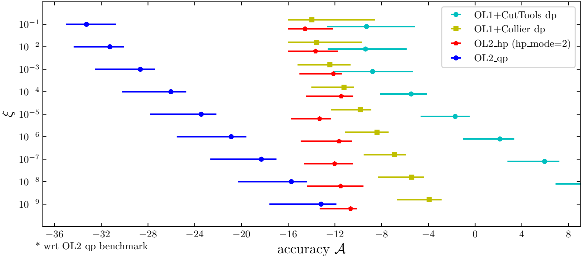
- $t = \frac{t_{\texttt{hp\_mode=1}}}{t_{\texttt{hp\_mode=0}}} \approx 1.5 - 2.5$
- $t = \frac{t_{\texttt{hp\_mode=2}}}{t_{\texttt{hp\_mode=0}}} \approx 3. - 7.$

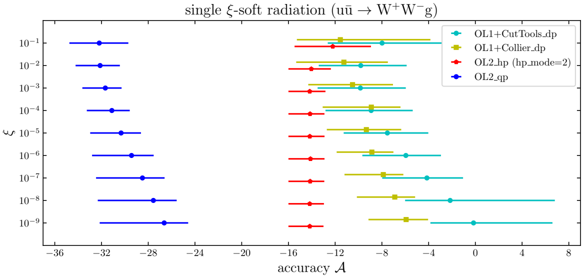
for ξ values between 10^{-3} and 10^{-9}.

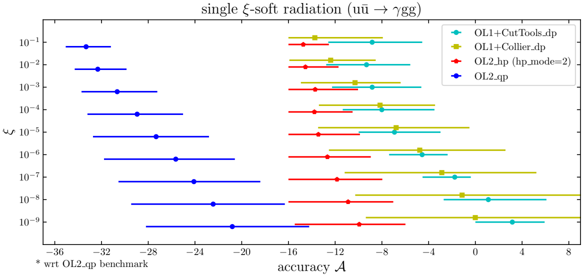
single intial-state ξ-collinear radiation ($gg \to t\bar{t}g$)



single final-state ξ-collinear radiation ($u\bar{u} \to \gamma gg$)







* wrt OL2_qp benchmark

Conclusion

- Release of OpenLoops2 featuring complete NLO corrections
  - New automated construction of one-loop amplitudes and reduction to scalar integrals
  - Major improvements in stability and performance
- Numerical instabilities addressed via:
  - Permutations/all-order expansions to cure single GD2 instabilities and avoid GD3 instabilities to the box reduction
  - Stable computation of invariants and use of analytic 2-point vertex functions to improve IR regions
  - Efficient hybrid precision mode to cure residual dominant numerical instabilities

Outlook

- NNLO subtraction tests with relaxed cutoffs to test deep IR cancellations
- Stability studies for $2 \rightarrow 4$ with one unresolved parton

Backup slides

Any-order expansions - technical details

In the any-order expansions we define the functions

$$S_{0,n}(\delta, \ldots) := \sum_{m=n}^{\infty} \delta^{m-n} \left[\frac{1}{m!} \left(\frac{\partial}{\partial \delta} \right)^m S_0(\delta, \ldots) \right]_{\delta=0},$$

with S_0 being t-channel 2- and 3-point functions

- $B_0\left(-p^2(1+\delta), m_0^2, m_1^2\right)$,
- $C_0\left(-p^2, -p^2(1+\delta), 0, m_0^2, m_1^2, m_2^2\right)$:
- $\sqrt{\Delta} = \frac{p^2}{2}\delta$

Deriving $S_{0,n}$ is a simple linear algebra task

$$C_0 = \frac{1}{p^2} \sum_{n=0}^{\infty} \delta^n C_n, \quad C_n = \int_0^1 \mathrm{d}y \; c_n,$$

c_n is a rational number in y and by partial fraction:

$$c_n = \frac{P(y)}{Q(y)} = \sum_{j=1}^{n+1} \left[\frac{d_1(j,n)}{(y-y_1)^j} + \frac{d_2(j,n)}{(y-y_2)^j} \right]$$

The direct reduction to scalar integrals yields

$$C^{\mu}\left(p_1, p_2, 0, 0, 0)\right) = \int \mathrm{d}^D q \, \frac{q^{\mu}}{D_0 D_1 D_2} = p_1^{\mu} C_1 + (p_1 - p_2)^{\mu} C_2,$$

$$C_1 = \frac{B_0(-p^2(1+\delta)) - B_0(-p^2)}{p^2 \delta} - C_0(-p^2, -p^2(1+\delta)),$$

$$C_2 = -2\frac{B_0(-p^2) - B_0(-p^2(1+\delta))}{p^2\delta^2} + \frac{B_0(-p^2(1+\delta)) - p^2 C_0(-p^2, -p^2(1+\delta))}{p^2\delta},$$

Any-order expansions yields

$$C_1 = \left[\frac{B_{0,1}}{p^2} - C_0\right], \quad C_2 = \frac{B_{0,1} + 2\,B_{0,2}}{p^2} - C_{0,1}, \tag{1}$$

- δ^n poles cancel, result numerically stable
- QCD/EW cases implemented in library TrRed as a multi-precision library (dp and qp)

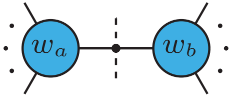
Computation of tree-level diagrams OpenLoops

Amplitudes are decomposed into a colour basis

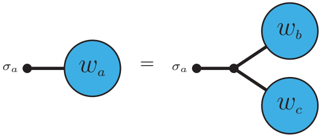
$$\mathcal{M} = \sum_d \mathcal{M}^{(d)}, \quad \mathcal{M}^{(d)} = C^{(d)} A^{(d)}$$

and recusion is applied to colour-stripped amplitudes:

$A_i =$



Reconstruct subtrees via Berends-Giele-like recursion

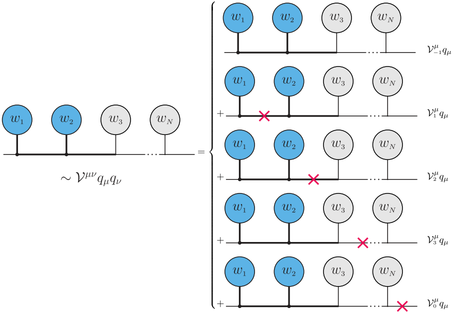


$$= w_a^\alpha(k_a, h_a) = \frac{X^\alpha_{\beta\gamma}(k_b, k_c)}{k_a^2 - m_a^2} w_b^\beta(k_b, h_b) w_c^\gamma(k_c, h_c)$$

- Recursion kernels derived from SM Lorentz structure
- Subtrees recycled whenever possible

On-the-fly reduction - technical details

Reduction proliferates subtopologies



Efficient implementation requires merging: *on-the-fly merging*