Numerical Multi-loop Computations: HJ and HH production at the LHC

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Borowka, Greiner, Heinrich, Jahn, Kerner, Luisoni, Schlenk, Schubert, Zirke

[1811.11720]
PRL 120 (2018) 162001 [1802.00349]
CPC 22 (2018) 313 [1703.09692]
JHEP 10 (2016) 107 [1608.04798]
PRL 117 (2016) 012001, Erratum 079901 [1604.06447]

Outline

Motivation

Framework & the heavy top-quark limit

Gluon fusion

- (HJ) Higgs boson + jet production history
- (HH) Di-Higgs boson production history

NLO (2-loop) calculation including the full top-quark mass

Numerical multi-loop calculations

- Sector decomposition
- Quasi-Monte Carlo integration

Results & comparisons

- HJ & HH

Motivation - Precision Higgs Physics

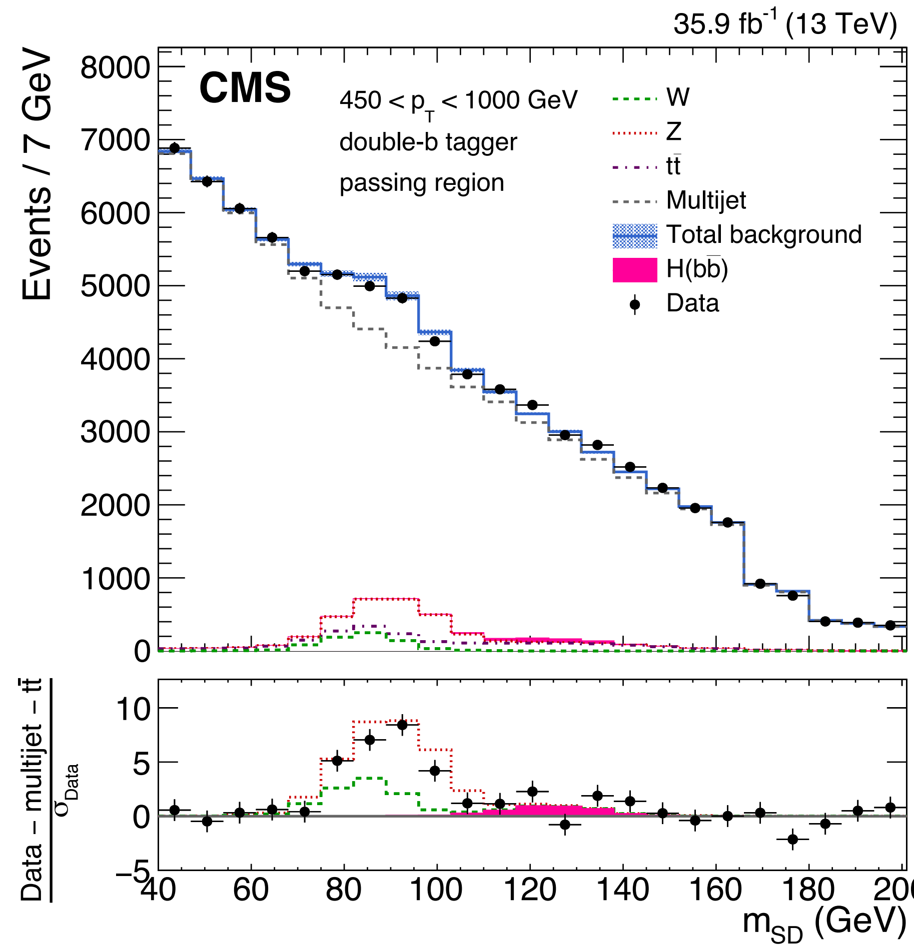
Higgs physics has transformed from discovery to precision study

Data are becoming more precise also for more differential observables

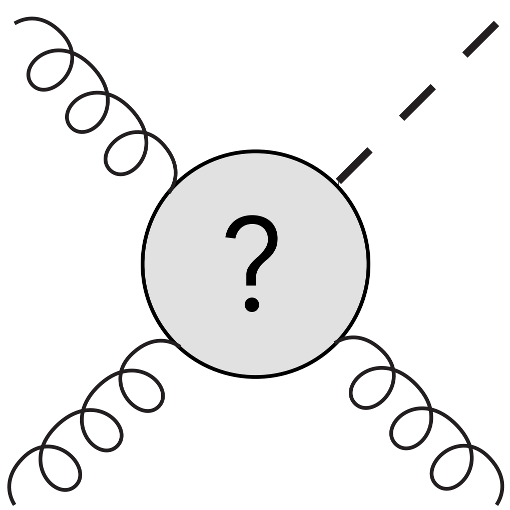
CMS 17: Search for boosted $H \to b\bar{b}$

$$\boxed{p_T > 450 \text{ GeV}}$$

$$\sigma(H \to b\bar{b}) = 74 \pm 48(\text{stat})^{-10}_{+17}(\text{syst}) \text{ fb}$$



CMS 17



At moderate/large pT:

- Particles in the loop can be resolved
- May disentangle modified top quark Yukawa coupling from (BSM) point-like ggH coupling

Motivation - Higgs self-coupling

Standard Model Higgs Lagrangian:

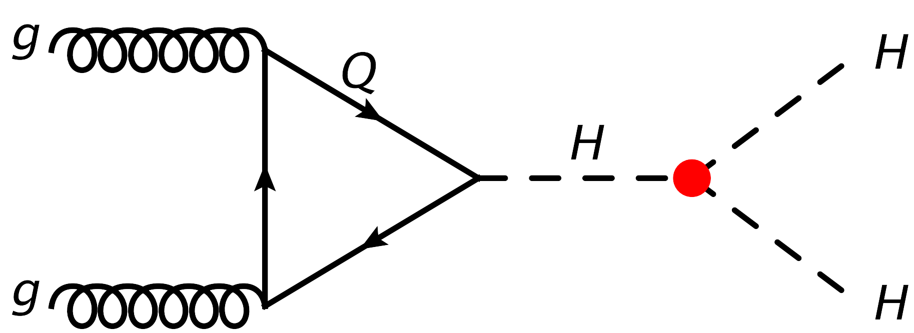
$$\mathcal{L} \supset -V(\phi), \quad V(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2$$

EW symmetry breaking

$$V(H) = \frac{1}{2}m_H^2 H^2 + \boxed{\lambda v H^3} + \frac{\lambda}{4}H^4, \qquad \mu^2 = \lambda v^2, \quad m_H^2 = 2\lambda v^2$$

SM: self-couplings determined by m_H, v

Higgs pair production probes triple-Higgs coupling



Higgs self-coupling not yet measured!
Extremely challenging to measure at LHC due to $\mathcal{O}(\mathrm{fb})$ cross section and difficult backgrounds

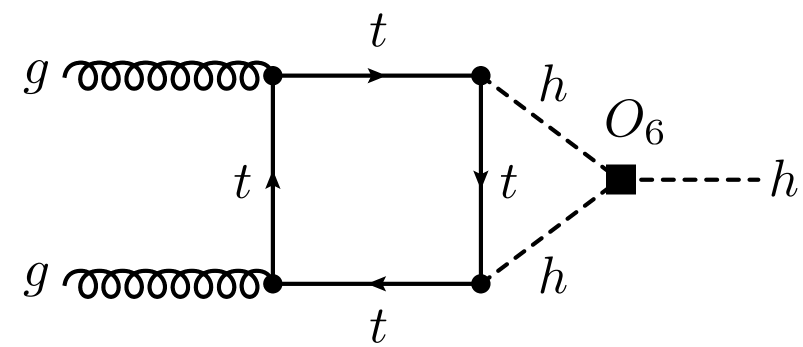
Data & Constraints

HH extremely challenging to measure, combining $b\bar{b}b\bar{b}, b\bar{b}\tau^+\tau^-, b\bar{b}\gamma\gamma$

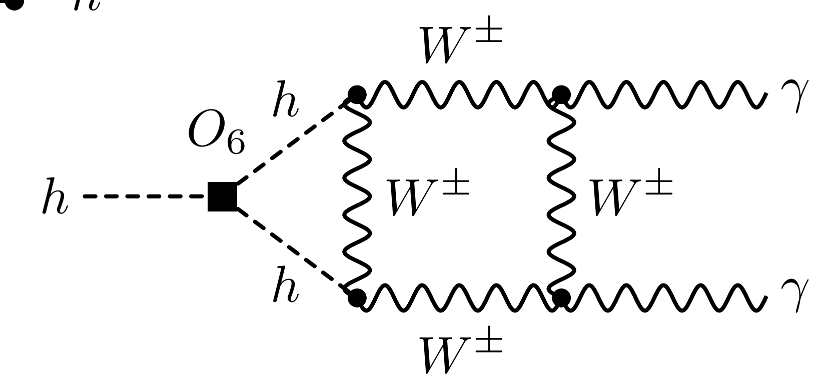
$\leq 6.7\ \sigma_{\rm SM}$ ATLAS-CONF-2018-043

Several other promising ideas to obtain limits on λ_3:

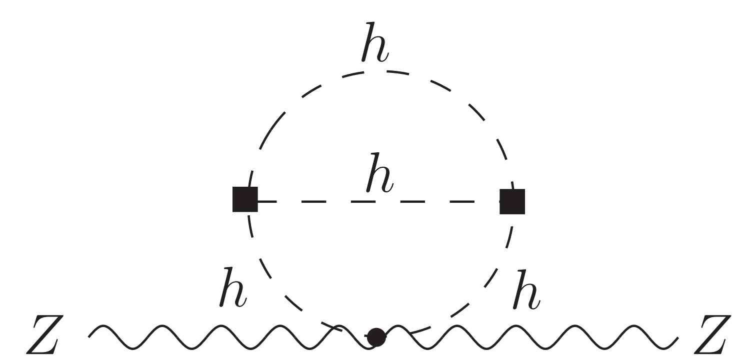
Electroweak corrections
to single H production
(also VBF, VH)



Gorbahn, Haisch 16; Bizoń, Gorbahn, Haisch, Zanderighi 16; Degrassi, Giardino, Maltoni, Pagani 16; Maltoni, Pagani, Shivaji, Zhao 17; Di Vita, Grojean, Panico, Riembau, Vantalon 17



Modification of precision EW observables
(EW oblique corrections) S, T



Degrassi, Fedele, Giardino 17;
Kribs, Maier, Rzehak, Spannowsky, Waite 17;

Limits on λ_4: from (partial) EW corrections to HH

Bizon, Haisch, Rottoli 18; Borowka, Duhr, Maltoni, Pagani, Shivaji, Zhao 18

QCD Factorisation

Focus of this talk will be on the computation of higher order perturbative QCD corrections to HJ and HH production at the LHC

$$d\sigma = \int dx_a dx_b f(x_a) f(x_b) d\hat{\sigma}_{ab}(x_a, x_b) F_J + \mathcal{O}\left((\Lambda/Q)^m\right)$$

PDFs/ Input parameters

Hard Scattering Matrix Element (Focus of this talk)

Non-perturbative effects ~ few %

With $\alpha_s \sim 0.1$
Typically: NLO ~ 10% correction, NNLO ~ 1% correction

However, there are important exceptions:

- Higgs Boson production (NLO ~100%, NNLO ~10%, N3LO ~ 2%)
- New partonic channels can open (e.g. di-boson production)

and

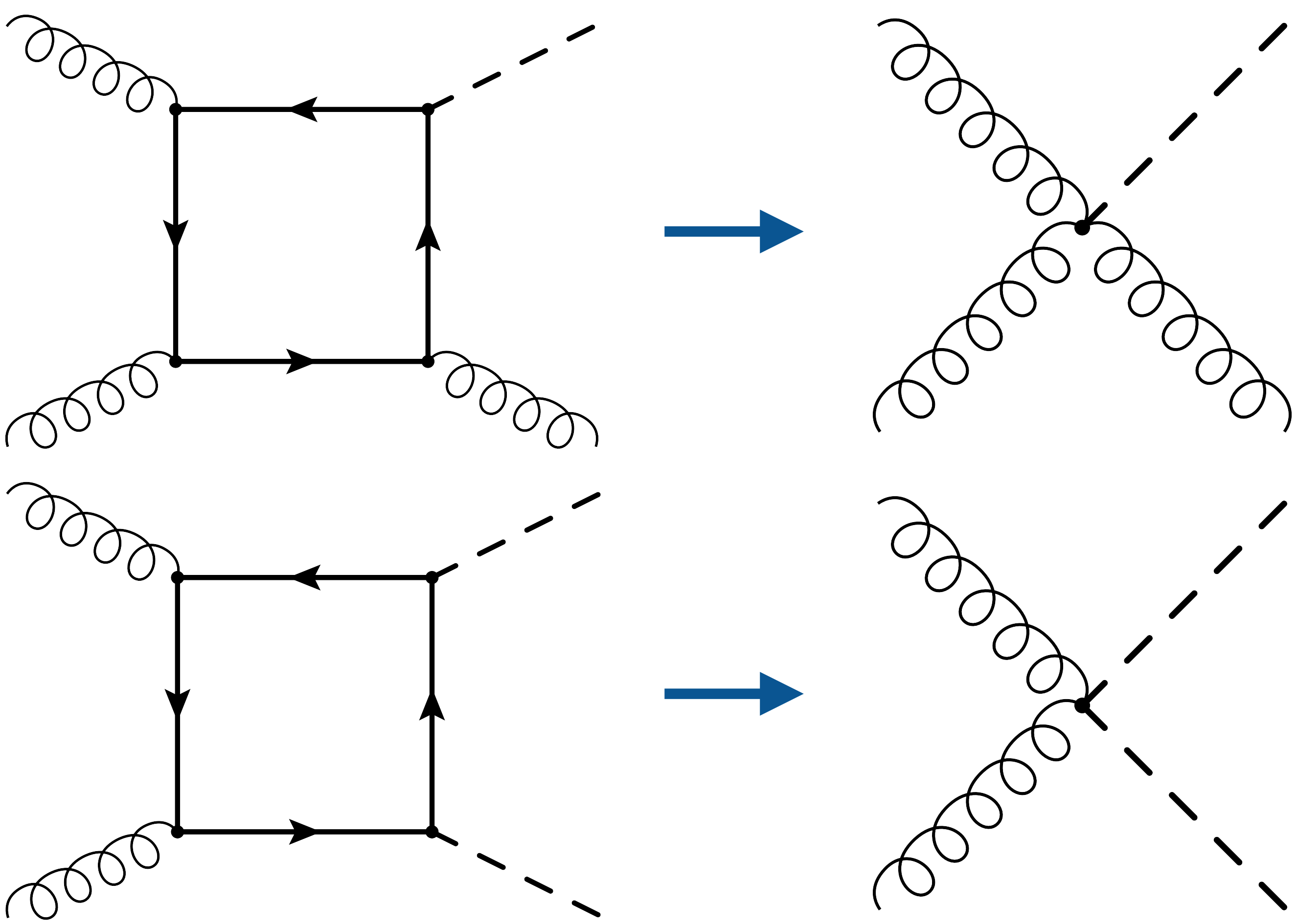
- Distributions can be modified substantially (even if σ_{tot} is stable)

Heavy top quark limit

Heavy top quark limit (HTL): $m_T \to \infty$

Introduces effective tree-level coupling between Higgs and gluons

Lowers the number of loops by 1



$$\mathcal{L}_{\text{eff}} \supset -\frac{\lambda}{4} H G^a_{\mu\nu} G^{a,\mu\nu}$$

$$\lambda = -\frac{\alpha_s}{3\pi v} + \mathcal{O}(\alpha_s^2)$$

HTL valid for: $\sqrt{\hat{s}} \ll 2m_T$

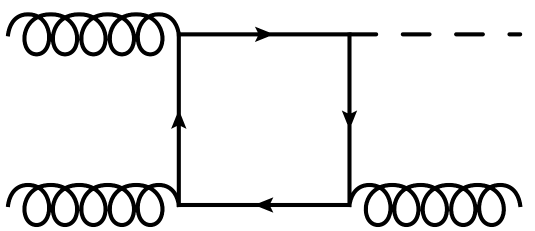
HJ: Does not describe well high p_T region

HH: $2m_H < \sqrt{\hat{s}}$

HJ Production History

1. LO (full m_T dependence)

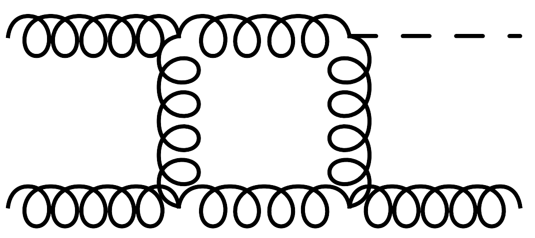
   Ellis, Hinchliffe, Soldate, van der Bij 87
   Baur, Glover 89



2. NLO **K≈1.8**

   Heavy Top Quark Limit

   de Florian, Grazzini, Kunszt 99; Glosser, Schmidt 02;
   Ravindran, Smith, van Neerven 02



   Approximate m_T dependence

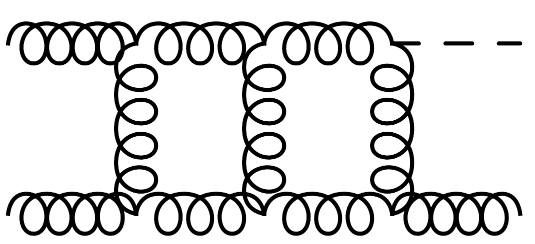
   Harlander, Neumann, Ozeren, Wiesemann 12; Neumann, Wiesemann 14; Buschmann, Goncalves, Kuttimalai, Schonherr, Krauss, Plehn 14; Frederix, Frixione, Vryonidou, Wiesemann 16; Neumann, Williams 16; Caola, Forte, Marzani, Muselliand, Vita 16; Braaten, Zhang, Zhang 17; Lindert, Kudashkin, Melnikov, Wever 18; Neumann 18;

   Top quark/Bottom quark interference

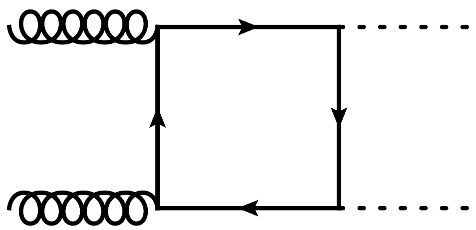
   (Lindert,) Melnikov, Tancredi, Wever 16, 17;
   Caola, Lindert, Melnikov, Monni, Tancredi, Wever 18

3. NNLO Heavy Top Quark Limit **K≈1.2**

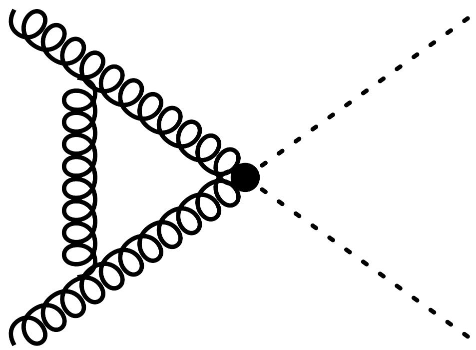
   Boughezal, Caola, Melnikov, Petriello, Schulze 13, 14; Chen, (Martinez,) Gehrmann, Glover, Jaquier 14, 16; Boughezal, Focke, Giele, Liu, Petriello 15; Bizon, Chen, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli 18



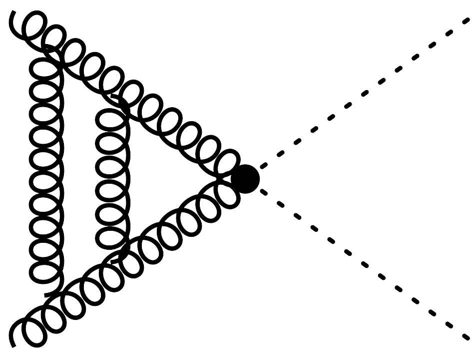
HH Production History



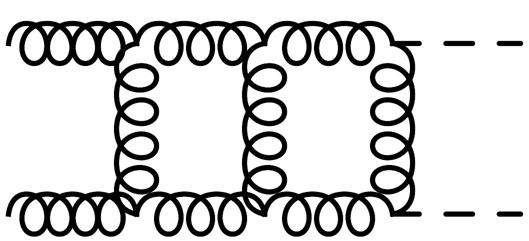
1. LO (full m_T dependence)
   Glover, van der Bij 88
2. NLO **K≈1.9**



   Heavy Top Quark Limit
   Dawson, Dittmaier, Spira 98
   Approximate m_T dependence
   Grigo, Hoff, Melnikov, Steinhauser 13; Grigo, Hoff 14; Grigo, Hoff, Steinhauser 15; Maltoni, Vryonidou, Zaro 14;
3. NNLO Heavy Top Quark Limit **K≈1.2**



   de Florian, Mazzitelli 13; Grigo, Melnikov, Steinhauser 14; Shao, Li, Li, Wang 13; de Florian, Mazzitelli 15; de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16
   Approximate m_T dependence
   Grigo, Hoff, Steinhauser 15;
4. (Partial) N3LO Heavy Top Quark Limit



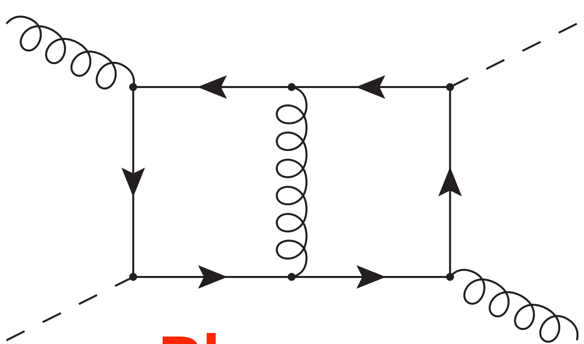
   Banerjee, Borowka, Dhani, Gehrmann, Ravindran 18

Calculation

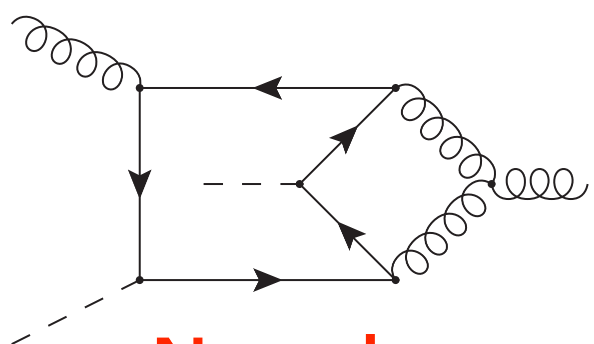
The Goal

NLO QCD corrections to HJ/HH production with full top quark mass: ≤7 propagator, 4-point, 2-loop diagrams, 4 mass scales (s, t, m_T, m_H)

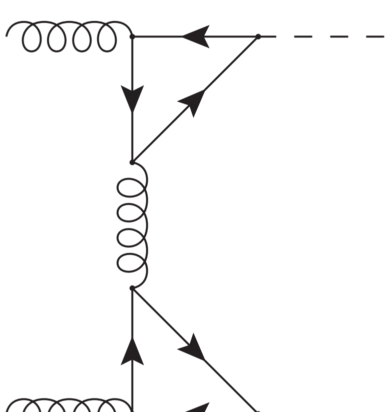
HH:



Planar

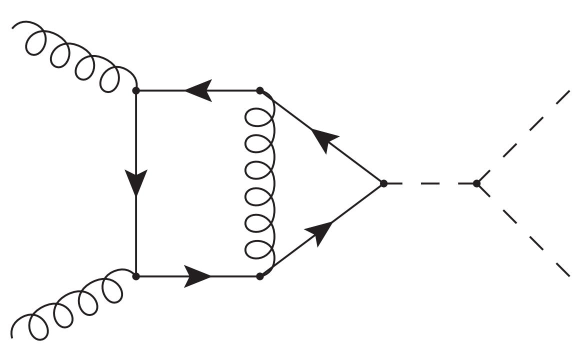


Non-planar



Reducible

Degrassi, Giardino, Gröber 16

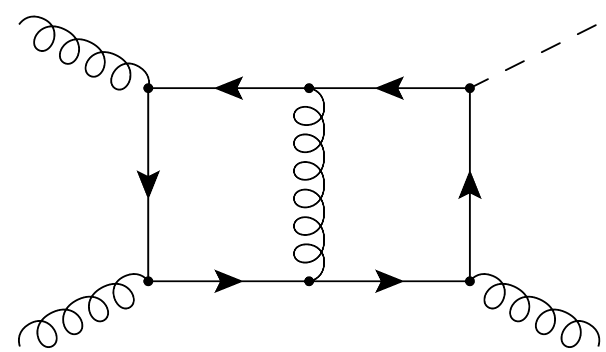


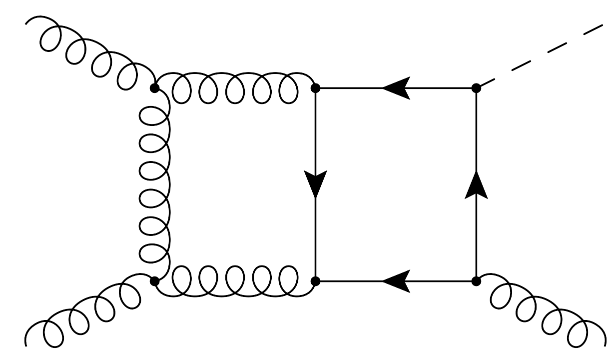
$gg \to H$

Spira, Djouadi et al. 93, 95;
Bonciani, P. Mastrolia 03,04;
Anastasiou, Beerli et al. 06;

$H \to Z\gamma$ Gehrmann, Guns, Kara 15;

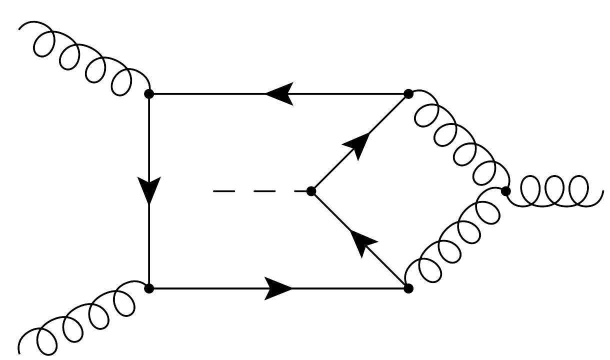
HJ:

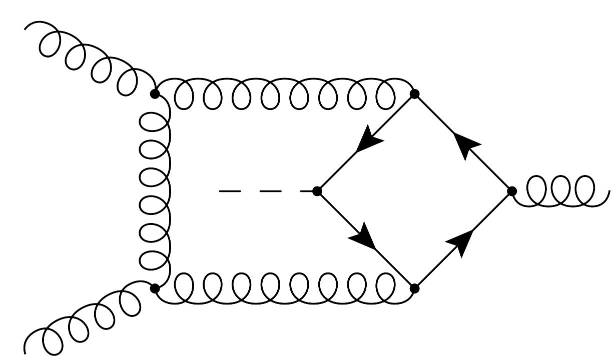




Planar

Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov 16; (See also Primo, Tancredi 16)





Non-planar

The Difficulty

What makes these processes so difficult to calculate?

One traditional method:

1. Decompose amplitude into form factors & construct projectors (**minutes**)
2. Generate Feynman diagrams (**seconds**)
3. Apply projectors & compute amplitude (**hours/days**)
4. Integral reduction (**6+ months**)
5. Compute master integrals (**analytic: challenging, numeric: ~seconds/hours**)
6. Generate events & compute (differential) cross-section

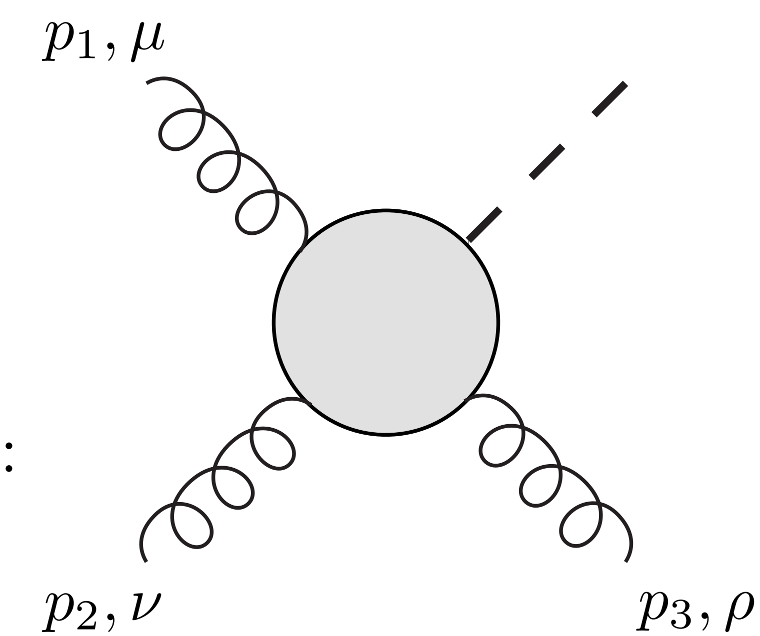
   (**analytic: seconds?, numeric: ~hours/days**)

We applied this method (computing the integrals numerically) to both HH and HJ.

Let us take a closer look at these steps for the case of HJ…

Form Factor Decomposition (HJ Gluon)

Expose tensor structure: $\mathcal{M} = \epsilon_\mu(p_1)\epsilon_\nu(p_2)\epsilon_\tau(p_3)\mathcal{M}^{\mu\nu\tau}$



$s_{ij} = (p_i + p_j)^2$

Form Factors (Contain integrals)

$$\mathcal{M}^{\mu\nu\tau}_{\text{physical}} = F_{212}T^{\mu\nu\tau}_{212} + F_{332}T^{\mu\nu\tau}_{332} + F_{311}T^{\mu\nu\tau}_{311} + F_{312}T^{\mu\nu\tau}_{312}$$

Choose tensor basis (constructed from external momenta & metric):

$$\begin{aligned}
T^{\mu\nu\tau}_{212} &= (s_{12}g^{\mu\nu} - 2p_2^\mu p_1^\nu)(s_{23}p_1^\tau - s_{13}p_2^\tau)/(2s_{13}) \\
T^{\mu\nu\tau}_{332} &= (s_{23}g^{\nu\tau} - 2p_3^\nu p_2^\tau)(s_{13}p_2^\mu - s_{12}p_3^\mu)/(2s_{12}) \\
T^{\mu\nu\tau}_{311} &= (s_{13}g^{\tau\mu} - 2p_1^\tau p_3^\mu)(s_{12}p_3^\nu - s_{23}p_1^\nu)/(2s_{23})
\end{aligned}$$

$$T^{\mu\nu\tau}_{312} = \Big(g^{\mu\nu}(s_{23}p_1^\tau - s_{13}p_2^\tau) + g^{\nu\tau}(s_{23}p_2^\mu - s_{12}p_3^\mu) + g^{\tau\mu}(s_{12}p_3^\nu - s_{23}p_1^\nu) + 2p_3^\mu p_1^\nu p_2^\tau - 2p_2^\mu p_3^\nu p_1^\tau\Big)/2$$

Build projectors P such that: $P^{212}_{\mu\nu\tau}\mathcal{M}^{\mu\nu\tau} = F_{212}$, etc…

Thanks: Glover, Frellesvig

With this choice form factors are gauge invariant and related by cyclic permutations:

$$\boxed{\begin{aligned}
F_{311}(s_{23}, s_{12}, s_{13}) &= F_{332}(s_{13}, s_{23}, s_{12}) = F_{212}(s_{12}, s_{13}, s_{23}) \\
F_{312}(s_{23}, s_{12}, s_{13}) &= F_{312}(s_{13}, s_{23}, s_{12}) = F_{312}(s_{12}, s_{13}, s_{23})
\end{aligned}}$$

Note: Need to generate code only for 2 form factors and use it to compute all
In reality we generate code for all form factors and use this symmetry as a cross-check

Reduction (IBPs)

Integration by parts Identities: **loop/external momentum**

$$\int \mathrm{d}^d p_i \frac{\partial}{\partial p_i^\mu} \left[q^\mu \, \mathbf{I}'(p_1, \ldots, p_l; k_1, \ldots, k_m) \right] = 0$$

Produce linear relations between integrals Tkachov 81; Chetyrkin 81

Can perform e.g. Gaussian elimination on system of equations

Relate integrals to a smaller set of **Master integrals**

Example:

$$I_{\alpha_1 \alpha_2 \alpha_3} = \int \frac{\mathrm{d}^d p}{i\pi^{d/2}} \frac{1}{D_1^{\alpha_1} D_2^{\alpha_2} D_3^{\alpha_3}}$$

$$k_1^2 = k_2^2 = 0, k_3^2 = s$$

$$D_1 = p^2$$

$$D_2 = (p + k_1)^2$$

$$D_3 = (p + k_1 + k_2)^2$$

IBP Identities:

$$(d-4) I_{111} - I_{102} - I_{201} = 0$$

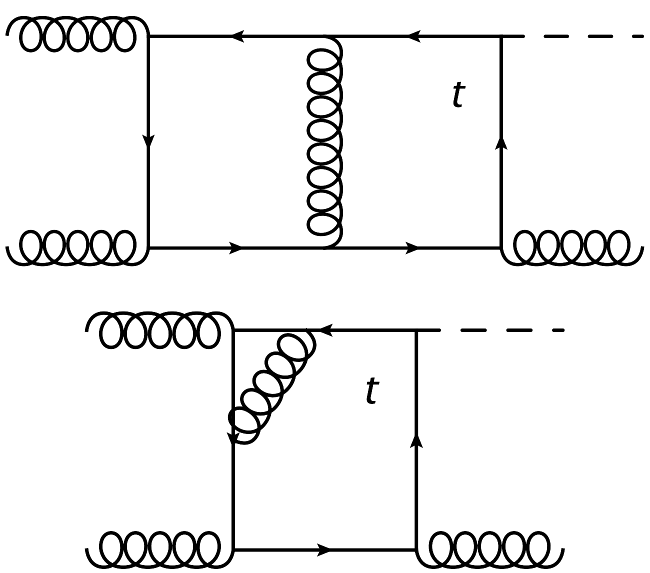
$$s I_{102} + (d-3) I_{101} = 0$$

$$s I_{201} + (d-3) I_{101} = 0$$

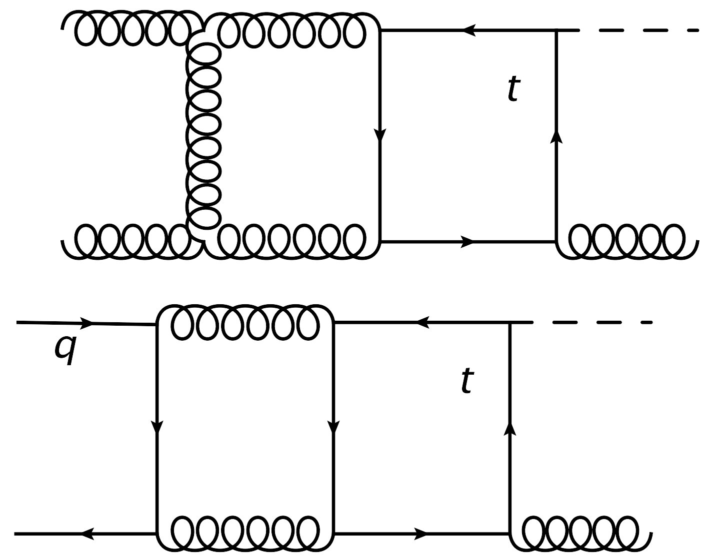
$$\therefore I_{111} = \frac{-2(d-3)}{s(d-4)} I_{101}$$

HJ Integral Families

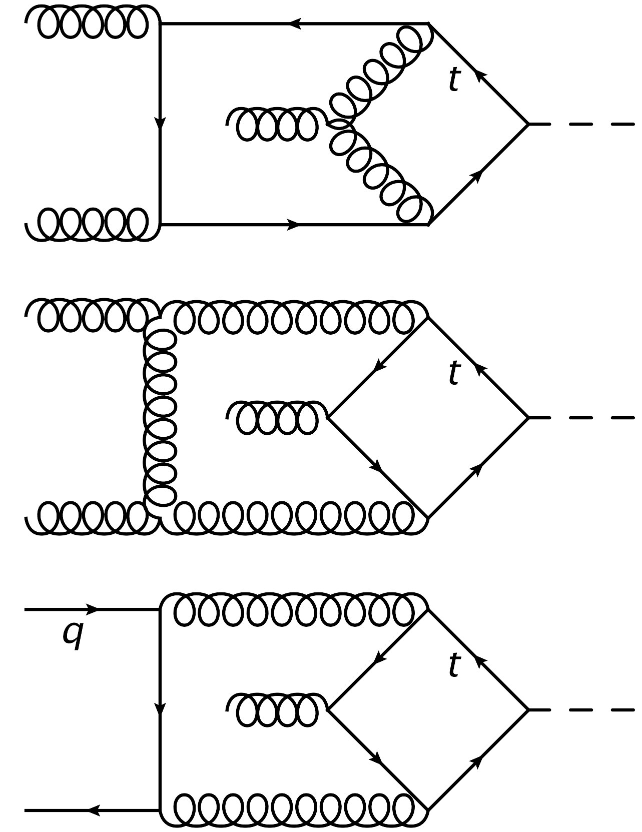
F1



F2



F3



Channel	#Diagrams @ 2-loop
$gggH$	354
$q\bar{q}gH$	57

Quark and gluon channel integrals belong to same integral families

HJ Integral Families (II)

All 2-loop HJ integrals can be written in terms of 3 integral families:

	F1	F2	F3
D_1	$k_1^2 - m_T^2$	k_2^2	$k_1^2 - m_T^2$
D_2	$(k_1 + p_1)^2 - m_T^2$	$(k_2 + p_1)^2$	$(k_1 + p_1)^2 - m_T^2$
D_3	$(k_1 - p_2)^2 - m_T^2$	$(k_2 - p_2)^2$	$(k_1 - p_2 - p_3)^2 - m_T^2$
D_4	$(k_1 - p_2 - p_3)^2 - m_T^2$	$(k_2 - p_2 - p_3)^2$	$k_2^2 - m_T^2$
D_5	$k_2^2 - m_T^2$	$k_1^2 - m_T^2$	$(k_2 + p_1)^2 - m_T^2$
D_6	$(k_2 + p_1)^2 - m_T^2$	$(k_1 + p_1)^2 - m_T^2$	$(k_2 - p_3)^2 - m_T^2$
D_7	$(k_2 - p_2)^2 - m_T^2$	$(k_1 - p_2)^2 - m_T^2$	$(k_1 - k_2)^2$
D_8	$(k_2 - p_2 - p_3)^2 - m_T^2$	$(k_1 - p_2 - p_3)^2 - m_T^2$	$(k_1 - k_2 - p_2)^2$
D_9	$(k_1 - k_2)^2$	$(k_1 - k_2)^2 - m_T^2$	$(k_1 - k_2 - p_2 - p_3)^2$

Melnikov, Tancredi, Wever 16

Integrals written as:

$$I^{F_j}_{\alpha_1,\ldots,\alpha_9} = \int \mathrm{d}^d k_1 \int \mathrm{d}^d k_2 \frac{1}{D_1^{\alpha_1} \cdots D_9^{\alpha_9}}$$

HJ Integral Reduction

Full IBP reduction achieved with (modified) Reduze2

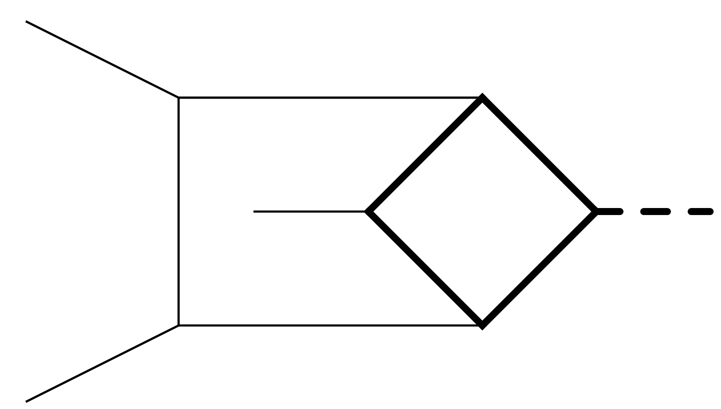
Chetyrkin, Tkachov 81; Laporta 01; von Manteuffel, Studerus 12

Unreduced Amplitude

- 3767 Integrals
- Up to 3 inverse propagators for 7-propagator integrals
- Up to 4 inverse propagators for factoring 6-propagator integrals

Reduced Amplitude

- 458 Integrals
- Up to 6 master integrals per sector, e.g:



Sector also known to be elliptic

Frellesvig, Loops & Legs 2018

HJ Integral Reduction (II)

Reduze2 Modifications:

- Change order of solving system of equations, sort by number of unreduced integrals (prefer fewer)
- Specify list of required integrals, consider only equations containing these integrals
- Improve mechanism for pausing/resuming reductions

Reduction achieved using 2 different setups:

Current calculation uses only this result

Symbolic mass dependence

Reduction directory size: **1.1 TB**

Can include m_B, Γ_T

Fix mass ratio

$$\frac{m_H^2}{m_T^2} = \frac{12}{23} \to \begin{aligned} m_H &= 125 \text{ GeV} \\ m_T &= 173.055 \text{ GeV} \end{aligned}$$

Reduction directory size: **0.25 TB**

Can not include m_B, Γ_T

Finite Basis

Always possible to pick finite basis of integrals using:

- Dimension Shifts Tarasov 96; Lee 10
- Dots

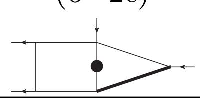
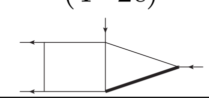
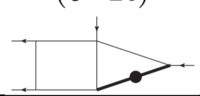
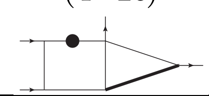
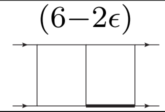
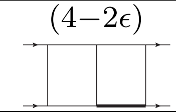
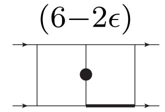
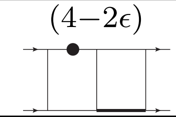
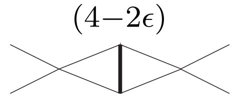
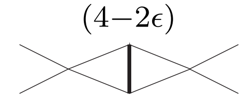
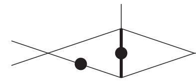
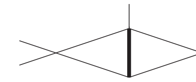
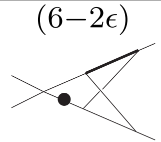
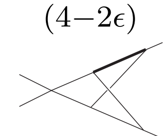
Panzer 14; von Manteuffel, Panzer, Schabinger 15

Finding finite integrals and basis change implemented in Reduze2.1

Finite basis greatly improves numerical performance
(but requires reduction of integrals with up to 2 inverse props and 2 dots)

Two-loop EW-QCD Drell-Yan

von Manteuffel, Schabinger 17

Finite Basis...			Conventional...		
$(6-2\epsilon)$  (s,t)	201 s	2.34×10^{-4}	$(4-2\epsilon)$  (s,t)	384 s	8.12×10^{-4}
$(6-2\epsilon)$  (s,t)	150 s	4.83×10^{-4}	$(4-2\epsilon)$  (s,t)	56538 s	1.67×10^{-2}
$(6-2\epsilon)$  (s,t)	280 s	1.00×10^{-3}	$(4-2\epsilon)$  (s,t)	214135 s	8.29×10^{-3}
$(6-2\epsilon)$  (s,t)	294 s	1.21×10^{-3}	$(4-2\epsilon)$  (s,t)	3484378 s	30.9
$(4-2\epsilon)$  (s)	91 s	3.76×10^{-4}	$(4-2\epsilon)$  (s)	87 s	3.76×10^{-4}
$(6-2\epsilon)$  (s)	17 s	5.15×10^{-4}	$(4-2\epsilon)$  (s)	20 s	1.95×10^{-4}
$(6-2\epsilon)$  (s)	119 s	2.32×10^{-3}	$(4-2\epsilon)$  (s)	118 s	2.12×10^{-3}
Total/Max:	3995 s	5.84×10^{-3}	Total/Max:	5136862 s	30.9

Rel. Err.

Computing the integrals:
numerical multi-loop computations

Analytic Results (so far)

Planar integrals contributing to HJ at 2-loop are known analytically

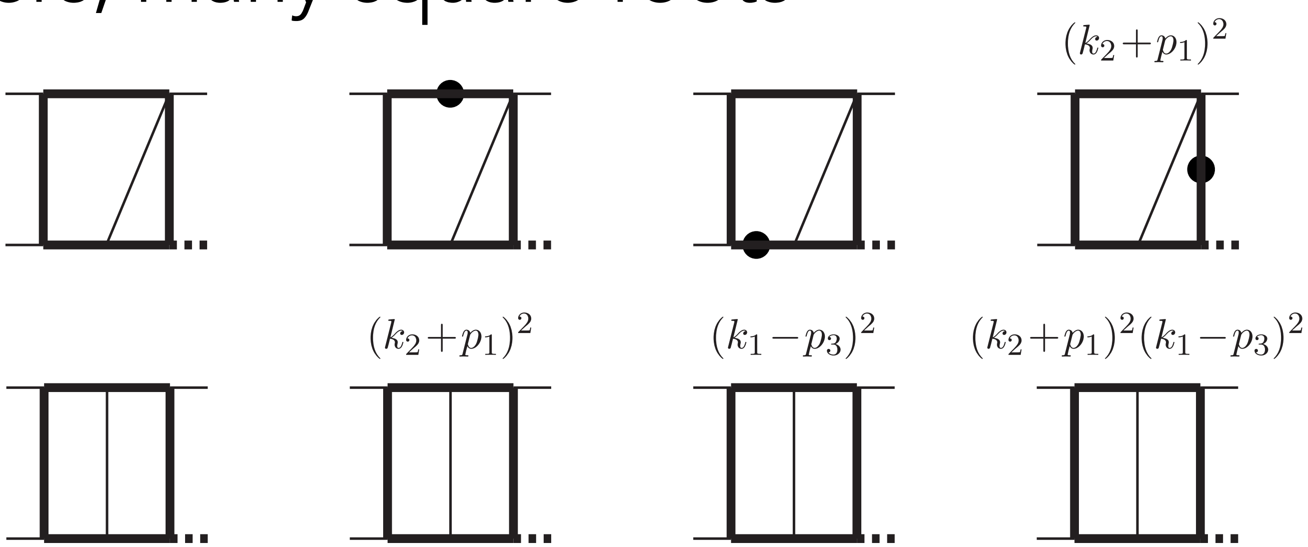
Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov 16; (See also Primo, Tancredi 16)

Most integrals:

- Canonical basis found
- $\log, \mathrm{Li}_2$ up to weight 2
- 1-fold integrals at weights 3,4
- Alphabet with 3 variables, 49 letters, many square roots

Two sectors expressed in terms of elliptic functions:

- 2- and 3-fold iterated integrals
- Elliptic kernel



Non-planar integrals not currently known analytically, but subject to ongoing work by other group(s)

Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov, …

Integration

Many methods to evaluate Feynman integrals

Here I will focus on the method of **Sector Decomposition**, a technique which enables dimensionally regulated parameter integrals to be integrated **numerically**

1) Feynman Parametrise integral and compute momentum integrals

$$G = (-1)^{N_\nu} \frac{\Gamma(N_\nu - LD/2)}{\prod_{j=1}^{N} \Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} \mathrm{d}x_j \; x_j^{\nu_j - 1} \delta(1 - \sum_{i=1}^{N} x_i) \frac{\mathcal{U}^{N_\nu - (L+1)D/2}(\vec{x})}{\mathcal{F}^{N_\nu - LD/2}(\vec{x}, s_{ij})}$$

Here $\mathcal{U}, \mathcal{F}$ are 1st, 2nd Symanzik Polynomials

Have exchanged L D-dim. momentum integrals for N param. integrals

Sector Decomposition

2) After integrating out δ we are faced with integrals of the form:

$$G_i = \int_0^1 \left(\prod_{j=1}^{N-1} \mathrm{d}x_j x_j^{\nu_j - 1} \right) \frac{\mathcal{U}_i(\vec{x})^{\mathrm{expo}\mathcal{U}(\epsilon)}}{\mathcal{F}_i(\vec{x}, s_{ij})^{\mathrm{expo}\mathcal{F}(\epsilon)}}$$

Powers depending on ϵ

Polynomials in F.P

Which may contain overlapping singularities which appear when several $x_j \to 0$ simultaneously (corresponding to UV/IR singularities)

Sector decomposition maps each integral into integrals of the form:

$$G_{ik} = \int_0^1 \left(\prod_{j=1}^{N-1} \mathrm{d}x_j x_j^{a_j - b_j \epsilon} \right) \frac{\mathcal{U}_{ik}(\vec{x})^{\mathrm{expo}\mathcal{U}(\epsilon)}}{\mathcal{F}_{ik}(\vec{x}, s_{ij})^{\mathrm{expo}\mathcal{F}(\epsilon)}}$$

Singularity structure can be read off

$$\mathcal{U}_{ik}(\vec{x}) = 1 + u(\vec{x})$$

$$\mathcal{F}_{ik}(\vec{x}) = -s_0 + f(\vec{x})$$

$u(\vec{x}), f(\vec{x})$ **have no constant term**

Hepp 66; Denner, Roth 96; Binoth, Heinrich 00

Sector Decomposition (II)

One technique **Iterated Sector Decomposition** repeat:

Binoth, Heinrich 00

$$\int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \frac{1}{(x_1+x_2)^{2+\epsilon}} \longleftarrow \textbf{Overlapping singularity for } x_1, x_2 \to 0$$

$$= \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \frac{1}{(x_1+x_2)^{2+\epsilon}} \left(\theta(x_1 - x_2) + \theta(x_2 - x_1)\right)$$

$$= \int_0^1 \mathrm{d}x_1 \int_0^{x_1} \mathrm{d}x_2 \frac{1}{(x_1+x_2)^{2+\epsilon}} + \int_0^1 \mathrm{d}x_2 \int_0^{x_2} \mathrm{d}x_1 \frac{1}{(x_1+x_2)^{2+\epsilon}}$$

$$= \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}t_2 \frac{x_1}{(x_1+x_1 t_2)^{2+\epsilon}} + \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}t_1 \frac{x_2}{(x_2 t_1+x_2)^{2+\epsilon}}$$

$$= \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}t_2 \frac{x_1^{-1-\epsilon}}{(1+t_2)^{2+\epsilon}} + \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}t_1 \frac{x_2^{-1-\epsilon}}{(t_1+1)^{2+\epsilon}} \longleftarrow \textbf{Singularities factorised}$$

If this procedure terminates depends on order of decomposition steps

An alternative strategy **Geometric Sector Decomposition** always terminates; both strategies are implemented in `pySecDec`.

Kaneko, Ueda 10; See also: Bogner, Weinzierl 08; Smirnov, Tentyukov 09

Sector Decomposition (III)

3) Expand in ϵ (simple case $a = -1$ "Logarithmic Divergence"):

$$\int_0^1 \mathrm{d}x^{-1-b\epsilon} g(x) = \underbrace{\frac{g(0)}{-b\epsilon}}_{\textbf{Poles}} + \int_0^1 \mathrm{d}x x^{-b\epsilon} \left[\frac{g(x) - g(0)}{x}\right] \longleftarrow \textbf{Finite}$$

`subtraction' of $g(0)$

By Definition: $g(0) \neq 0, g(0)$ finite

4) Numerically integrate

Key Point: Sector Decomposed integrals can be expanded in ϵ and numerically integrated

Numerical Integration

$$I[f] \equiv \int_{[0,1]^d} \mathrm{d}\mathbf{x}\ f(\mathbf{x}) \quad \approx \quad Q[f] = \frac{1}{N} \sum_{i=1}^{N} w_i\ f(\mathbf{x}_i)$$

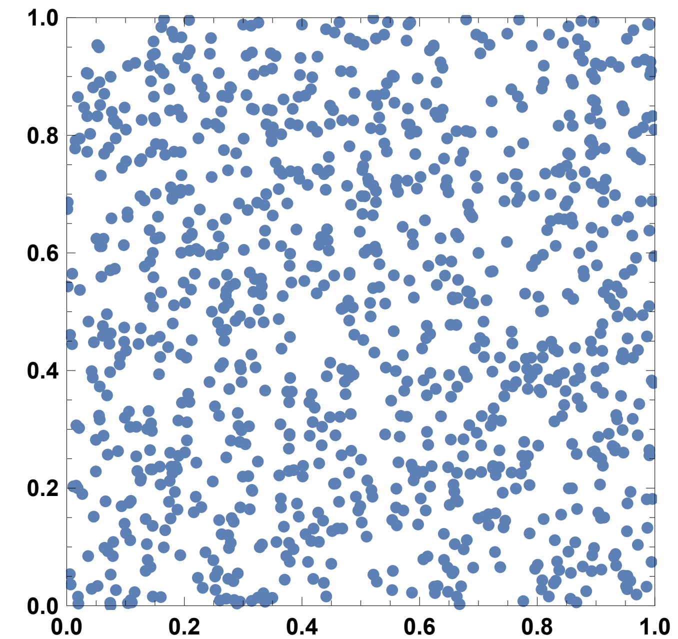
Goal: select points to minimise integration error $\varepsilon \equiv |I[f] - Q[f]|$

Monte Carlo:

Randomly select sampling points

$\varepsilon \approx \mathrm{Var}[f]/\sqrt{N}, \quad \varepsilon \sim \mathcal{O}(N^{-1/2})$

Improves slowly with N

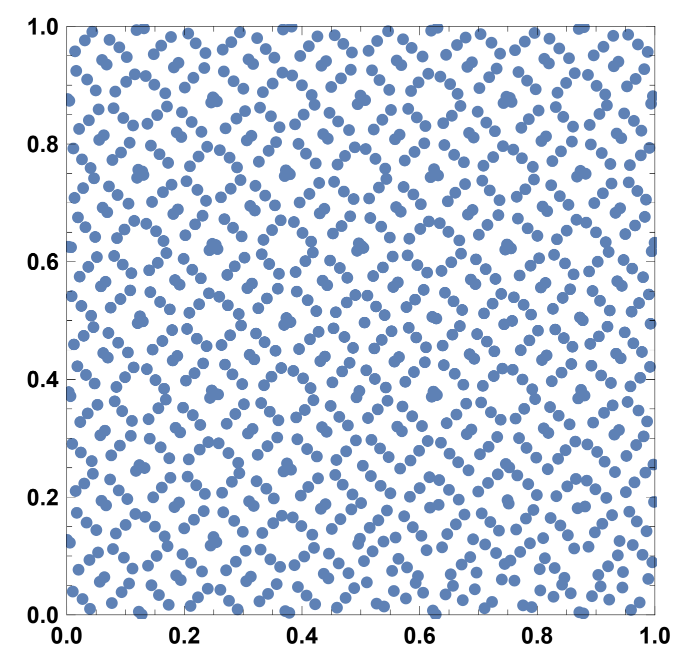


Quasi-Monte Carlo

Select points with low discrepancy D_N

$\varepsilon \leq D_N \cdot \mathrm{V}[f], \quad \varepsilon \sim \mathcal{O}(\log^d(N)/N)$

Poor performance for large d



Quasi-Monte Carlo (Rank 1 Lattices)

Quasi-Monte Carlo In a Weighted Function Space (QMC)

First application to sector-decomposed loop integrals: Li, Wang, Yan, Zhao 15

$\varepsilon \leq e_{\boldsymbol{\gamma}} \cdot ||f||_{\boldsymbol{\gamma}}, \quad \varepsilon \sim \mathcal{O}(N^{-1})$ or better

$$I[f] \approx \bar{Q}_{n,m}[f] \equiv \frac{1}{m} \sum_{k=0}^{m-1} Q_n^{(k)}[f], \qquad Q_n^{(k)}[f] \equiv \frac{1}{n} \sum_{i=0}^{n-1} f\left(\left\{ \frac{i\mathbf{z}}{n} + \boldsymbol{\Delta}_k \right\}\right)$$

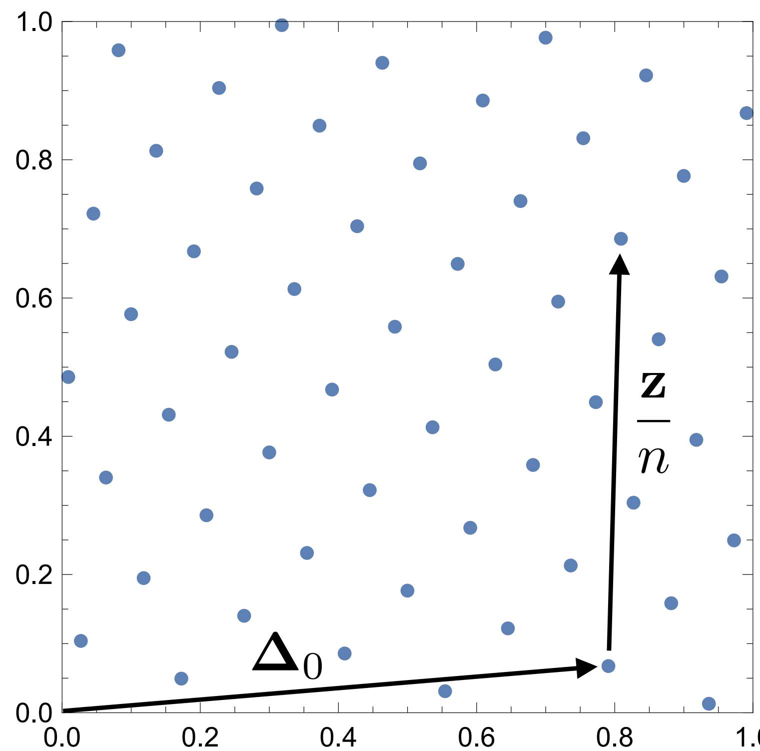
$\mathbf{z}$ - Generating vec.

$\boldsymbol{\Delta}_k$ - Random shift vec.

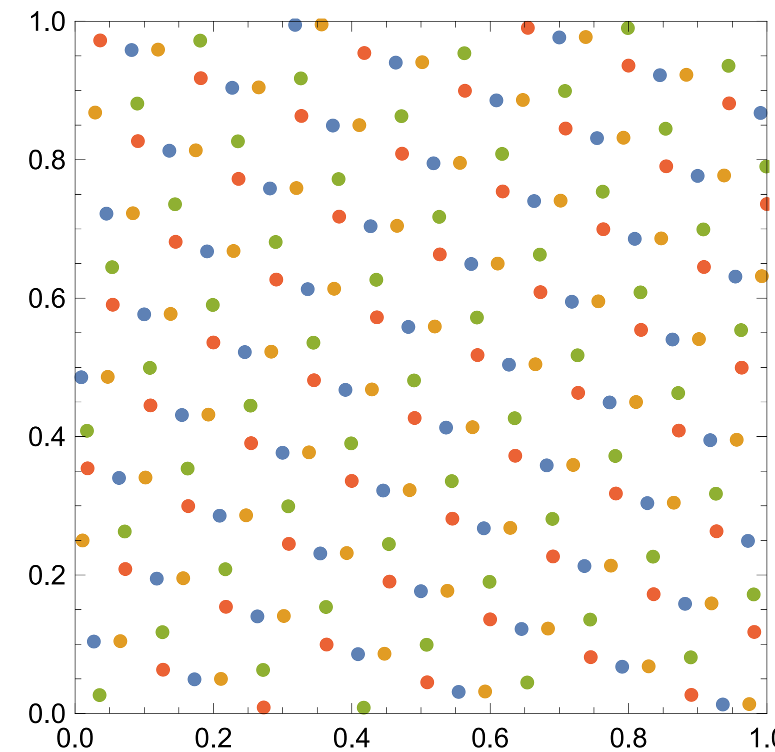
$\{\}$ - Fractional part

n - # Lattice points

m - # Random shifts



1 Shift



4 Shifts

Unbiased error estimate computed using (10-50) random shifts

Weighted Function Spaces

Review: Dick, Kuo, Sloan 13

Assign weights $\gamma_{\mathfrak{u}}$ to each subset of dimension $\mathfrak{u} \subseteq \{1, \dots, d\}$

Sobolev Space

Functions with square integrable first derivatives

Norm $$||f||_{\gamma}^{2} = \sum_{\mathfrak{u} \subseteq \{1,\dots,d\}} \frac{1}{\gamma_{\mathfrak{u}}} \int_{[0,1]^{|\mathfrak{u}|}} \left(\int_{[0,1]^{d-|\mathfrak{u}|}} \frac{\partial^{|\mathfrak{u}|} f(\mathbf{x})}{\partial \mathbf{x}_{\mathfrak{u}}} \mathrm{d}\mathbf{x}_{-\mathfrak{u}} \right)^{2} \mathrm{d}\mathbf{x}_{\mathfrak{u}}$$

Worst-case error $$e_{\gamma}^{2} \leq \left(\frac{1}{\psi(n)} \sum_{\emptyset \neq \mathfrak{u} \subseteq \{1,\dots,d\}} \gamma_{\mathfrak{u}}^{\lambda} \left(\frac{2\zeta(2\lambda)}{(2\pi^{2})^{\lambda}} \right)^{|\mathfrak{u}|} \right)^{\frac{1}{\lambda}}$$

$\lambda \in (1/2, 1]$

$$\boxed{\varepsilon \sim \mathcal{O}(N^{-1})}$$

Korobov Space

Periodic functions which are α times differentiable in each variable

$$||f||_{\gamma}^{2} = \sum_{\mathbf{h} \in \mathbb{Z}^{d}} \frac{\prod_{j \in \mathfrak{u}(\mathbf{h})} |h_j|^{2\alpha}}{\gamma_{\mathfrak{u}(\mathbf{h})}} |\hat{f}(\mathbf{h})|^{2}$$

Fourier Coefficient

$$e_{\gamma}^{2} \leq \left(\frac{1}{\psi(n)} \sum_{\emptyset \neq \mathfrak{u} \subseteq \{1,\dots,d\}} \gamma_{\mathfrak{u}}^{\lambda} \left(2\zeta(2\alpha\lambda)\right)^{|\mathfrak{u}|} \right)^{\frac{1}{\lambda}}$$

$\lambda \in (1/(2\alpha), 1]$, smoothness α

$$\boxed{\varepsilon \sim \mathcal{O}(N^{-\alpha})}$$

Generating vector $\mathbf{z}$ precomputed for a **fixed** number of lattice points, chosen to minimise worst-case error Nuyens 07

Periodizing Transforms

Sector decomposed functions are typically continuous and smooth but not periodic
Functions can be periodized by a suitable change of variables: $\mathbf{x} = \phi(\mathbf{u})$

$$I[f] \equiv \int_{[0,1]^d} \mathrm{d}\mathbf{x}\; f(\mathbf{x}) = \int_{[0,1]^d} \mathrm{d}\mathbf{u}\; \omega_d(\mathbf{u}) f(\phi(\mathbf{u}))$$

$$\phi(\mathbf{u}) = (\phi(u_1), \ldots, \phi(u_d)), \qquad \omega_d(\mathbf{u}) = \prod_{j=1}^{d} \omega(u_j) \qquad \text{and} \qquad \omega(u) = \phi'(u)$$

Korobov transform: $\omega(u) = 6u(1-u), \quad \phi(u) = 3u^2 - 2u^3$

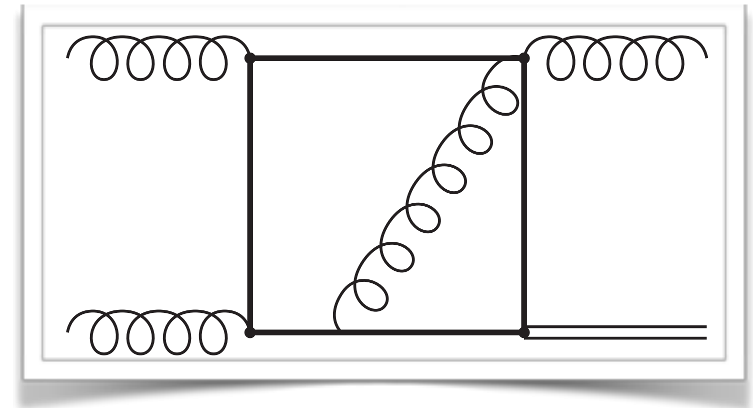
Sidi transform: $\omega(u) = \pi/2 \sin(\pi u), \quad \phi(u) = 1/2(1 - \cos \pi t)$

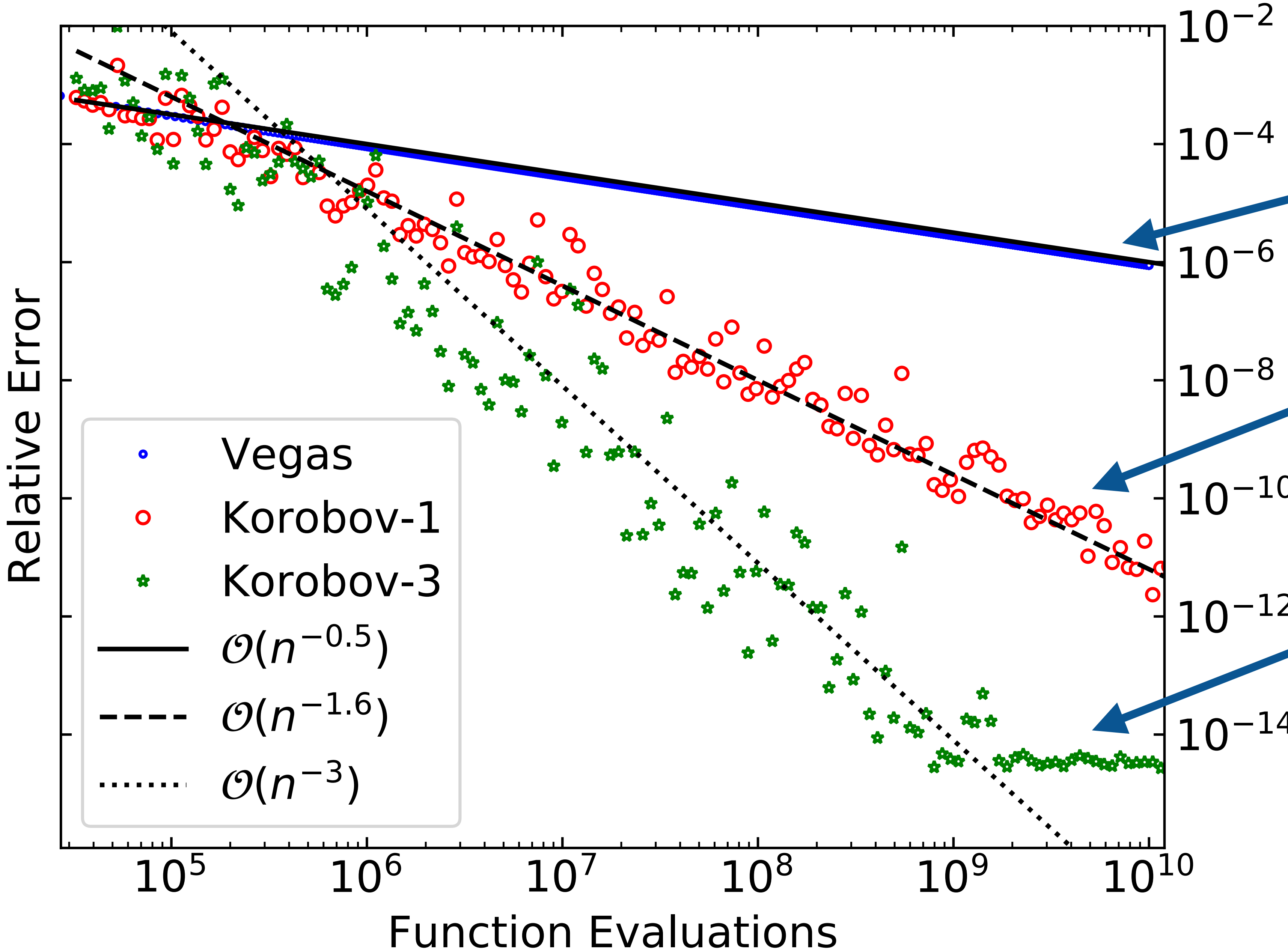
Baker transform: $\phi(u) = 1 - |2u - 1|$

Scaling

Example:

Sector Decomposed HJ Integral





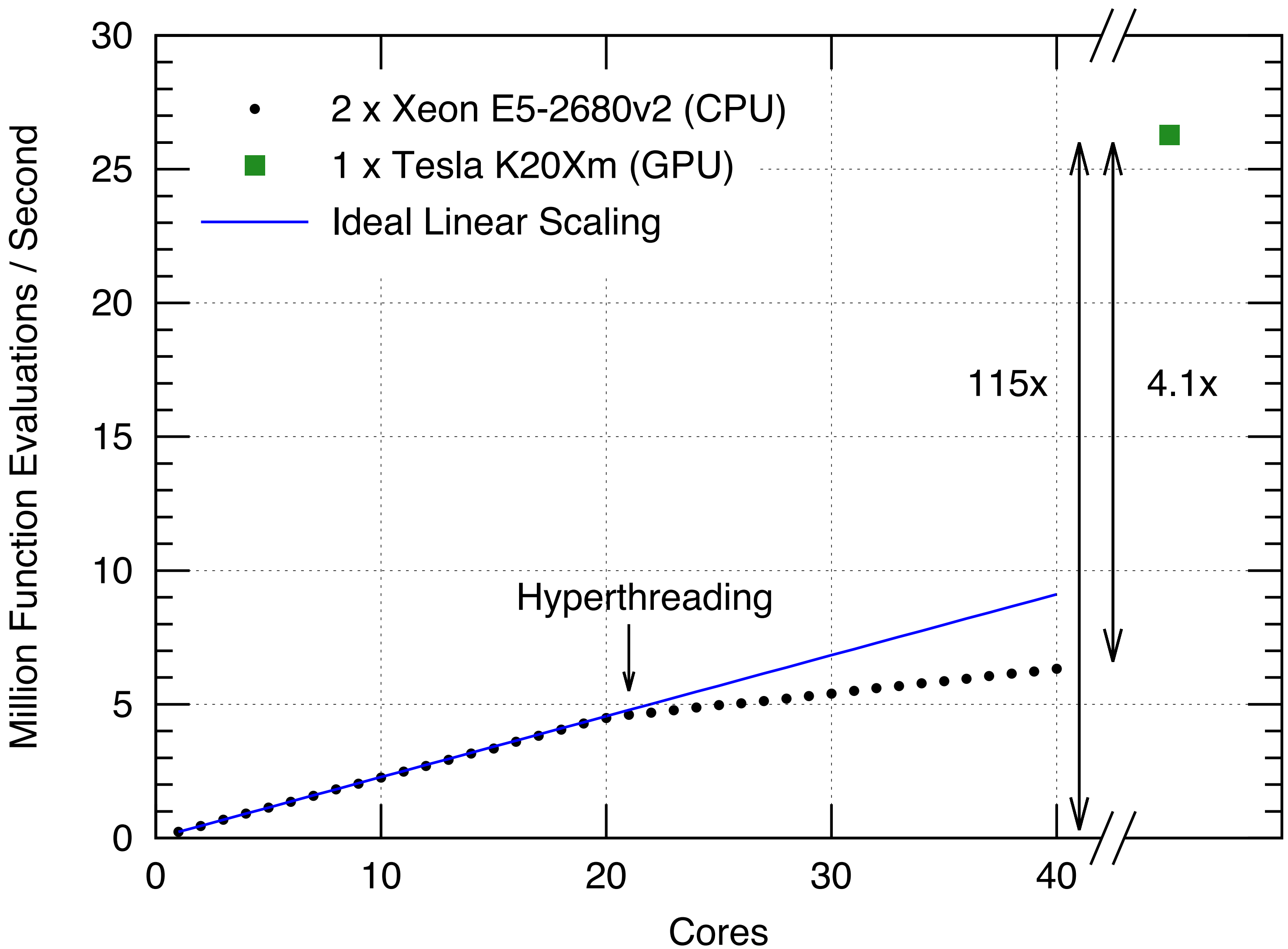
Monte Carlo scaling

Better than ``guaranteed" n^{-1} scaling

Limited by machine precision (double)

High Performance Computing

Accuracy limited by number of function evaluations
Can accelerate this using Graphics Processing Units (GPUs)



2 CPUs (20 Cores + HT)

1 GPU

n	CPU (s)	GPU(s)	C/G
655357	6.63	1.60	**4.1**
7208951	72.3	16.4	**4.4**
67264993	674.2	152.2	**4.4**

Note:

1) Plot made for old OpenCL implementation (new CUDA impl. similar)
2) Performance gain highly dependent on integrand & hardware!

Variance Reduction

Can improve performance of numerical integrator by flattening integrand via a variable transformation $y = p(x)$

$$I = \int_0^1 \mathrm{d}y \; f(y) = \int_0^1 \mathrm{d}x \; p'(x) f(p(x)) \qquad \text{s.t.} \qquad p'(x) \propto |f(p(x))|^{-1}$$

VEGAS:

Assume integrand separable $f(\mathbf{x}) = g(x_1)g(x_2)\cdots g(x_d)$

Iteratively approximate $p(x)$ with piecewise linear function

But: Spoils smoothness of integrand (bad for QMC)

Alternative:

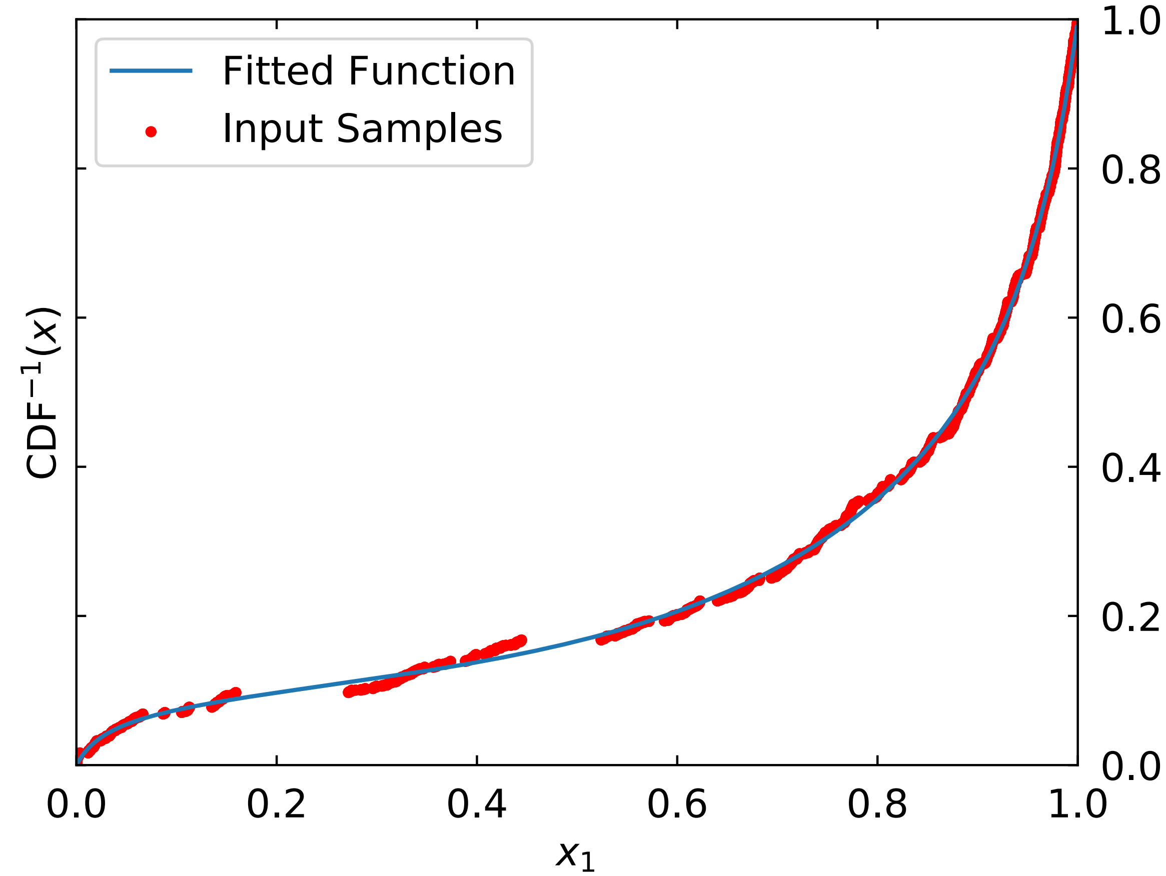
Choose a smooth function for $p(x)$, for example:

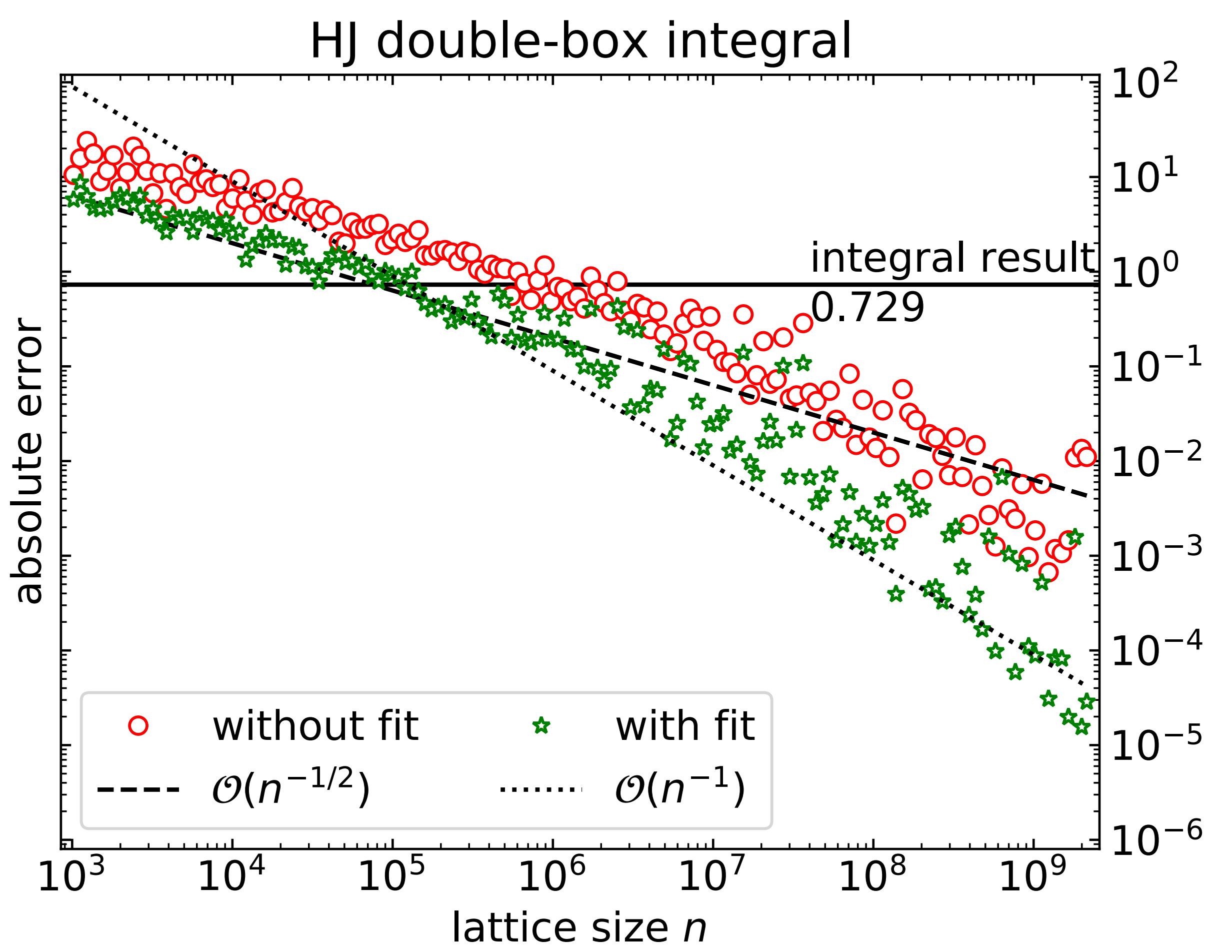
$$p(x) = a_2 \cdot x \, \frac{a_0 - 1}{a_0 - x} + a_3 \cdot x \, \frac{a_1 - 1}{a_1 - x} + a_4 \cdot x + a_5 \cdot x^2 + \left(1 - \sum_{i=2}^{5} a_i\right) \cdot x^3$$

Fit parameters a_i to inverse cumulative distribution function (CDF)

Variance Reduction (II)

1) Pre-sample integrand
2) In each dimension: fit inverse CDF
3) Use fit as variable transformation, obtain flatter integrand without spoiling smoothness





Example: HJ Integral

Small n: ~ 3x improvement
Large n: ~ 10x improvement

QMC Scaling preserved

pySecDec

pySecDec: a program to numerically evaluate dimensionally regulated parameter integrals (written in python, FORM & c++)

Vermaseren 00; Kuipers, Ueda, Vermaseren 13; Ruijl, Ueda, Vermaseren 17

Code: https://github.com/mppmu/secdec/releases

Docs: https://secdec.readthedocs.io

Borowka, Heinrich, Jahn, SJ, Kerner, Schlenk, Zirke

Supports:

Contour deformation, Arbitrary loops/legs (within reason)

Soper 99; Binoth, Guillet, Heinrich, Pilon, Schubert 05; Nagy, Soper 06; Anastasiou, Beerli, Daleo 07; Beerli 08; Borowka, Carter, Heinrich 12; Borowka 14;

General parameter integrals (not just loop integrals)

Arbitrary number of regulators

Flexible numerators (contracted Lorentz vectors, inverse propagators)

Generates c++ Library (can be linked to your own program)

New: Quasi-Monte Carlo integration & CUDA GPU Support

1811.11720; Li, Wang, Yan, Zhao 15; Review: Dick, Kuo, Sloan 13;

Putting it all together…

HJ Amplitude Structure

Write integrals with r propagators and s inverse propagators as

Arbitrary scale

$$I_{r,s}(\hat{s},\hat{t},m_h^2,m_t^2) = (M^2)^{-L\epsilon}(M^2)^{2L-r+s} I_{r,s}\left(\frac{\hat{s}}{M^2},\frac{\hat{t}}{M^2},\frac{m_h^2}{M^2},\frac{m_t^2}{M^2}\right)$$

We renormalize strong coupling, a, in $\overline{\mathrm{MS}}$ scheme and top quark mass in OS scheme, each renormalized form factor can be written as:

$$\mathrm{F} = a^{\frac{3}{2}}\left[\mathrm{F}^{(1)} + a(\frac{n_g}{2}\delta Z_A + \frac{3}{2}\delta Z_a)\mathrm{F}^{(1)} + a\delta m_t^2 \mathrm{F}^{ct,(1)} + a\mathrm{F}^{(2)} + O(a^2)\right]$$

$$\mathrm{F}^{(1)} = \left(\frac{\mu_R^2}{M^2}\right)^{\epsilon}\left[b_0^{(1)} + b_1^{(1)}\epsilon + b_2^{(1)}\epsilon^2 + \mathcal{O}(\epsilon^3)\right]$$ ⟵ 1-loop

$$\mathrm{F}^{ct,(1)} = \left(\frac{\mu_R^2}{M^2}\right)^{\epsilon}\left[c_0^{(1)} + c_1^{(1)}\epsilon + \mathcal{O}(\epsilon^2)\right]$$ ⟵ Mass Counter-Terms

$$\mathrm{F}^{(2)} = \left(\frac{\mu_R^2}{M^2}\right)^{2\epsilon}\left[\frac{b_{-2}^{(2)}}{\epsilon^2} + \frac{b_{-1}^{(2)}}{\epsilon} + b_0^{(2)} + \mathcal{O}(\epsilon)\right]$$ ⟵ 2-loop

Scale variations do not require any re-computation of **red** terms

Amplitude Evaluation

Use Quasi-Monte Carlo (QMC) integration $\mathcal{O}(n^{-1})$ error scaling

Li, Wang, Yan, Zhao 15; Review: Dick, Kuo, Sloan 13;

Implemented in OpenCL, evaluated on GPUs

Entire 2-loop amplitude evaluated with a single code

$$F = \sum_i \underbrace{\left(\sum_j C_{i,j} \epsilon^j \right)}_{\text{coeff.}} \underbrace{\left(\sum_k I_{i,k} \epsilon^k \right)}_{\text{integral}} = \epsilon^{-2} \left[C^{(L)}_{1,-2} {\color{red} I^{(L)}_{1,0}} + \ldots \right] + \epsilon^{-1} \left[C^{(L)}_{1,-1} {\color{red} I^{(L)}_{1,0}} + \ldots \right] + \ldots$$

compute once

Dynamically set target precision for each sector, minimising time:

$$T = \sum_i t_i + \bar{\lambda} \left(\sigma^2 - \sum_i \sigma_i^2 \right), \quad \sigma_i \sim t_i^{-e}$$

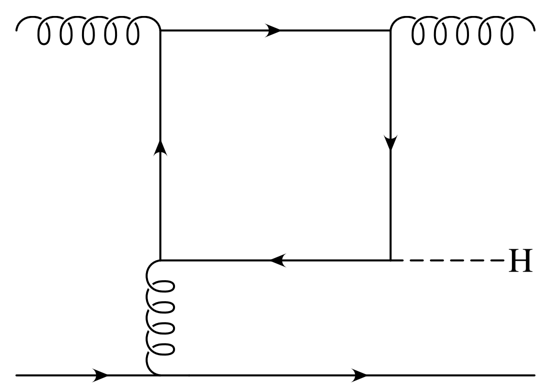
$\bar{\lambda}$ – Lagrange multiplier

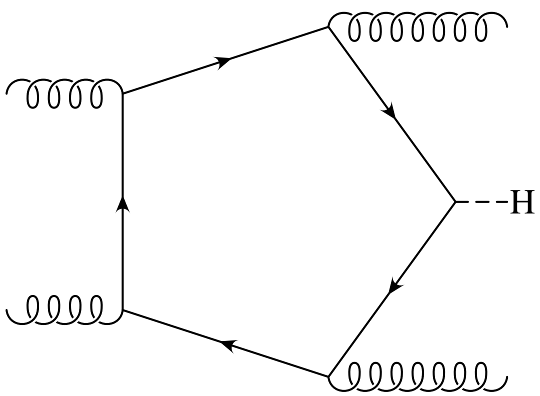
σ – precision goal

σ_i – error estimate

HJ Phase-Space & Real Radiation

Real Radiation





For HJ Known analytically

Del Duca, Kilgore, Oleari, Schmidt, Zeppenfeld 01

We use an upgraded **`GoSam`** setup:
Cullen et al. 14

- Generate quadruple precision copy of the code
- Rescues unstable points on-the-fly with **`Ninja`** (quad) Mastrolia, Mirabella, Peraro 12; Peraro 14
- **`OneLoop`** for scalar integrals van Hameren 11

Implemented in **`POWHEG-BOX-V2`**, uses FKS subtraction
Nason 04; Frixione, Nason Oleari 07; Alioli, Nason, Oleari, Re 10; Frixione, Kunszt, Signer 96

Virtual Phase Space

1. Apply VEGAS algorithm to LO matrix element
2. Using LO events generate unweighted events via accept/reject

For HJ p_T distribution we include a reweighing factor to sample sufficiently also at large transverse momenta

Comparison of HJ and HH

	HJ production	**HH production**
#Form factors	4+2	2
Full reduction	✓	only planar
(quasi-) finite basis	✓	only planar
#Master integrals including crossings	458	327*
#Master integrals neglecting crossings	120	215*
#Integrals after sector decomposition and expansion in ε	22675	11244
Code size coefficients	~340 MB	~80 MB
Code size integrals	~330 MB	~580 MB
Compile time coefficients	~2 weeks	few days
Compile time integrals	~4 hours	~1-2 days
Time for linking the program	~3-4 days	few hours

Slide: Matthias Kerner, Radcor 2017

* HH non-planar not fully reduced

HJ Results

HJ Results: Total Cross Section

$$m_H = 125 \text{ GeV},$$
$$m_T = \sqrt{23/12}\, m_H \approx 173.05 \text{ GeV}$$
$$p_{T,j} > 30 \text{ GeV}, \quad \text{anti}-k_T \;\; R = 0.4,$$
$$\mu = \frac{H_T}{2} = \frac{1}{2}\left(\sqrt{m_H^2 + p_{t,H}^2} + \sum_i |p_{t,i}|\right)$$

`PDF4LHC15_nlo_30_pdfas`

Butterworth et al. 16; Dulat et al. 16;
Harland-Lang et al. 15; Ball et al. 15

$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\mathrm{FT_{approx}}} = \int \mathrm{d}\phi_2 \left(\mathrm{d}\sigma_B + \frac{\mathrm{d}\sigma_{\mathrm{B}}}{\mathrm{d}\sigma_{\mathrm{B}}^{\mathrm{HEFT}}} \mathrm{d}\sigma_{\mathrm{V}}^{\mathrm{HEFT}} \right) + \int \mathrm{d}\phi_3 \; \mathrm{d}\sigma_{\mathrm{R}}$$

$\mathrm{FT_{approx}}$:
Full Born & Reals
Reweight virtuals event-by-event

Theory	LO [pb]	NLO [pb]
HEFT:	$\sigma_{\mathrm{LO}} = 8.22^{+3.17}_{-2.15}$	$\sigma_{\mathrm{NLO}} = 14.63^{+3.30}_{-2.54}$
$\mathrm{FT_{approx}}$:	$\sigma_{\mathrm{LO}} = 8.57^{+3.31}_{-2.24}$	$\sigma_{\mathrm{NLO}} = 15.07^{+2.89}_{-2.54}$
Full:	$\sigma_{\mathrm{LO}} = 8.57^{+3.31}_{-2.24}$	$\sigma_{\mathrm{NLO}} = 16.01^{+1.59}_{-3.73}$

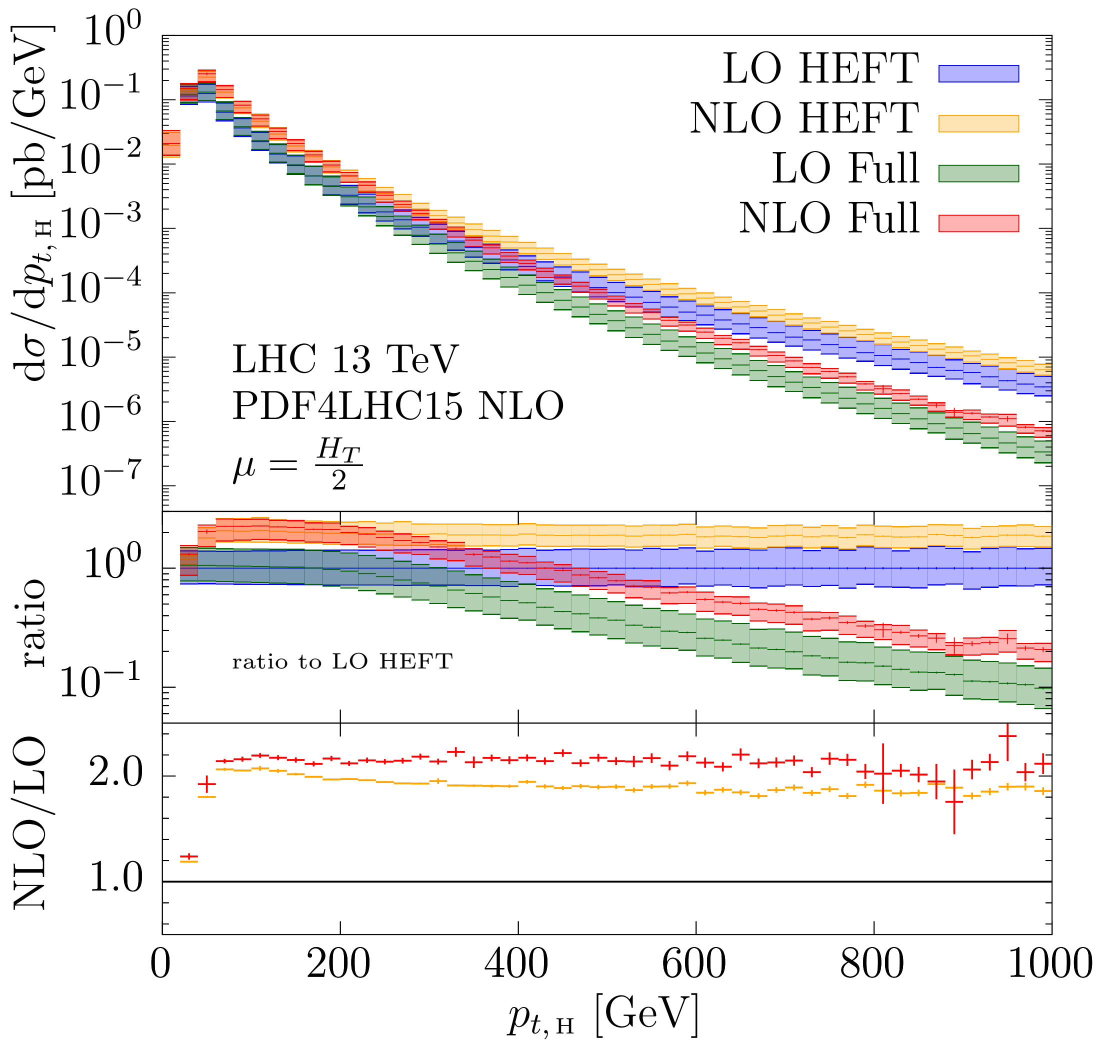
+6%

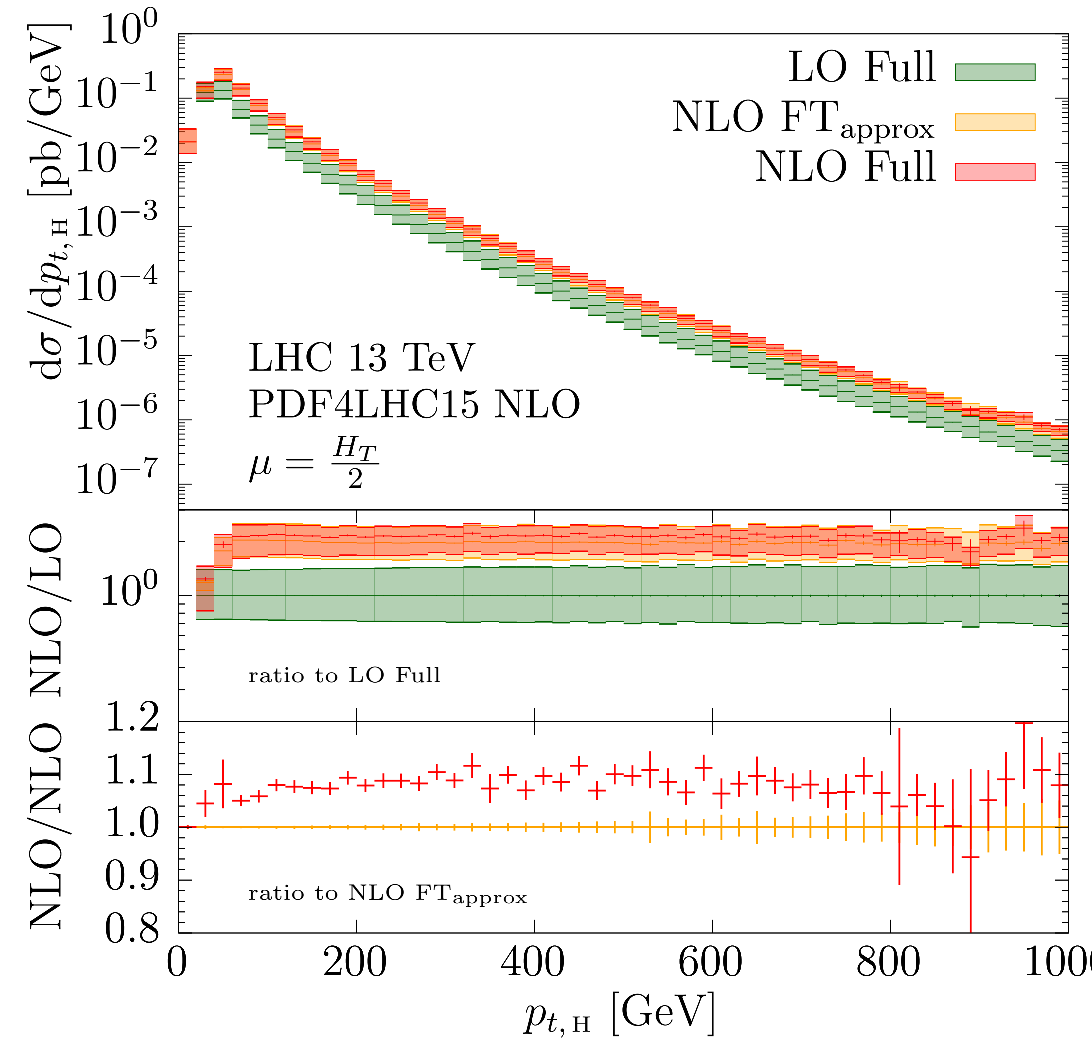
+9%

Note: Non-negligible contribution from top-bottom interference known at NLO but not included here

(Lindert,) Melnikov, Tancredi, Wever 16, 17

HJ Results (II): Higgs Boson pT





Confirm expected scaling of $\mathrm{d}\sigma/\mathrm{d}p_T^2$ in HEFT and full theory at NLO

$\sim p_T^{-2}$ in HEFT

$\sim p_T^{-4}$ in full theory

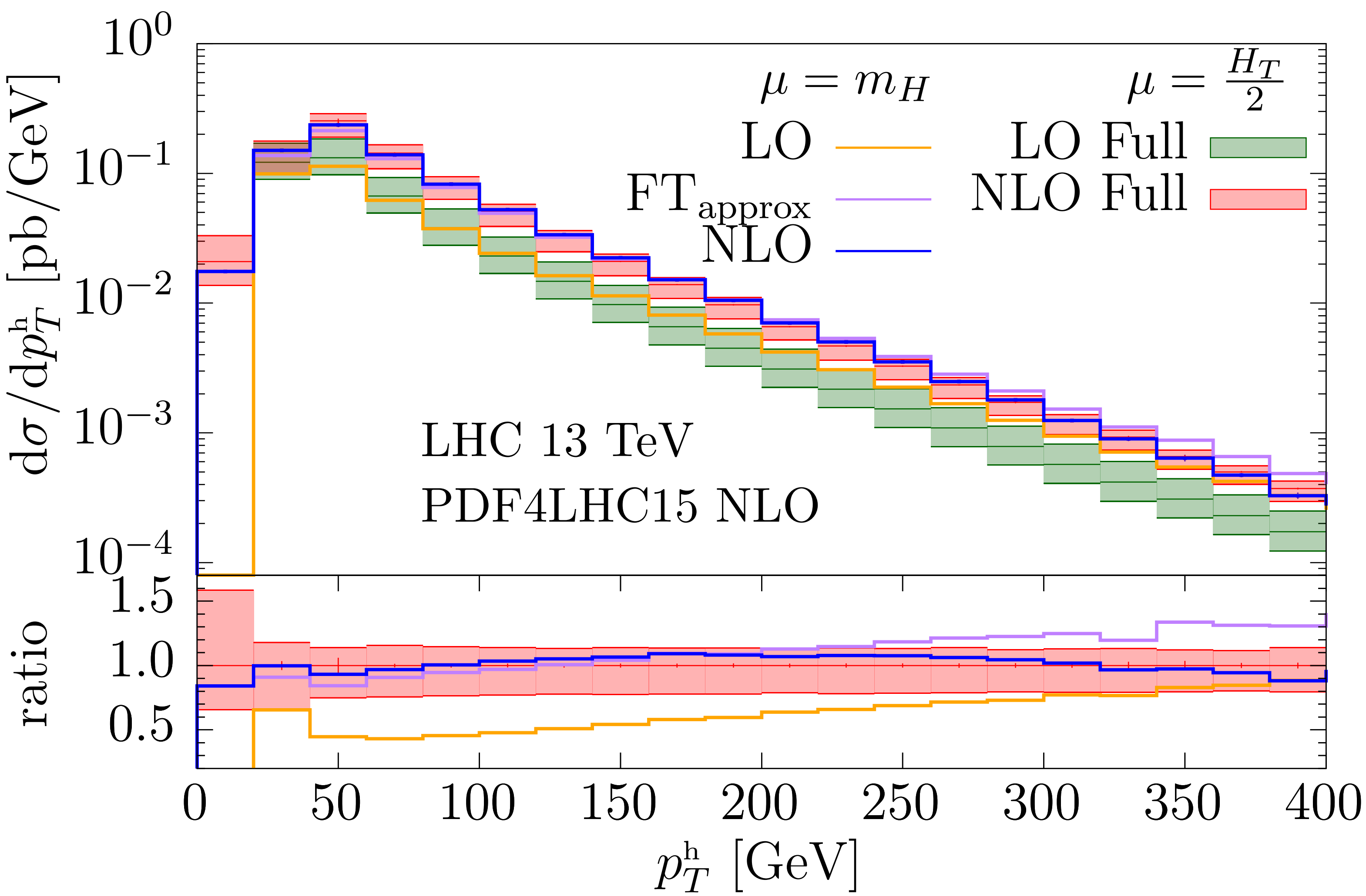
Forte, Muselli 15; Caola, Forte, Marzani, Muselli, Vita 16;

$\mathrm{FT}_{\mathrm{approx}}$ predicts similar p_T distribution shape to full theory

Full theory predicts nearly flat K factor at large p_T

~8% increase in tail by including top quark mass dependence in virtuals

HJ Results (III): Different Scale Choices



Full NLO result with $\mu = m_H$ and $\mu = H_T/2$ in good agreement

With fixed scale $\mathrm{FT_{approx}}$ has different shape to full theory (overestimates tail)

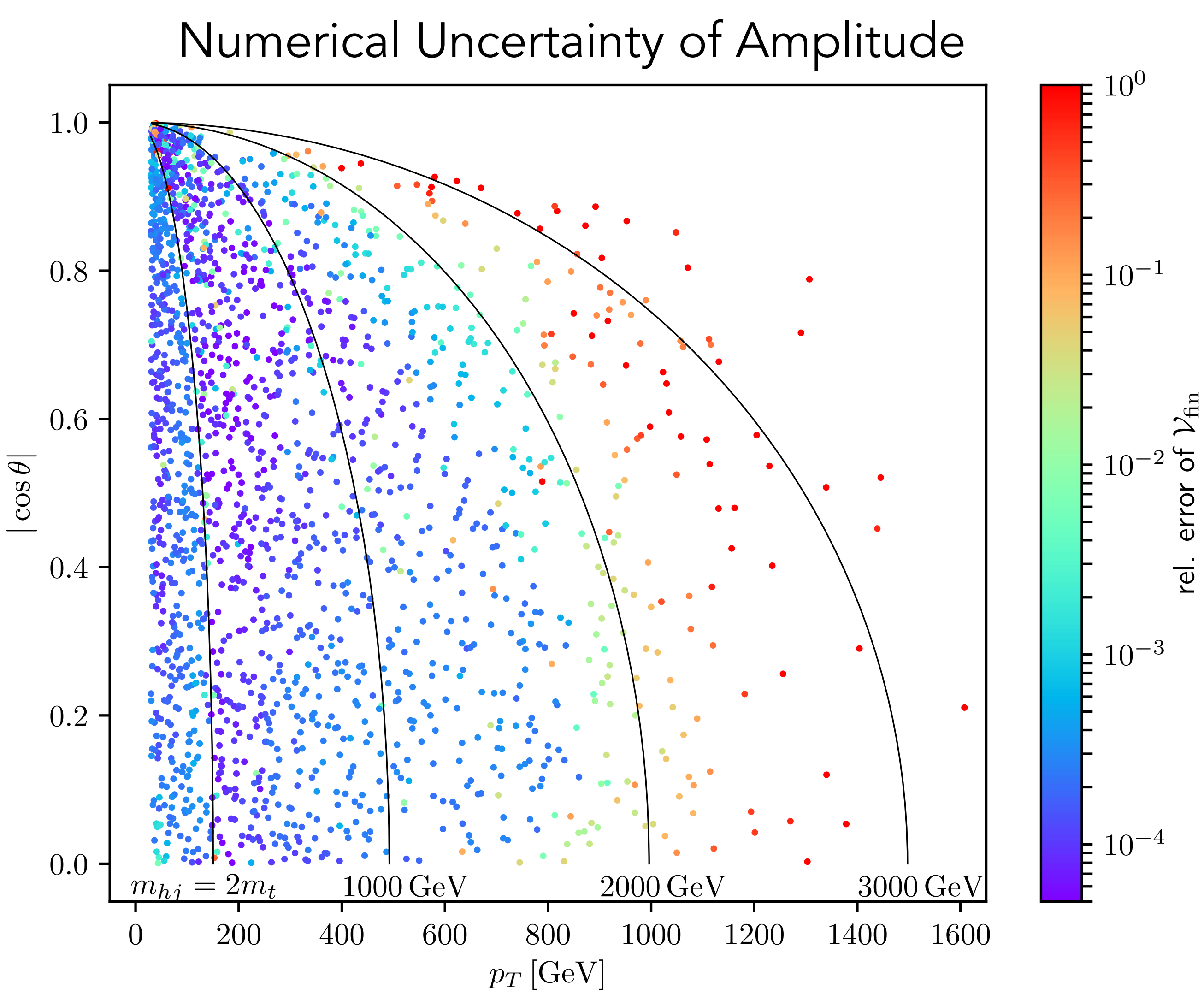
Note: K-factor only flat for dynamic scale choice, not for fixed scale

HJ Numerical Stability

Numerical evaluation of virtual amplitude:

- accuracy goal: 0.5% for each form factor
- wall-clock limit: 2d GPU-time (Tesla K20X GPU)

Thanks to MPCDF for compute resources



Accuracy reached for $|\mathcal{M}|^2$

- Better than per-mill for most points below $m_{hj} = 1.5$ TeV
- Region $m_{hj} \geq 2$ TeV numerically challenging
- Forward region challenging

Run time per point:

- Minimum: 1.3h
- Median: 15h

HJ Numerical Stability (II)

MI Basis Change:

- Consider quasi-finite integrals (prefer finite integrals)
- Brute force combinations of masters, factor denominators and check that dimension factorises (achieved)
- Prefer simple denominator factors
- Prefer computing fewer orders in epsilon for each master
- Prefer simpler numerators (check number of terms/file size)

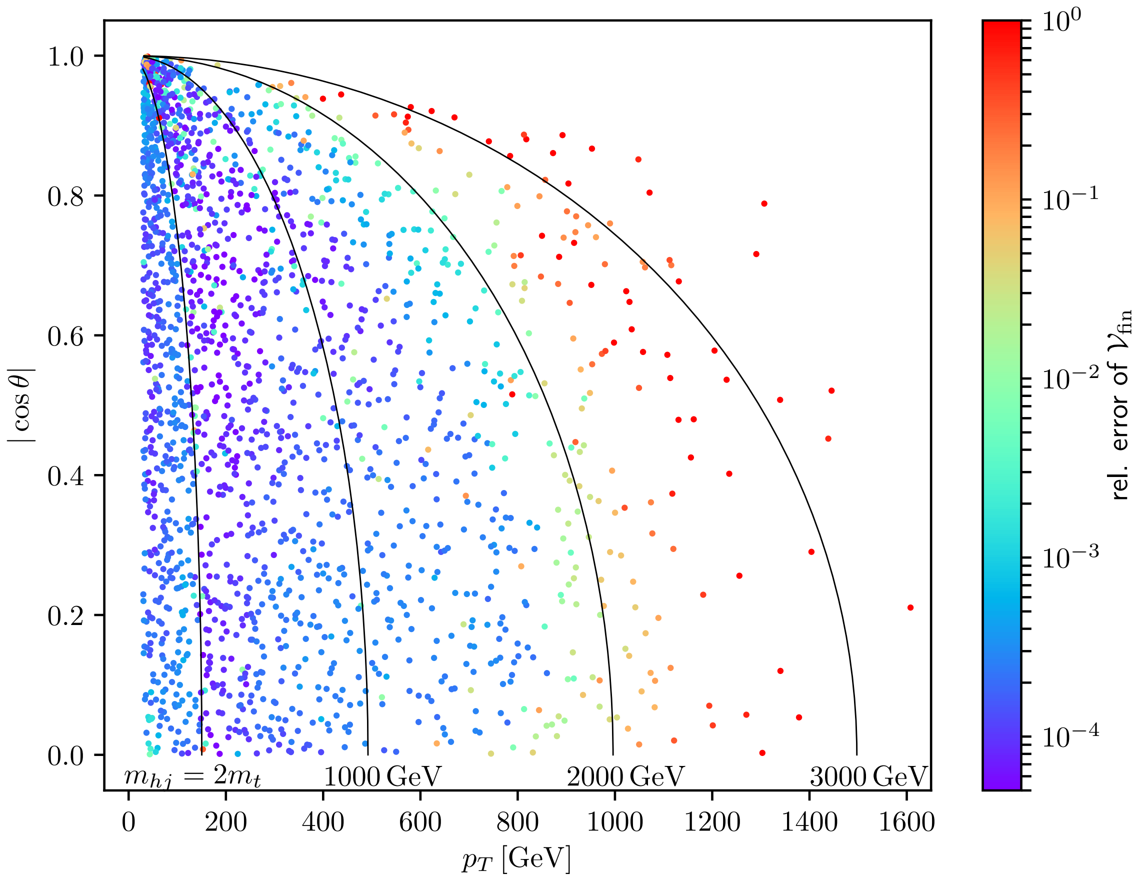
See: Matthias Kerner, Loops and Legs Proceedings 2018

Numerical Improvements:

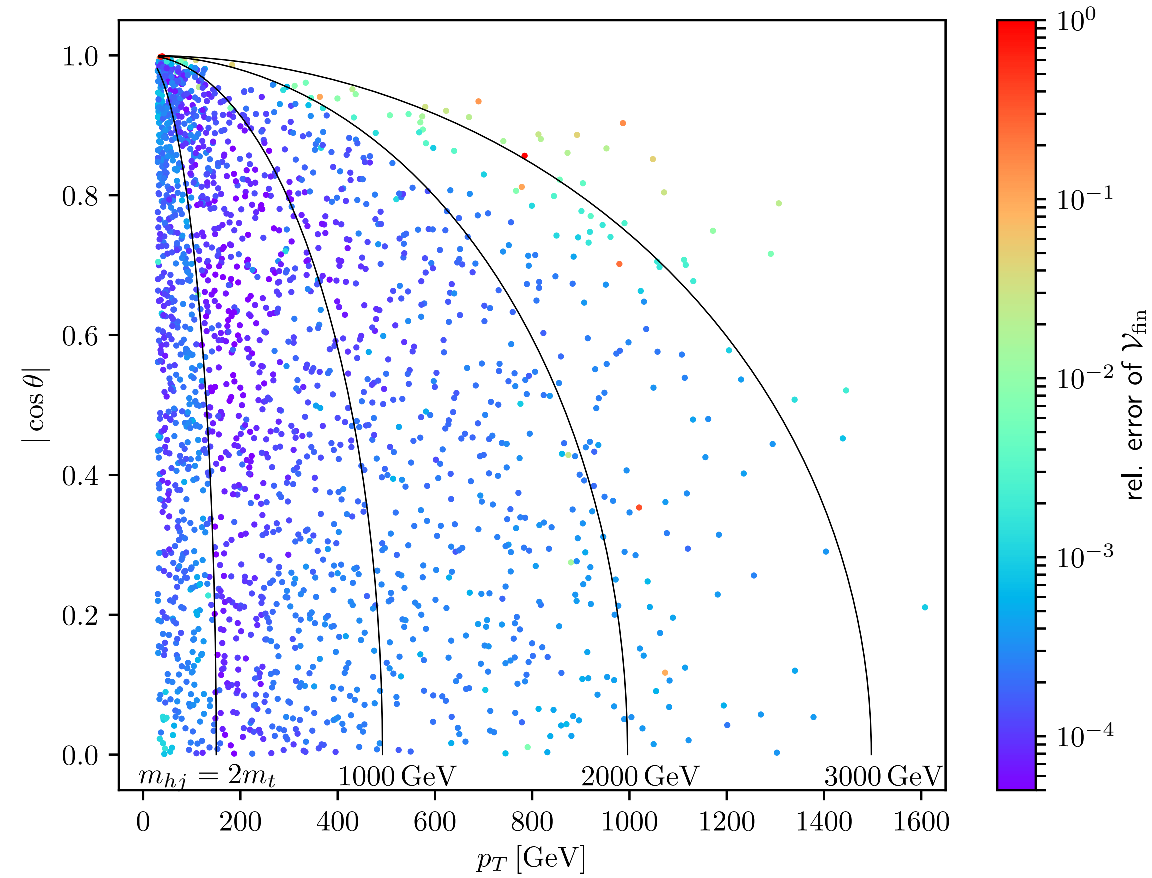
- Improve modular arithmetic implementation (needed for larger lattices)
- Do not try to further evaluate integrals if $\mathrm{rel.\ err.} < 10^{-14}$
- Adjust time spent integrating when iteration may exceed wall clock limit

HJ Numerical Stability (III)

Before basis change:



After basis change:



Phase-space points significantly more stable:

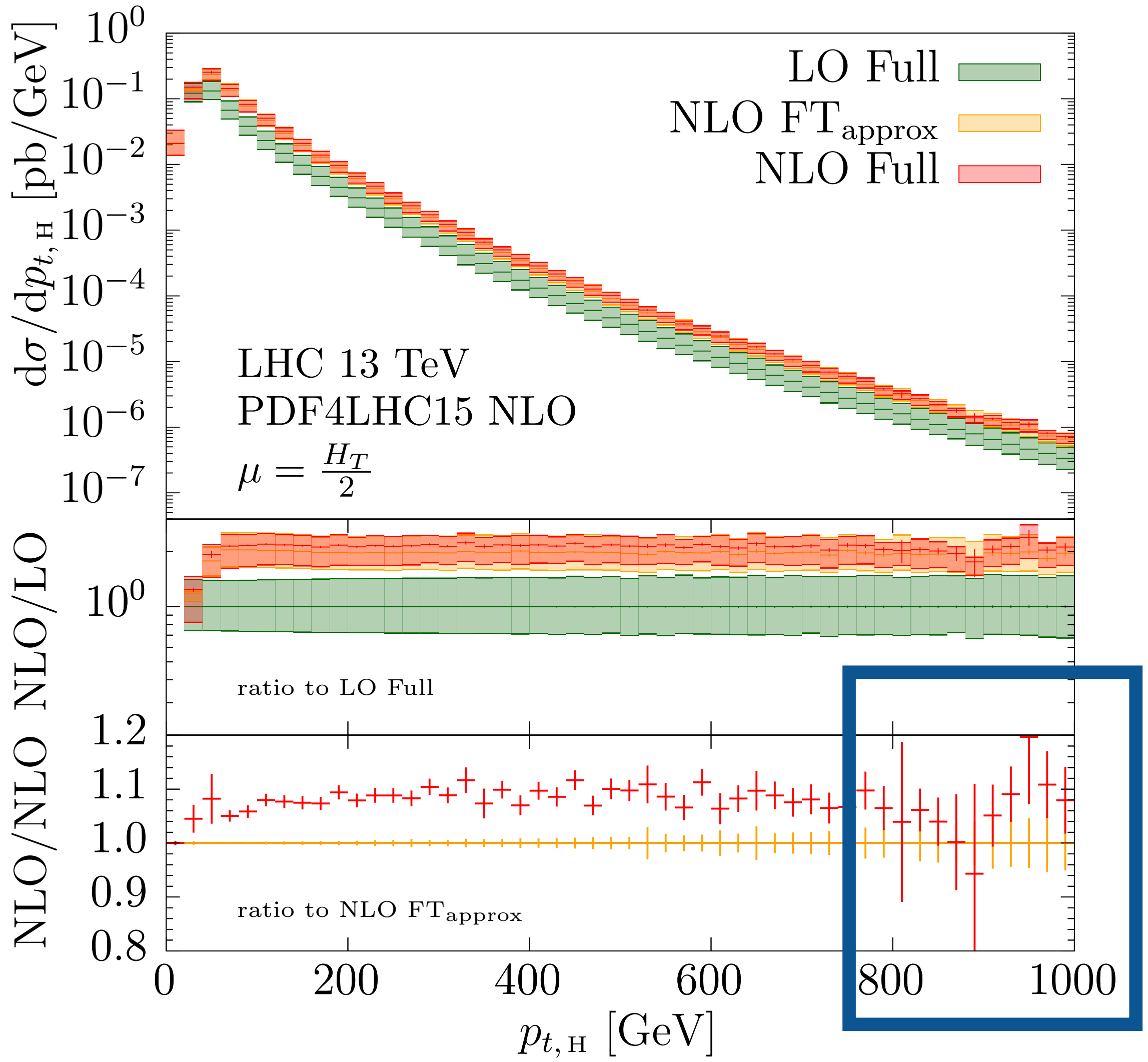
- Good accuracy around top quark threshold
- Huge improvement in accuracy at larger invariant mass (2-3 TeV)
- Improvement in the forward region

Coefficient code size: 340 MB → 100 MB

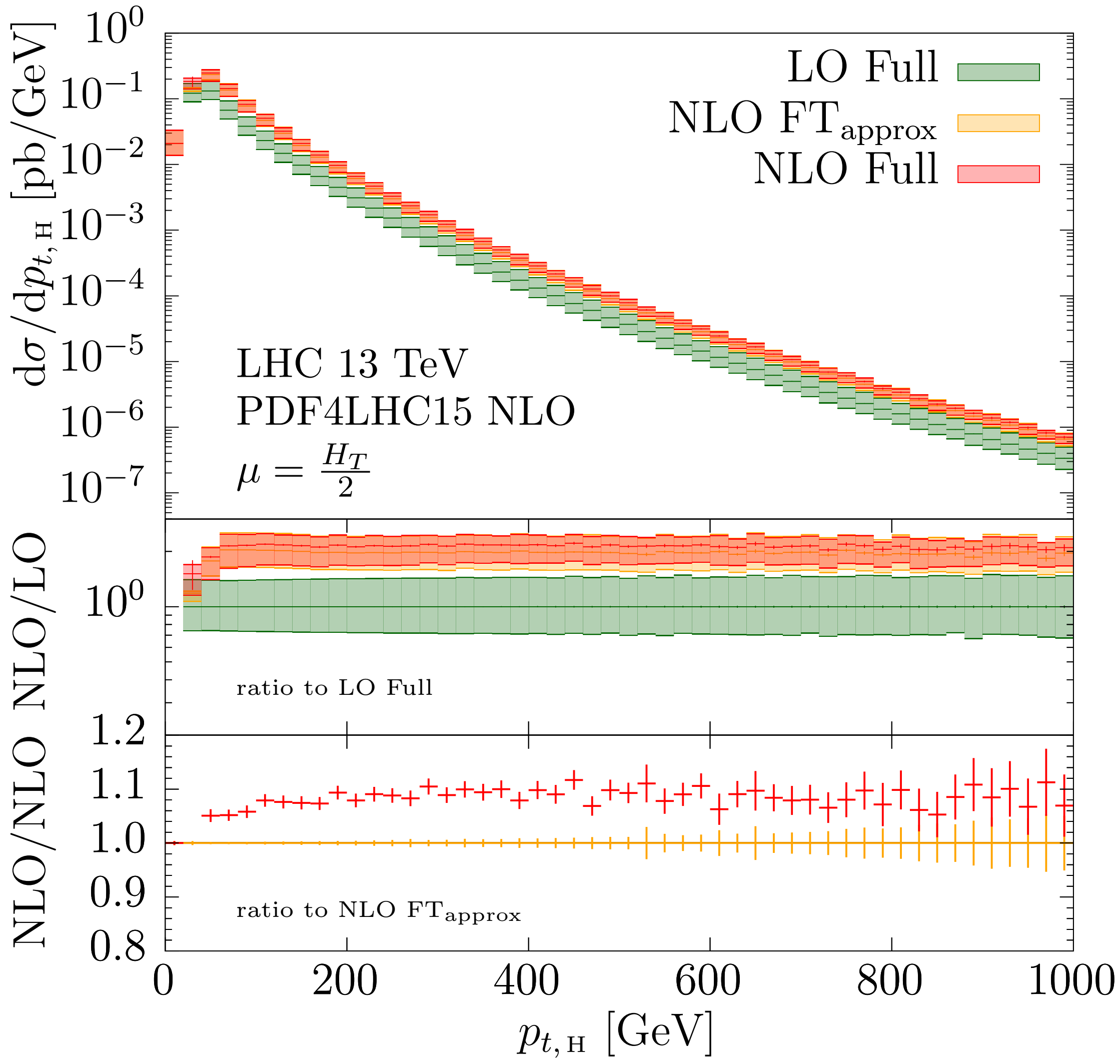
Median runtime 15h → <2h

HJ Results (IV): Higgs Boson pT

Before basis change:



After basis change:



Recomputing unstable points improves fluctuations in tail

Low fraction of points excluded 3/2004

HJ Expansion

Alternatively can consider Higgs boson & top quark masses as small

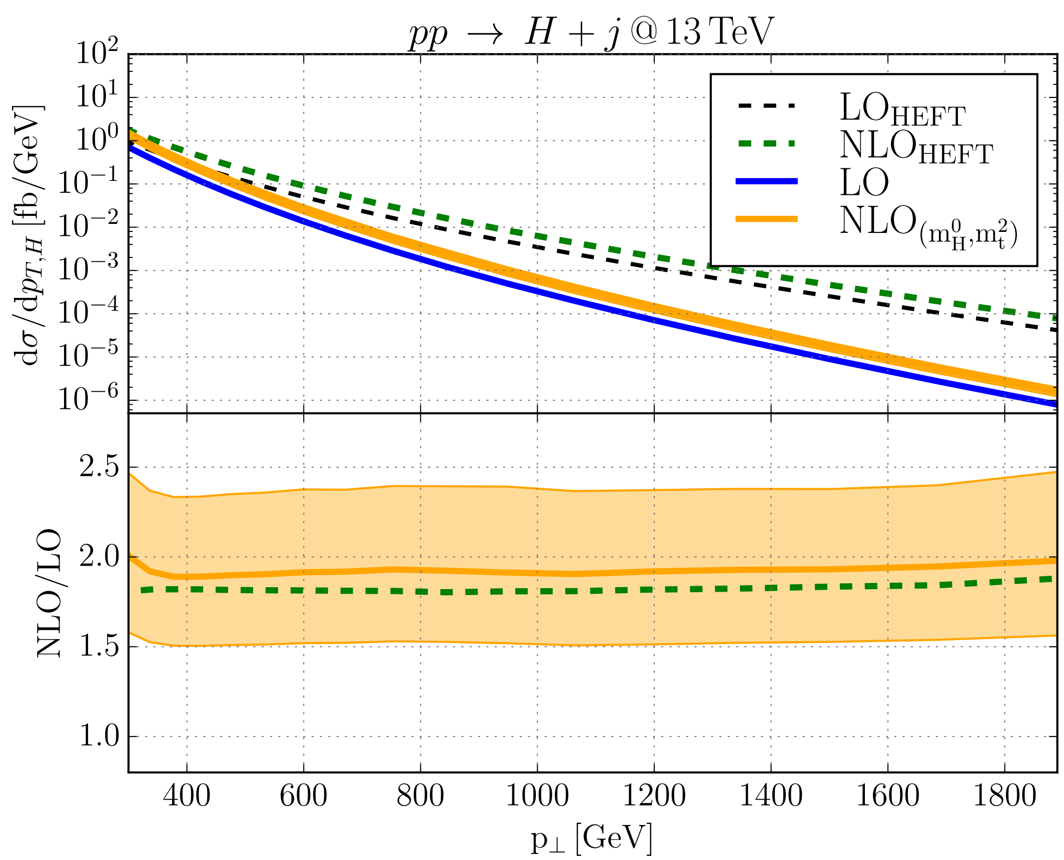
Introduce variables:

$$\eta = -\frac{m_H^2}{4m_T^2}, \quad \kappa = -\frac{m_T^2}{s}, \quad z = \frac{u}{s}$$

Expand integrals to $\mathcal{O}(\eta^0\kappa^1)$ justified for $m_H^2, m_T^2 \ll |s| \sim |t| \sim |u|$,

For example at large $p_T^2 = ut/s$

Kudashkin, Melnikov, Wever 17



Expanded 2-loop virtuals can be combined with full reals to predict Higgs boson p_T distribution above top threshold

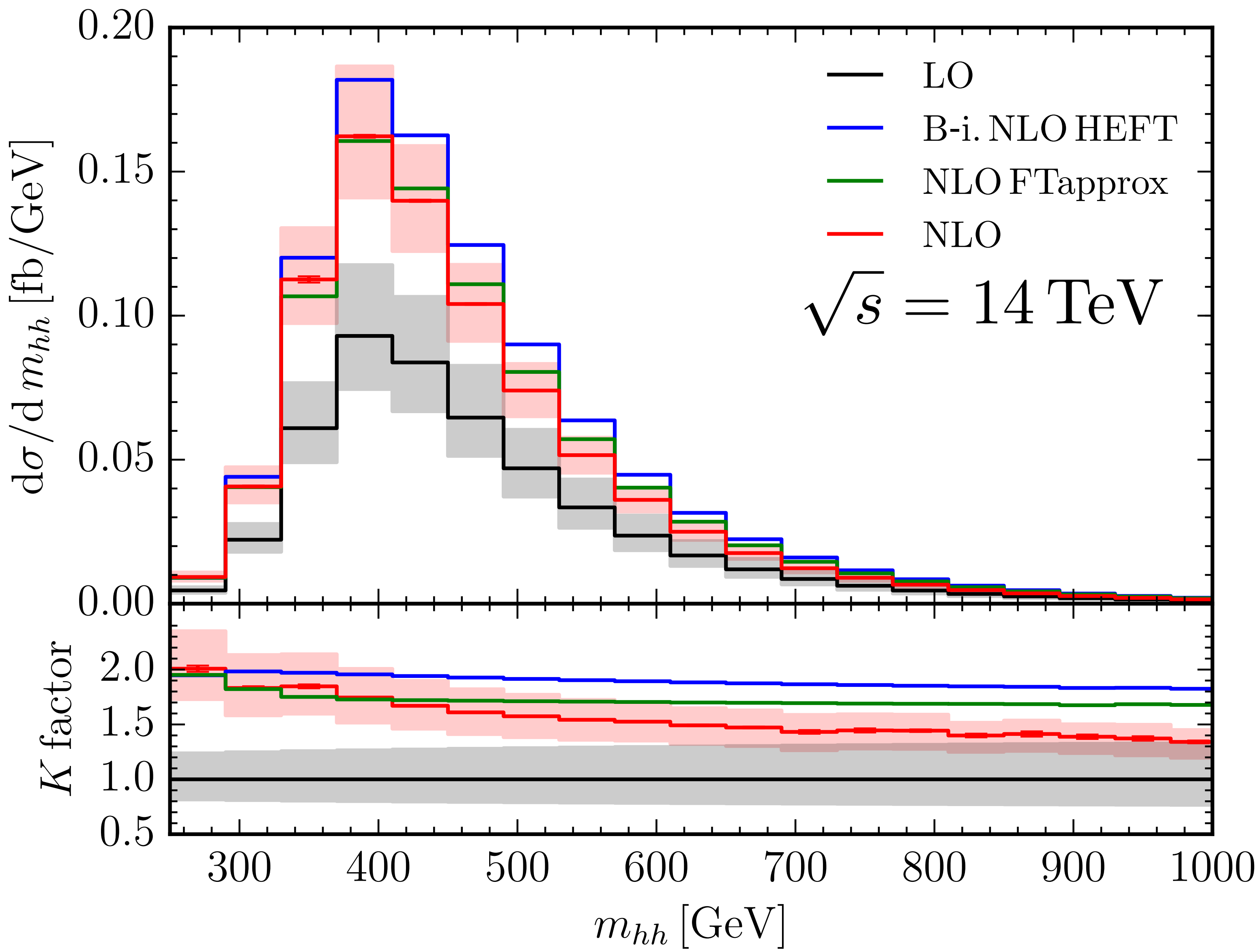
Lindert, Kudashkin, Melnikov, Wever 18; Neumann 18

$$\frac{K^{\mathrm{SM}}}{K^{\mathrm{HTL}}} = 1.04 \ldots 1.06$$

Minor difference to full result, due to missing $\mathcal{O}(\eta^1)$ terms?

HH Results

HH Results (I): Invariant Mass



PDF4LHC15_nlo_30_pdfas

$m_H = 125$ GeV

$m_T = 173$ GeV

Uncertainty:

$\mu_R = \mu_F = \frac{m_{HH}}{2}$

$\mu \in \left[\frac{\mu_0}{2}, 2\mu_0\right] \quad (7-\text{point})$

HTL: Outside scale var.

$m_{hh} > 420$ GeV

FTapp: Outside scale var.

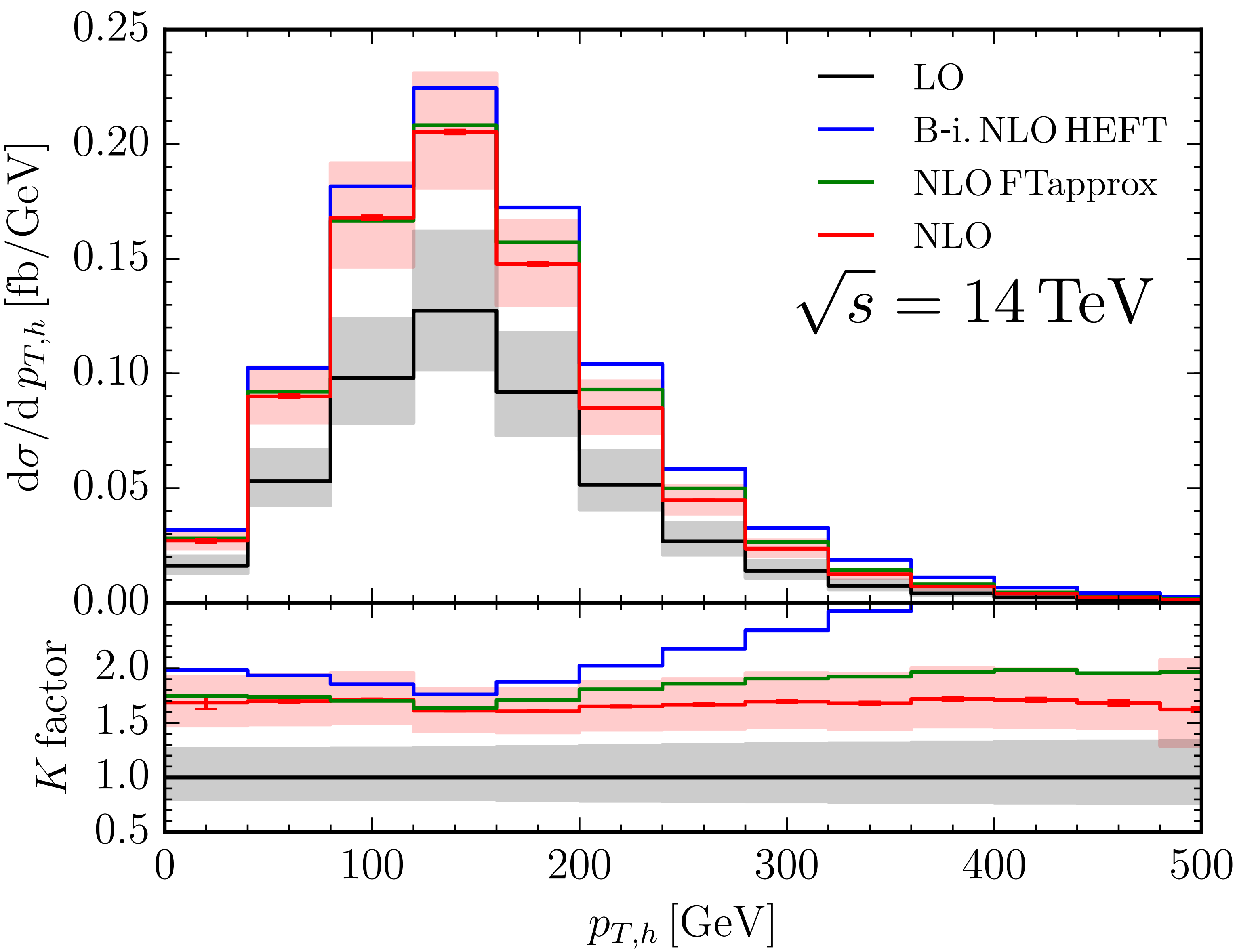
$m_{hh} > 620$ GeV

	σ_{LO} (fb)	σ_{NLO} (fb)
B.I. HEFT	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$
FTapprox	$19.85^{+27.6\%}_{-20.5\%}$	$34.26^{+14.7\%}_{-13.2\%}$
Full Theory	$19.85^{+27.6\%}_{-20.5\%}$	$\mathbf{32.91^{+13.6\%}_{-12.6\%} \pm 0.3\%(\text{stat.}) \pm 0.1\%(\text{int.})}$

HTL overestimates by 16%

FTap. overestimates by 4%

HH Results (II): pT either Higgs



HTL: Can poor approx. for larger $p_{T,h}$

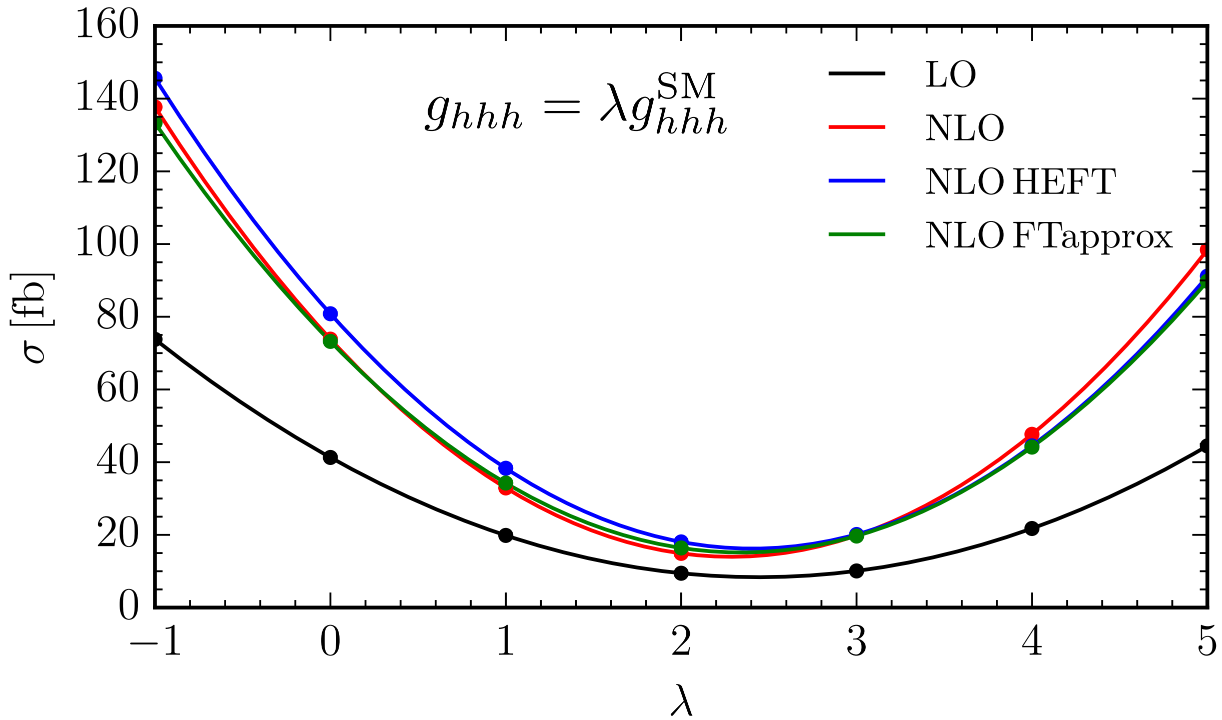
Note: ambiguous how to rescale HTL reals by full LO born differentially

FTapp: Significantly better but still overestimating

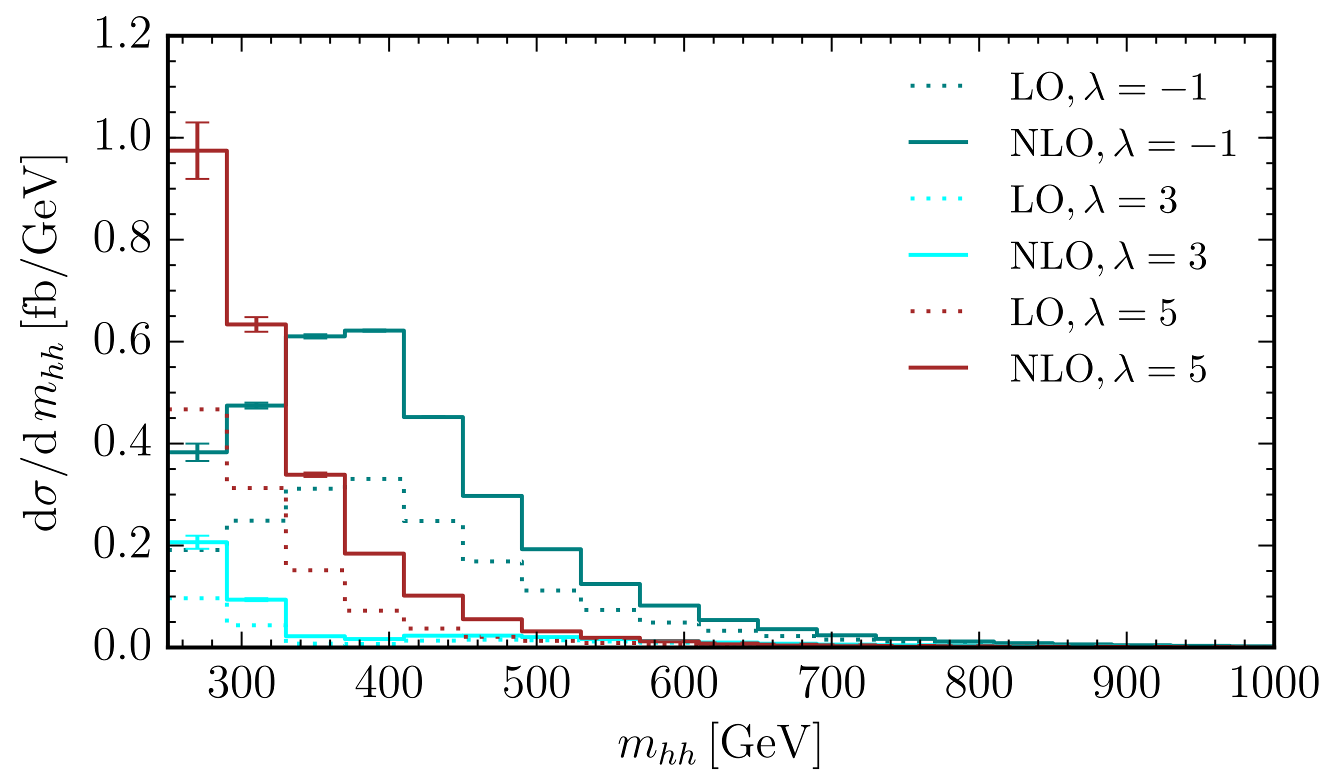
Real radiation plays larger role for large $p_{T,h}$

(As hoped) Including full reals does improve over HTL in tails

Variation of the Higgs Self Coupling



SM: Destructive interference between g_{hhh} and y_T^2 contrib.



Distributions: Can help to distinguish between λ values

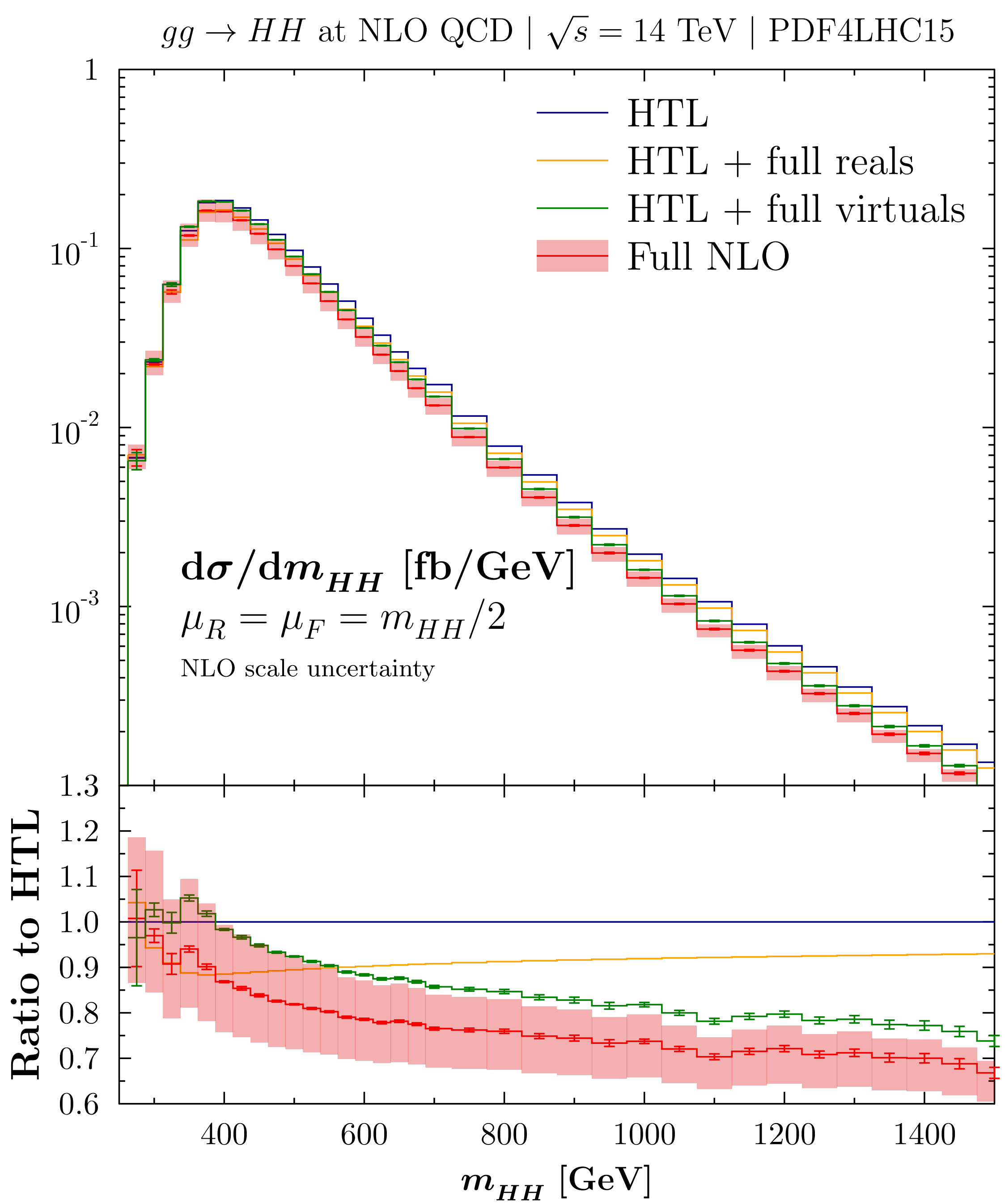
Result has also been used in a full NLO QCD EFT analysis

Buchalla, Capozi, Celis, Heinrich, Scyboz 18

HH Top Quark Mass Scheme Uncertainties

HH recently recomputed by another group (also using numerical methods)

Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18



Mutual agreement with our result

Studied top quark mass scheme/scale uncertainties:

$$\left.\frac{d\sigma(gg\to HH)}{dQ}\right|_{Q=300\text{ GeV}} = 0.0312(5)^{+9\%}_{-23\%}\,\text{fb/GeV},$$

$$\left.\frac{d\sigma(gg\to HH)}{dQ}\right|_{Q=400\text{ GeV}} = 0.1609(4)^{+7\%}_{-7\%}\,\text{fb/GeV},$$

$$\left.\frac{d\sigma(gg\to HH)}{dQ}\right|_{Q=600\text{ GeV}} = 0.03204(9)^{+0\%}_{-26\%}\,\text{fb/GeV},$$

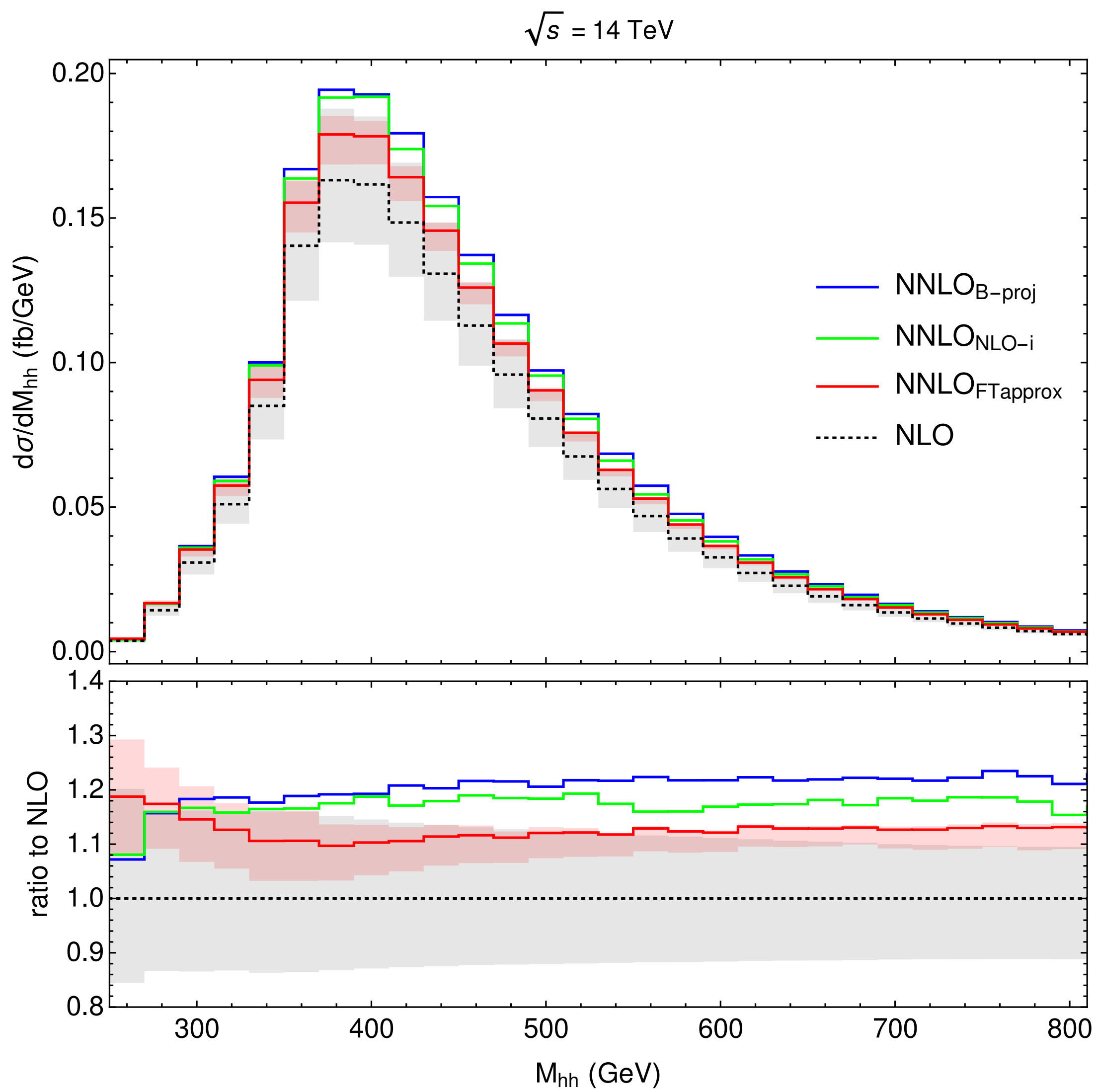
$$\left.\frac{d\sigma(gg\to HH)}{dQ}\right|_{Q=1200\text{ GeV}} = 0.000435(4)^{+0\%}_{-30\%}\,\text{fb/GeV},$$

Large uncertainty obtained varying scale of top quark mass (in $\overline{\text{MS}}$) by factor 2 up/down

HH: NNLO EFT Combined with NLO SM

Differential NNLO HTL + NLO SM

Top quark mass effects studied using 3 different approximations



Grazzini, Heinrich, SJ, Kallweit, Kerner, Lindert, Mazzitelli 18; (+NNLL) de Florian, Mazzitelli 18;

$\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
NLO [fb]	$27.78^{+13.8\%}_{-12.8\%}$	$32.88^{+13.5\%}_{-12.5\%}$	$127.7^{+11.5\%}_{-10.4\%}$	$1147^{+10.7\%}_{-9.9\%}$
$\text{NLO}_{\text{FTapprox}}$ [fb]	$28.91^{+15.0\%}_{-13.4\%}$	$34.25^{+14.7\%}_{-13.2\%}$	$134.1^{+12.7\%}_{-11.1\%}$	$1220^{+11.9\%}_{-10.6\%}$
$\text{NNLO}_{\text{NLO-i}}$ [fb]	$32.69^{+5.3\%}_{-7.7\%}$	$38.66^{+5.3\%}_{-7.7\%}$	$149.3^{+4.8\%}_{-6.7\%}$	$1337^{+4.1\%}_{-5.4\%}$
$\text{NNLO}_{\text{B-proj}}$ [fb]	$33.42^{+1.5\%}_{-4.8\%}$	$39.58^{+1.4\%}_{-4.7\%}$	$154.2^{+0.7\%}_{-3.8\%}$	$1406^{+0.5\%}_{-2.8\%}$
$\text{NNLO}_{\text{FTapprox}}$ [fb]	$31.05^{+2.2\%}_{-5.0\%}$	$36.69^{+2.1\%}_{-4.9\%}$	$139.9^{+1.3\%}_{-3.9\%}$	$1224^{+0.9\%}_{-3.2\%}$
M_t unc. $\text{NNLO}_{\text{FTapprox}}$	$\pm 2.6\%$	$\pm 2.7\%$	$\pm 3.4\%$	$\pm 4.6\%$
$\text{NNLO}_{\text{FTapprox}}/\text{NLO}$	1.118	1.116	1.096	1.067

1) $\text{NNLO}_{\text{NLO-i}}$

Rescale NLO by $K_{\text{NNLO}} = \text{NNLO}_{\text{HTL}}/\text{NLO}_{\text{HTL}}$

2) $\text{NNLO}_{\text{B-proj}}$

Project real radiation contributions to Born configurations, rescale by $\text{LO}/\text{LO}_{\text{HEFT}}$

3) $\text{NNLO}_{\text{FTapprox}}$

NNLO EFT correction rescaled for each multiplicity by:

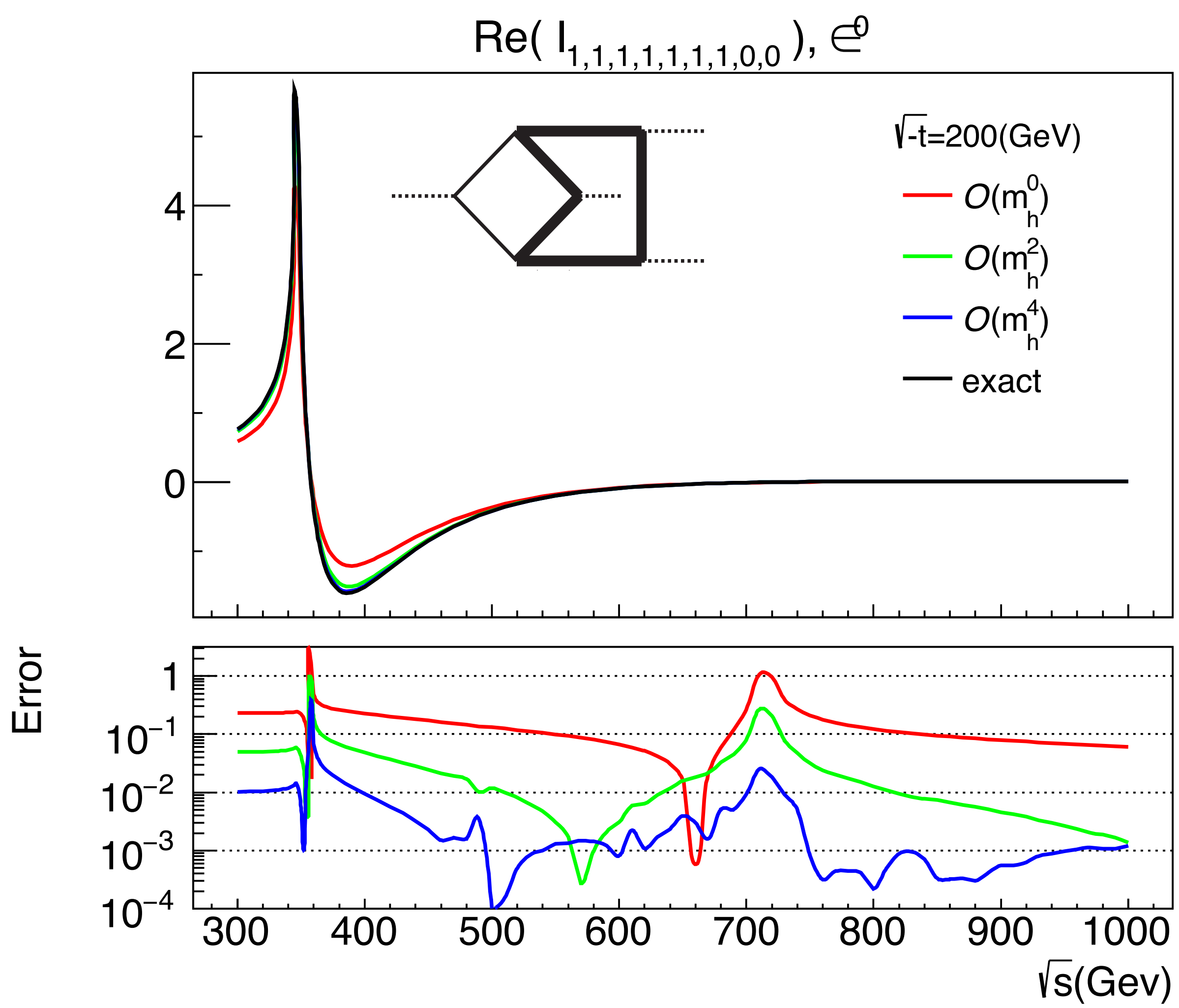
$$\mathcal{R}(ij \to HH + X) = \frac{\mathcal{A}^{\text{Born}}_{\text{Full}}(ij \to HH + X)}{\mathcal{A}^{(0)}_{\text{HEFT}}(ij \to HH + X)}$$

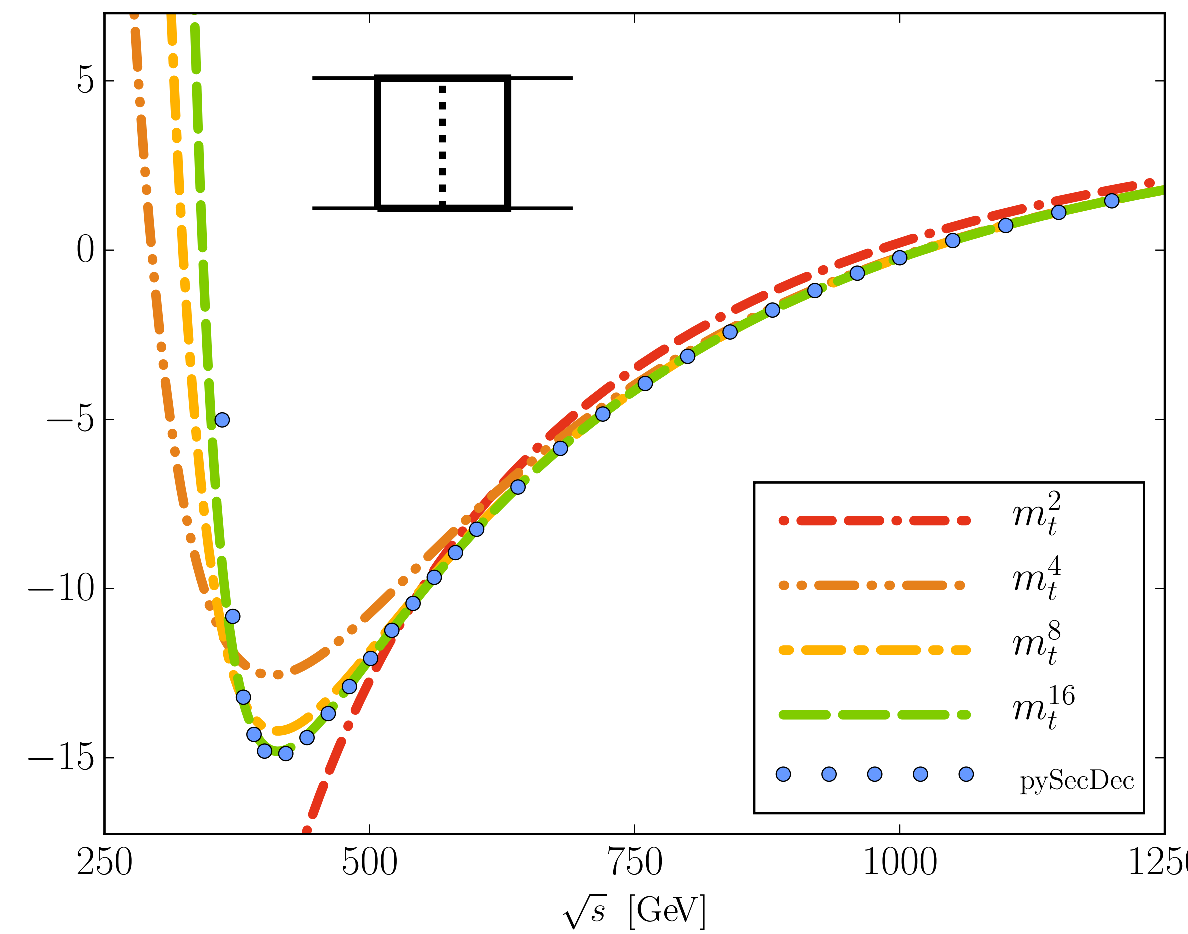
HH Production via Expansions

Expansion about small $m_H^2, m_T^2 \ll |s|, |t|$ can be applied to HH

Davies, Mishima, Steinhauser, Wellmann 18, 18;

Agreement between expanded integrals and numerical results (where expansion is valid)





Alternatively:

Expand only about small m_H^2

Larger range of validity, some integrals significantly more involved (Elliptic) Xu, Yang 18

HH Production via Expansions (II)

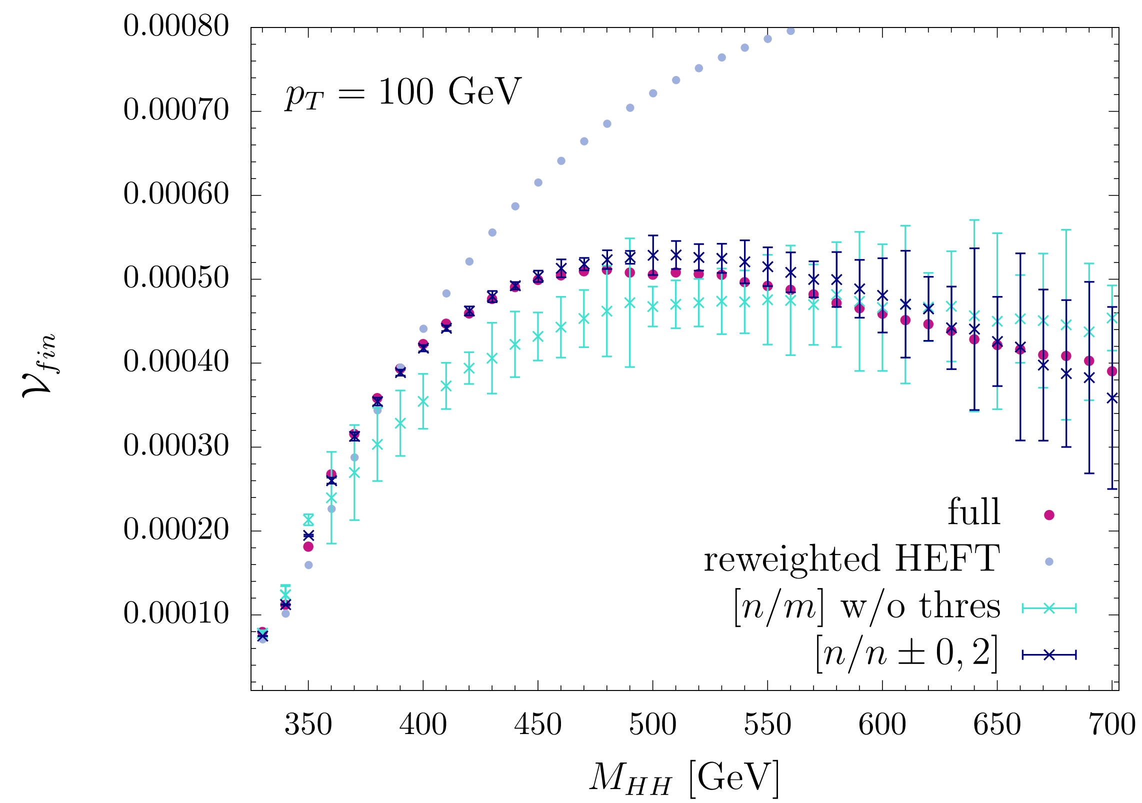
But, there is more than one way to skin a cat…

Produce a Padé approximation using:
Large- m_T expansion + threshold expansion

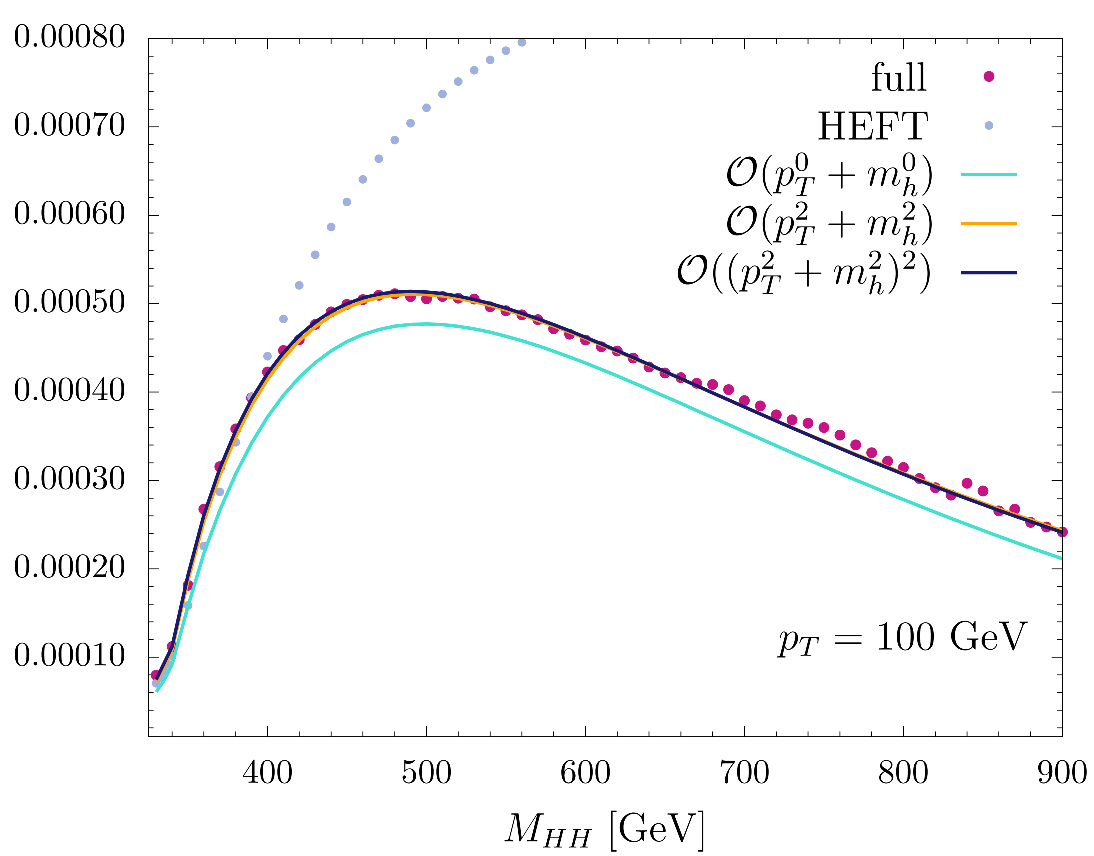
Incorporate non-analytic threshold corrections into approximation

Method applicable to more processes:

$$gg \to H^{(*)}, HZ, ZZ$$



Gröber, Maier, Rauh 17



Have: $p_T^2 + m_H^2 \leq \hat{s}/4$

Expand in: $p_T^2 + m_H^2$

Solve remaining dependence on $\hat{s}, m_T$

Invariant mass distribution agrees well with full (numerical) result up to ~900 GeV

Bonciani, Degrassi, Giardino, Gröber 18

Conclusion

Higgs+Jet & Di-Higgs Production at the LHC

- Computed at NLO with full top quark mass dependence
- 2-loop virtual amplitude calculated numerically

Numerical Multi-loop Calculations

- Discussed basis choice which improves numerical performance
- Described sector decomposition procedure and pySecDec
- Public release of QMC integration code with CUDA GPU support

Future

- Complete study of HJ (more distributions, grid of results, combination with parton shower and NNLO HTL)
- Attack more multi-scale 2→2 processes and refine our technique

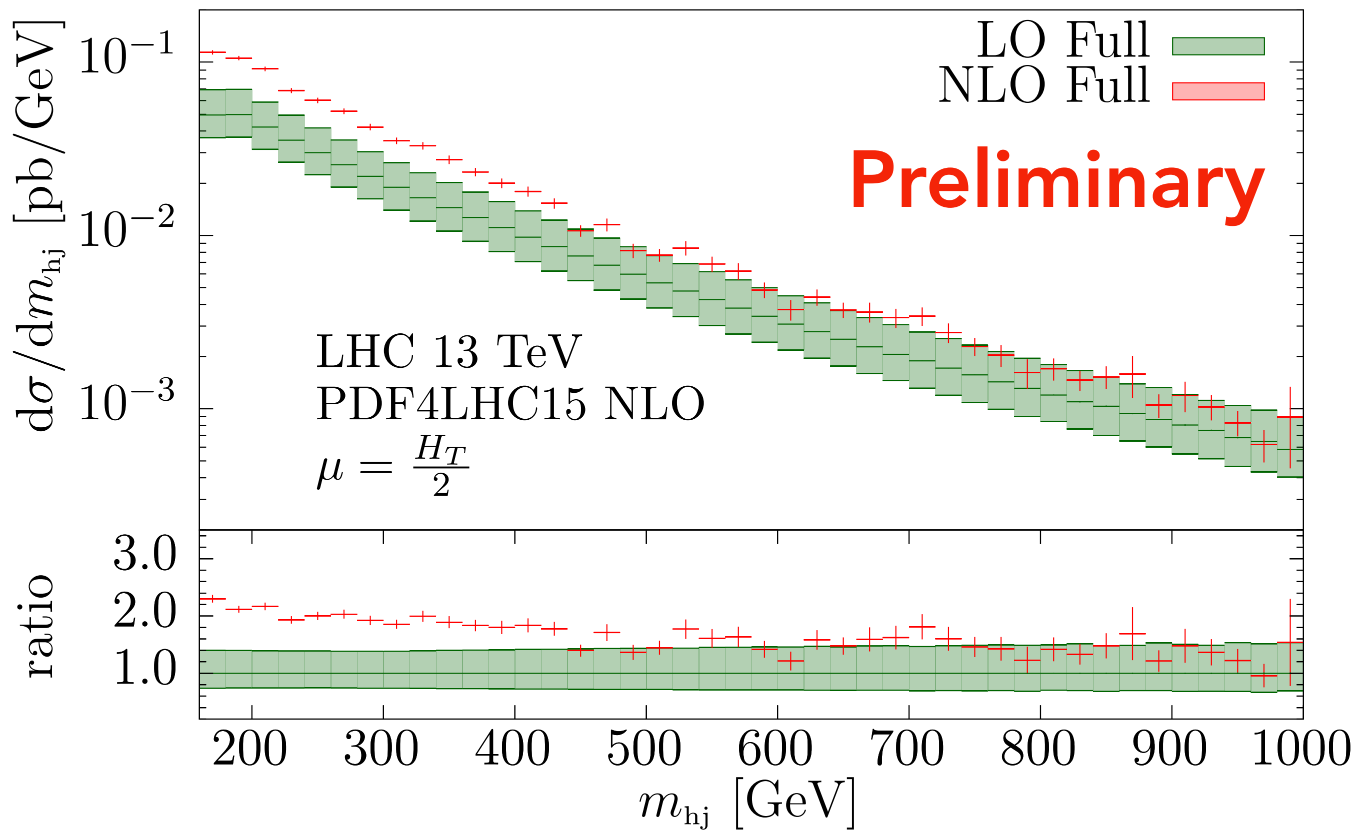
Thank you for listening!

Backup

HJ Ongoing Work

Starting to look at other distributions…

Basis change improves numerical stability of invariant mass distribution



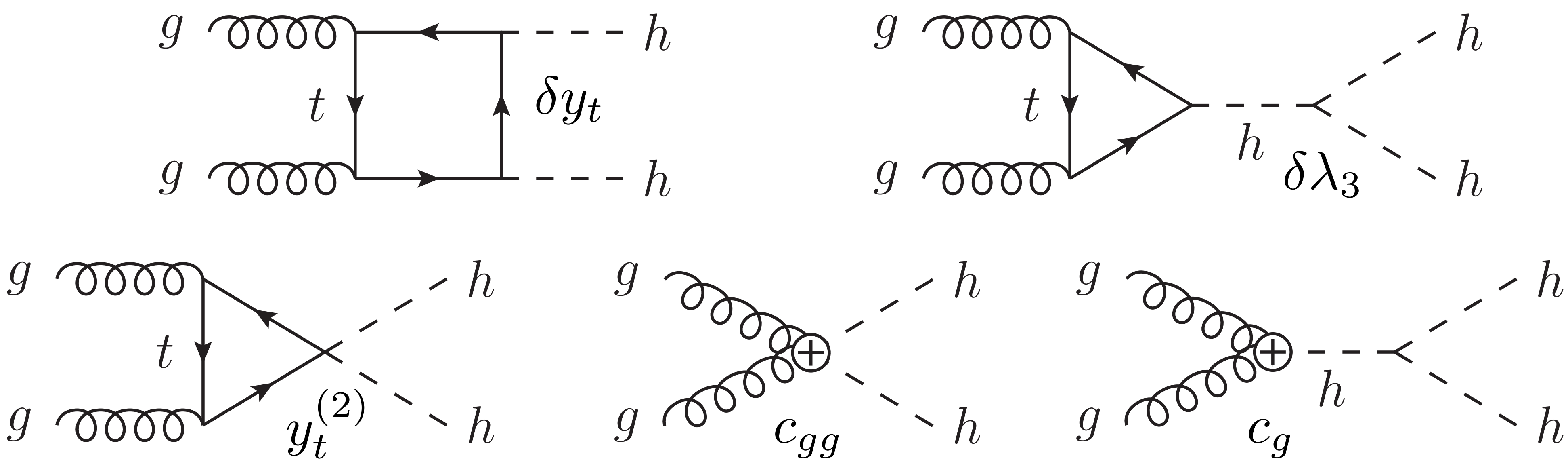
Note:

- Produced with fairly low statistics (817 points)
- No scale variations yet

BSM EFT

But: Just varying λ: one ``direction'' in EFT parameter space

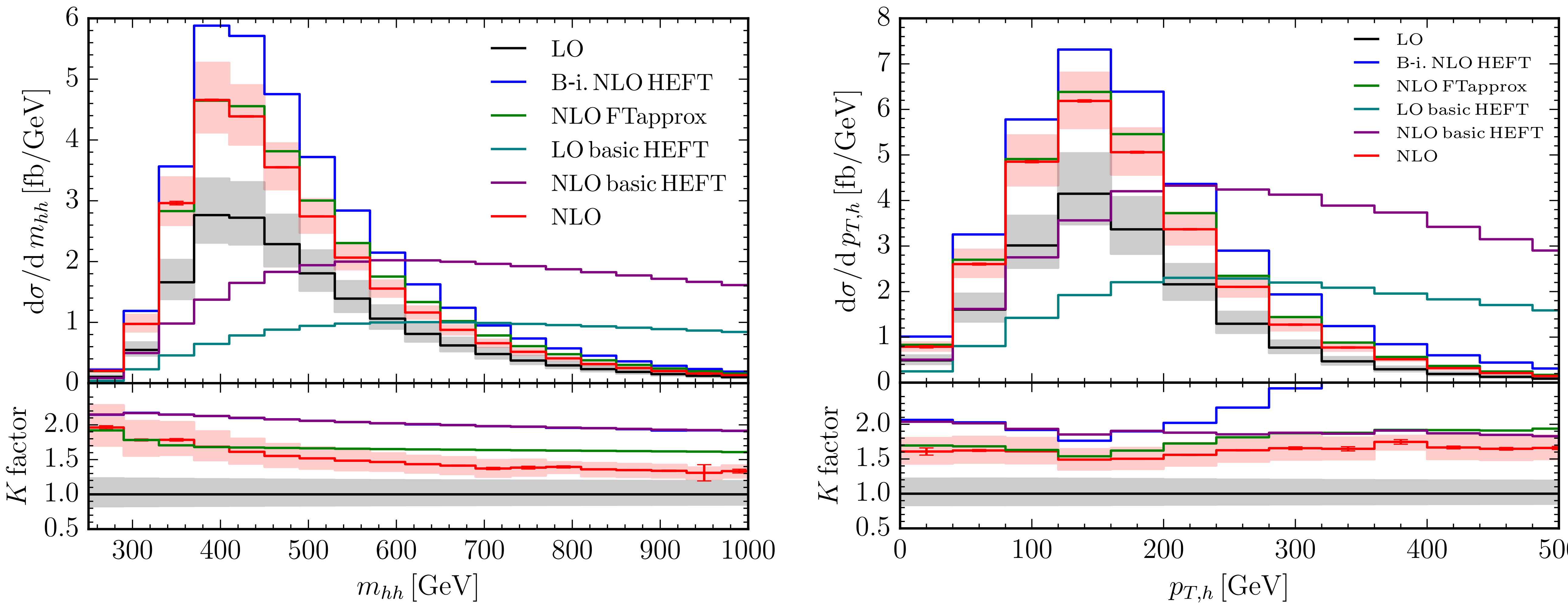
Parametrise **non-resonant** new physics with EFT (5 parameters):



Azatov, Contino, Panico, Son 15;
Buchalla, Cata, Celis, Krause 15;
(B.I. NLO HEFT) Gröber, Mühlleitner, Spira, Streicher 15;
(B.I. NNLO HEFT) de Florian, Fabre, Mazzitelli 17;
(Cluster analysis) Dall'Osso, Dorigo, Gottardo, Oliveira, Tosi, Goertz 15;
+ Carvalho, Manzano, Dorigo, Gouzevich 16;
Kim, Sakaki, Son 18;
(NLO) Buchalla, Capozi, Celis, Heinrich, Scyboz 18

← 12 representative ``clusters''

HH Results: 100 TeV



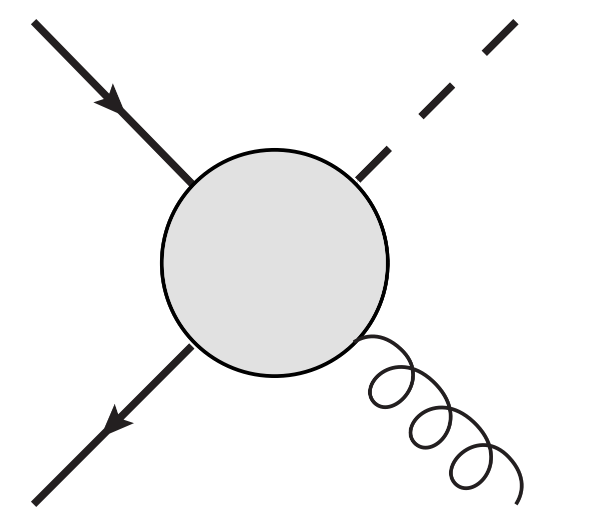
	σ_{LO} (fb)	σ_{NLO} (fb)
B.I. HEFT	–	$1511^{+16.0\%}_{-13.0\%}$
FTapprox	–	$1220^{+11.9\%}_{-10.7\%}$
Full Theory	$731.3^{+20.9\%}_{-15.9\%}$	$1149^{+10.8\%}_{-10.0\%}$

HEFT overestimates by 32%
FTap. overestimates by 6%

Difference between full theory and HEFT more pronounced

Form Factor Decomposition (HJ Quark)

We compute $q\bar{q}gH$ and obtain other channels by crossing



Form Factors (Contain integrals)

$$\mathcal{M}^{\mu}_{\beta\alpha} = F_1 T^{\mu}_{1,\beta\alpha} + F_2 T^{\mu}_{2,\beta\alpha}$$

Choose basis:

$$T^{\mu}_{1,\beta\alpha} = \left(\bar{v}_\beta(p_2)\not{p}_3 u_\alpha(p_1) p_1^\mu - \bar{v}_\beta(p_2)\gamma^\mu u_\alpha(p_1) p_1 \cdot p_3\right)$$
$$T^{\mu}_{2,\beta\alpha} = \left(\bar{v}_\beta(p_2)\not{p}_3 u_\alpha(p_1) p_2^\mu - \bar{v}_\beta(p_2)\gamma^\mu u_\alpha(p_1) p_2 \cdot p_3\right)$$

Build projectors P such that: $(P^1_{\alpha\beta})_\mu \mathcal{M}^{\mu}_{\beta\alpha} = F_1, \ (P^2_{\alpha\beta})_\mu \mathcal{M}^{\mu}_{\beta\alpha} = F_2$

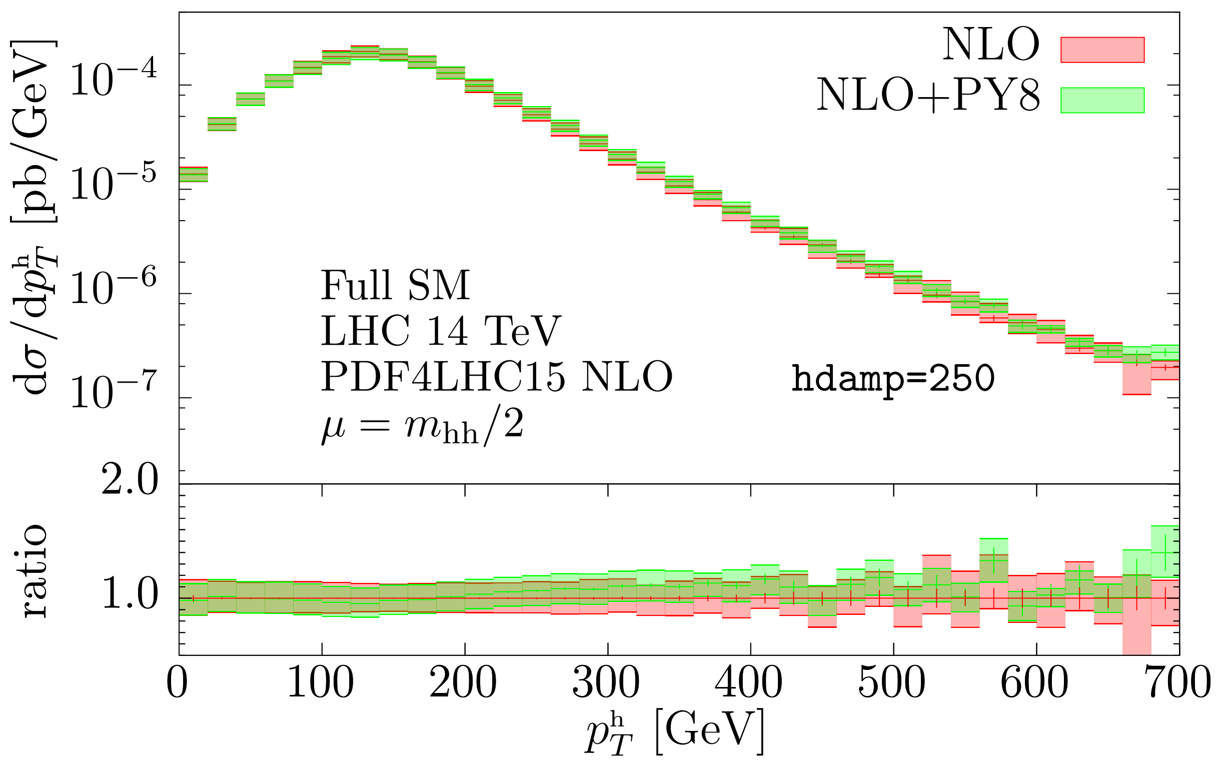
Gehrmann, Glover, Jaquier, Koukoutsakis 11

NLO Showered Results

No Higgs decay or hadronization included

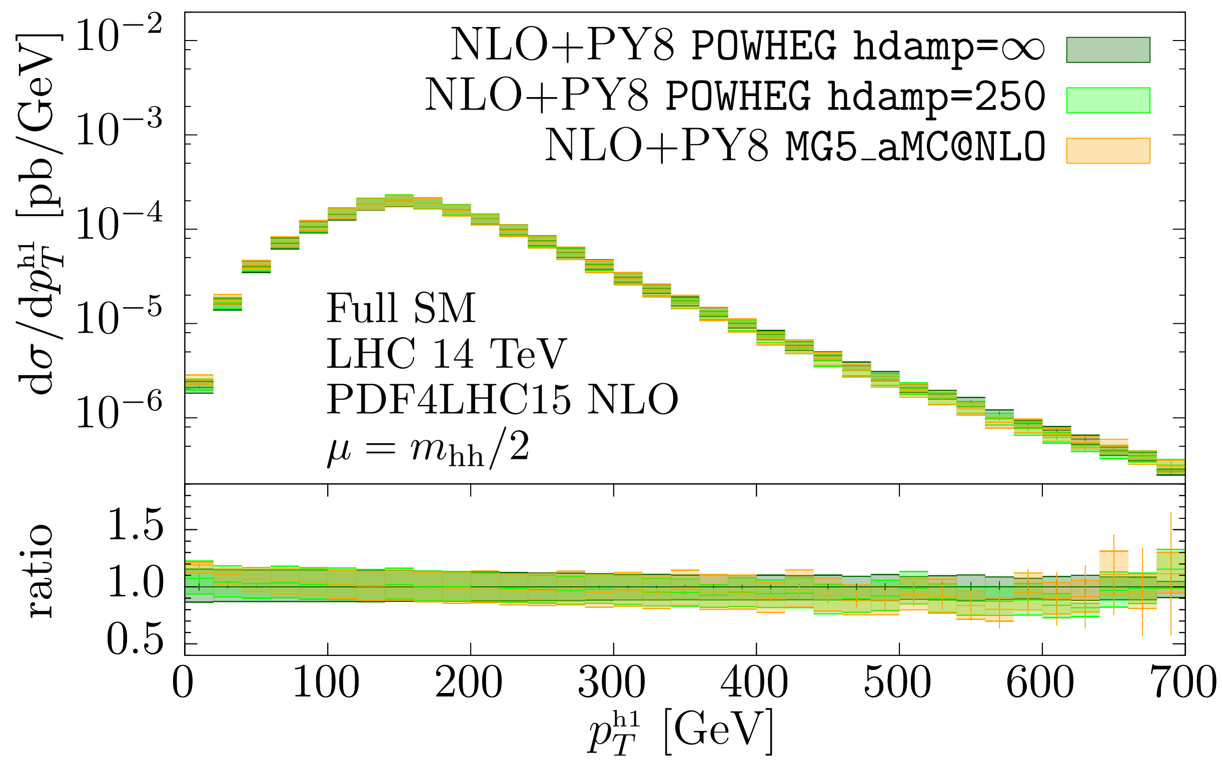
Assume $\Gamma_h = 0$ (decay can be attached e.g. in narrow width approx.)

POWHEG-BOX



Shower has moderate impact on NLO accurate observables

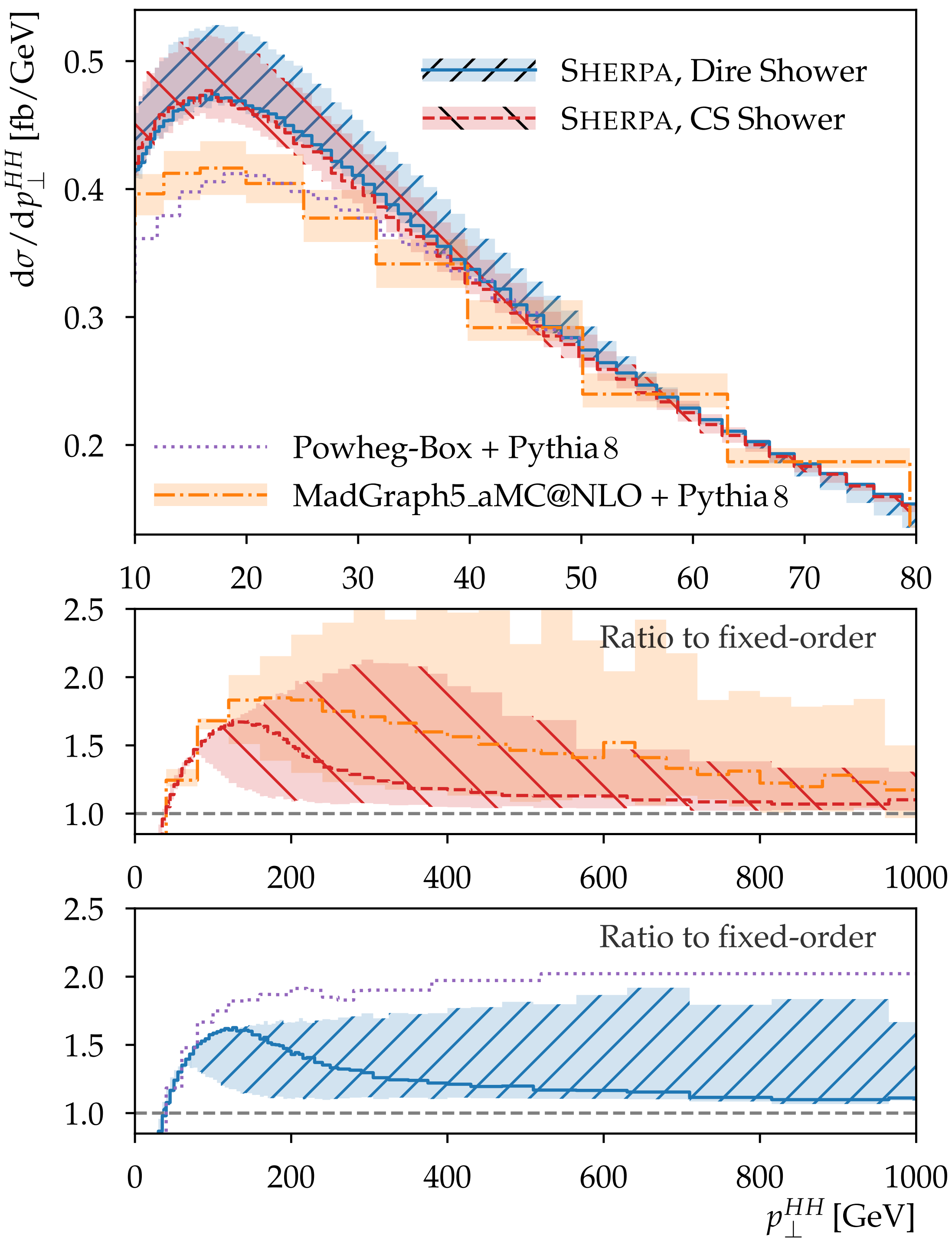
POWHEG-BOX/MG5_aMC@NLO



NLO accurate observables ($m_{hh}, p_T^h, p_T^{h_1}, p_T^{h_2}$) only moderately sensitive to matching procedure

Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17

NLO Showered Results (II)



μ_{PS} variation

SPJ, Kuttimalai 17

Matching/shower has significant impact on LO accurate observables

Can have large matching uncertainties

$$\langle \mathcal{O} \rangle = \int \left[\bar{B}(\phi_B) - B(\phi_B)\right] \frac{D(\phi_B, \phi_1)}{B(\phi_B)} \Theta(\mu_{\mathrm{PS}}^2 - t) \mathcal{O}(\phi_R)\, \mathrm{d}\phi_B\, \mathrm{d}\phi_1 + \int R(\phi_R) \mathcal{O}(\phi_R)\, \mathrm{d}\phi_R.$$

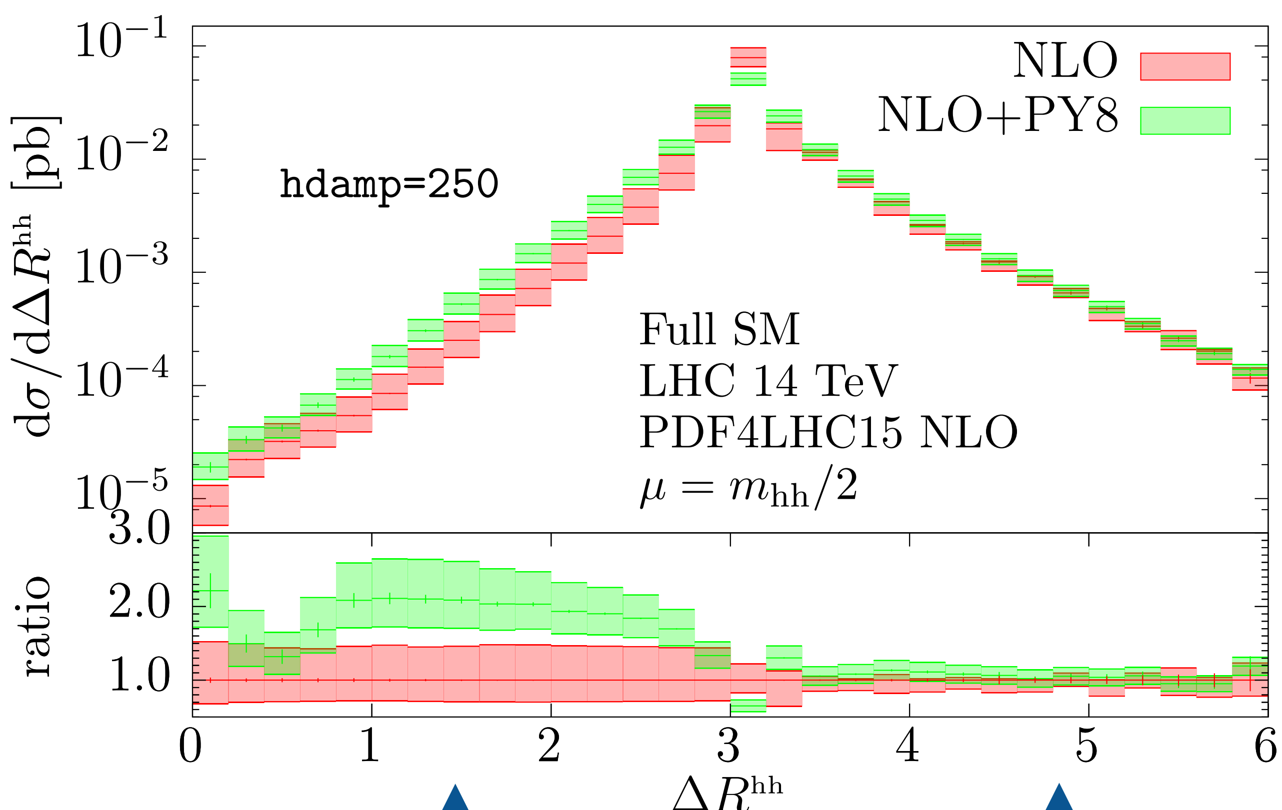
Cancellation spoiled if:

- Large NLO corrections ($\bar{B} - B$)
- Splitting kernels over Born (D/B) numerically large compared to real radiation
- Phase space accessible to the PS (depends on scale μ_{PS} and PS evolution variable t)

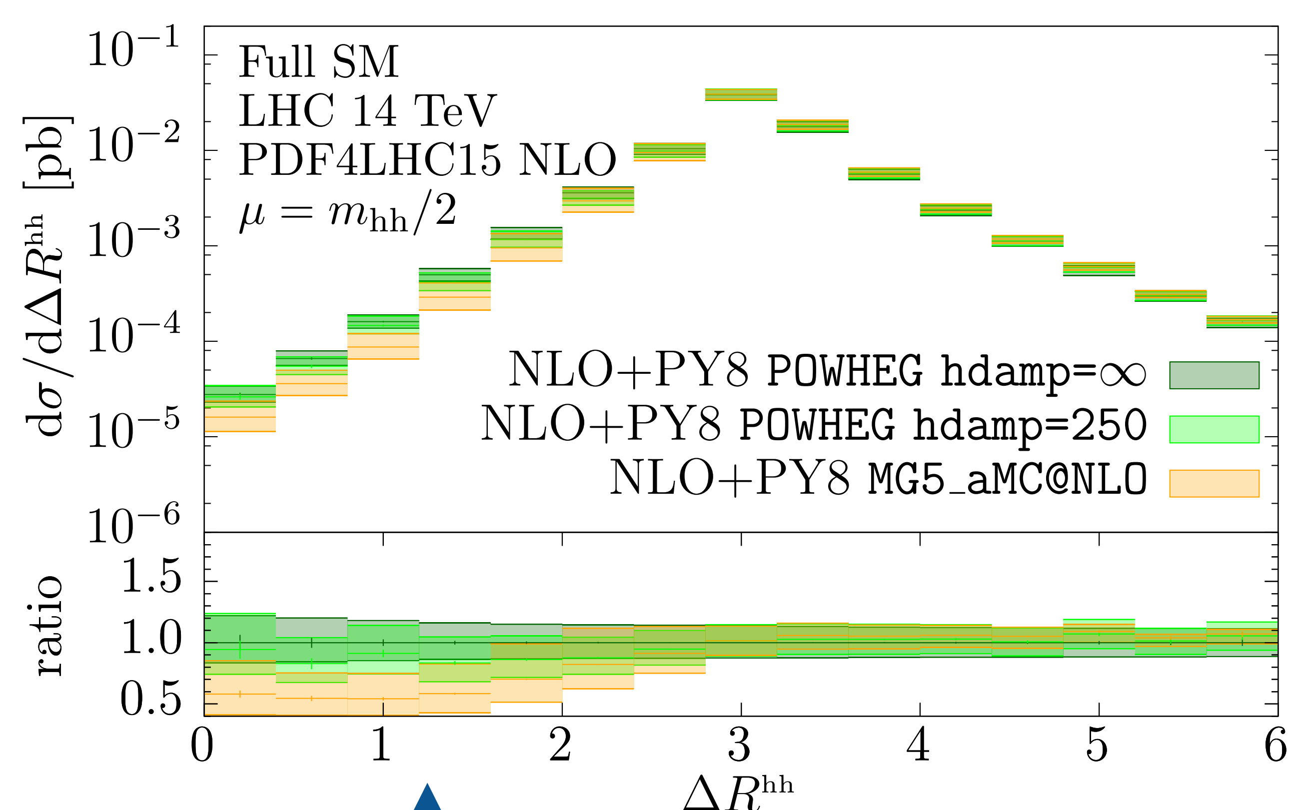
NLO Showered Results (III)

Radial separation: $\Delta R^{\mathrm{hh}} = \sqrt{(\eta_1 - \eta_2)^2 + (\Phi_1 - \Phi_2)^2}$

POWHEG



POWHEG/MG5_aMC@NLO



LO accurate

NLO accurate

Accordingly, matching scheme uncertainties larger for small radial separation

Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17