Heavy physics contributions to neutrinoless double beta decay from QCD

Nicolas Garron

University of Cambridge, HEP seminar, 19^{th} of October, 2018

CalLat (California Lattice)

red = postdoc and blue = grad student

- Jülich: Evan Berkowitz
- LBL/UCB: Davd Brantley, Chia Cheng (Jason) Chang, Thosrsten Kurth, Henry Monge-Camacho, André Walker-Loud
- NVIDIA: K Clark
- Liverpool: Nicolas Garron
- JLab: Balint Joó
- Rutgers: Chris Monahan
- North Carolina: Amy Nicholson
- City College of New York: Brian Tiburzi
- RIKEN/BNL: Enrico Rinaldi
- LLNL: Pavlos Vranas

Introduction

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- Observation of Neutrino oscillations, accumulation of evidences since the late 60's: solar ν, atmospheric ν, ν beam, ...

  2015 Nobel prize in physics: Kajita and McDonald

  $\Rightarrow$ Neutrinos have non-zero mass

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- Experimental searches for neutrinoless double β decay ($0\nu\beta\beta$)

  If measured $\rightarrow$ Majorana particle, probe of new physics, ...

  Huge experimental effort

Neutrinoless double beta decay

- β-decay

$$n \longrightarrow p + e^- + \bar{\nu}_e$$

- and a ν_e can be absorbed in the process

$$\nu_e + n \longrightarrow p + e^-$$

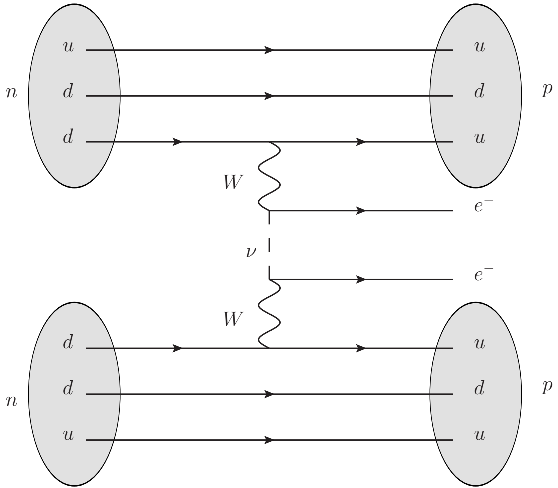
- so that if $\nu_e = \bar{\nu}_e$ it is possible to have

$$n + n \longrightarrow p + p + e^- + e^-$$

$\Rightarrow$ Neutrinoless double beta decay

Neutrinoless double beta decay

Neutrinoless double β decay: $n + n \longrightarrow p + p + e^- + e^-$



Yet to be measured (LFV)

Neutrinoless double beta decay

- $0\nu\beta\beta$ violates Lepton-number conservation $\Rightarrow$ New Physics
- Can be related to leptogensis and Matter-Antimatter asymmetry
- Can probe the absolute scale of neutrino mass (or of new physics)
- Related to dark matter ?
- Worldwide experimental effort

Neutrinoless double beta decay

- Completed experiments:
  - Gotthard TPC
  - Heidelberg-Moscow, ^{76}Ge detectors (1997–2001)
  - IGEX, ^{76}Ge detectors (1999–2002)[17]
  - NEMO, various isotopes using tracking calorimeters (2003–2011)
  - Cuoricino, ^{130}Te in ultracold TeO_2 crystals (2003–2008)[18]
- Experiments taking data as of November 2017:
  - COBRA, ^{116}Cd in room temperature CdZnTe crystals
  - CUORE, ^{130}Te in ultracold TeO_2 crystals
  - EXO, a ^{136}Xe and ^{134}Xe search
  - GERDA, a ^{76}Ge detector
  - KamLAND-Zen, a ^{136}Xe search. Data collection from 2011.[18]
  - MAJORANA, using high purity ^{76}Ge p-type point-contact detectors.[19]
  - XMASS using liquid Xe
- Proposed/future experiments:
  - CANDLES, ^{48}Ca in CaF_2, at Kamioka Observatory
  - MOON, developing ^{100}Mo detectors
  - AMoRE, ^{100}Mo enriched $CaMoO_4$ crystals at YangYang underground laboratory[20]
  - nEXO, using liquid ^{136}Xe in a time projection chamber [21]
  - LEGEND, Neutrinoless Double-beta Decay of ^{76}Ge.
  - LUMINEU, exploring ^{100}Mo enriched $ZnMoO_4$ crystals at LSM, France.
  - NEXT, a Xenon TPC. NEXT-DEMO ran and NEXT-100 will run in 2016.
  - SNO+, a liquid scintillator, will study ^{130}Te
  - SuperNEMO, a NEMO upgrade, will study ^{82}Se
  - TIN.TIN, a ^{124}Sn detector at INO

(source: Wikipedia)

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- Relating possible experimental signatures to New-Physics model requires the knowledge of QCD contributions

Neutrinoless double beta decay

Computing the full process is very ambitious

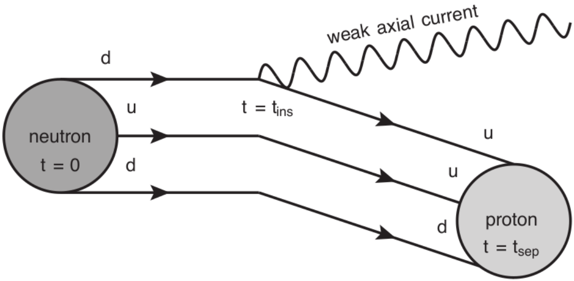
- Different scales, different interactions
- Multi-particles in initial and final states
- Nucleon $\Rightarrow$ Signal-to-noise problem

Very hard task in Lattice QCD

g_A

The axial coupling of the nucleon

Nuclear β decay: $n \longrightarrow p + e^- + \bar{\nu}_e$



we find

$$g_A^{QCD} = 1.271(13)$$

vs experiment

$$g_A^{PDG} = 1.2723(23)$$

[C Chang, A Nicholson, E Rinaldi, E Berkowitz, NG, D Brantley, H Monge-Camacho, C Monahan, C Bouchard, M Clark, B Joó, T Kurth, K Orginos, P Vranas, A Walker-Loud]

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$0\nu\beta\beta$ and EFT

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  and its proportional to the mass of the light neutrino

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- Standard seesaw $m_l \sim M_D^2/M_R \ll m_h \sim M_R$

$0\nu\beta\beta$ and EFT

Consider "heavy" particles contributions, integrate out heavy d.o.f.

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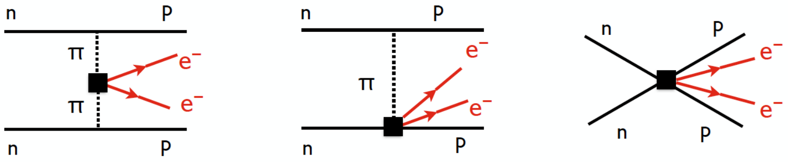
EFT framework, see e.g. [Prézeau, Ramsey-Musolf, Vogel '03], the LO contributions are

- $\pi^- \longrightarrow \pi^+ + e^- + e^-$
- $n \longrightarrow p + \pi^+ + e^- + e^-$
- $n + n \longrightarrow p + p + e^- + e^-$



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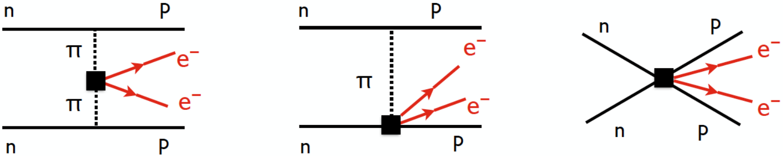

In this work we focus on the $\pi^- \longrightarrow \pi^+$ matrix elements

$0\nu\beta\beta$ and EFT

- On the lattice, compute the Matrix elements of $\pi^- \longrightarrow \pi^+$ transitions
- Extract the LEC through Chiral fits
- Use the LEC in the EFT framework to estimate a physical amplitude

Lattice Computation of $\pi^- \to \pi^+$ matrix elements

4-quark operators

We only consider light valence quarks $q = u, d$

the operators of interest are

$$\mathcal{O}_{1+}^{++} = \left(\bar{q}_L \tau^+ \gamma^\mu q_L\right) \left[\bar{q}_R \tau^+ \gamma_\mu q_R\right]$$

$$\mathcal{O}_{2+}^{++} = \left(\bar{q}_R \tau^+ q_L\right) \left[\bar{q}_R \tau^+ q_L\right] + \left(\bar{q}_L \tau^+ q_R\right) \left[\bar{q}_L \tau^+ q_R\right]$$

$$\mathcal{O}_{3+}^{++} = \left(\bar{q}_L \tau^+ \gamma^\mu q_L\right) \left[\bar{q}_L \tau^+ \gamma_\mu q_L\right] + \left(\bar{q}_R \tau^+ \gamma^\mu q_R\right) \left[\bar{q}_R \tau^+ \gamma_\mu q_R\right]$$

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\end{aligned}$$

and the colour partner

$$\begin{aligned}
\mathcal{O}_{1+}^{\prime ++} &= \left(\bar{q}_L \tau^+ \gamma^\mu q_L\right] \left[\bar{q}_R \tau^+ \gamma_\mu q_R\right) \\
\mathcal{O}_{2+}^{\prime ++} &= \left(\bar{q}_L \tau^+ q_L\right] \left[\bar{q}_L \tau^+ q_L\right) + \left(\bar{q}_R \tau^+ q_R\right] \left[\bar{q}_R \tau^+ q_R\right)
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where $()\,[] \equiv$ color unmix and $(]\,[) \equiv$ color unmix

4-quark operators (II)

In a slightly more *human readable* way

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The colour unmix are

$$\mathcal{O}_{3+}^{++} \sim \gamma_L^\mu \times \gamma_L^\mu + \gamma_R^\mu \times \gamma_R^\mu \longrightarrow VV + AA$$

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$$\begin{aligned}
\mathcal{O}_{1+}^{\prime ++} &\longrightarrow (VV - AA)_{mix} \sim (SS - PP)_{unmix}\\
\mathcal{O}_{2+}^{\prime ++} &\longrightarrow (SS + PP)_{mix} \sim (SS + PP)_{unmix} + c(TT)_{unmix}
\end{aligned}$$

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The computation goes along the lines of $\Delta F = 2$ ME:

- Extract the bare ME by fitting 3p and 2p functions or ratios
- Non-Perturbative Renormalisation
- Global Fit, extrapolation to physical pion mass and continuum limit

Lattice QCD in a nutshell

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- Lattice QCD is a discretised version of Euclidean QCD
- Well-defined regularisation of the theory
- Gauge invariant (Wilson) at finite lattice spacing
- Continuum Euclidean QCD is recovered in the lmit $a \to 0$

$$\langle O \rangle_{continuum} = \lim_{a \to 0} \lim_{V \to \infty} \langle O \rangle_{latt}$$

Allows for non-perturbative and first-principle determinations of QCD observables

Lattice QCD in a nutshell

Various steps of a Lattice computation (schematically)

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- Compute Wick contractions $\leftrightarrow$ bare Green functions
- Determine Z factors (if needed) $\leftrightarrow$ renormalised Green functions
- Continuum & physical pion mass extrapolations $\leftrightarrow$ physical observables

Remarks

Different discretizations of the Dirac operators are possible: Wilson, staggered, Twisted-mass, etc.

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$\Rightarrow$ choose the discretization adapted to the situation you want to describe

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In particular chiral symmetry is notoriously difficult to maintain

We consider here Domain-Wall fermions, a type of discretisation which respects chiral and flavour symmetry almost exactly.

The price to pay is a high numerical cost

This computation

The setup

The main features of our computation are:

- Mixed-action: Möbius Domain-Wall on $N_f = 2 + 1 + 1$ HISQ configurations
- 3 lattice spacings, pion mass down to the physical value

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- Chiral-flavour symmetry maintained (in the valence sector)
- Lattice artefact of order $\mathcal{O}(a^2)$
- Good control over the chiral behaviour, continuum limit, finite volume effects
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In addition we perform the renormalisation non-perturbatively
Only perturbative errors come from the conversion to $\overline{\mathrm{MS}}$

HISQ gauge configuration parameters						
abbr.	$N_{\rm cfg}$	volume	$\sim a$ [fm]	m_l/m_s	$\sim m_{\pi_5}$ [MeV]	$\sim m_{\pi_5} L$
a15m400	1000	$16^3 \times 48$	0.15	0.334	400	4.8
a15m350	1000	$16^3 \times 48$	0.15	0.255	350	4.2
a15m310	1960	$16^3 \times 48$	0.15	0.2	310	3.8
a15m220	1000	$24^3 \times 48$	0.15	0.1	220	4.0
a15m130	1000	$32^3 \times 48$	0.15	0.036	130	3.2
a12m400	1000	$24^3 \times 64$	0.12	0.334	400	5.8
a12m350	1000	$24^3 \times 64$	0.12	0.255	350	5.1
a12m310	1053	$24^3 \times 64$	0.12	0.2	310	4.5
a12m220S	1000	$24^3 \times 64$	0.12	0.1	220	3.2
a12m220	1000	$32^3 \times 64$	0.12	0.1	220	4.3
a12m220L	1000	$40^3 \times 64$	0.12	0.1	220	5.4
a12m130	1000	$48^3 \times 64$	0.12	0.036	130	3.9
a09m400	1201	$32^3 \times 64$	0.09	0.335	400	5.8
a09m350	1201	$32^3 \times 64$	0.09	0.255	350	5.1
a09m310	784	$32^3 \times 96$	0.09	0.2	310	4.5
a09m220	1001	$48^3 \times 96$	0.09	0.1	220	4.7

The setup (II)

For this analysis we only consider

	$m_\pi \sim 310$ MeV		$m_\pi \sim 220$ MeV		$m_\pi \sim 130$ MeV	
a(fm)	V	$m_\pi L$	V	$m_\pi L$	V	$m_\pi L$
0.15	$16^3 \times 48$	3.78	$24^3 \times 48$	3.99		
0.12			$24^3 \times 64$	3.22		
0.12	$24^3 \times 64$	4.54	$32^3 \times 64$	4.29	$48^3 \times 64$	3.91
0.12			$40^3 \times 64$	5.36		
0.09	$32^3 \times 96$	4.50	$48^3 \times 96$	4.73		

Bare results

Define usual 2p and 3p functions

$$C_\pi(t) = \sum_{\mathbf{x}} \sum_{\alpha} \langle \alpha | \Pi^+(t, \mathbf{x}) \Pi^-(0, \mathbf{0}) | \alpha \rangle$$
$$= \sum_n \frac{|z_n^\pi|^2}{2E_n^\pi} \left(e^{-E_n^\pi t} + e^{-E_n^\pi (T-t)} \right) + \cdots$$

where $z_n^\pi = \langle \Omega | \Pi^+ | n \rangle$, $\Omega = vaccum$ and

$$C_i^{\mathrm{3pt}}(t_f, t_i) = \sum_{\mathbf{x}, \mathbf{y}, \alpha} \langle \alpha | \Pi^+(t_f, \mathbf{x}) \mathcal{O}_i(0, \mathbf{0}) \Pi^+(t_i, \mathbf{y}) | \alpha \rangle$$

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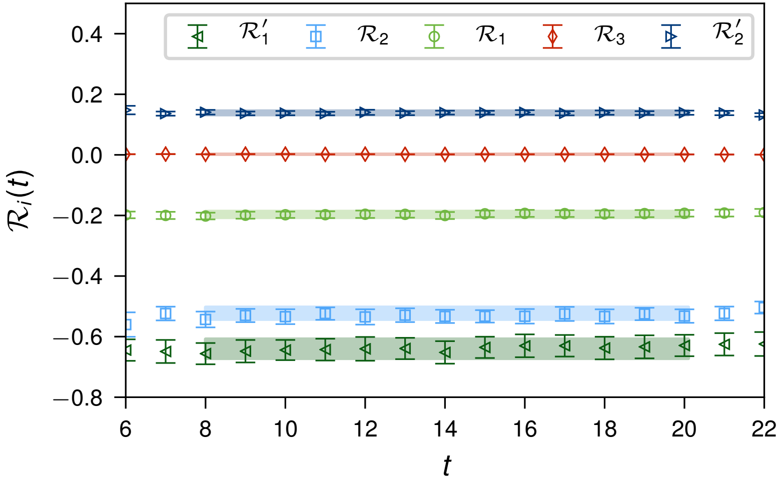
where $z_n^\pi = \langle \Omega | \Pi^+ | n \rangle$, $\Omega = vaccum$ and

$$C_i^{3\mathrm{pt}}(t_f, t_i) = \sum_{\mathbf{x},\mathbf{y},\alpha} \langle \alpha | \Pi^+(t_f, \mathbf{x}) \mathcal{O}_i(0, \mathbf{0}) \Pi^+(t_i, \mathbf{y}) | \alpha \rangle$$

for example fit ratio such as

$$\mathcal{R}_i(t) \equiv C_i^{3\mathrm{pt}}(t, T-t) / \left(C_\pi(t) C_\pi(T-t) \right)$$
$$\longrightarrow \frac{a^4 \langle \pi | \mathcal{O}_{i+}^{++} | \pi \rangle}{(a^2 z_0^\pi)^2} + \ldots$$

Bare results



Example of results for $a \simeq 0.12$ fm , near physical pion mass ensemble

Non Perturbative Renormalisation (NPR)

A few words on the renormalisation

First step: remove the divergences

Non-perturbative Renormalisation à la Rome-Southampton [Martinelli et al '95]

$$Q_i^{lat}(a) \to Q_i^{MOM}(\mu, a) = Z(\mu, a)_{ij} Q_j^{lat}(a)$$

and take the continuum limit

$$Q_i^{MOM}(\mu, 0) = \lim_{a^2 \to 0} Q_i^{MOM}(\mu, a)$$

Second step: Matching to $\overline{\mathrm{MS}}$, done in perturbation theory [Sturm et al., Lehner and Sturm, Gorbahn and Jäger, Gracey, ...]

$$Q_i^{MOM}(\mu, 0) \to Q_i^{\overline{\mathrm{MS}}}(\mu) = (1 + r_1 \alpha_S(\mu) + r_2 \alpha_S(\mu)^2 + \ldots)_{ij} Q_j^{MOM}(\mu, 0)$$

The Rome Southampon method [Martinelli et al '95]

Consider a quark bilinear $O_\Gamma = \bar{\psi}_2 \Gamma \psi_1$

Define

$$\Pi(x_2, x_1) = \langle \psi_2(x_2) O_\Gamma(0) \bar{\psi}_1(x_1) \rangle = \langle S_2(x_2, 0) \Gamma S_1(0, x_1) \rangle$$

In Fourier space $S(p) = \sum_x S(x, 0) e^{ip.x}$

$$\Pi(p_2, p_1) = \langle S_2(p_2) \Gamma S_1(p_1)^\dagger) \rangle$$

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Amputated Green function

$$\Lambda(p_2, p_1) = \langle S_2(p_2)^{-1} \rangle \langle S_2(p_2) \Gamma S_1(p_1)^\dagger) \rangle \langle (S_2(p_1)^{\dagger\,-1}) \rangle$$

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$$\Pi(p_2, p_1) = \langle S_2(p_2) \Gamma S_1(p_1)^\dagger) \rangle$$

Amputated Green function

$$\Lambda(p_2, p_1) = \langle S_2(p_2)^{-1} \rangle \langle S_2(p_2) \Gamma S_1(p_1)^\dagger) \rangle \langle (S_2(p_1)^{\dagger\,-1}) \rangle$$

Rome Southampton original scheme (RI-MOM), $p_1 = p_2 = p$ and $\mu = \sqrt{p^2}$

$$Z(\mu, a) \times \lim_{m \to 0} \mathrm{Tr}(\Gamma \Lambda(p, p))_{\mu^2 = p^2} = \mathrm{Tree}$$

The Rome Southampon method [Martinelli et al '95]

Remarks

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- Can use $\not{q}$ as projector
- In principle the results should agree after conversion to $\overline{\text{MS}}$, and extrapolation to the continuum limit

Renormalisation basis of the $\Delta F = 2$ operators

As for BSM neutral meson mixing one needs to renormalise 5 operators ,

$$(27,1) \qquad O_1^{\Delta S=2} = \gamma_\mu \times \gamma_\mu + \gamma_\mu\gamma_5 \times \gamma_\mu\gamma_5$$

$$(8,8) \qquad \left\{ \begin{array}{lcl} O_2^{\Delta s=2} & = & \gamma_\mu \times \gamma_\mu - \gamma_\mu\gamma_5 \times \gamma_\mu\gamma_5 \\ O_3^{\Delta s=2} & = & 1 \times 1 - \gamma_5 \times \gamma_5 \end{array} \right.$$

$$(6,\bar{6}) \qquad \left\{ \begin{array}{lcl} O_4^{\Delta s=2} & = & 1 \times 1 + \gamma_5 \times \gamma_5 \\ O_5^{\Delta s=2} & = & \sigma_{\mu\nu} \times \sigma_{\mu\nu} \end{array} \right.$$

So the renormalisation matrix has the form

$$\mathcal{Z}^{\Delta S=2} = \begin{pmatrix} \mathcal{Z}_{11} & & & & \\ & \mathcal{Z}_{22} & \mathcal{Z}_{23} & & \\ & \mathcal{Z}_{32} & \mathcal{Z}_{33} & & \\ & & & \mathcal{Z}_{44} & \mathcal{Z}_{45} \\ & & & \mathcal{Z}_{54} & \mathcal{Z}_{55} \end{pmatrix}$$

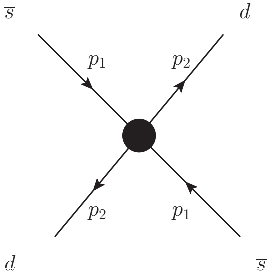
More details on NPR

- Setup is the similar to RBC-UKQCD

  In particular we follow [Arthur & Boyle '10]

- We implement momentum sources [Gockeler et al '98] to achieve high stat. accuracy

- Non exceptional kinematic with symmetric point $p_1^2 = p_2^2 = (p_2 - p_1)^2$



  to suppress IR contaminations [Sturm et al', RBC-UKQCD '09 '10]

Choice of SMOM scheme

- Orientation of the momenta kept fixed

$$p_1 = \frac{2\pi}{L}[n\,,0\,,n\,,0] \qquad p_2 = \frac{2\pi}{L}[0\,,n\,,n\,,0]$$

$\Rightarrow$ Well defined continuum limit

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- We chose γ_μ projectors, for example

$$P^{(\gamma_\mu)} \quad \leftrightarrow \quad \gamma_\mu \times \gamma_\mu + \gamma_\mu\gamma_5 \times \gamma_\mu\gamma_5$$

$\Rightarrow$ Z factor of a four quark operator O in the scheme (γ_μ, γ_μ) defined by

$$\lim_{m\to 0} \frac{Z_O^{(\gamma_\mu,\gamma_\mu)}}{Z_V^2} \left. \frac{P^{(\gamma_\mu)}\{\Lambda_O\}}{\left(P^{(\gamma_\mu)}\{\Lambda_V\}\right)^2} \right|_{\mu^2=p^2} = Tree$$

- Note that this defines an off-shell massless scheme

Step-scaling

- Rome-Southampton method requires a *windows*

$$\Lambda_{QCD}^2 \ll \mu^2 \ll (\pi/a)^2$$

- And our lattice spacings are $a^{-1} \sim 2.2, 1.7, 1.3 GeV$

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- we follow [Arthur & Boyle '10] and [Arthur, Boyle, NG, Kelly, Lytle '11] and define

$$\sigma(\mu_2, \mu_1) = \lim_{a^2 \to 0} \lim_{m \to 0} \left[(P\Lambda(\mu_2, a))^{-1} P\Lambda(\mu_1, a) \right] = \lim_{a^2 \to 0} Z(\mu_2, a) Z(\mu_1, a)^{-1}$$

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- We use 3 lattice spacings to compute $\sigma(2\ \text{GeV}, 1.5\ \text{GeV})$ but only the two finest to compute $\sigma(3\ \text{GeV}, 2\ \text{GeV})$ and get

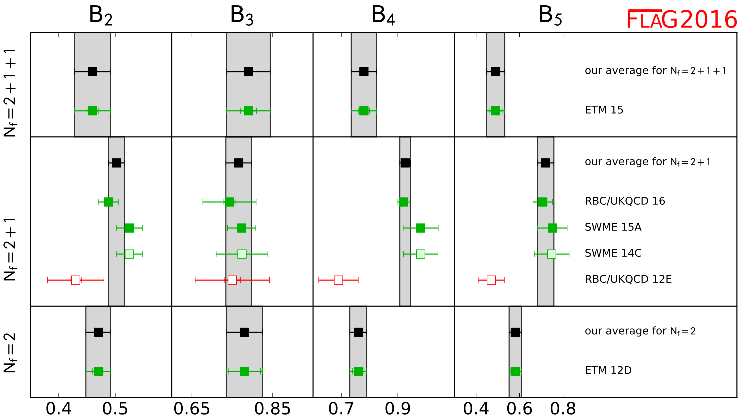
$$Z(3\ \text{GeV}, a) = \sigma(3\ \text{GeV}, 2\ \text{GeV})\, \sigma(2\ \text{GeV}, 1.5\ \text{GeV})\, Z(1.5\ \text{GeV}, a)$$

Intermezzo: the importance of SMOM schemes

based on RBC-UKQCD 2010-now

. . . [NG Hudspith Lytle'16] , [Boyle NG Hudspith Lehner Lytle '17] [. . . Kettle, Khamseh, Tsang 17-18]

BSM kaon mixing - Results



Pole subtraction

- The Green functions might suffer from IR poles, $\sim 1/p^2$, or $\sim 1/m_\pi^2$ which can pollute the signal
- In principle these poles are suppressed at high μ but they appear to be quite important at $\mu \sim 3$ GeV for some quantities which allow for pion exchanges
- The traditional way is to "subtract " these contamination by hand

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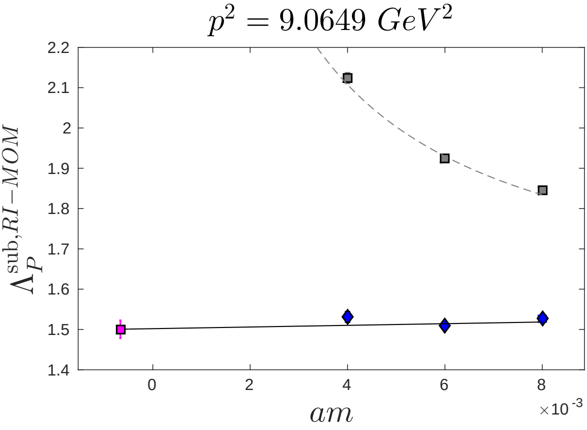
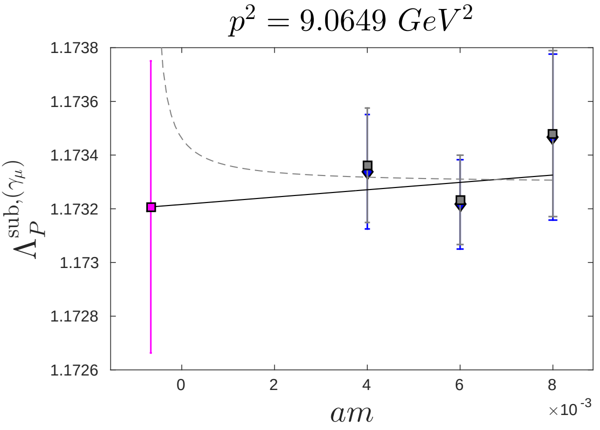
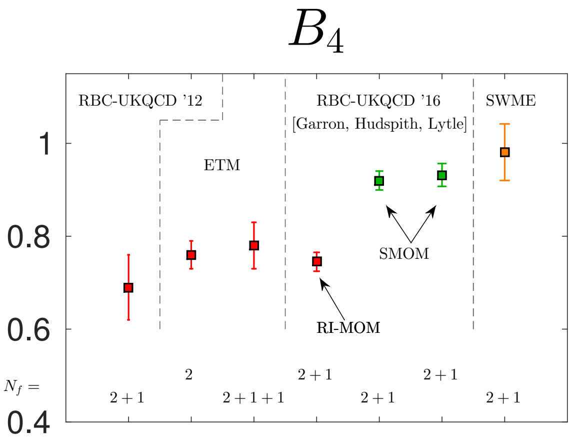
- The traditional way is to "subtract " these contamination by hand
- However these contaminations are highly suppressed in a SMOM scheme, with non-exceptional kinematics
- We argue that this pion pole subtractions is not-well under control and that schemes with exceptional kinematics should be discarded

Pole subtraction

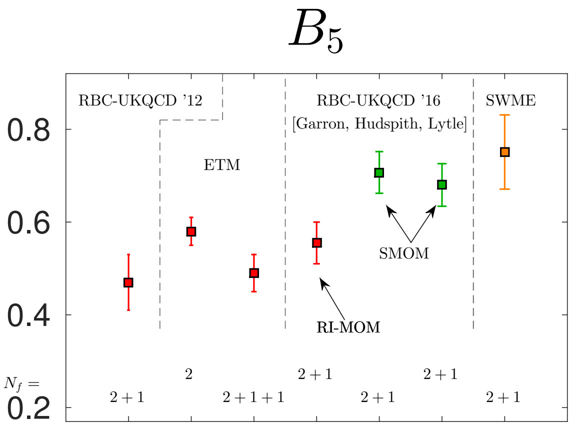



$p^2 = 9.0649\ GeV^2$
1.1738
1.1736
1.1734
1.1732
1.173
1.1728
1.1726
$\Lambda_P^{\mathrm{sub},(\gamma_\mu)}$
0
2
4
6
8
$\times 10^{-3}$
am

BSM kaon mixing - Results



BSM kaon mixing - Results



Better MOM schemes ?

More MOM schemes

Renormalisation scale is μ, given by the choice of kinematics

- Original RI-MOM scheme

$$p_1 = p_2 \text{ and } \mu^2 \equiv p_1^2 = p_2^2$$

But this lead to "exceptional kinematics' and bad IR poles

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Much better IR behaviour [Sturm et al., Lehner and Sturm, Gorbahn and Jäger, Gracey, ...]

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Much better IR behaviour [Sturm et al., Lehner and Sturm, Gorbahn and Jäger, Gracey, ...]

- We are now studying a generalisation (see also [Bell and Gracey])

$$p_1 \neq p_2 \text{ and } \mu^2 \equiv p_1^2 = p_2^2, \quad (p_1 - p_2)^2 = \omega\mu^2 \text{ where } \omega \in [0, 4]$$

Note that $\omega = 0 \leftrightarrow RI - MOM$ and $\omega = 1 \leftrightarrow RI - SMOM$

In collaboration with [...,Cahill, Gorbahn, Gracey, Perlt , Rakow, ...]

Back to $0\nu\beta\beta$: Physical results

Chiral extrapolations

With

$$\Lambda_\chi = 4\pi F_\pi\,, \qquad\qquad \epsilon_\pi = \frac{m_\pi}{\Lambda_\chi}\,,$$

we find in the continuum at NLO (β_i and c_i are free parameters)

$$O_1 = \frac{\beta_1 \Lambda_\chi^4}{(4\pi)^2}\left[1 + \frac{7}{3}\epsilon_\pi^2 \ln(\epsilon_\pi^2) + c_1\epsilon_\pi^2\right]$$
$$O_2 = \frac{\beta_2 \Lambda_\chi^4}{(4\pi)^2}\left[1 + \frac{7}{3}\epsilon_\pi^2 \ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2\right]$$
$$\frac{O_3}{\epsilon_\pi^2} = \frac{\beta_3 \Lambda_\chi^4}{(4\pi)^2}\left[1 + \frac{4}{3}\epsilon_\pi^2 \ln(\epsilon_\pi^2) + c_3\epsilon_\pi^2\right]$$

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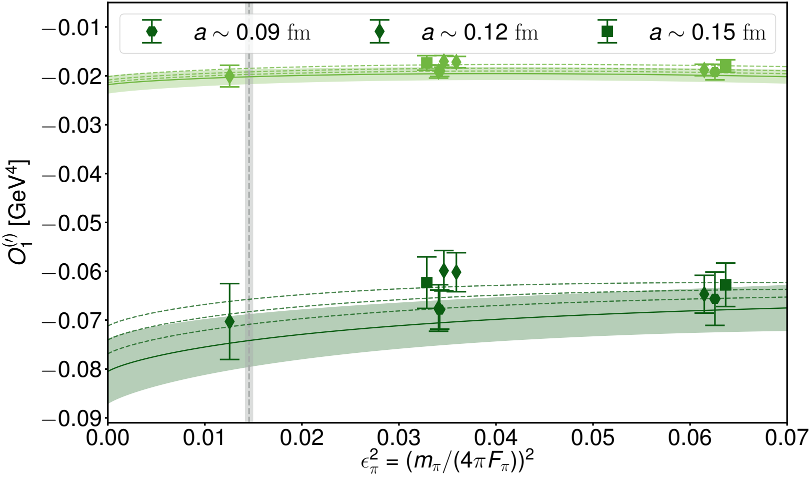
\end{aligned}$$

In practice, these expressions are modified to incorporate a^2, mixed-action effects and finite volume effects

Extrapolations to the physical point



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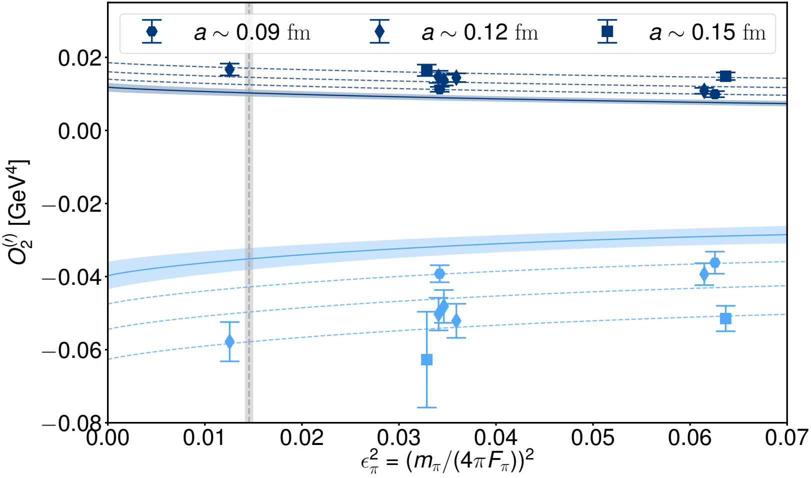

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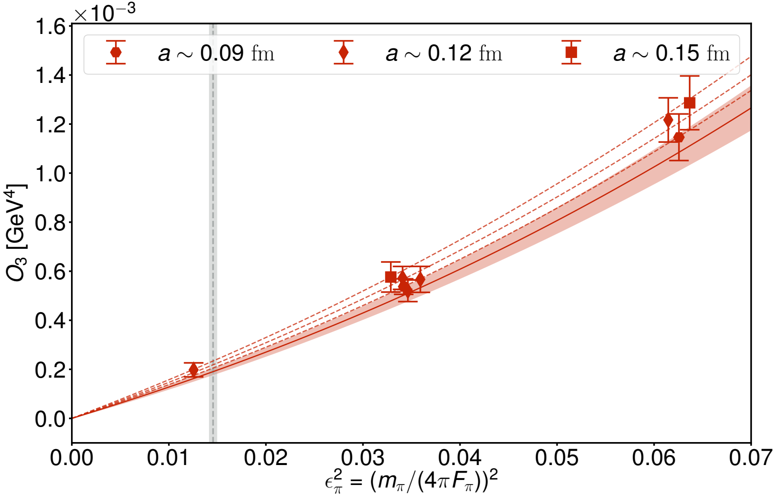
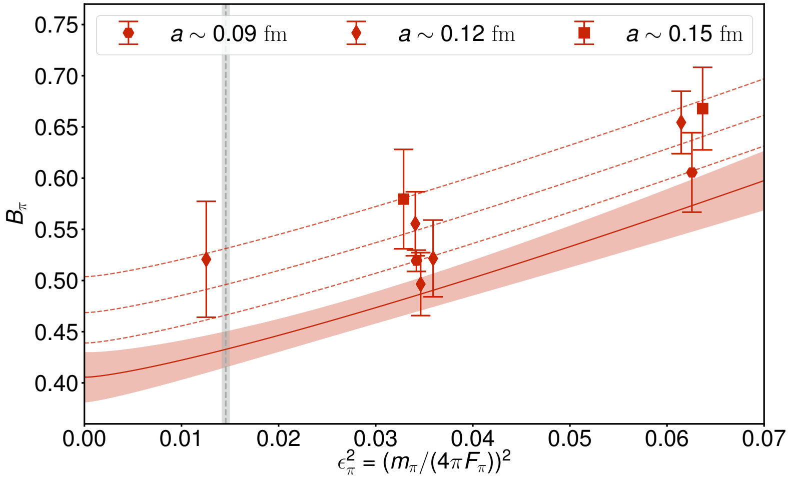


"Pion bag parameter"

We define $B_\pi = O_3/(\frac{8}{3} m_\pi^2 F_\pi^2)$



Physical results

$O_i[\mathrm{GeV}]^4$	RI/SMOM $\mu = 3$ GeV	$\overline{\mathrm{MS}}$ $\mu = 3$ GeV
O_1	$-1.96(14) \times 10^{-2}$	$-1.94(14) \times 10^{-2}$
O_1'	$-7.21(53) \times 10^{-2}$	$-7.81(57) \times 10^{-2}$
O_2	$-3.60(30) \times 10^{-2}$	$-3.69(31) \times 10^{-2}$
O_2'	$1.05(09) \times 10^{-2}$	$1.12(10) \times 10^{-2}$
O_3	$1.89(09) \times 10^{-4}$	$1.90(09) \times 10^{-4}$

$1 - 2\sigma$ agreement with [V. Cirigliano, W. Dekens, M. Graesser, E. Mereghetti 1701.01443]
where they use an estimate from $SU(3)$ χPT

but the uncertainty decreases from $20 - 40\%$ to $5 - 8\%$

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and $B_\pi = 0.430(16)[0.432(16)]$

low value, far from 1 as anticipated for example by [Pich & de Rafael]

Conclusions

Conclusions and outlook

- First computation of heavy contributions to $0\nu\beta\beta$, the $\langle\pi^+|O|\pi^-\rangle$ MEs
- Accepted by PRL [A. Nicholson, E. Berkowitz, H. Monge-Camacho, D. Brantley, N.G., C.C. Chang, E. Rinaldi, M.A. Clark, B. Joo, T. Kurth, B. Tiburzi, P. Vranas, A. Walker-Loud] arXiv:1805.02634

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Our computation features

- Good Chiral symmetry
- Non-perturbative renormalisation
- Physical pion masses, three lattice spacings

As for BSM neutral meson meson mixing, chiral symmetry and SMOM schemes are crucial !

Conclusions and outlook

There is still some work to do:

- Compute contributions within nuclei $\langle N|O|N\rangle$
- Other unknown short-distance contributions
- Long-distance contributions ?

Backup

g_A

g_A the Nucleon axial coupling

Insertion of the axial current between two nucleon state,

$$\langle N(p')|\overline{\psi}\gamma_\mu\gamma_5\psi|N(p)\rangle = \bar{u}(p')\left[\gamma_\mu\gamma_5 G_A(q^2) + \gamma_5\frac{q_\mu}{2m_N}G_P(q^2)\right]\bar{u}(p)$$

where q is the momentum transfer $q = p' - p$

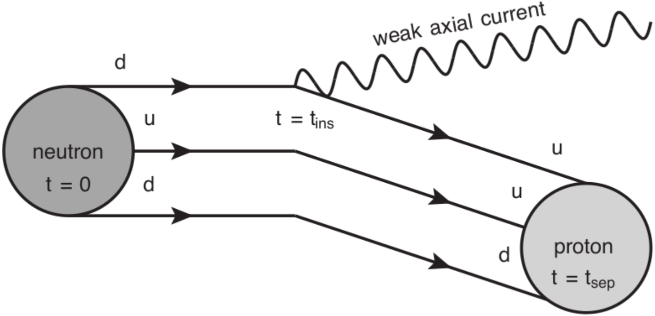
The nucleon axial coupling is then

$$g_A = G_A(0)$$

g_A is the strength at which the nucleon couples to the axial current

g_A the Nucleon axial coupling

Nuclear β decay: $n \longrightarrow p + e^- + \bar{\nu}_e$

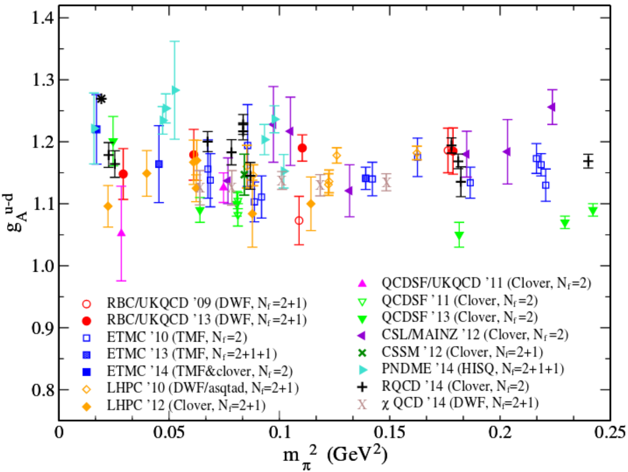


$\longrightarrow$ Well-measured experimentally $g_A = 1.2723(23)$ error $< 0.2\%$

A problem on the lattice

- It should be a relatively “simple” quantity
- But turned out to be a long standing puzzle
- Can we believe in lattice results for nucleons ?
- Or is there a problem with QCD ?

A problem on the lattice



Summary plot from [Martha Constantinou @ Lat2014]

Our computation

With CalLat (California Lattice) Collaboration

- Möbius fermions on $N_f = 2 + 1 + 1$ HISQ ensembles
  $\Rightarrow$ Chiral symmetry
- 3 lattice spacings $a \sim 0.15, 0.012, 0.09\ \mathrm{fm}$, several volumes
- Multiple pion mass
  and physical pion mass on $a \sim 0.15, 0.012$ ensembles

$\Rightarrow$ Good control over Chiral/cont./ infinite Vol. extrapolations

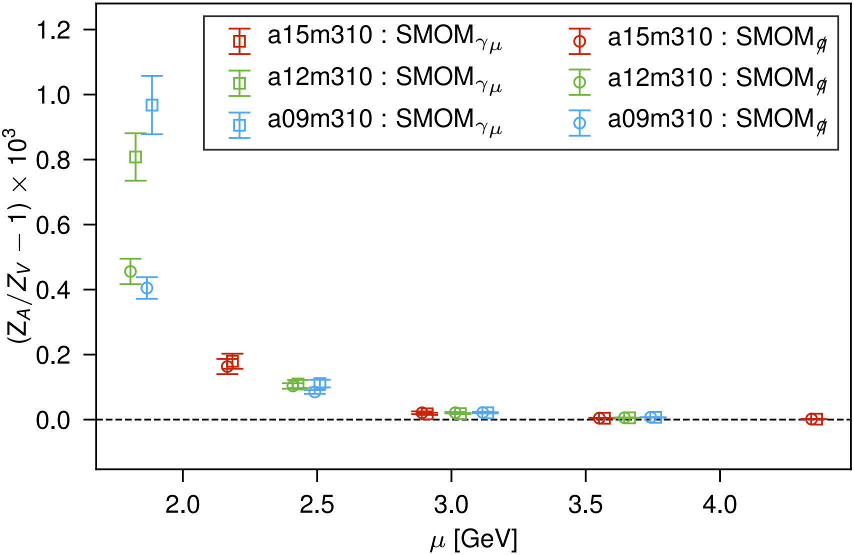
Our computation

Main improvements (compared to recent computations)

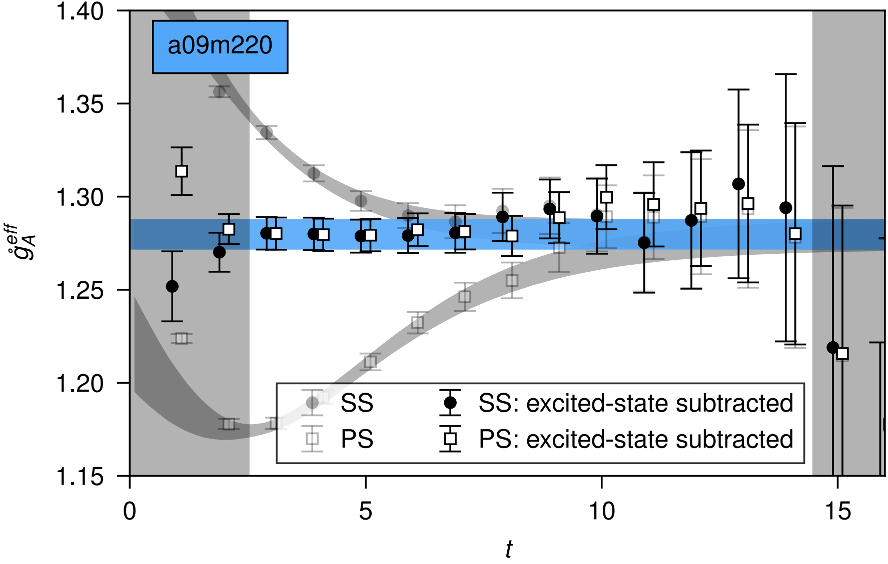
- New method to extract the signal “kills” the noise problem
- Chiral fermions, so dominant Lattice artefacts are a^2 and a^4
- Non-perturbative renormalisation $Z_A/Z_V = 1$

$$g_A = \frac{Z_A}{Z_V}\left(\frac{g_A}{g_V}\right)^{\mathrm{bare}}$$

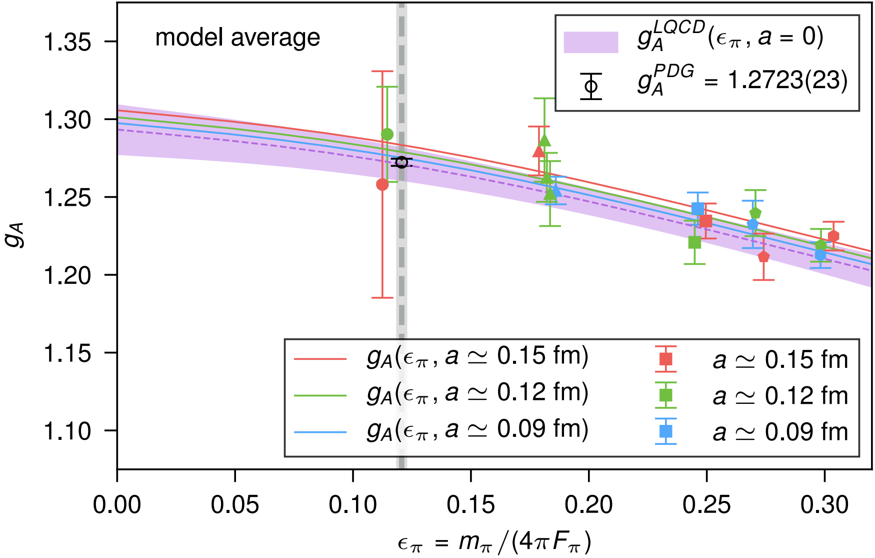
Results



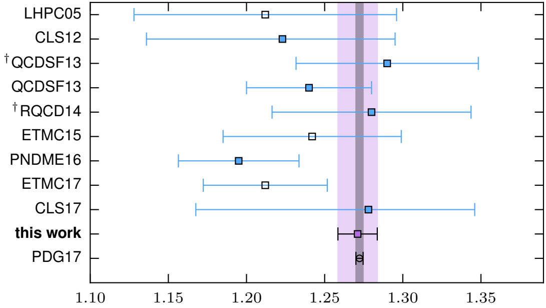
a15m310 : SMOM$_{\gamma_\mu}$
a12m310 : SMOM$_{\gamma_\mu}$
a09m310 : SMOM$_{\gamma_\mu}$
a15m310 : SMOM$_{\not{q}}$
a12m310 : SMOM$_{\not{q}}$
a09m310 : SMOM$_{\not{q}}$
$(Z_A/Z_V - 1) \times 10^3$
μ [GeV]
1.2
1.0
0.8
0.6
0.4
0.2
0.0
2.0
2.5
3.0
3.5
4.0


a09m220
1.40
1.35
1.30
1.25
1.20
1.15
$\mathring{g}_A^{eff}$
0
5
10
15
t
SS
PS
SS: excited-state subtracted
PS: excited-state subtracted

Results


model average
$g_A^{LQCD}(\epsilon_\pi, a = 0)$
$g_A^{PDG} = 1.2723(23)$
$g_A(\epsilon_\pi, a \simeq 0.15\text{ fm})$
$g_A(\epsilon_\pi, a \simeq 0.12\text{ fm})$
$g_A(\epsilon_\pi, a \simeq 0.09\text{ fm})$
$a \simeq 0.15$ fm
$a \simeq 0.12$ fm
$a \simeq 0.09$ fm
1.35
1.30
1.25
1.20
1.15
1.10
g_A
0.00
0.05
0.10
0.15
0.20
0.25
0.30
$\epsilon_\pi = m_\pi/(4\pi F_\pi)$

Results



$g_A^{QCD} = 1.271(13)$ $g_A^{PDG} = 1.2723(23)$

[Chang, Nicholson, Rinaldi, Berkowitz, N.G., Brantley, Monge-Camacho, Monahan, Bouchard, Clark, Joó, Kurth, Orginos, Vranas, Walker-Loud]

Published in Nature 558 (2018) no.7708

Error budget

$$g_A = 1.2711(103)^{\mathrm{s}}(39)^{\chi}(15)^{\mathrm{a}}(19)^{\mathrm{v}}(04)^{\mathrm{I}}(55)^{\mathrm{M}}$$

where the errors are statistical (s), chiral (χ), continuum (a), infinite volume (v), isospin breaking (I) and model-selection (M)

To be compare to the experimental value $g_A = 1.2723(23)$