Quasi PDF as observables

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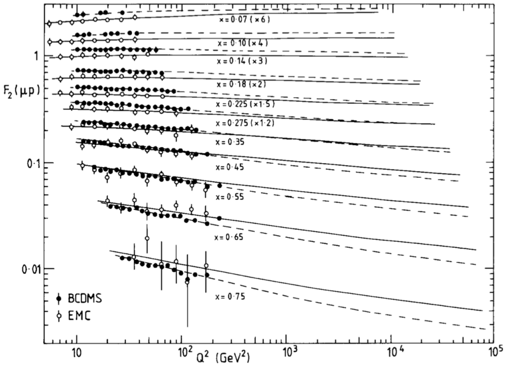
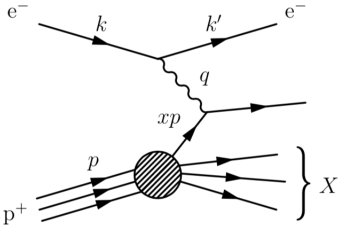


in progress/collaboration with K Cichy, T Giani, C Monahan

based on Collins 1980

DIS



$$s = (p+k)^2, \quad Q^2 = -q^2 = -(k-k')^2, \quad x = Q^2/(2p\cdot q)$$

$$d\sigma = \kappa \left(\frac{\alpha}{Q^2}\right)^2 L^{\mu\nu} H_{\mu\nu}$$

$$H_{\mu\nu} = \sum_X (2\pi)^D \delta\left(p - p_X - q\right) \, \langle p|J_\mu(0)|X\rangle\langle X|J_\nu(0)|p\rangle$$

hadronic tensor

$$
\begin{aligned}
H_{\mu\nu} &= \sum_X (2\pi)^D \delta\left(p - p_X - q\right)\, \langle p|J_\mu(0)|X\rangle\langle X|J_\nu(0)|p\rangle \\
&= \int d^D y\, e^{iq\cdot y} \langle p|J_\mu(y) J_\nu(0)|p\rangle \\
&= \int d^D y\, e^{iq\cdot y} \langle p|\left[J_\mu(y), J_\nu(0)\right]|p\rangle
\end{aligned}
$$

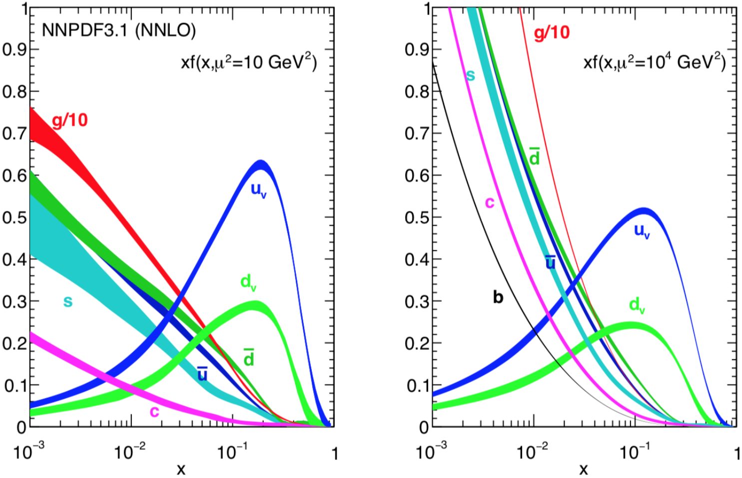
nucleon dynamics encoded in Lorentz-invariant form factors

$$
H_{\mu\nu} = F_1(x,Q^2)\left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) + F_2(x,Q^2)\ldots
$$

factorization and (universal) PDFs

$$
F_i(x,Q^2) = \int_x^1 \frac{d\xi}{\xi}\, C_{i,a}(\xi, Q^2, \mu^2) f_R^a(x/\xi, \mu^2) + \ldots
$$

NNPDF3.1



NNPDF3.1 (NNLO)
xf(x,μ²=10 GeV²)
g/10
u_v
d_v
s
d̄
ū
c
x
xf(x,μ²=10⁴ GeV²)
g/10
s
d̄
c
ū
u_v
b
d_v
x

PDFs from field theory

$$\boxed{\mathcal{M}^i(\zeta, P) = \langle P|\bar{\psi}(\zeta)\Gamma^i \,\mathrm{P}\exp\left(-ig\int_0^\zeta d\eta\, A(\eta)\right)\psi(0)|P\rangle}$$

light-cone PDF: $\zeta = (0, y^-, \vec{0}_\perp)$:

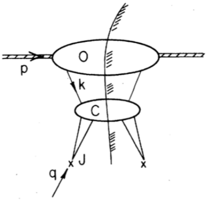
$$f(x,\mu) = \int \frac{dy^-}{4\pi}\, e^{-i(xP^+)y^-}\, \mathcal{M}^+(y^-, P^+)$$

quasi-PDF, time-independent quantity: $\zeta = (0,0,0,z)$:

$$q(x,\mu,M_N,P_z) = \int \frac{dz}{4\pi}\, e^{-i(xP_z)z}\, \mathcal{M}^z(z,P_z) \overset{P_z\to\infty}{\longrightarrow} f(x,\mu)$$

toy model: DIS in ϕ_6^3

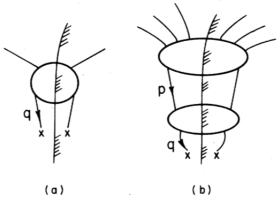
$$F(x,Q^2) = \int dy\, e^{iqy}\, \langle P | j_R(y) j_R(0) | P \rangle$$



$$\begin{aligned} F(x,Q^2) &= \int_1^\infty \frac{d\eta}{\eta}\, C(\eta, Q^2/\mu^2, g_R)\, f_R(\eta x, g_R, m_R, \mu) + \ldots \\ &= \int_0^1 \frac{d\xi}{\xi}\, \mathcal{C}(\xi, Q^2/\mu^2, g_R)\, f_R(x/\xi, g_R, m_R, \mu) + \ldots \\ &= (\mathcal{C} \otimes f_R)(x) + \ldots \end{aligned}$$

light-cone bilocal operators

$$\mathcal{F}(q^+) = q^+ \int \frac{dy^-}{2\pi}\, e^{-iq^+ y^-}\, \phi(y)\phi(0) \quad \longrightarrow \quad f_R(x) = \langle P|\mathcal{F}_R(xP^+)|P\rangle$$

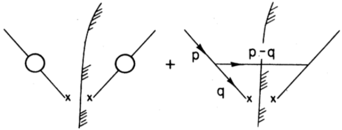


$$\begin{aligned}
\mathcal{F}_R(q^+) &= \int_1^\infty \frac{d\eta}{\eta}\, K(\eta, g_R, \epsilon)\, \mathcal{F}(\eta q^+, g_R, m_R, \mu, \epsilon) \\
&= \int_0^1 \frac{d\eta}{\eta}\, \mathcal{K}(\eta, g_R, \epsilon)\, \mathcal{F}(q^+/\eta, g_R, m_R, \mu, \epsilon) \\
&= (\mathcal{K} \otimes \mathcal{F})\, (q^+)
\end{aligned}$$

renormalization at 1-loop

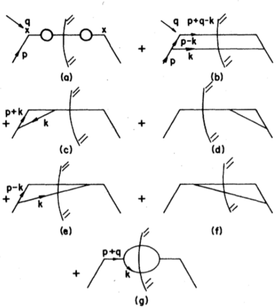
$$\hat{f}_R(x) = \left(\mathcal{K} \otimes \hat{f}\right)(x)$$
$$K(\eta, \alpha) = (1 + \alpha\,\kappa)\,\delta(1 - \eta) + \alpha\,K^{(1)}(\eta) + \mathcal{O}\left(\alpha^2\right)$$

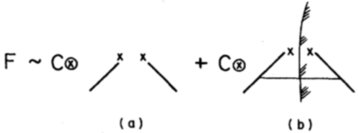
$$\hat{f}\left(x, \mu^2, \epsilon\right) = z\left(\mu^2, \epsilon\right)\,\delta(1 - x) + \alpha\,\hat{f}^{(1)}\left(x, \mu^2, \epsilon\right) + \mathcal{O}\left(\alpha^2\right)$$



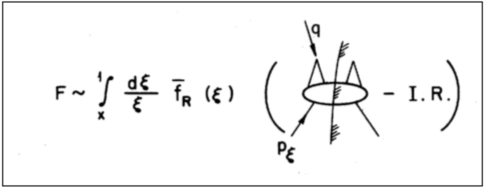
$$K(\eta, \alpha) = \left(1 - \frac{\alpha}{12}\frac{1}{\epsilon}\right)\delta(1 - \eta) - \alpha\,\frac{1}{\epsilon}\frac{\eta - 1}{\eta^2} + \mathcal{O}\left(\alpha^2\right).$$

DIS at 1-loop





IR picture



$$
\begin{aligned}
F(x,Q^2) &\sim \left(f\otimes \hat{F}\right)(x) \\
&\sim \left(f\otimes \bar{\mathcal{K}}\otimes \mathcal{C}\right)(x) \\
&\sim \left(\bar{f}_R\otimes \mathcal{C}\right)(x)
\end{aligned}
$$

where

$$\bar{\mathcal{K}}\otimes\mathcal{C} = \hat{F}$$

$\mathcal{C}$ is IR-finite

IR picture at 1-loop

$$F\left(x,Q^2\right) = \int_x^1 \frac{d\xi}{\xi} \bar{f}_R\left(\xi\right) \left(\hat{F}\left(\frac{x}{\xi},Q^2\right) - \mathsf{IR}\right)$$

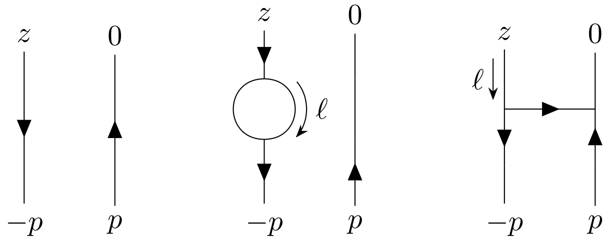
$$\hat{F} = \bar{\mathcal{K}} \otimes \left(\hat{F} - \mathsf{IR}\right)$$

$$\bar{K}\left(\eta,\alpha\right) = \left(1 - \frac{\alpha}{12}\frac{1}{\epsilon}\right)\,\delta\left(1-\eta\right) - \alpha\,\frac{1}{\epsilon}\frac{\eta-1}{\eta^2} = K(\eta,\alpha)$$

$$\boxed{\bar{f}_R\left(x,Q^2\right) = f_R\left(x,Q^2\right)}$$

position space matrix element

$$\mathcal{M}\left(\nu, z^2\right) = \langle P|\phi(z)\phi(0)|P\rangle \,.$$



perturbative computation

$$\hat{\mathcal{M}}^{(0)}(\nu, z^2) = \exp[-ip \cdot z] = \exp[-i\nu] = \hat{\mathcal{M}}^{(0)}(\nu, 0)$$

$$\hat{\mathcal{M}}^{(1)}_{\text{self}}\left(\nu, z^2\right) = \left[1 + \frac{\alpha}{6}\left(\frac{1}{\epsilon} + \log\frac{m^2}{\mu^2} + b\right)\right] \hat{\mathcal{M}}^{(0)}\left(\nu, z^2\right)$$

$$\hat{\mathcal{M}}^{(1)}_{\text{op}}\left(\nu, z^2\right) = g^2 \int_0^1 dx\ (1-x)\left(\frac{M^2}{\mu^2}\right)^{D/2-3} K\left(z^2 M^2\right)\ \hat{\mathcal{M}}^{(0)}(x\nu, 0)$$

where

$$M^2 = m^2\left(1 - x + x^2\right)$$

$$K(z^2M^2) = \int_{\hat{q}_E} \frac{e^{i\hat{q}_E\cdot\hat{\zeta}_E}}{\left(\hat{q}_E + 1\right)^3}$$

$$= \frac{1}{(4\pi)^3}\,(4\pi)^{3-D/2} \int_0^\infty \frac{dT}{T}\, T^{3-D/2} e^{-T} e^{-\hat{\zeta}_E^2/(4T)}$$

$\hat{\zeta}^\mu_E = M z^\mu_E\,,\ \hat{\zeta}^2_E = -M^2 z^2\,,\ \hat{q}^\mu_E = q^\mu_E/M$

light-like separation

$$\hat{\mathcal{M}}\left(\nu,0\right)=\left[1+\frac{\alpha}{6}\,\left(\frac{1}{\epsilon}+\log\frac{m^2}{\mu^2}+b\right)\right]\,\hat{\mathcal{M}}^{(0)}\left(\nu,0\right)$$
$$+\,\alpha\int_0^1 dx\,\left(1-x\right)\left(\frac{1}{\epsilon}+\log\frac{\mu^2}{m^2\left(1-x+x^2\right)}\right)\,\hat{\mathcal{M}}^{(0)}\left(x\nu,0\right).$$

renormalization kernel:

$$\hat{\mathcal{M}}_R\left(\nu,\mu^2\right)=\int_0^1 dy\,\mathcal{K}\left(y\right)\hat{\mathcal{M}}\left(y\nu,0\right).$$

$$\mathcal{K}\left(y\right)=\left[1-\frac{\alpha}{6}\frac{1}{\epsilon}\right]\,\delta\left(1-y\right)-\alpha\,\frac{1}{\epsilon}\left(1-y\right)$$

light-like separation

$$\hat{\mathcal{M}}_R\left(\nu;\mu^2\right) = \left[1 + \frac{\alpha}{6}\left(\log\frac{m^2}{\mu^2} + b\right)\right]\hat{\mathcal{M}}^{(0)}\left(\nu,0\right) + \alpha\int_0^1 dx\ (1-x)\ \log\frac{\mu^2}{m^2\left(1-x+x^2\right)}\hat{\mathcal{M}}^{(0)}\left(x\nu,0\right).$$

$$\mu^2\frac{d}{d\mu^2}\,\hat{\mathcal{M}}_R\left(\nu;\mu^2\right) = \alpha\int_0^1 dx\,P\left(x\right)\ \hat{\mathcal{M}}_R\left(x\nu;\mu^2\right) + \mathcal{O}\left(\alpha\right),$$

with the $\mathcal{O}\left(\alpha\right)$ splitting kernel given by

$$P\left(x\right) = \left(1-x\right) - \frac{1}{6}\delta\left(1-x\right) = \left(1-x\right)_+ + \frac{1}{3}\delta\left(1-x\right).$$

spatial separation

no UV divergences in K for $z_3^2 \neq 0$

$$K\left(-z_3^2, M^2\right) = \frac{1}{(4\pi)^3} \int_0^\infty \frac{dT}{T}\, e^{-TM^2} e^{-\frac{z_3^2}{4T}} = \frac{1}{(4\pi)^3}\, 2K_0\left(M z_3\right)$$

renormalized operator

$$\hat{\mathcal{M}}_R\left(\nu, z_3^2; \mu^2\right) = \left[1 + \frac{\alpha}{6}\left(\log\frac{m^2}{\mu^2} + b\right)\right] \hat{\mathcal{M}}^{(0)}\left(\nu, z_3^2\right) + \alpha \int_0^1 dx\, \left(1 - x\right)\, 2K_0\left(M z_3\right) \hat{\mathcal{M}}^{(0)}\left(x\nu, z_3^2\right)$$

factorization theorem

$$\hat{\mathcal{M}}_R\left(\nu, -z_3^2;\, \mu^2\right) = \hat{\mathcal{M}}_R\left(\nu, \mu^2\right) + \\ + \alpha \int_0^1 dx\; (1-x) \left(2K_0\left(Mz_3\right) - \log\frac{\mu^2}{M^2}\right)\, \hat{\mathcal{M}}_R\left(x\nu, \mu^2\right)$$

using

$$\hat{\mathcal{M}}_R\left(\nu, \mu^2\right) = \int_{-1}^1 d\xi\, e^{i\xi\nu}\, \hat{f}_R\left(\xi, \mu^2\right)$$

yields

$$\hat{\mathcal{M}}_R\left(\nu, -z_3^2; \mu^2\right) = \int_{-1}^1 d\xi\, \tilde{C}\left(\xi\nu, mz_3, \frac{\mu^2}{m^2}\right)\, \hat{f}_R\left(\xi, \mu^2\right)$$

$$\tilde{C}\left(\xi\nu, mz_3, \frac{\mu^2}{m^2}\right) = e^{i\xi\nu} - \alpha \int_0^1 dx\; (1-x) \left(2K_0\left(Mz_3\right) - \log\frac{\mu^2}{M^2}\right) e^{ix\xi\nu}$$

IR behaviour & scaling of pseudo PDF

for $Mz_3 \ll 1$

$$2K_0\left(Mz_3\right) = -\log\left(M^2 z_3^2\right) + 2\log\left(2e^{-\gamma_E}\right) + \mathcal{O}\left(M^2 z_3^2\right)$$

and therefore

$$\hat{\mathcal{M}}_R\left(\nu, -z_3^2;\, \mu^2\right) = \int_{-1}^{1} d\xi\, \tilde{C}\left(\xi\nu, \mu^2 z_3^2\right) \hat{f}_R\left(\xi, \mu^2\right) + O(m^2 z_3^2)$$

$$\tilde{C}\left(\xi\nu, \mu^2 z_3^2\right) = e^{i\xi\nu} - \alpha \int_0^1 dx\, \left(1-x\right)\log\left(\mu^2 z_3^2 \frac{e^{2\gamma_E}}{4}\right) e^{ix\xi\nu}$$

Wilson coefficient is IR-safe

z^2 depence only at $\mathcal{O}(\alpha)$

quasi PDF

Fourier transform to momentum space

$$q_R\left(y, \mu^2, P_3^2\right) = \frac{P_3}{2\pi} \int_{-\infty}^{\infty} dz_3\, e^{-iyP_3 z_3} \hat{\mathcal{M}}_R\left(P_3 z_3, -z_3^2\right)$$

we get

$$q_R\left(y, \mu^2, P_3^2\right) = \int_{-1}^{1} \frac{d\xi}{|\xi|} C\left(\frac{y}{\xi}, \frac{m^2}{\xi^2 P_3^2}, \frac{\mu^2}{m^2}\right) f_R\left(\xi, \mu^2\right)$$

$$C\left(\eta, \frac{m^2}{\xi^2 P_3^2}, \frac{\mu^2}{m^2}\right) = \int_0^1 dx\, (1-x) \times$$
$$\times \left[\frac{1}{\sqrt{(\eta - x)^2 + \frac{M^2}{\xi^2 P_3^2}}} - \delta\left(x - \eta\right) \log \frac{\mu^2}{M^2}\right].$$

large momentum limit

for $M^2/(\xi^2 P_3^2) \ll 1$

$$\lim_{\frac{M^2}{\xi^2 P_3^2}\to 0} C\left(\eta, \frac{M^2}{\xi^2 P_3^2}, \frac{\mu^2}{M^2}\right) = C\left(\eta, \frac{\mu^2}{\xi^2 P_3^2}\right) =$$

$$= \delta\left(1-\eta\right) + \alpha \begin{cases} (1-\eta)\log\frac{\eta}{\eta-1} + 1 \\ (1-\eta)\log\left[4\eta\left(1-\eta\right)\frac{\xi^2 P_3^2}{\mu^2}\right] + 2\eta - 1 \\ -\left(1-\eta\right)\log\frac{\eta}{\eta-1} - 1 \end{cases}$$

Wilson coefficient is IR-safe

QCD matrix elements

$$\boxed{\mathcal{M}_{\Gamma,A}(z) = \bar{\psi}(z)\Gamma\lambda_A \, \mathrm{P}\exp\left(-ig\int_0^z d\eta\, A(\eta)\right)\psi(0)}$$

Ioffe time distributions

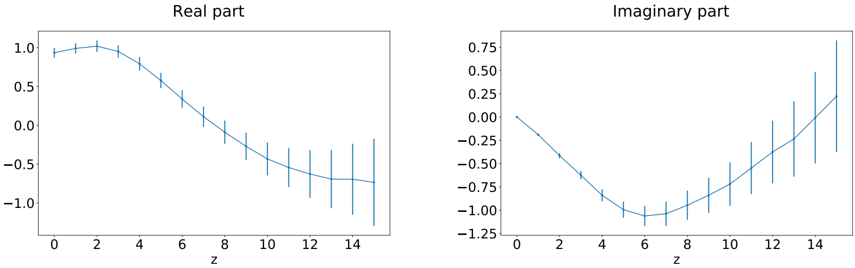
$$M_{\gamma^\mu,A}(z,P) = \langle P|\mathcal{M}_{\gamma^\mu,A}(z)|P\rangle$$

Lorentz covariance

$$M_{\gamma^\mu,A}(z,P) = P^\mu h_{\gamma^\mu,A}(z\cdot P, z^2) + z^\mu h'_{\gamma^\mu,A}(z\cdot P, z^2)$$

lattice observables - 1

$$\mathcal{O}^{\mathsf{Re}}_{\gamma^0}\left(zP_z, z^2\right) \equiv \mathsf{Re}\left[\mathsf{h}_{\gamma^0,3}\left(zP_z, z^2\right)\right] \qquad \mathcal{O}^{\mathsf{Im}}_{\gamma^0}\left(zP_z, z^2\right) \equiv \mathsf{Im}\left[\mathsf{h}_{\gamma^0,3}\left(zP_z, z^2\right)\right]$$



[C Alexandrou et al 18]

lattice observables - 2

inverse Fourier transform

$$\mathcal{O}^{\mathsf{Re}}_{\gamma^0}\left(zP_z, z^2, \mu^2\right) = \int_{-\infty}^{\infty} dx\, \cos\left(xP_z z\right) \int_{-1}^{+1} \frac{dy}{|y|}\, C_3\left(\frac{x}{y}, \frac{\mu}{|y|P_z}\right) f_3\left(y, \mu^2\right)$$
$$\mathcal{O}^{\mathsf{Im}}_{\gamma^0}\left(zP_z, z^2, \mu^2\right) = \int_{-\infty}^{\infty} dx\, \sin\left(xP_z z\right) \int_{-1}^{+1} \frac{dy}{|y|}\, C_3\left(\frac{x}{y}, \frac{\mu}{|y|P_z}\right) f_3\left(y, \mu^2\right)$$

$$f_3\left(x, \mu^2\right) = \begin{cases} u\left(x, \mu^2\right) - d\left(x, \mu^2\right) & \text{if} \quad x > 0 \\ -\bar{u}\left(-x, \mu^2\right) + \bar{d}\left(-x, \mu^2\right) & \text{if} \quad x < 0 \end{cases}$$

factorization formula for lattice observables

using the explicit expressions for C_3

$$\mathcal{O}^{\mathsf{Re}}_{\gamma^0} = \int_0^1 dx\ \mathcal{C}_3^{\mathsf{Re}}\left(x, z, \frac{\mu}{P_z}\right) V_3\left(x, \mu\right) = \mathcal{C}_3^{\mathsf{Re}}\left(z, \frac{\mu}{P_z}\right) \circledast V_3\left(\mu^2\right)$$

$$\mathcal{O}^{\mathsf{Im}}_{\gamma^0} = \int_0^1 dx\ \mathcal{C}_3^{\mathsf{Im}}\left(x, z, \frac{\mu}{P_z}\right) T_3\left(x, \mu\right) = \mathcal{C}_3^{\mathsf{Im}}\left(z, \frac{\mu}{P_z}\right) \circledast T_3\left(\mu^2\right)$$

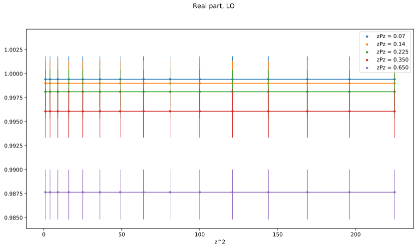
where V_3 and T_3 are the nonsinglet distributions defined by

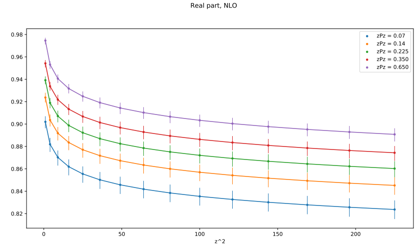
$$V_3\left(x\right) = u\left(x\right) - \bar{u}\left(x\right) - \left[d\left(x\right) - \bar{d}\left(x\right)\right]$$

$$T_3\left(x\right) = u\left(x\right) + \bar{u}\left(x\right) - \left[d\left(x\right) + \bar{d}\left(x\right)\right]$$

$$\mathrm{LO}: \quad \mathcal{O}^{\mathsf{Re}}_{\gamma^0}\left(zP_z, z^2, \mu^2\right) = \int dx \cos(zP_z x) V_3(x, \mu^2)$$

Bjorken scaling of ME



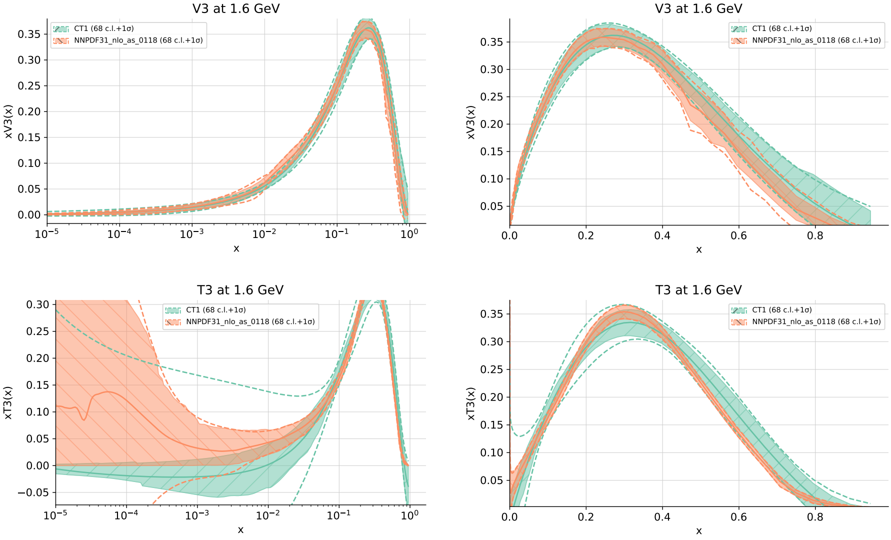


systematic errors

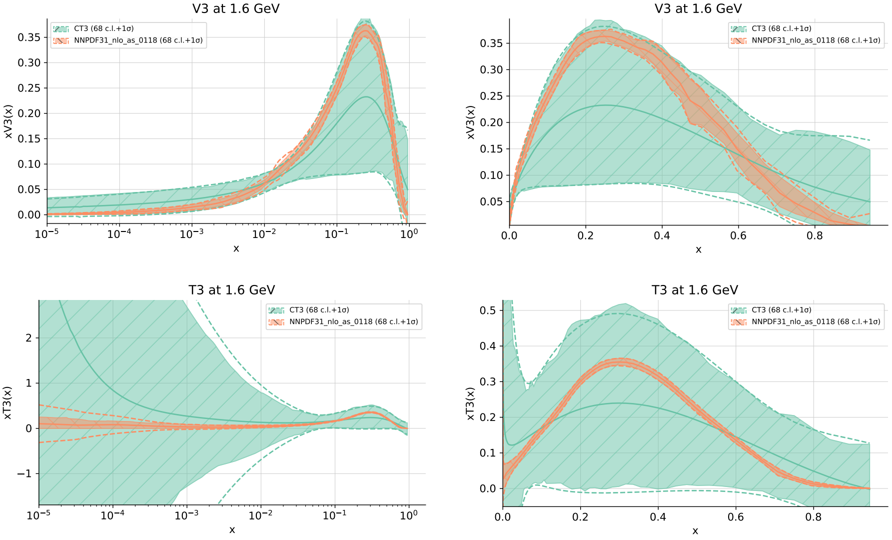
- cut-off effects
- finite volume effects
- excited states contamination
- truncation effects
- higher-twist terms
- isospin breaking

Scenario	Cut-off	FVE	Excited states	Truncation
S1	10%	2.5%	5%	10%
S2	20%	5%	10%	20%
S3	30%	$e^{-3+0.062z/a}\%$	15%	30%
S4	0.1	0.025	0.05	0.1
S5	0.2	0.05	0.1	0.2
S6	0.3	$e^{-3+0.062z/a}$	0.15	0.3

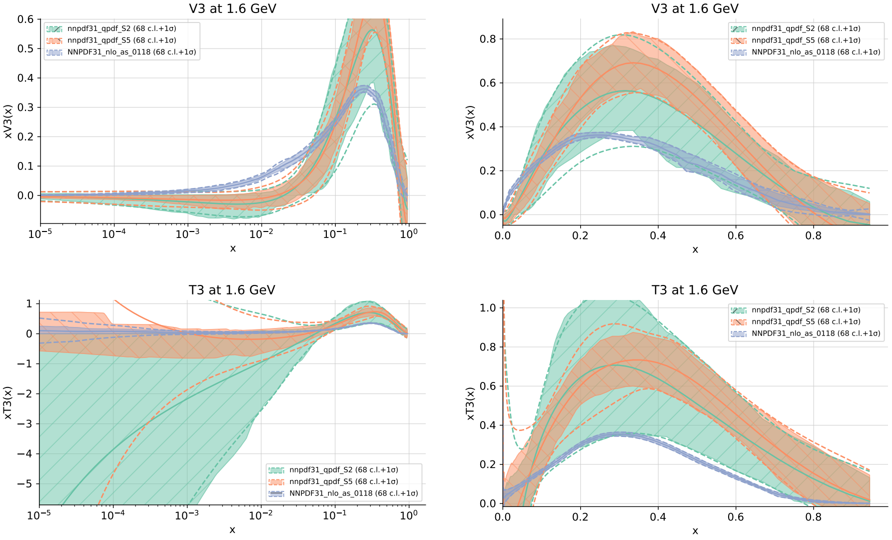
closure test – 1



closure test – 2



fit results


V3 at 1.6 GeV
nnpdf31_qpdf_S2 (68 c.l.+1σ)
nnpdf31_qpdf_S5 (68 c.l.+1σ)
NNPDF31_nlo_as_0118 (68 c.l.+1σ)
xV3(x)
x
V3 at 1.6 GeV
xV3(x)
x
T3 at 1.6 GeV
xT3(x)
x
T3 at 1.6 GeV
xT3(x)
x

outlook

- light-cone PDFs + factorization describe the structure of the proton
- necessary input for the exploitation of LHC, HL-LHC
- current extraction from data is very precise + improving
- lattice data provide complementary information, can be included in global fits like any other data
- identify the areas where a significant phenomenological impact from lattice QCD is possible