Quantum Gravity as an EFT, Black Hole Thermodynamics and an Excursion into the UV

Basem Kamal El-Menoufi

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University of Sussex

What is this talk about?

- Perhaps in quantum gravity we should start by …

The question is: what is the question?

- The question of this talk is …

What is the infrared limit of quantum gravity?

- Why infrared …

We probe the IR. we understand QM and GR in the IR.

- How about the ultraviolet …

Perhaps the IR limit can teach us lessons about the UV

Prelude

- Conceptual problem: Einstein told us that gravity is not a ‘force’.
- Perhaps QG prompts us to .….
- BUT … we observe a continuum space-time around us: *‘small’* fluctuations in the geometry should be amenable to quantization.
- Technical problem: QFT of the metric field is not re-normalizable.
- Is re-normalizability a principle of nature?
- Surely not … New physics naturally manifests in non-renormalizable interactions.

Philosophy of effective theories

- All our theories are provisional - only tested over a finite range of energies.
- New interactions and d.o.f appear as we probe higher energies.
- Why can we make predictions if we lack an ultimate description?
- Ultra-violet (UV) fluctuations look local when viewed at low energies.
- The UV either renormalize the parameters of the original Lagrangian or induce local interactions suppressed by a heavy scale. **Appelquist & Carazzone**
- Low-energy physics is shielded - separation of scales.

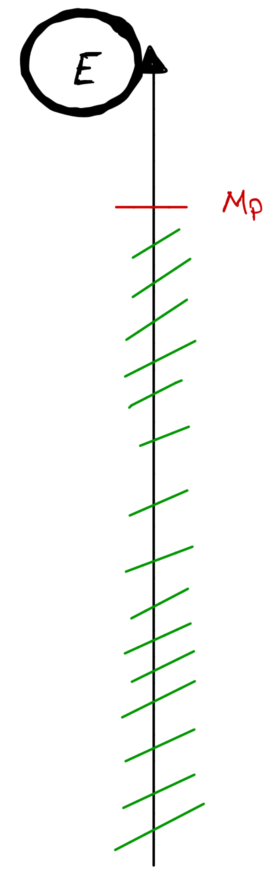
EFT Recipe

- Identify the relevant d.o.f for the problem and their symmetries.
- Identify a counting parameter for the effective theory.
- Write down the most general local Lagrangian as an expansion in the counting parameter.
- Compute any observable to any desired accuracy.
- Wilson coefficients are subject to renormalization - match or measure.
- The low-energy portion of the effective theory must be identical to that in the full theory.

EFT for Quantum Gravity

Donoghue

Not very covariant!



Add your favorite UV approach

Graviton

$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}$

Diff. Invariance

Power counting $\frac{E}{M_{\rm P}}$

Wilson Coefficients

$$\mathcal{S}_{\rm GEFT} = \int d^{\rm d}x \sqrt{g} \left(\frac{M_{\rm P}^2}{2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} + c_4 \nabla^2 R \right)$$

Derivative expansion

QG in everyday life

- How much can we achieve with the EFT in everyday life?

- Ultimate power of EFTs: extract long-range physics that dominate observables at low energy - non-analytic functions in amplitudes.

- Leading correction to the Newtonian potential energy:

$$V_{\rm N}(r) = -\frac{GMm}{r}\left[1 + \frac{3G(M+m)}{rc^2} + \frac{41}{10\pi^2}\frac{L_{\rm P}^2}{r^2}\right]$$

Long-distance

Bjerrum-Bohr et. al. ('03)

- In this example, Wilson coefficients do not play any role at long distance.

- This is genuine prediction of QG - any UV completion must reproduce this result.

Beyond scattering amplitudes

- Utility of gravitational amplitudes is ultimately very limited.
- A systematic framework is needed to tackle problems in cosmology and black hole physics.
- The effective action of QG lies at the frontier - it retains diffeomorphism invariance.
- How does the IR manifest in the effective action? **Non-locality**
- Non-local effective actions offer the best pathway to capture quantum effects at long distance.
- QG is non-local in the IR.

Curvature expansion & form factors

- The BFM offers the natural set-up to compute the effective action in non-trivial background geometries.

- The one-loop result looks as follows:

$$\Gamma[\bar{g}] = \Gamma_{\text{Local}}[c_i(\mu)] + \Gamma_{\text{n-Local}}[\mu]$$

- The local part is the local Lagrangian with renormalized constants.

- The non-local portion is parameter free and is dependent on the mass-less particle in the loop.

$$\Gamma_{\text{n-Local}}[\mu] = \int d^4x\sqrt{g}\left(\alpha R \log\left(\frac{\nabla^2}{\mu^2}\right) R + \beta R_{\mu\nu} \log\left(\frac{\nabla^2}{\mu^2}\right) R^{\mu\nu} + \gamma R_{\mu\nu\alpha\beta} \log\left(\frac{\nabla^2}{\mu^2}\right) R^{\mu\nu\alpha\beta}\right)$$

- Notice: the operator in the logarithm is the covariant d' Alembertian. The logarithm is an example of a 'form factor'.

Come on, that's not even causal!

- The logarithm is a genuine bi-local tensor distribution.
- Causality and reality are ensured using the in-in formalism.
- We compute expectation values not matrix elements.

$$\langle \mathcal{O}(t) \rangle = \; _I\langle \Phi(-\infty)| S^\dagger(\infty,-\infty) T\left[\mathcal{O}_I(t) S(\infty,-\infty)\right] |\Phi(-\infty)\rangle_I$$

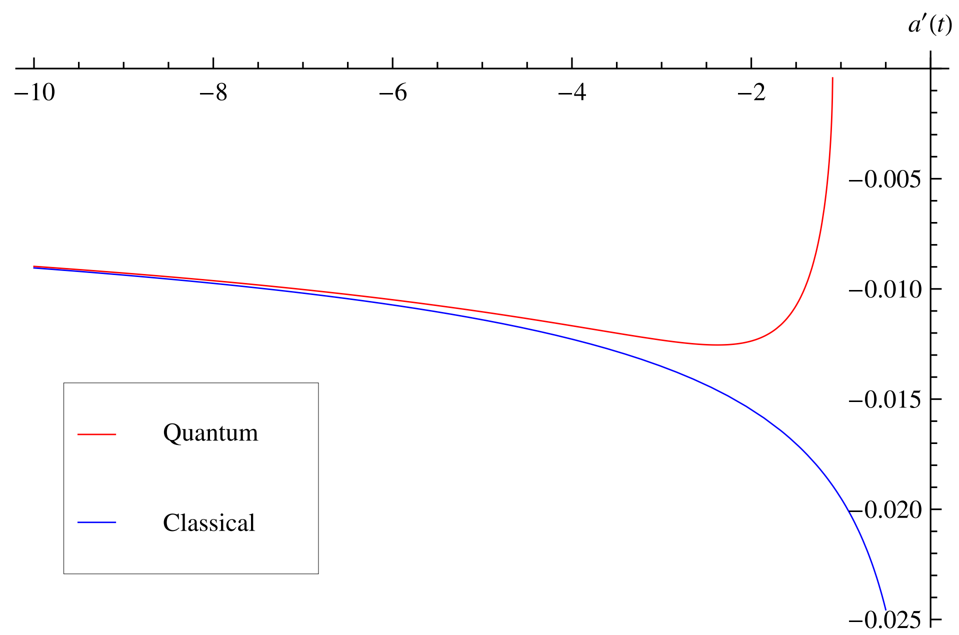
- In an FRW background - ignore derivatives of the scale factor:

$$\mathfrak{L}(t-t') = \lim_{\epsilon\to 0}\left[\frac{\theta(t-t'-\epsilon)}{t-t'} + \delta(t-t')\left(\log(\mu\epsilon)+\gamma\right)\right]$$ **Donoghue, BKE ('14)**

- A short cut is to enforce retarded boundary conditions on the form factors.

App I: Singularity Avoidance

- The quantum-corrected Friedmann equation becomes intero-differential.



Donoghue, BKE ('14)

- Conformal SM leads to an UV-independent bounce below the Planck scale.

- *Lesson*: the system exhibits a 'quantum memory', and quantum effects build up over long times to become appreciable below the Planck scale.

App II: Black holes

stay tuned

- How about quantum corrections to black holes?
- At second order in curvature, the Schwarzschild black hole does not receive any quantum correction!!
- A highly non-trivial result given the previous statements claimed in the literature relying on loop effects around point mass sources. **Duff ('74), ...**
- But Birkhoff theorem breaks down, and the field around a constant-density star is quantum corrected.

$$\delta g_{tt} = \frac{18\alpha l_{\rm P}^2}{R_S^2}\frac{r_h}{r}, \qquad \delta g_{rr} = \frac{6\alpha l_{\rm P}^2}{r^2}\frac{r_h}{r}$$

- Notice the power-law of the correction on the (tt) component. Only the fully covariant effective action could catch this.

App III: Grav. radiation

stay tuned

- The radiation field of a binary system receives a quantum correction.
- The non-local kernel is now more complicated:

$$\mathfrak{L}(x-x') = \lim_{\delta \to 0} \left[\frac{i}{\pi^2} \left(\frac{\Theta(t-t')\Theta((x-x')^2)}{((t-t'+i\delta)^2 - (\vec{x}-\vec{x}')^2)^2} - \frac{\Theta(t-t')\Theta((x-x')^2)}{((t-t'-i\delta)^2 - (\vec{x}-\vec{x}')^2)^2} \right) - \delta^{(4)}(x-x') \ln(\delta\mu)^2 \right]$$

- Analytically, the best one can do is to look for *small* corrections to the quadropole radiation of GR

Sure! It's highly suppressed

$$\mathfrak{h}_{xx} = -\mathfrak{h}_{yy} = \frac{\kappa^4 \, (\beta + 4\gamma) \, \mu (d\omega_s)^2}{8\pi r^2} \left(2\omega_s \sin(2\omega_s t_r) - \frac{1}{r} \cos(2\omega_s t_r) \right)$$

$$\mathfrak{h}_{xy} = \mathfrak{h}_{yx} = -\frac{\kappa^4 \, (\beta + 4\gamma) \, \mu (d\omega_s)^2}{8\pi r^2} \left(\frac{1}{r} \sin(2\omega_s t_r) + 2\omega_s \cos(2\omega_s t_r) \right)$$

- Notice that the Wilson coefficients of the NLO Lagrangian do not appear, similar to the Newtonian potential energy!

App IV: conformal anomaly and B fields

BKE ('15)

- Consider massless QED in curved space, the effective action is non-local and encapsulates the trace anomaly.

beta function coeff.

$$\Gamma_{anom.}[g, A] = S_{EM} - \frac{b_i e^2}{12} \int d^4x \, \sqrt{g} F_{\mu\nu} F^{\mu\nu} \frac{1}{\nabla^2} R$$

- This is a starting point to investigate the mechanism of inflationary magnetogenesis.

- Some partial success is achieved. Experimental bounds are achieved iff the reheating temperature is quite low:

$$T_{\rm RH} \leq 100 \, {\rm GeV}$$

Black Hole Thermodynamics

- Bekenstein suggested the analogy between black hole mechanics and the laws of thermodynamics is more profound.

- The discovery of Hawking radiation put black hole thermodynamics on a firm ground.

$$T_{\mathrm{H}} = \frac{\kappa}{2\pi} \qquad\qquad S_{\mathrm{BH}} = \frac{A}{4L_{\mathrm{P}}^2}$$

- What microscopic d.o.f give rise to BH entropy? UV

- Could we learn more from the macroscopic effective theory? IR

- This turns out to be a very fruitful direction! IR $\longrightarrow$ UV

Euclidean quantum gravity

- The Euclidean approach stands out as a self-consistent framework to study the thermal properties of black holes.

- Hawking and Gibbons ('77) gave a prescription for the partition function of quantum gravity:

$$Z(\beta) = \int \mathcal{D}\Psi \mathcal{D}g \, e^{-\mathcal{S}_E - \mathcal{S}_\partial}$$

- The Euclidean action contains the HGY boundary term.

- The metrics in the path integral are Euclidean, periodic in time and asymptote to the flat metric on $\mathbb{R}^3 \times S^1$ **Gross et. al., ('83)**

- Evaluated on the Euclidean section of Schwarzschild black hole, all thermodynamic relations are recovered.

Intro to Kerr-Schild geometries

- Surprise!! Further work uncovered a problem: the operator in the logarithm is not covariant - does not appear in an expansion around flat space.
- Still, the action could be made covariant using 'counter-term' method - leads to complicated structures. **Donoghue, BKE ('15)**
- Question: is there any background geometry where the effective action could be explicitly constructed?
- Kerr-Schild geometries offer the solution:

$$g_{\mu\nu} = \eta_{\mu\nu} + k_\mu k_\nu, \quad k \cdot k = 0, \quad \sqrt{g} = 1$$

- This class of metrics cover all 4D black holes in Einstein's gravity!

Technology: non-local heat kernel

- Vilkovisky and collaborators developed a useful technique to compute non-local effective actions.
- The formalism uses the *heat kernel* as the central object.
- The heat kernel satisfies the heat diffusion equation:

$$(\partial_s + \mathcal{D}_x) H(x, y; s) = 0, \quad H(x, y; 0) = \delta^{(\mathrm{d})}(x - y)$$

- The operator of interest is second order but otherwise arbitrary - it depends on background fields including the metric.
- There exists a perturbative, non-covariant, method to solve the heat kernel:

Massless operator $\mathcal{D} = \partial^2 + V$ **'potential'**

Perturbation theory contd.

- Using the previous decomposition, the heat kernel is expressed as an expansion in the interaction V(x): **Codello, Zanusso ('12)**

$$H(s) = H_0(x) \times U(s), \quad U(s) = \mathrm{T}\exp\left(-\int_0^1 dt\, H_0(-st) \times V \times H_0(st)\right)$$

- This is very similar to real time evolution operator in QFT - notice that 'proper time' plays the role of (it).

- The flat space kernel plays the role of the propagator:

$$H_0(x,y;s) = \frac{i}{(4\pi s)^{\mathrm{d}/2}} \exp\left[\frac{(x-y)^2}{4s}\right]$$

- <u>It is not clear so far how to make the result covariant!</u>

Prelude to curvature expansion

- How to retain covariance? write down the heat kernel as an expansion in curvatures.

- There are two ways to accomplish this: non-linear completion (as done before) vs. 'covariant perturbation theory'. Both procedures yield identical results.

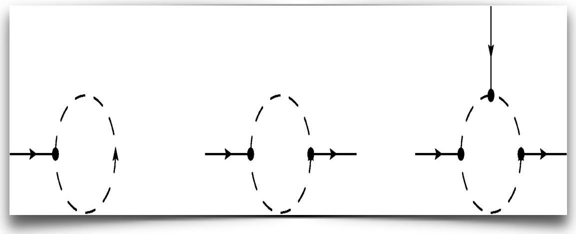
- Covariance remains an issue for form factors (e.g. the logarithm).

- How useful is it to work with generic geometries? The KS geometry offers an example where the heat kernel can be computed exactly.

- The simplest operator is the covariant d' Alembertian. In KS:

$$V = \lambda \left(k^\mu k^\nu \partial_\mu \partial_\nu + \frac{1}{2} \partial_\mu (k^\mu k^\nu) \partial_\nu + \frac{1}{2} \partial_\nu (k^\mu k^\nu) \partial_\mu \right)$$

- Feynman rules for KS perturbation theory.



Curvature expansion KS

BKE ('16)

- Introduce the curvature expansion: **Not necessary**

$$\mathcal{H}(s) = \frac{i}{(4\pi s)^{\mathrm{d}/2}} \int d^{\mathrm{d}}x \left[\mathcal{E}_0 + s\,\mathcal{G}_R(s\Box)\,R + s^2\,R\,\mathcal{F}_R(s\Box)\,R + s^2\,R_{\mu\nu}\,\mathcal{F}_{Ric}(s\Box)\,R^{\mu\nu} + s^2\,R_{\mu\nu\alpha\beta}\,\mathcal{F}_{Riem}(s\Box)\,R^{\mu\nu\alpha\beta} + \mathcal{O}(R^3)\right]$$

- Through matching onto the diagrams, one can determine the form factors:

$$\mathcal{G}_R(s\Box) = \frac{1}{4} f(s\Box) + \frac{1}{2s\Box}[f(s\Box) - 1], \quad f(s\Box) = \int_0^1 d\sigma\, e^{-\sigma(1-\sigma)s\Box}$$

- This is a genuine curvature expansion - no term proportional to $\Box R$

- Now the effective action is immediate:

Flat-space operator

$$\Gamma_{\ln}[\bar{g}] = -\hbar \int d^4x \left(\alpha R \ln\left(\frac{\Box}{\mu^2}\right) R + \beta R_{\mu\nu} \ln\left(\frac{\Box}{\mu^2}\right) R^{\mu\nu} + \gamma R_{\mu\nu\alpha\beta} \ln\left(\frac{\Box}{\mu^2}\right) R^{\mu\nu\alpha\beta} + \Theta \ln\left(\frac{\Box}{\mu^2}\right) \Box R\right)$$

KS spacetime

Effective action contd.

- The coefficient of the logarithm <u>tracks the divergences</u>:

	α	β	γ	Θ
Scalar	5	-2	2	-6
Fermion	-5	8	7	–
U(1)boson	-50	176	-26	–
Graviton	430	-1444	424	–

- RG invariance is guaranteed:

$$c_1^r(\mu) = c_1^r(\mu_\star) - \alpha \ln\left(\frac{\mu^2}{\mu_\star^2}\right) \qquad c_2^r(\mu) = c_2^r(\mu_\star) - \beta \ln\left(\frac{\mu^2}{\mu_\star^2}\right)$$

$$c_3^r(\mu) = c_3^r(\mu_\star) - \gamma \ln\left(\frac{\mu^2}{\mu_\star^2}\right) \qquad c_4^r(\mu) = c_4^r(\mu_\star) - \Theta \ln\left(\frac{\mu^2}{\mu_\star^2}\right)$$

- *Lesson*: <u>It is perhaps much better (yet less economical) to break diff. and search for clever ways to obtain the effective action in fixed geometries.</u>

Back to black holes!

- Recall that a Schwarzschild black hole is a Kerr-Schild spacetime. Partition function of Euclidean path integral!

$$k_\mu = \sqrt{\frac{2GM}{r}}\,(1, \vec{x}/r)$$

- The black hole appears as an extremal point of the Euclidean action, allowing for a semiclassical evaluation $\hbar \to 0$

$$\ln Z_0 = -\frac{\beta^2}{16\pi G}, \quad \beta = 8\pi GM$$

- All known thermodynamics is then derived - the area law is recovered.

- The temperature is fixed by requiring no conical singularity in the Euclidean section.

- The *Kerr-Schild* Euclidean section is complex, but the metric is positive definite and no problem arises.

Partition function at one-loop

BKE ('17)

- Analytic continuation of the EA in KS geometry yields the one-loop partition function accurate to second order in curvatures: $\Box \to -\Delta$

- Here one can impose thermal boundary conditions, but what is the log?

$$\ln\left(\frac{-\Delta}{\mu^2}\right) = -\int_0^\infty dm^2\left[(-\Delta+m^2)^{-1} - (\mu^2+m^2)^{-1}\right]$$

$$\mathfrak{L}(\vec{x}-\vec{x}') = -\frac{1}{2\pi}\lim_{\epsilon\to 0}\left[\mathcal{P}\left(\frac{1}{|\vec{x}-\vec{x}'|^3}\right) + 4\pi(\ln(\mu\epsilon)+\gamma_E-1)\delta^{(3)}(\vec{x}-\vec{x}')\right]$$

Principal Value

- An explicit calculation, *not so simple,* gives the free energy correct up to one-loop:

Notice the sign

$$F(\beta) = \frac{\beta}{16\pi G} - \frac{64\pi^2}{\beta}\Big(c_3(\mu) + 2\,\Xi\ln(\mu\beta)\Big) \qquad \Xi = \frac{1}{11520\pi^2}\left(2N_s + 7N_f - 26N_V + 424\right)$$

- The UV scale is not predicted from the EFT, determined by either matching onto the full theory or via a measurement: $c_3(\mu) = -2\,\Xi\ln(\mu\beta_{\rm QG})$

Thermodynamic stability

- For an isolated system to be stable, the second law requires concavity of the entropy. For a stable system in contact with a reservoir, second law requires:

$$\frac{\partial^2 F}{\partial T^2} \leq 0$$

- Let us apply this criterion to the free energy; two cases emerge.

- Case I: theory with a large number of gauge fields: $\Xi < 0$

- Stability is achieved at:

$$T_{\mathrm{C}} = \sqrt{\frac{90}{(26N_V - 7N_f - 2N_s - 424)}}\, T_{\mathrm{P}}$$

- Two remarkable features:

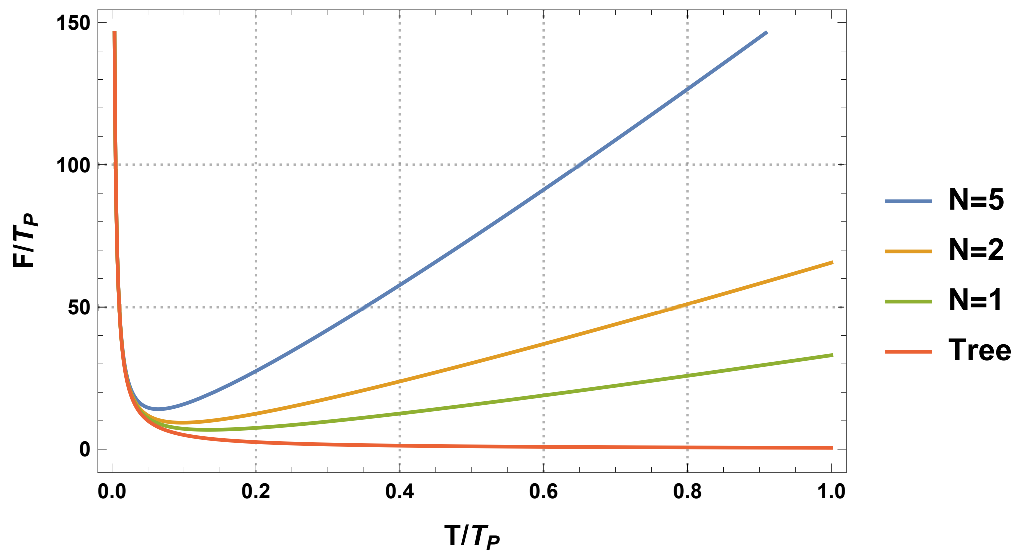
  - The black hole mass scales as $\sqrt{N_V}$ in Planck units

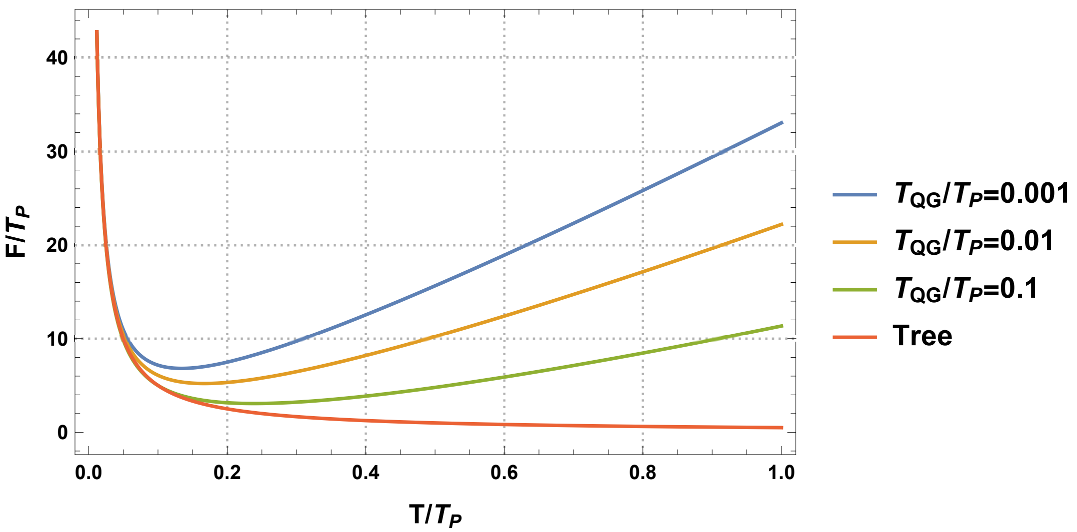
  - Insensitive to the UV scale: β_{QG}

Thermodynamics contd.

- Thermodynamic stability is not achieved if $\Xi > 0$

- Free energy develops a minimum!





- The minimum corresponds to vanishing entropy - interpretation?

- Is this hinting to a zero entropy *remnant*?

Missing corrections?

- How much should we trust this analysis?

- In the large N limit, quantum gravity effects are subleading. MMC fields are one-loop exact.

- Only fourth-order invariants get renormalized at one-loop (RG fixed).

- Higher curvature non-local operators appear in the partition function.

$$\ln Z \subset \int d^4x \, R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\gamma\beta} \frac{1}{\Box} R_{\gamma\beta}{}^{\mu\nu}$$

Scale invariant

- Scaling arguments uncovers the contribution of this operator: $\bar{g}_{\mu\nu} \to \Lambda^2 \bar{g}_{\mu\nu}$

- Stability analysis is intact while entropy changes by an inconsequential constant.

Critique

- Gross, Perry and Yaffe ('83) have analyzed the spectrum of gravitational fluctuations in Euclidean Schwarzschild.

- They have found that TT metric perturbations possess a negative mode.

Where is the negative mode in my gravity result?

- So far, no success in solving this issue. Perhaps I need to go third order in curvatures to extract the negative mode.

Linking the IR and UV

- Can we learn anything about the UV from these EFT computations?

- A central result of black hole thermodynamics is the area-law. What is the quantum correction?

$$S_{\rm bh} = S_{\rm BH} + 64\pi^2\,\Xi\,\ln\left(\frac{\mathcal{A}}{\mathcal{A}_{\rm QG}}\right)$$

See Refs in BKE ('16)

- Bekenstein & Mukhanov proposed a quantization rule for the area of a black hole:

$$\mathcal{A}_n = \gamma_0 n L_{\rm P}^2, \quad n = 1, 2, ...$$

- Area quantization is present in LQG, although levels are not equidistant.

Linking the IR and UV contd.

- Let us demand that the exponential of black hole entropy yields the number of micro states defined and counted in your favorite UV theory:

$$g(n) = e^{S_{\rm bh}(n)}$$

Hod ('04)

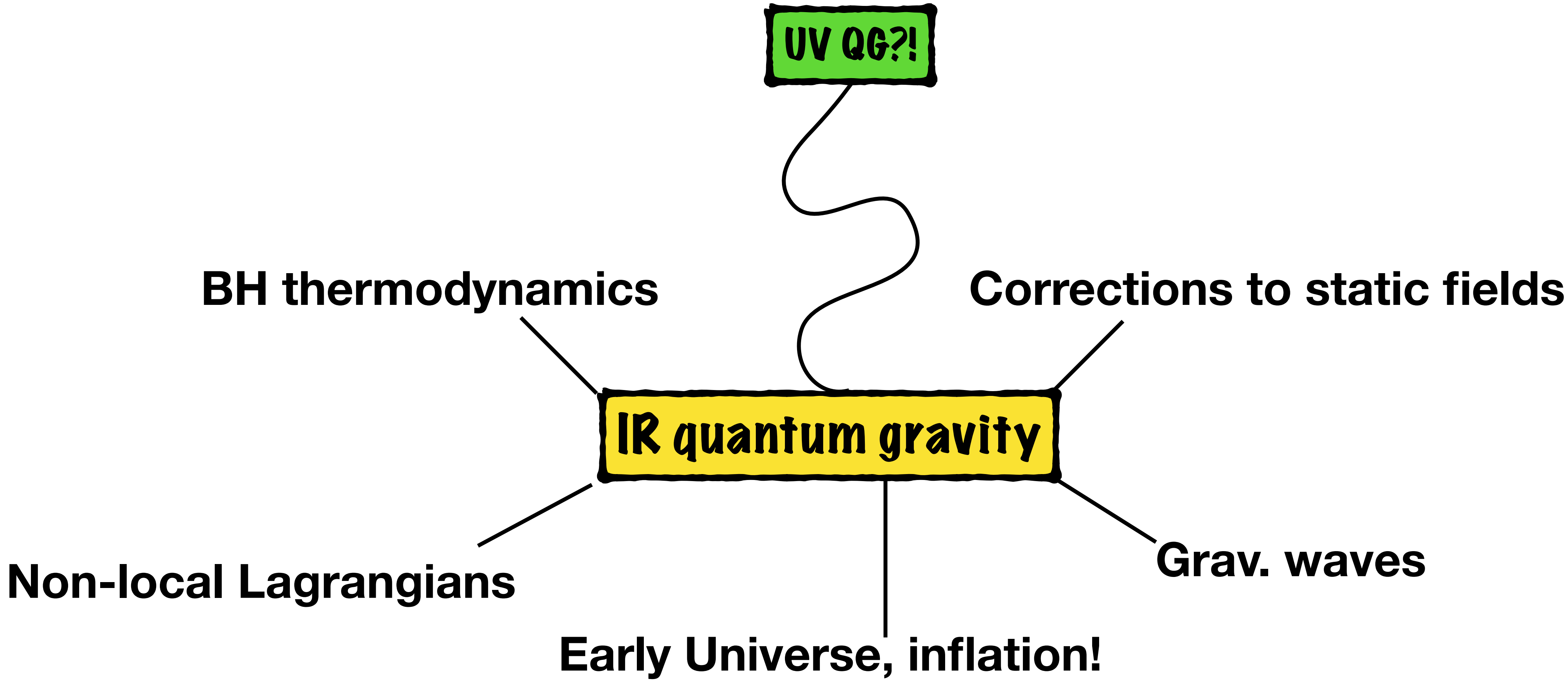
- The number of states must be an integer. Few steps reveal two quantization rules:

$$2N_s + \frac{7}{2}N_f - 26N_V + 424 = 180 \cdot l, \quad l \in \mathbb{N}$$

Related to c3

$$\frac{\mathcal{A}_{\rm QG}}{L_{\rm P}^2} = \frac{\gamma_0}{m^{1/l}}, \quad m \in \mathbb{N}$$

Outlook


UV QG?!
BH thermodynamics
Corrections to static fields
IR quantum gravity
Non-local Lagrangians
Grav. waves
Early Universe, inflation!