Improving QCD Event Generators

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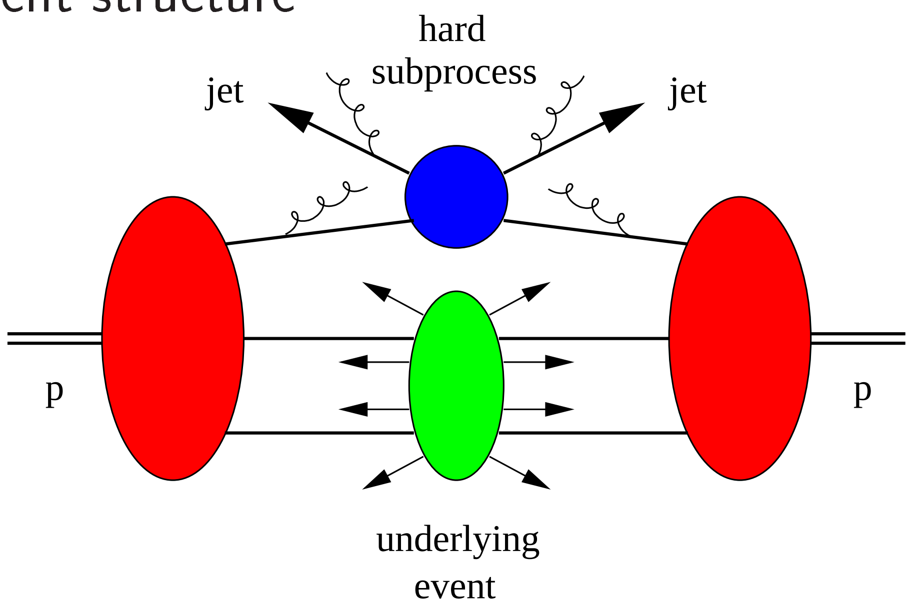
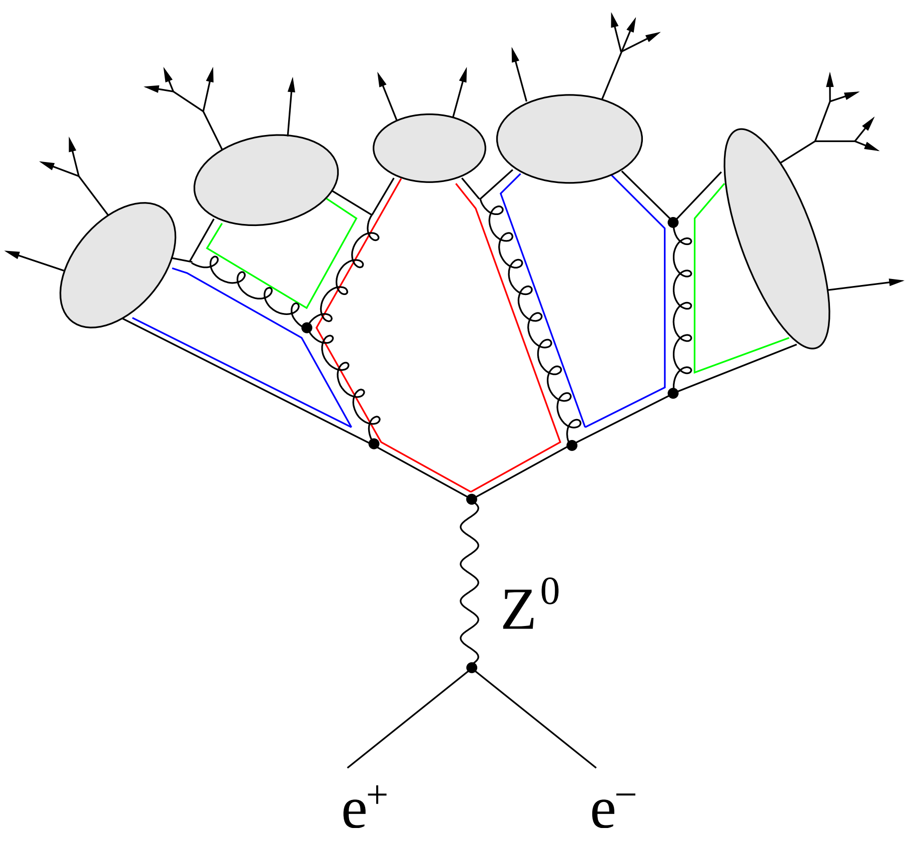
- QCD Event Generators
  - Basic event structure
  - Parton showers and hadronization
- Matrix Element Corrections
  - ‘Hard’ corrections
  - ‘Soft’ corrections
  - Weights vs. events
- Multi-Jet Final States
  - Modified matrix elements
  - Vetoed parton showers
  - Comments/results

QCD Event Generators

- Basic event structure

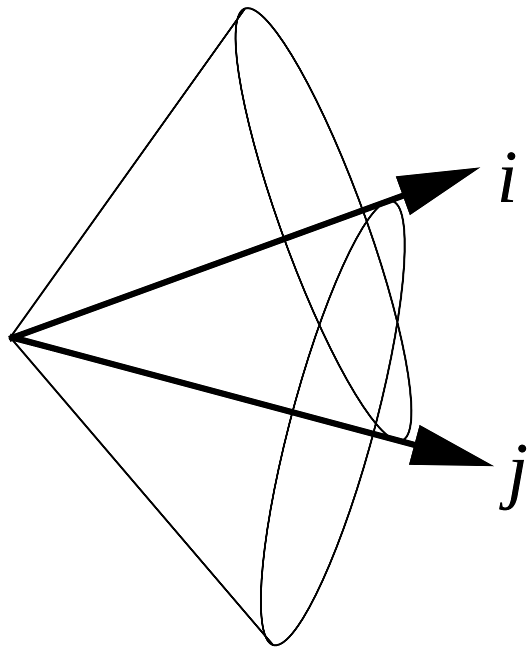


- Parton showers and hadronization

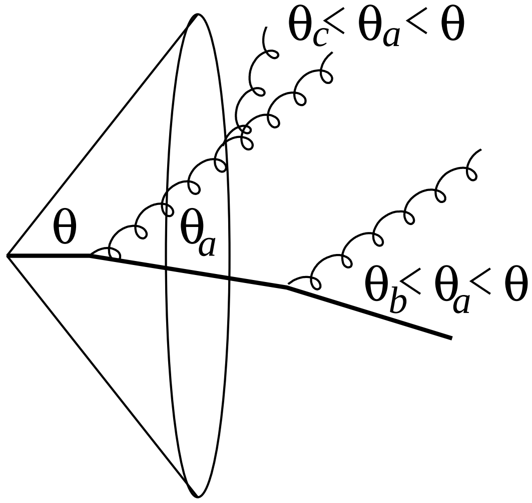


- Colour coherence is simulated by angular ordering:

  - Angular region for shower from parton i is cone bounded by direction of j (and vice-versa), where i and j are colour-connected:

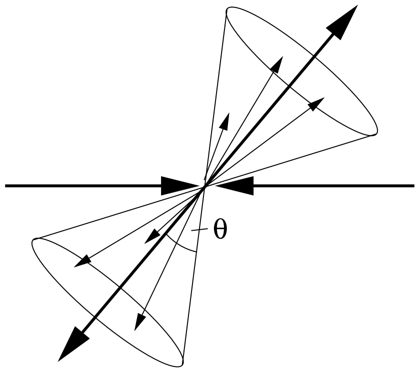


  - Angles within shower decrease away from hard subprocess:

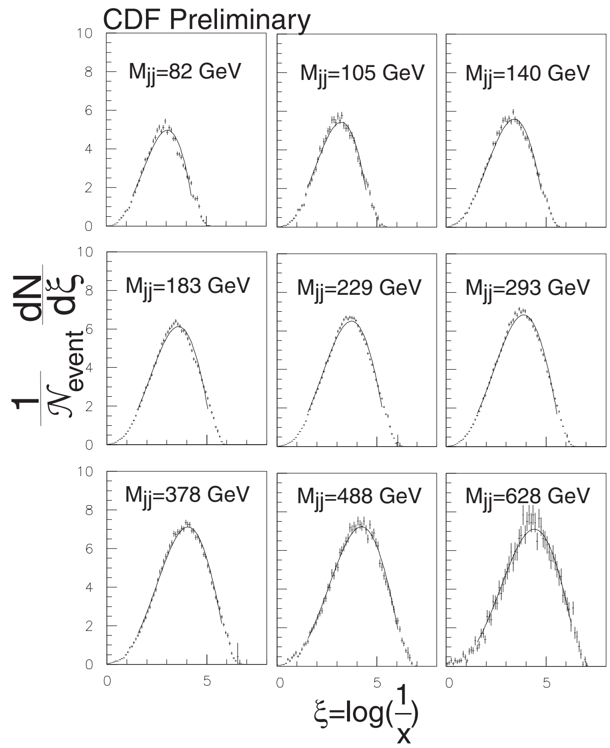


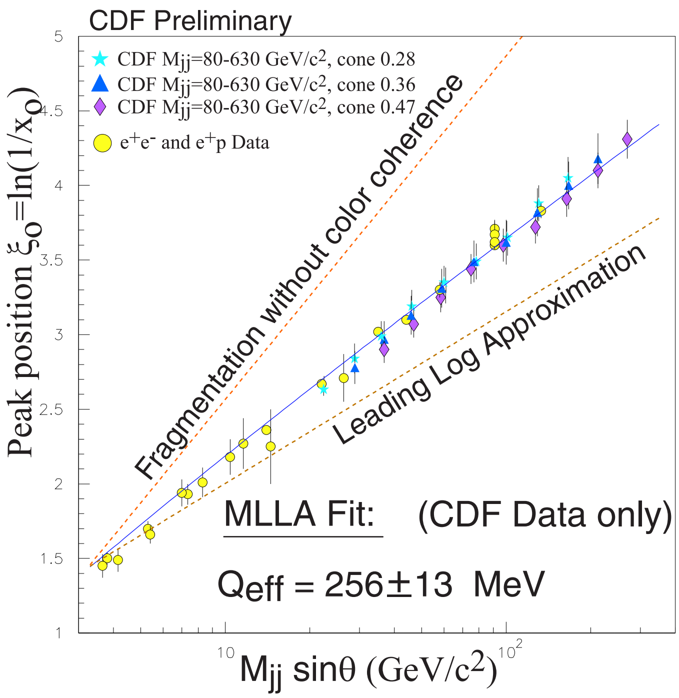
  - This suppresses soft gluon/hadron production (MLLA/LPHD).

- Beautifully confirmed by CDF dijet data:



$$x = p_{\mathrm{had}}/E_{\mathrm{jet}}$$

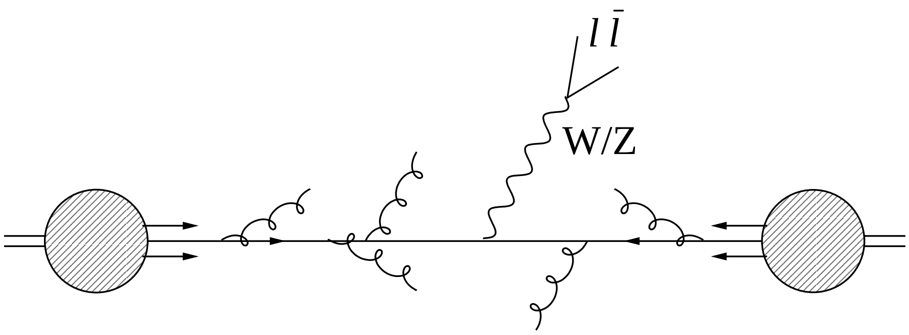




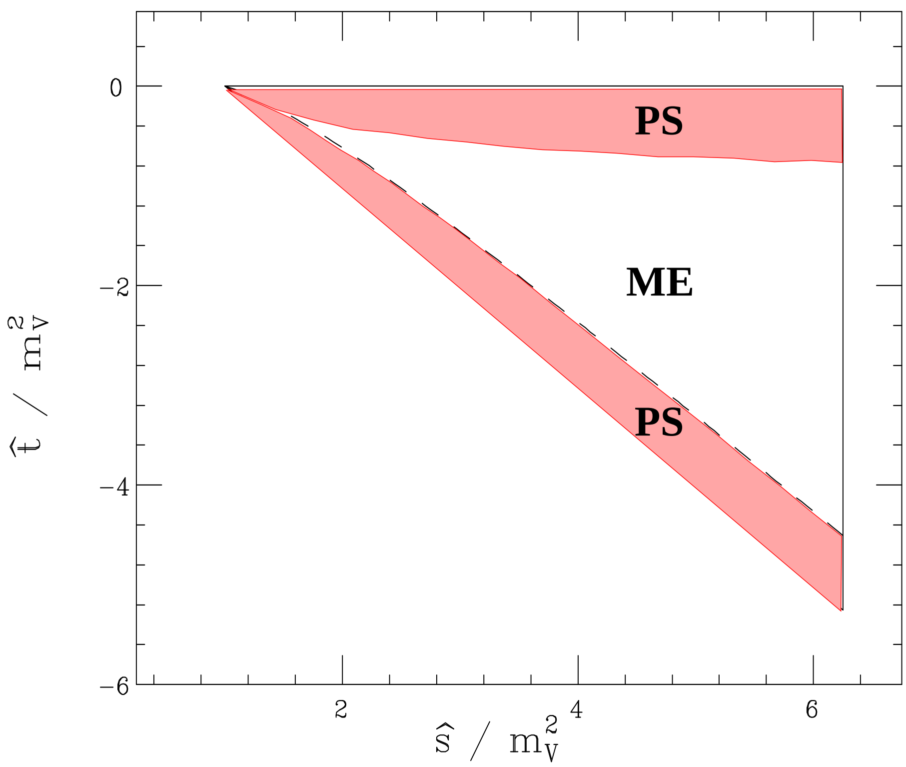
Matrix Element Corrections

- Parton showers inside cones do not populate whole phase space. We also have to include (less singular) matrix element corrections

- For example, in W/Z hadroproduction

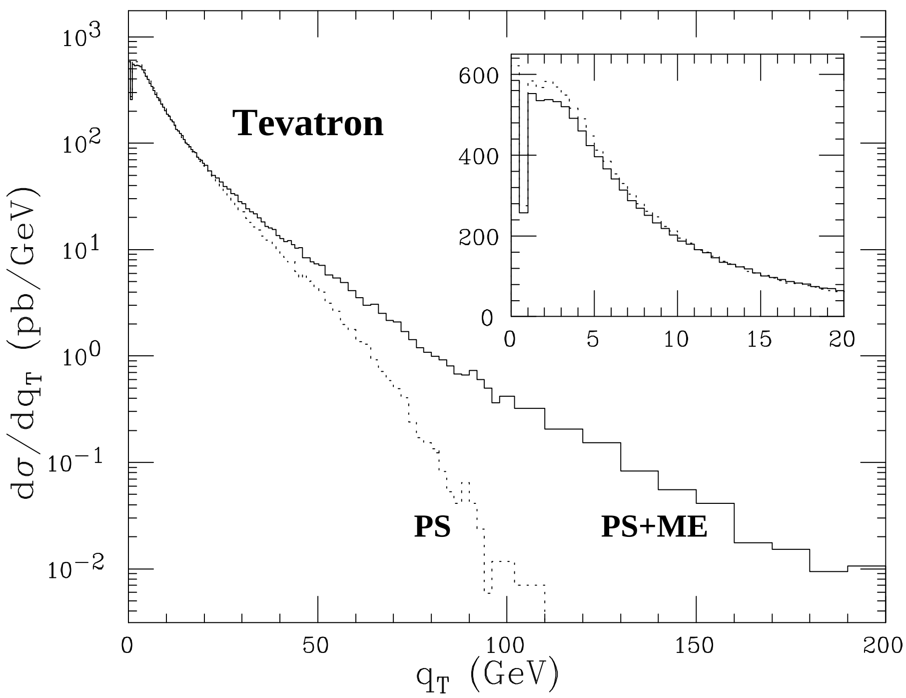


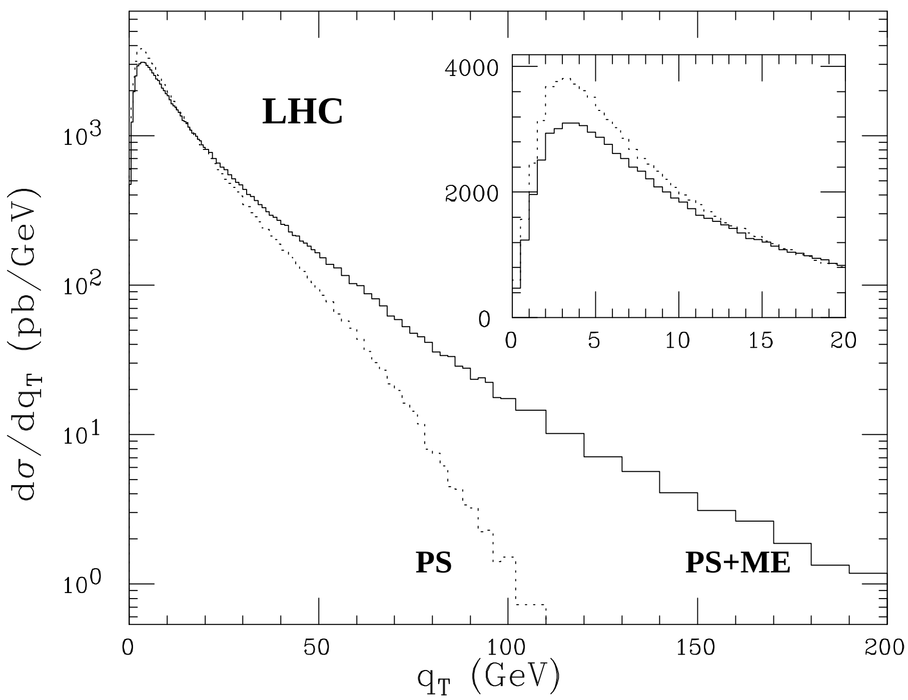
- Phase space for W + jet



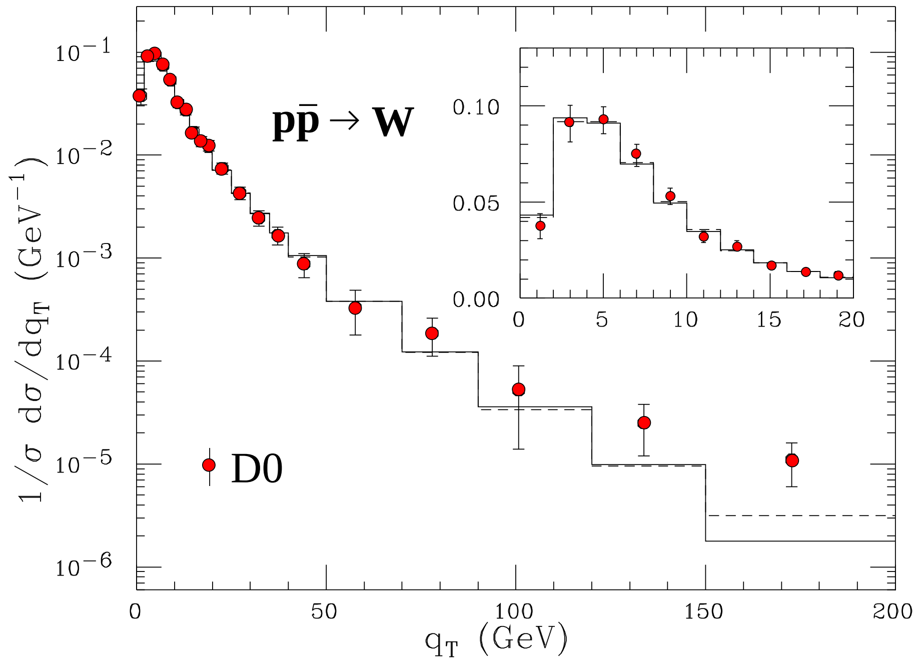
- ME corrections essential for W/Z production at large transverse momentum (important background to new physics):

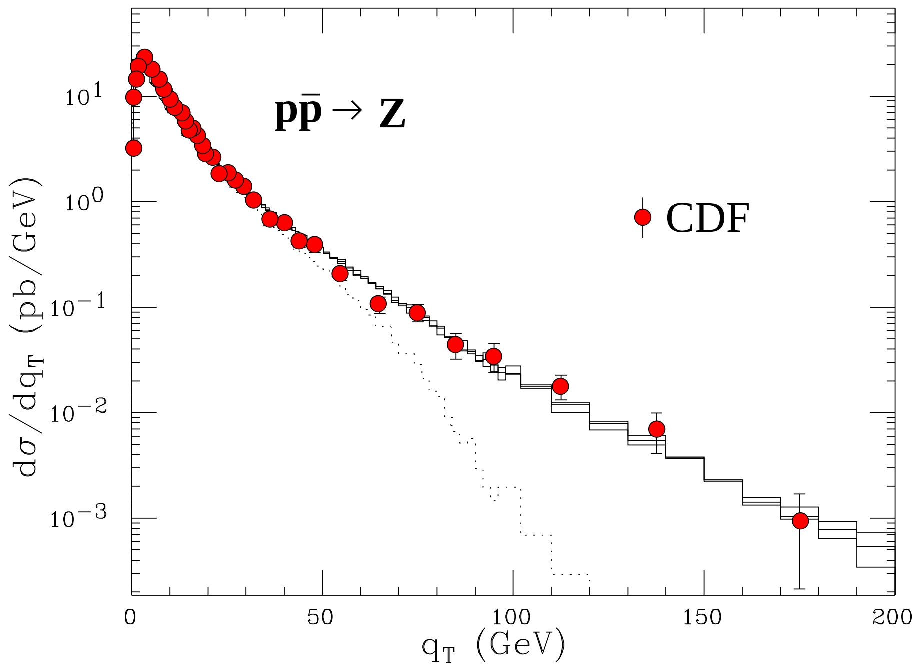
G Corcella & MH Seymour, hep-ph/9908388, 0007295





- Comparisons with Tevatron data:





Weights vs. Events

- Monte Carlo generates weights

$$\sigma = \frac{1}{N_w} \sum_{i=1}^{N_w} w_i \equiv \overline{w}$$

- Error depends on width of weight distribution

$$\frac{\delta\sigma}{\sigma} = \frac{1}{\sqrt{N_w}} \frac{\delta w_{\mathsf{rms}}}{\overline{w}}$$

- Unweighted events generated by 'hit and miss': $w'_i = 0$ or 1 with $P(w'_i = 1) = w_i / w_{\mathsf{max}}$. Then

$$\frac{\delta\sigma}{\sigma} = \sqrt{\frac{w_{\mathsf{max}} - \overline{w}}{N_w \overline{w}}} = \sqrt{\frac{w_{\mathsf{max}} - \overline{w}}{N_e w_{\mathsf{max}}}}$$

  where $N_e = N_w \overline{w} / w_{\mathsf{max}}$ is number of events generated. Time needed (especially for detector simulation) depends mainly on number of events. Hence inefficiency of 'hit and miss' is not a disaster.

- Negative weights can be generated by subtraction procedure for matrix element corrections. These are not a problem of principle but could lead to huge errors and prevent naive 'hit and miss'. Consider creating two samples and using $|w_i|$ for 'hit and miss'?

Multi-Jet Final States

- Simulation of multi-jet final states is a challenging problem in QCD and important for new physics searches.

- Two extreme approaches can be formulated:
  - Use corresponding matrix elements with bare partons representing jets. Must add model for conversion of partons into hadrons; any realistic model will include parton showering, hence extra jet production and potential double counting.
  - Use parton model to generate simplest final state (e.g. $e^+e^- \to q\bar{q}$) and produce addition jets by parton showering. This gives poor simulation of configurations with several widely separated jets.

- Consider combined approach in which matrix element and parton shower domains are separated at some value y_1 of k_T (Durham) jet resolution

$$y_{ij} \equiv 2\min\{E_i^2, E_j^2\}(1 - \cos\theta_{ij})/Q^2$$

- Proposed method has the following features:
  - At $y_{ij} > y_1$ exclusive multi-jet cross sections and distributions are given by matrix elements modified by Sudakov form factors
  - At $y_{ij} < y_1$ they are given by vetoed parton showers
  - To NLL accuracy, y_1 dependence cancels between modified matrix elements and vetoed showers.

- Terminology:
  - NLL means $\alpha_S^n \ln^m y_1$ for all n and $m = 2n, 2n-1$
  - NLO means α_S^n, α_S^{n+1} for lowest relevant n

- Note that we do not claim a complete description of any configuration to NLO in α_S, although this might be possible after subtracting NLO terms from Sudakov form factors (see Collins).

  S Catani, F Krauss, R Kuhn & BRW, in prep.

  See also:

  M Seymour, Comp Phys Comm 90(1995)95

  J Andre and T Sjostrand, Phys Rev D57(1998)5767

  C Friberg and T Sjostrand, hep-ph/9906316

  J Collins, hep-ph/0001040

Modified Matrix Elements

- Exclusive e^+e^- n-jet fractions at c.m. energy Q and k_T-resolution $y_1 = Q_1^2/Q^2$ are given to NLL accuracy by

$$R_2(Q_1,Q) = [\Delta_q(Q_1,Q)]^2$$

$$R_3(Q_1,Q) = 2\,[\Delta_q(Q_1,Q)]^2 \int_{Q_1}^{Q} dq\,\Gamma_q(q,Q)\Delta_g(Q_1,q)$$

$$R_4(Q_1,Q) = 2\,[\Delta_q(Q_1,Q)]^2 \Bigg\{ \int_{Q_1}^{Q} dq\,\Gamma_q(q,Q)\Delta_g(Q_1,q) \int_{Q_1}^{Q} dq'\,\Gamma_q(q',Q)\Delta_g(Q_1,q')$$
$$+ \int_{Q_1}^{Q} dq\,\Gamma_q(q,Q)\Delta_g(Q_1,q) \int_{Q_1}^{q} dq'\,\Gamma_g(q',q)\Delta_g(Q_1,q')$$
$$+ \int_{Q_1}^{Q} dq\,\Gamma_q(q,Q)\Delta_g(Q_1,q) \int_{Q_1}^{q} dq'\,\Gamma_f(q')\Delta_f(Q_1,q') \Bigg\}$$

etc., where

$$\Gamma_q(q,Q) = \frac{2C_F}{\pi}\frac{\alpha_S(q)}{q}\left(\ln\frac{Q}{q} - \frac{3}{4}\right)$$

$$\Gamma_g(q,Q) = \frac{2C_A}{\pi}\frac{\alpha_S(q)}{q}\left(\ln\frac{Q}{q} - \frac{11}{12}\right)$$

$$\Gamma_f(q) = \frac{N_f}{3\pi}\frac{\alpha_S(q)}{q}$$

- $\Delta_{q,g}$ are quark and gluon Sudakov form factors:

$$\begin{aligned}
\Delta_q(Q_1,Q) &= \exp\left(-\int_{Q_1}^{Q} dq\,\Gamma_q(q,Q)\right) \\
\Delta_g(Q_1,Q) &= \exp\left(-\int_{Q_1}^{Q} dq\,\left[\Gamma_g(q,Q)+\Gamma_f(q)\right]\right)
\end{aligned}$$

with

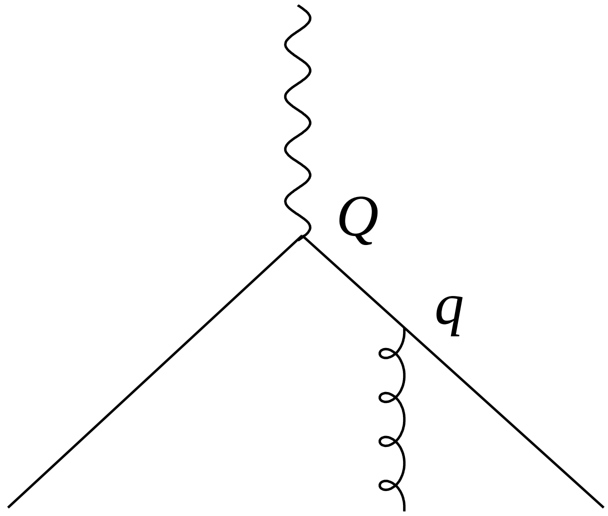
$$\Delta_f(Q_1,Q) = \left[\Delta_q(Q_1,Q)\right]^2 / \Delta_g(Q_1,Q)\ .$$

- Sudakov form factor $\Delta_i(Q_1,Q)$ represents probability for parton of type i to evolve from scale Q to scale Q_1 without any branching (resolvable at scale Q_1). Thus R_2 is probability that produced quark and antiquark both evolve without branching.

- More generally, probability to evolve from scale Q to scale q without branching (resolvable at scale Q_1) is

$$\frac{\Delta_i(Q_1,Q)}{\Delta_i(Q_1,q)}$$

- In R_3, quark q (or antiquark $\bar{q}$) evolve from Q to Q_1 without branching, while $\bar{q}$ (or q) evolves from Q to q then branches, and resulting partons evolve from q to Q_1 without further branching.



- Thus overall NLL probability is

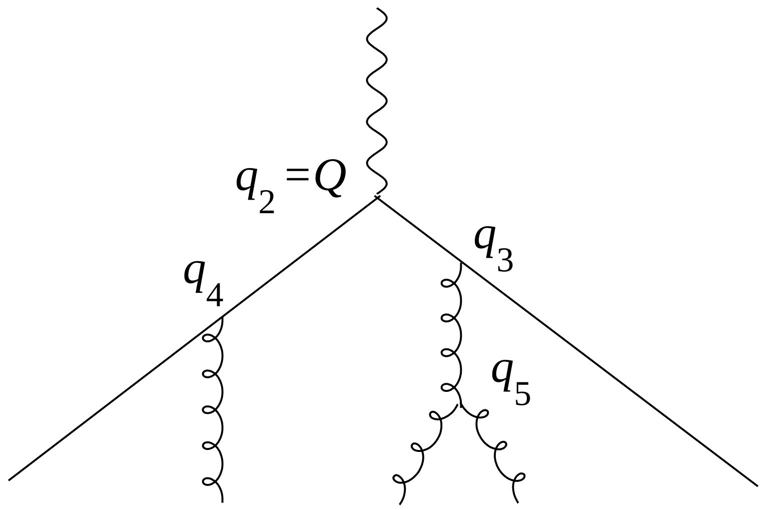
$$2\Delta_q(Q_1,Q)\frac{\Delta_q(Q_1,Q)}{\Delta_q(Q_1,q)}\Gamma_q(q,Q)\Delta_q(Q_1,q)\Delta_g(Q_1,q)$$

  This gives R_3 after integration over $Q_1 < q < Q$.

- Can improve description of 3-jet distributions throughout region $y_{qg}, y_{\bar{q}g} > y_1$ by using full tree-level matrix element squared $|\mathcal{M}_{q\bar{q}g}|^2$ in place of NLL branching probability $\Gamma_q(q,Q)$.

- General procedure for weighting any parton configuration produced by n-parton matrix element squared $|\mathcal{M}_n|^2$:

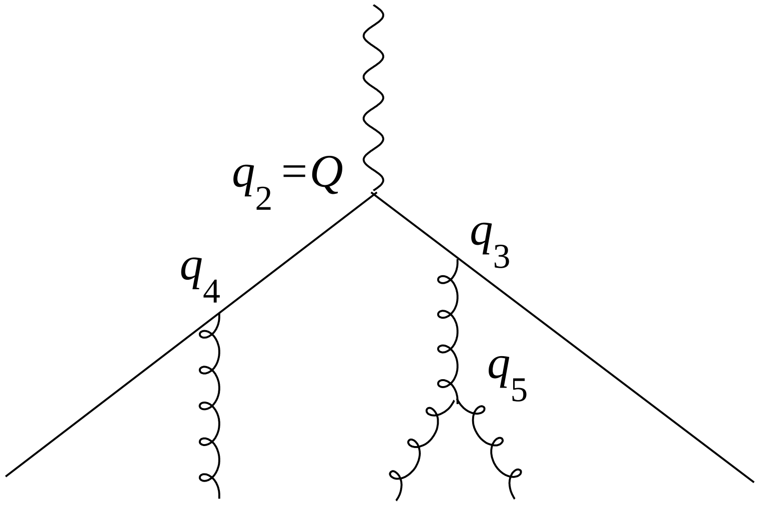
  - First apply k_T-clustering algorithm to determine resolution values $y_2 = 1 > y_3 > \ldots, > y_n > y_1$ at which $2, 3, \ldots, n$ jets are resolved. These give nodal values of $q_j = Q\sqrt{y_j}$ for tree diagram that specifies k_T-clustering sequence for that configuration.



  - For for parton type i joining nodes at q_j and $q_k < q_j$, apply Sudakov weight factor

$$\frac{\Delta_i(Q_1, q_j)}{\Delta_i(Q_1, q_k)}$$

  - Weight for external line from node at q_j is $\Delta_i(Q_1, q_j)$.
  - Arguments of α_S in $|\mathcal{M}_n|^2$ are $\alpha_S(q_3)\alpha_S(q_4)\cdots\alpha_S(q_n)$.



- For example above clustering sequence for $e^+e^- \to q\bar{q}ggg$ has Sudakov weight factor

$$
\begin{aligned}
& \frac{\Delta_q(Q_1,Q)}{\Delta_q(Q_1,q_3)}\frac{\Delta_q(Q_1,Q)}{\Delta_q(Q_1,q_4)}\frac{\Delta_g(Q_1,q_3)}{\Delta_g(Q_1,q_5)} \\
& \times\Delta_q(Q_1,q_3)\Delta_q(Q_1,q_4)\Delta_g(Q_1,q_4)[\Delta_g(Q_1,q_5)]^2 \\
& = \ [\Delta_q(Q_1,Q)]^2\Delta_g(Q_1,q_3)\Delta_g(Q_1,q_4)\Delta_g(Q_1,q_5)
\end{aligned}
$$

- Note that this is independent of topology and colour structure of graph – depends only on nodal scale values $\{q_i\}$, which are purely kinematic.

Vetoed Parton Showers

- Having generated multijet distributions above resolution value y_1 according to matrix elements modified by form factors, it remains to generate distributions at lower values by means of parton showers.

- This should be done in such a way that dominant (LL and NLL) dependence on arbitrary parameter y_1 cancels. Residual dependence on y_1 could be used to tune less singular terms for optimal agreement with data.

- Note that y_1 must set upper limit on interparton separations y_{ij} generated in the showers. Otherwise exclusive jet rates at resolution y_1 could be changed by showering.

- At first sight, this might suggest we should evolve showers from scale $Q_1 = Q\sqrt{y_1}$ instead of Q. However, this would not lead to cancellation of $\log y_1$ dependence.

- Consider, for example, 2-jet rate at resolution $y_0 = Q_0^2/Q^2 < y_1$. If we start from R_2 at scale Q_1 and then evolve from Q_1 to Q_0, we obtain

$$[\Delta_q(Q_1, Q)\Delta_q(Q_0, Q_1)]^2$$

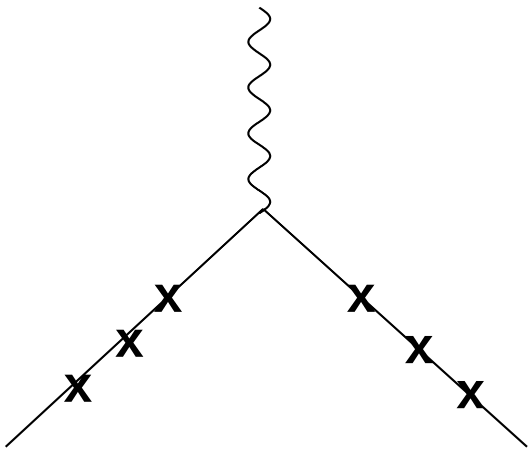
instead of correct result

$$R_2(Q_0, Q) = [\Delta_q(Q_0, Q)]^2$$

- This is because, although y_{ij} values in showers are limited by y_1, the angular regions in which they evolve still correspond to scale Q rather than Q_1.

- Consequently we should allow showers to evolve from scale Q but veto branchings with scales $q > Q_1$ — i.e. selected branching is forbidden but parton has its scale reset to q for subsequent branching.

- Two-jet rate at any scale $Q_0 < Q_1$ is now given by sum of probabilities of $0, 1, 2, \ldots$ vetoed branchings (and no actual resolved branchings):



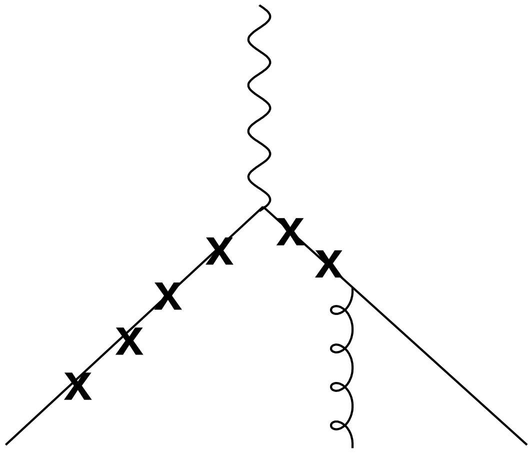
- Sum for quark is

$$\Delta_q(Q_1, Q)\Delta_q(Q_0, Q)\left\{1 + \int_{Q_1}^{Q} dq\,\Gamma_q(q, Q) + \int_{Q_1}^{Q} dq\,\Gamma_q(q, Q)\int_{Q_1}^{q} dq'\,\Gamma_q(q', Q) + \cdots\right\}$$

- Series sums to $1/\Delta_q(Q_1, Q)$, cancelling y_1 dependence and giving $\Delta_q(Q_0, Q)$. Similarly for antiquark.

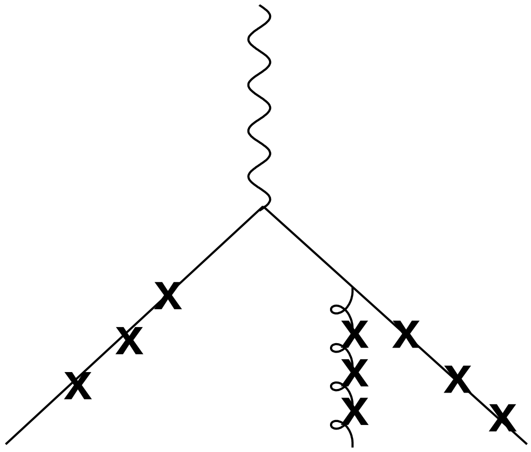
- For 3-jet rate at scale Q_0 there are two possibilities: either event was 2-jet at scale Q_1 and then has one branching resolved at scale Q_0, or it was 3-jet at scale Q_1 and remains so at scale Q_0.

- Probability of first case is



$$\left[\Delta_q(Q_1,Q)\frac{\Delta_q(Q_0,Q)}{\Delta_q(Q_1,Q)}\right]^2 \int_{Q_0}^{Q_1} dq\,\Gamma_q(q,Q)\Delta_g(Q_0,q)$$

- Probability of second case is

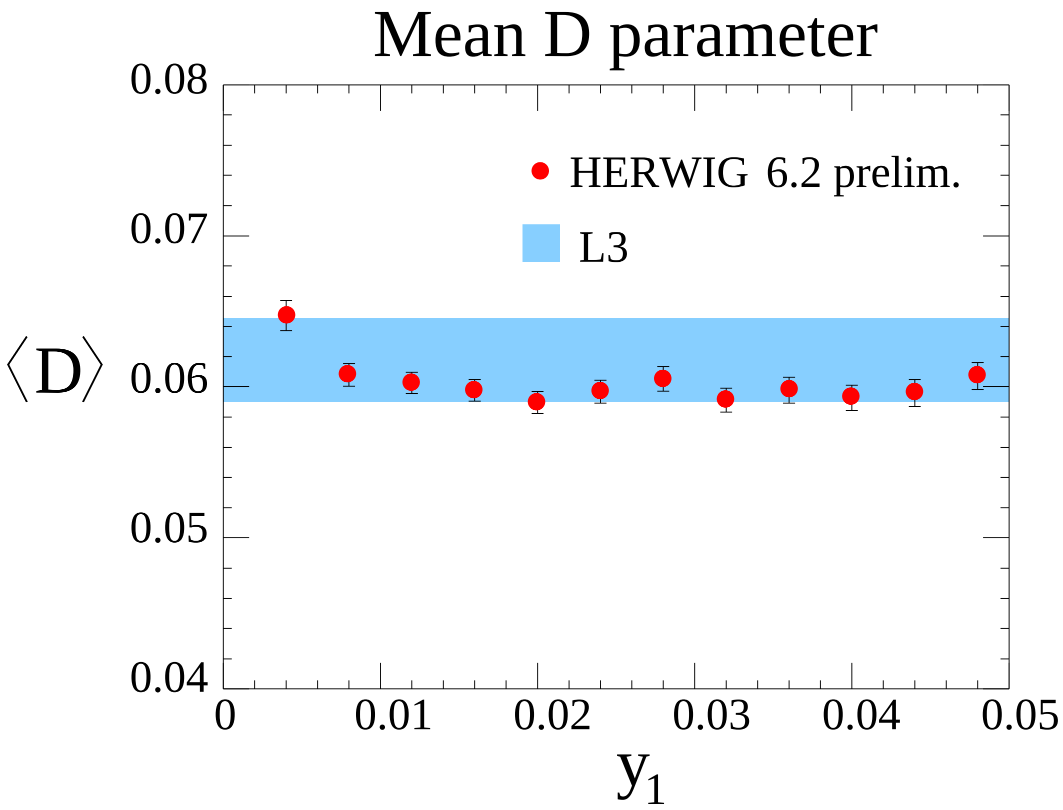


$$\left[\Delta_q(Q_1,Q)\frac{\Delta_q(Q_0,Q)}{\Delta_q(Q_1,Q)}\right]^2 \int_{Q_1}^{Q} dq\Gamma_q(q,Q)\Delta_g(Q_1,q)\frac{\Delta_g(Q_0,q)}{\Delta_g(Q_1,q)}$$

- Sum is indeed y_1-independent and equal to $R_3(Q_0,Q)$. Similarly for higher jet multiplicities.

Comments/Results on Multijets

- Four-jet event shapes insensitive to y_1 in pilot HERWIG study



- Note that in principle we need matrix elements $|\mathcal{M}_n|^2$ for $y_{ij} > y_1$ at all values of n. In practice, if we have $n \leq N$, then y_1 must be chosen large enough for $R_{n>N}(Q_1, Q)$ to be negligible ($y_1 > 0.004$ for $N = 4$, see above).

- This approach is being implemented with $N = 5$ in the e^+e^- event generator APACIC++

  F Krauss, R Kuhn and G Soff, hep-ph/9909572

- Should be possible to extend to lepton-hadron and hadron-hadron.

- Extension to NLO may be possible (Collins).

APACIC++ preliminary results: