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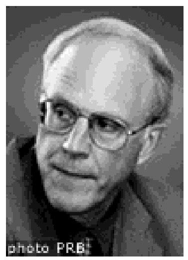
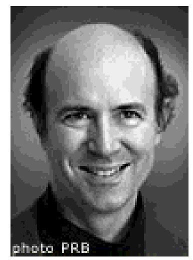
The Nobel Prize in Physics 2004

"for the discovery of asymptotic freedom in the theory of the strong interaction"

		
David J. Gross	**H. David Politzer**	**Frank Wilczek**
1/3 of the prize	1/3 of the prize	1/3 of the prize
USA	USA	USA
Kavli Institute for Theoretical Physics, University of California Santa Barbara, CA, USA	California Institute of Technology Pasadena, CA, USA	Massachusetts Institute of Technology (MIT) Cambridge, MA, USA
b. 1941	b. 1949	b. 1951

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2003

The 2004 Prize in:
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Asymptotic Freedom: History and Implications

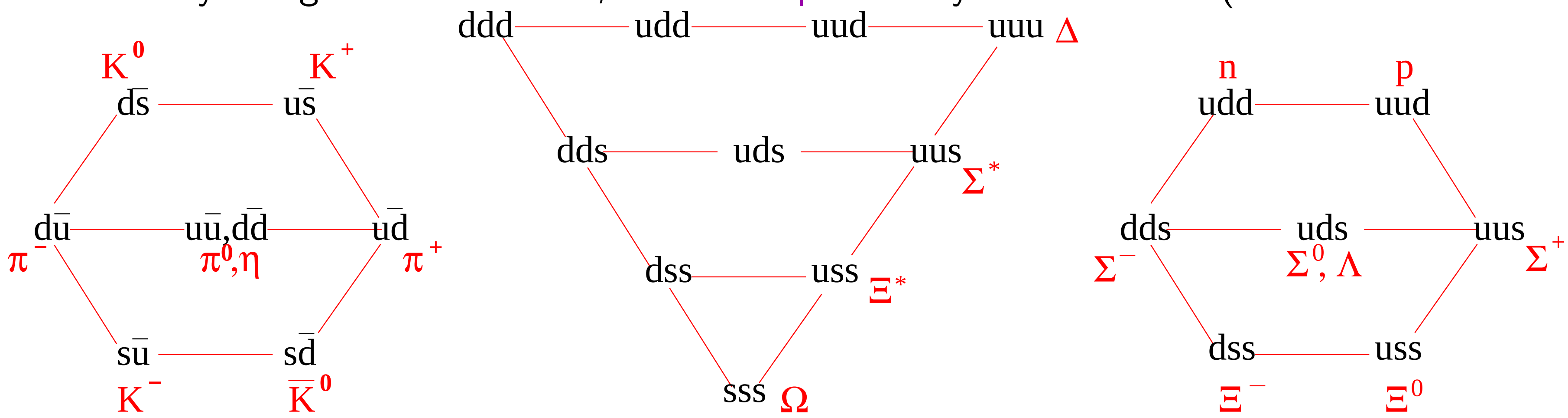
Bryan Webber
University of Cambridge
Cavendish Laboratory

- Quantum Chromodynamics
  - Quarks
  - Colour
- Asymptotic Freedom
  - Effective charge
  - Non-Abelian fields
  - History
- Implications
  - Confinement
  - Grand unification?
- Conclusions

RP Crease and CC Mann, *The Second Creation*, Macmillan 1986
L Hoddeson et al., *The Making of the Standard Model*, CUP 1997

Quark Model of Hadrons

- In the early 1960's, many new strongly-interacting particles (hadrons) were discovered, all apparently as 'fundamental' as the familiar proton, neutron and π-meson.
- In 1964, Gell-Mann and Zweig (independently) noticed that the quantum numbers of all the known hadrons corresponded to those of collections of 2 or 3 spin one-half, fractionally-charged constituents, called quarks by Gell-Mann (Nobel Prize 1969).



  - Zweig (unpublished CERN preprint) called them 'aces'.
  - Quarks were regarded as 'mathematical' entities because they were not seen individually.
  - Also they seemed to violate the spin-statistics theorem: some hadrons corresponded to totally symmetric combinations of 3 identical quarks.

Colour

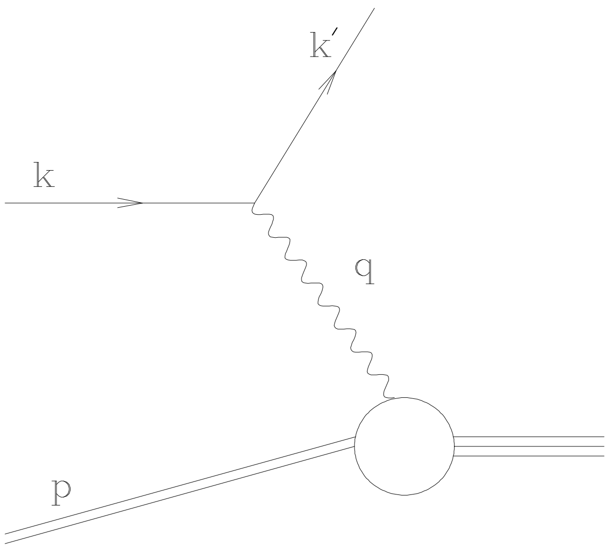
- Han and Nambu (1965, also Greenberg) solved the spin-statistics problem by proposing that quarks have a new "colour" degree of freedom which can take 3 values (red, green, blue), with respect to which the 'symmetrical' quark states are antisymmetric. Associated symmetry group SU(3).
  - Originally thought of as a *global* symmetry,

$$q_a \longrightarrow q_a' = \mathcal{U}_{ab}\, q_b$$

    where $a = 1, 2, 3$ and the symmetry transformations are represented by constant (i.e. space-time independent) 3×3 unitary matrices $\mathcal{U}$, which mix the colours while preserving the normalization.
  - Observed states $q\bar{q}$, qqq are colour singlets.
  - It was necessary to postulate that non-singlet states (q, qq, $(q\bar{q})_8$, . . .) are *forbidden*.
- The notion of quarks as 'real' rather than 'mathematical' constituents of hadrons was considered quite implausible . . . until amazing experimental data started arriving from the new Stanford Linear Accelerator Centre (SLAC).

Deep inelastic electron scattering

- The SLAC experiments (1967) probed the structure of the proton by scattering high-energy electrons



- Useful dimensionless variables are:

$$x = \frac{-q^2}{2p \cdot q} = \frac{Q^2}{2M(E - E')}\ , \qquad y = \frac{q \cdot p}{k \cdot p} = 1 - \frac{E'}{E}$$

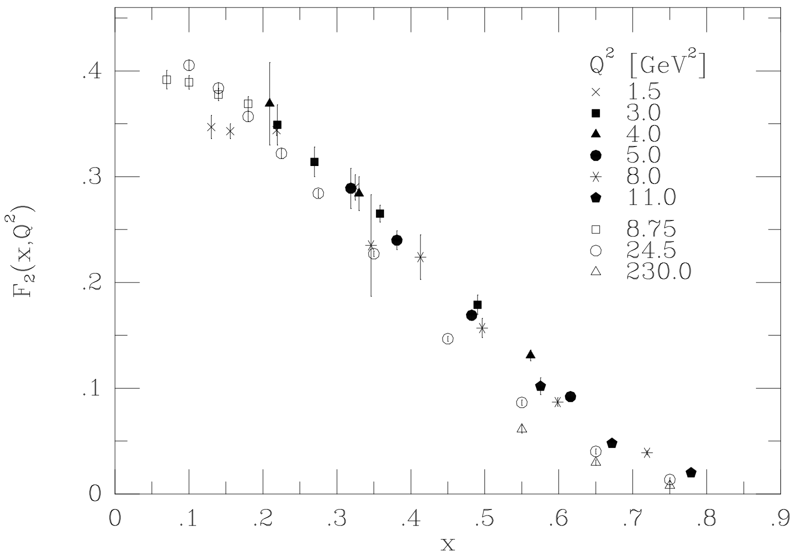
  where $Q^2 = -q^2 > 0$, $M =$ proton mass, and energies refer to target rest frame.
- Elastic scattering has $x = 1$. Deep inelastic scattering (DIS) means $Q^2 \gg M^2$ and $x < 1$.

- Structure functions $F_i(x, Q^2)$ parametrise target structure as 'seen' by virtual photon:

$$\frac{d^2\sigma}{dxdy} = \frac{8\pi\alpha^2 ME}{Q^4}\left[\left(\frac{1+(1-y)^2}{2}\right)2xF_1 + (1-y)(F_2 - 2xF_1) - (M/2E)xyF_2\right]$$

- Bjorken limit is $Q^2 \to \infty$ and $p \cdot q \to \infty$ with x fixed. In this limit structure functions were found to obey approximate Bjorken scaling (1969), i.e. they depend only on dimensionless variable x:

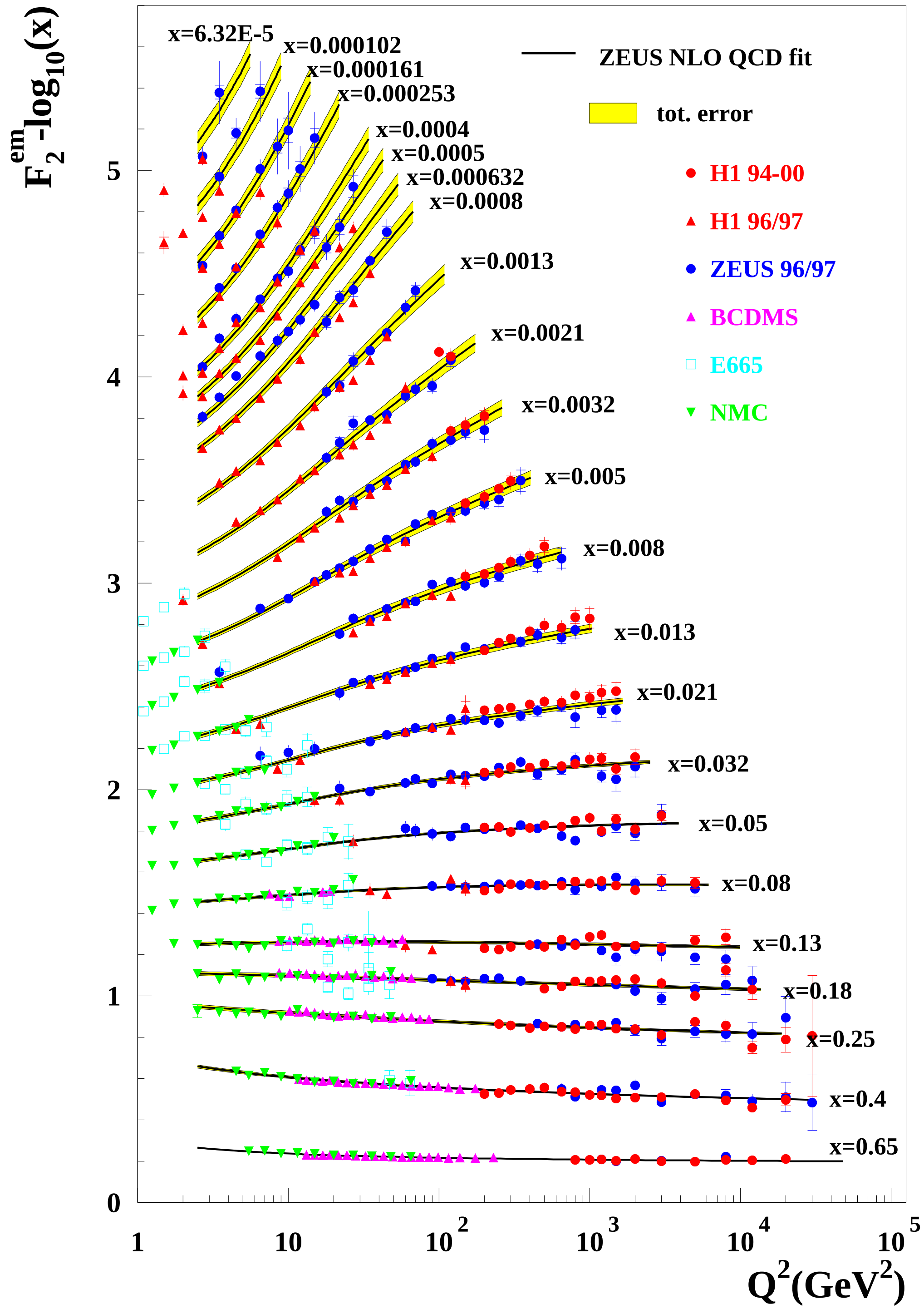
$$F_i(x, Q^2) \longrightarrow F_i(x)$$



- Although Q^2 varies by two orders of magnitude, in first approximation data lie on universal curve.

Implications of Bjorken scaling

- Bjorken scaling implies that virtual photon is scattered by *pointlike constituents* – partons (Feynman, 1969) — otherwise structure functions would depend on ratio Q/Q_0, with $1/Q_0$ a length scale characterizing size of constituents.
- Quantitative study over the next few years established that these pointlike partons had spin one-half and fractional charges, consistent with those expected for the quarks of Gell-Mann and Zweig.
  – However, the quark were found to carry only about one-half of the momentum of the target proton
  – The other half must be carried by strongly-interacting, neutral, bosonic constituents. . .
- Modern data show weak (logarithmic) scaling violation, understood as a calculable higher-order effect.



$F_2^{em}-\log_{10}(x)$
$Q^2(\mathrm{GeV}^2)$
ZEUS NLO QCD fit
tot. error
H1 94-00
H1 96/97
ZEUS 96/97
BCDMS
E665
NMC
x=6.32E-5
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65

Yang-Mills theory

- Back in 1954, Yang & Mills had shown that one could extend the notion of local gauge invariance from the *Abelian* U(1) symmetry group of QED:

$$\psi_e(x) \to \psi'_e(x) = U(x)\,\psi_e(x)$$

where $U(x) = e^{i\phi(x)}$, to a *non-Abelian* symmetry group:

$$\psi_a(x) \to \psi'_a(x) = \mathcal{U}_{ab}(x)\,\psi_b(x)$$

where $a = 1, \ldots$.N and $\mathcal{U}(x) \in$ SU(N) is a *space-time dependent* N$\times$N unitary matrix, which mixes the N states ψ_a while preserving the normalization.
  - As in QED, gauge invariance then requires the existence of a vector gauge field
  - Quanta of the gauge field would be massless spin-1 particles, analogous to the photon, communicating *long-range forces*
- In 1954, this did not fit with the properties of either strong or weak nuclear interactions. So Yang-Mills theory was ignored.
- Even QED was regarded as a sick theory (see later), and quantum field theory went out of fashion for about 15 years. . .

AND THEN . . .

- The SLAC results prompted renewed interest in a field theory of the strong interactions of pointlike quarks.
- Fritzsch & Gell-Mann (1971, also Weinberg) proposed that the strong interaction is an SU(3) Yang-Mills gauge theory: Quantum ChromoDynamics.
  - Quanta of the colour field (gluons) would communicate the strong force between quarks
  - Gluon exchange forces would be attractive in colour singlet states
  - Gluons could also carry the 'missing momentum' of the proton
- But why should quarks behave like almost-free particles in DIS?
- And why should colour non-singlet states be forbidden?

Quantum Chromodynamics

- QCD is an SU(3) gauge field theory: quarks come in 3 "colours" ($a = 1, 2, 3$) and the gauge transformations are represented by 3×3 unitary matrices $\mathcal{U}$, which mix the colours while preserving the normalization:

$$
\begin{aligned}
q_a(x) \to q'_a(x) &= \mathcal{U}_{ab}(x)\, q_b(x) \\
\mathcal{U}_{ab}(x) &= e^{i\boldsymbol{t}\cdot\boldsymbol{\theta}(x)} \\
\boldsymbol{t}\cdot\boldsymbol{\theta} &\equiv \sum_{A=1}^{8} t^A \theta^A
\end{aligned}
$$

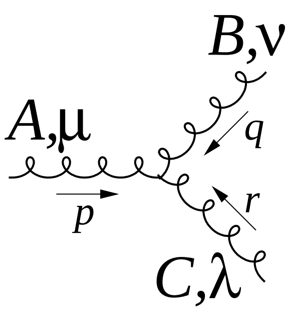
  where θ^A are 8 real parameters and t^A, the generators of SU(3), are the 8 linearly independent 3×3 traceless hermitian matrices.

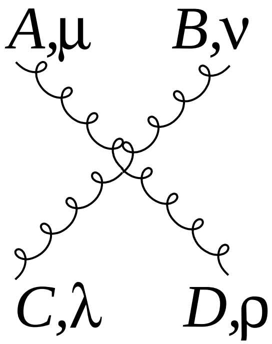
  - Conventional normalization is $\mathrm{Tr}\,(t^A t^B) = \frac{1}{2}\delta_{AB}$
  - The theory is non-Abelian, i.e., successive gauge transformations do not commute: $[t^A, t^B] = i f^{ABC} t^C$ where the structure constants of the gauge group, f^{ABC}, are totally antisymmetric.
  - $\boldsymbol{t}\cdot\boldsymbol{t} = C_F\, I$ where $C_F = \frac{4}{3}$ is the "quark colour charge squared"
  - $\sum_{A,B} f^{ABC} f^{ABD} = C_A\, \delta_{CD}$ where $C_A = 3$ is the "gluon colour charge squared"

- The QCD Lagrangian density is

$$\mathcal{L}_{\mathsf{QCD}} = -\tfrac{1}{4} F^A_{\mu\nu} F^{A\mu\nu} + \sum_{\text{flavours } f} \bar{q}^f_a \left[i\gamma^\mu (D_\mu)_{ab} - m_f\, \delta_{ab} \right] q^f_b$$

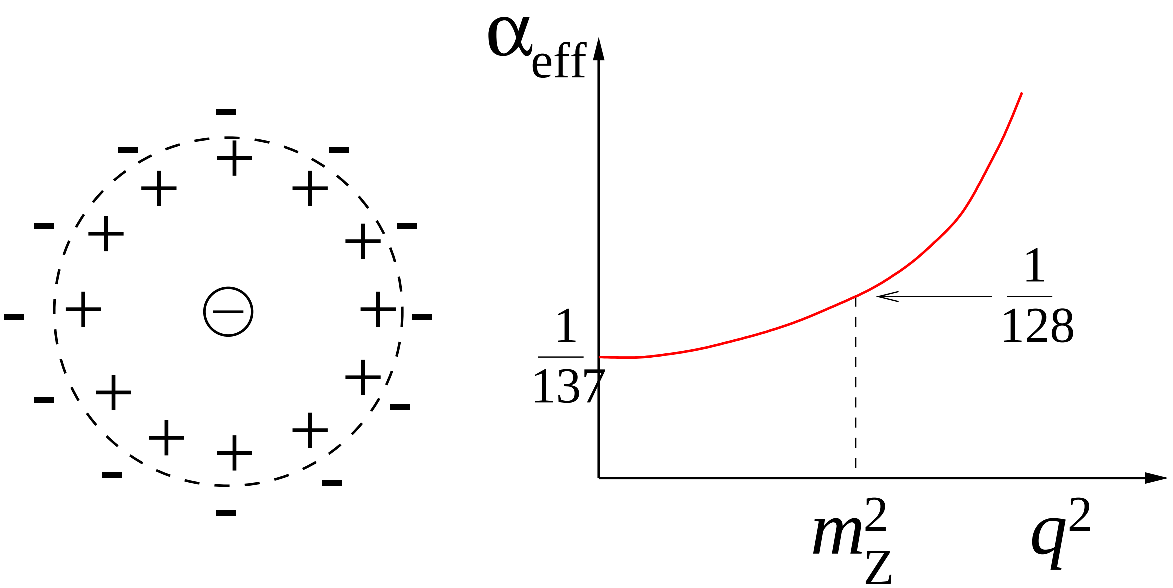
  - Gluon field strength tensor $F^A_{\mu\nu} = \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\mu - g_{\mathsf{S}} f^{ABC} \mathcal{A}^B_\mu \mathcal{A}^C_\nu$
  - Covariant derivative $(D_\mu)_{ab} = \partial_\mu\, \delta_{ab} + i g_{\mathsf{S}} \, (\boldsymbol{t} \cdot \boldsymbol{\mathcal{A}}_\mu)_{ab}$
  - $\mathcal{A}^A_\mu$ are the gluon fields $(A = 1, \ldots, 8)$
  - g_{S} is the QCD coupling; by analogy with QED we define $\alpha_{\mathsf{S}} \equiv g^2_{\mathsf{S}}/4\pi$
- The third term in the gluon field strength tensor is essential for gauge invariance of $\mathcal{L}_{\mathsf{QCD}}$. Notice that it would vanish for an Abelian (commuting) gauge group ($f^{ABC} = 0$).
- As a consequence of this term, there are 3-gluon and 4-gluon self-interactions:



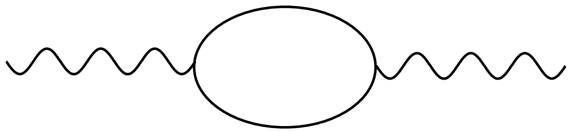


Effective Charge

- In QED the observed electron charge is distance-dependent ($\Rightarrow$ momentum transfer dependent) due to vacuum polarization:

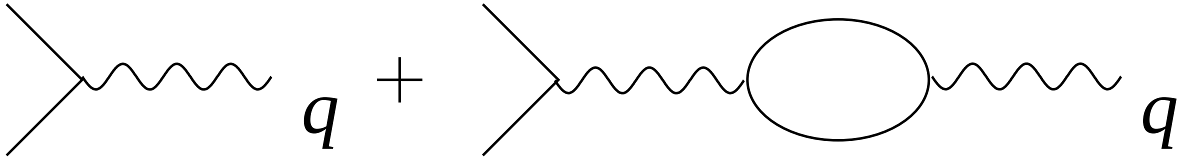


- The one-loop vacuum polarization diagram



is log-divergent and can be regularized by introducing a cut-off Λ.

- Then the observed coupling at momentum transfer q^μ is



$$\alpha(q^2) = \alpha_{\text{bare}} \left(1 + \frac{\alpha}{3\pi} \ln \frac{q^2}{\Lambda^2} + \cdots \right)$$

Thus

$$q^2 \frac{d\alpha}{dq^2} = \frac{\alpha^2}{3\pi} + \cdots \equiv \beta_{\text{QED}}(\alpha)$$

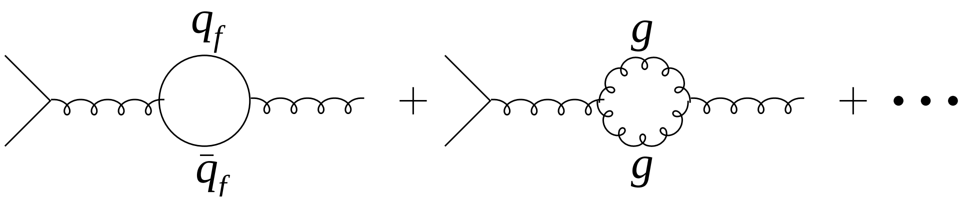
where the QED β-function $\beta_{\text{QED}}(\alpha) > 0$. At one-loop order, we find

$$\begin{aligned} \int_{1/137}^{\alpha(Q^2)} \frac{d\alpha}{\alpha^2} &= \frac{1}{3\pi} \int_{m_e^2}^{Q^2} \frac{dq^2}{q^2} = \frac{1}{3\pi} \ln \frac{Q^2}{m_e^2} \\ \Rightarrow \quad \alpha(Q^2) &= \frac{1}{137 - \frac{1}{3\pi} \ln \frac{Q^2}{m_e^2}} \end{aligned}$$

- Therefore $\alpha \to \infty$ at $Q^2 = e^{411\pi} m_e^2$ and is not defined at higher scales (shorter distances).
- This led to the view (Landau, 1954) that QED is not a well-defined theory, and more generally to a lack of confidence in quantum field theory, which lasted for about 15 years.

Asymptotic Freedom

- In QCD there are additional contributions from gluon self-interaction:



As a consequence, the β-function changes. Now

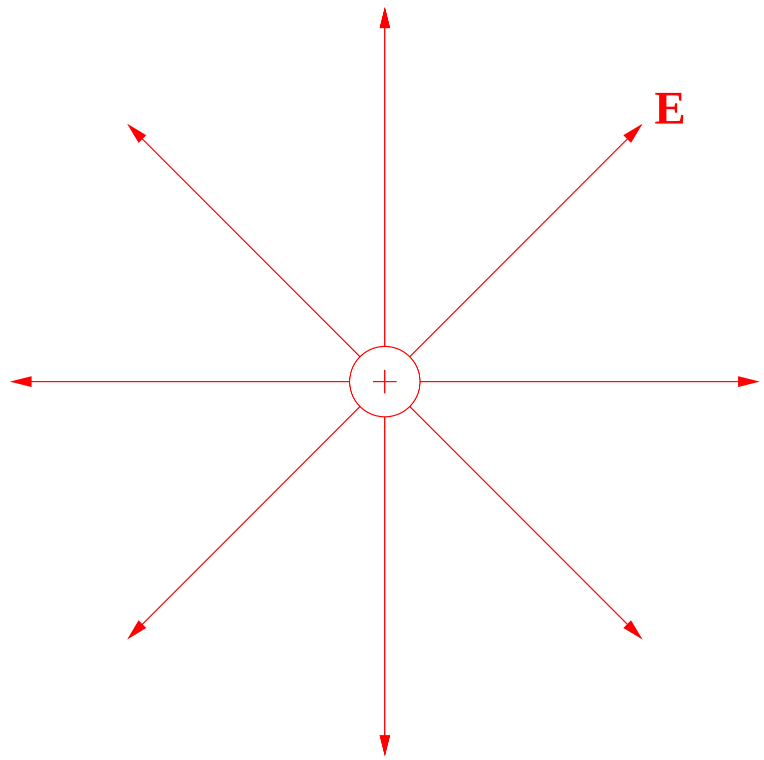
$$\alpha_S(q^2) = \alpha_{S,\text{bare}} \left(1 + \frac{n_f}{6\pi} \alpha_S \ln \frac{q^2}{\Lambda^2} - \frac{11}{12\pi} C_A \alpha_S \ln \frac{q^2}{\Lambda^2} - \cdots \right)$$

where n_f is the number of quark flavours. Hence $\beta_{\text{QCD}}(\alpha_S) = \beta_0 \alpha_S^2 - \cdots$ where

$$\beta_0 = \frac{1}{12\pi}(2n_f - 11C_A) < 0$$

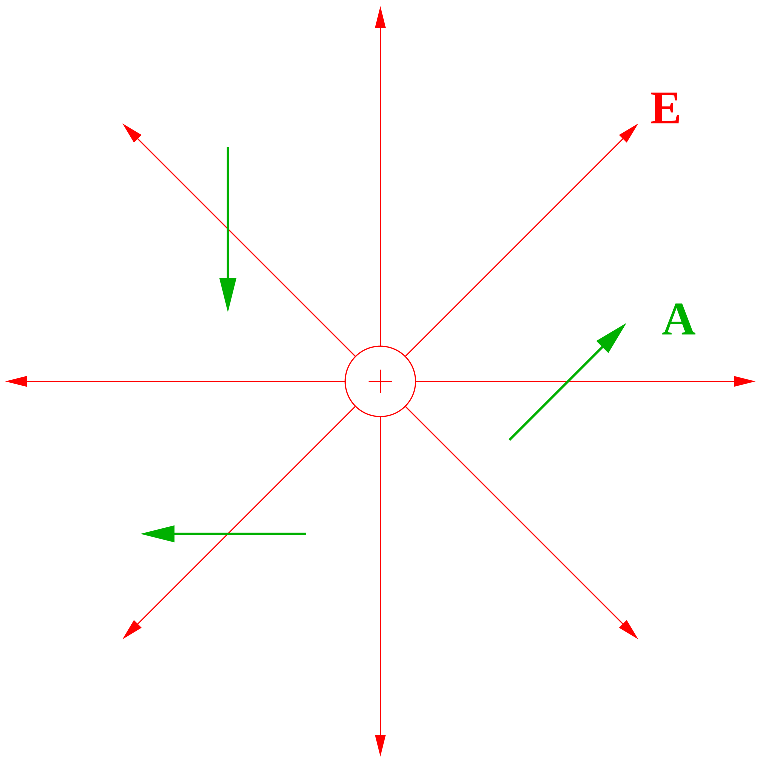
- The gluonic contribution to the vacuum polarization reverses the sign of the β-function, so that the strong coupling α_S decreases at large q^2 (short distances). This is called asymptotic freedom. It implies that quarks behave as (almost) free particles at short distances, and perturbation theory can be used for hard processes, i.e. processes involving large momentum transfers, such as DIS.

Non-Abelian Vacuum Polarization



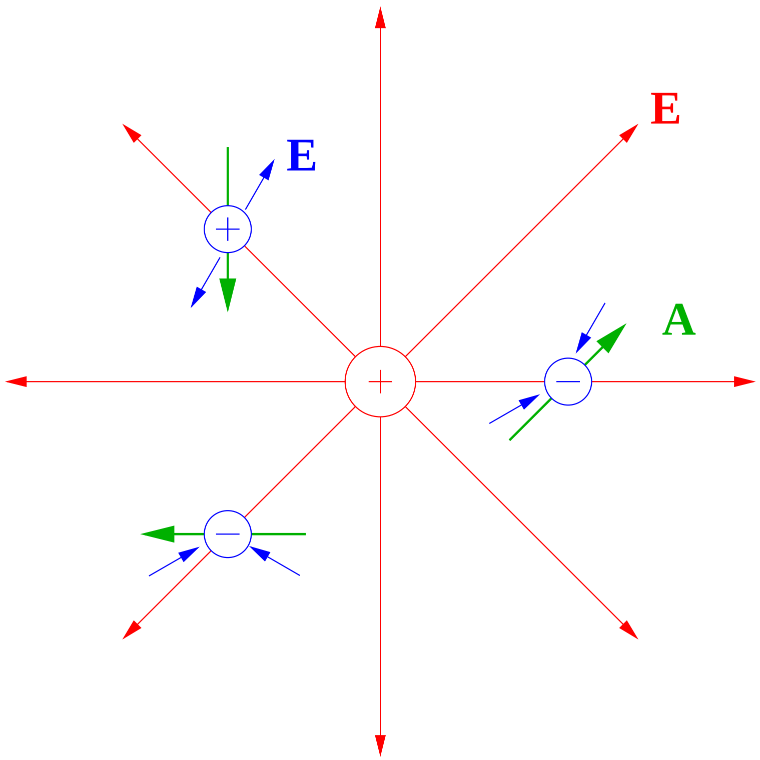
$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = g\,\delta^3(\boldsymbol{r}) + g\,(\boldsymbol{A} \cdot \boldsymbol{E} - \boldsymbol{A} \cdot \boldsymbol{E})$$

Non-Abelian Vacuum Polarization



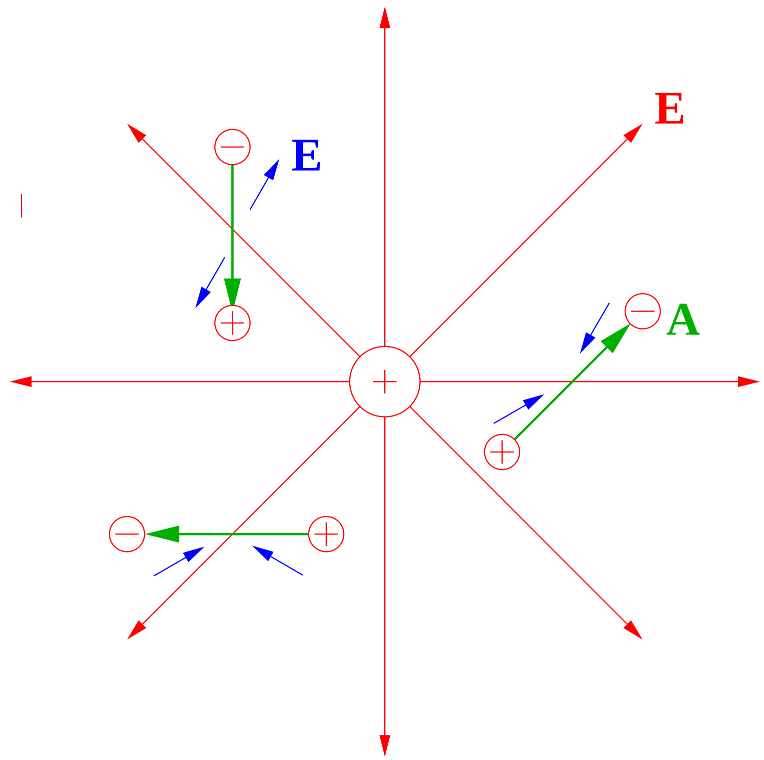
$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = g\,(\boldsymbol{A}\cdot\boldsymbol{E} - \boldsymbol{A}\cdot\boldsymbol{E})$$

Non-Abelian Vacuum Polarization



$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = g\,(\boldsymbol{A} \cdot \boldsymbol{E} - \boldsymbol{A} \cdot \boldsymbol{E})$$

Non-Abelian Vacuum Polarization



$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = g\,\delta^3(\boldsymbol{r}) + g\,(\boldsymbol{A} \cdot \boldsymbol{E} - \boldsymbol{A} \cdot \boldsymbol{E})$$

The Vacuum as a Magnetic Medium

- In a dielectric, screening of the effective charge corresponds to $\varepsilon > 1$, anti-screening to $\varepsilon < 1$.
- Since $\varepsilon\mu = 1$ for the vacuum, anti-screening implies magnetic susceptibility $\chi = \mu - 1 > 0$, i.e. a paramagnetic vacuum.
- Vacuum polarization due to quanta of spin S gives a vacuum susceptibility proportional to

$$(-1)^{2S}\left[(2S)^2 - \frac{1}{3}\right]$$

  Thus $S = 0$ and $\frac{1}{2}$ give *diamagnetic* (screening) contributions, but gauge bosons give anti-screening.
- The existence of large numbers of new spin-0 and/or spin-$\frac{1}{2}$ particles at higher energies could modify or even destroy asymptotic freedom.
  - This is exactly what happens in supersymmetric extensions of the Standard Model.

History of Asymptotic Freedom

1954 Yang & Mills study vector field theory with non-Abelian gauge invariance.

1965 Vanyashin & Terentyev compute vacuum polarization due to a massive charged vector field. In our notation, they found

$$\beta_0 = \frac{1}{12\pi}\left(-\frac{21}{2} = -11 + \frac{1}{2}\right)$$

- The $\frac{1}{2}$ comes from longitudinal polarization states (absent for massless gluons)
- They concluded that this result "... seems extremely undesirable".

1969 Khriplovich correctly computes the one-loop β-function in SU(2) Yang-Mills theory using the Coulomb gauge

$$\beta_0 = \frac{C_A}{12\pi}(-12 + 1 = -11)$$

- In Coulomb gauge the anti-screening (–12) is due to an instantaneous Coulomb interaction
- He did not make a connection with strong interactions

1971 't Hooft correctly computes the one-loop β-function for SU(3) gauge theory but does not publish it.

- He wrote it on the blackboard at a conference
- His supervisor (Veltman) told him it wasn't interesting
- 't Hooft & Veltman received the 1999 Nobel Prize for proving the *renormalizability* of QCD (and the whole Standard Model).

1972 Fritzsch & Gell-Mann propose that the strong interaction is an SU(3) gauge theory, later named QCD by Gell-Mann

1973 Gross & Wilczek, and independently Politzer, compute and publish the 1-loop β-function for QCD

$$\beta_0 = \frac{1}{12\pi}\left(2n_f - 11C_A\right)$$

$\Rightarrow$ 2004 Nobel Prize (now that 't Hooft has one anyway . . .)

1974 Caswell & Jones compute the 2-loop β-function for QCD

1980 Tarasov, Vladimirov & Zharkov compute the 3-loop β-function for QCD

1997 van Ritbergen, Vermaseren & Larin compute the 4-loop β-function for QCD ($\sim 50,000$ Feynman diagrams)

". . . We obtained in this way the following result for the 4-loop beta function in the $\overline{\mathrm{MS}}$-scheme:

$$q^2\frac{\partial a_s}{\partial q^2} = -\beta_0 a_s^2 - \beta_1 a_s^3 - \beta_2 a_s^4 - \beta_3 a_s^5 + O(a_s^6)$$

where $a_s = \alpha_S/4\pi$ and . . .

$$
\begin{aligned}
\beta_0 &= \frac{11}{3}C_A - \frac{4}{3}T_F n_f \\
\beta_1 &= \frac{34}{3}C_A^2 - 4C_F T_F n_f - \frac{20}{3}C_A T_F n_f \\
\beta_2 &= \frac{2857}{54}C_A^3 + 2C_F^2 T_F n_f - \frac{205}{9}C_F C_A T_F n_f \\
&\quad -\frac{1415}{27}C_A^2 T_F n_f + \frac{44}{9}C_F T_F^2 n_f^2 + \frac{158}{27}C_A T_F^2 n_f^2 \\
\beta_3 &= C_A^4\left(\frac{150653}{486} - \frac{44}{9}\zeta_3\right) + C_A^3 T_F n_f\left(-\frac{39143}{81} + \frac{136}{3}\zeta_3\right) \\
&\quad +C_A^2 C_F T_F n_f\left(\frac{7073}{243} - \frac{656}{9}\zeta_3\right) + C_A C_F^2 T_F n_f\left(-\frac{4204}{27} + \frac{352}{9}\zeta_3\right) \\
&\quad +46 C_F^3 T_F n_f + C_A^2 T_F^2 n_f^2\left(\frac{7930}{81} + \frac{224}{9}\zeta_3\right) + C_F^2 T_F^2 n_f^2\left(\frac{1352}{27} - \frac{704}{9}\zeta_3\right) \\
&\quad +C_A C_F T_F^2 n_f^2\left(\frac{17152}{243} + \frac{448}{9}\zeta_3\right) + \frac{424}{243}C_A T_F^3 n_f^3 + \frac{1232}{243}C_F T_F^3 n_f^3 \\
&\quad +\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(-\frac{80}{9} + \frac{704}{3}\zeta_3\right) + n_f\frac{d_F^{abcd}d_A^{abcd}}{N_A}\left(\frac{512}{9} - \frac{1664}{3}\zeta_3\right) \\
&\quad +n_f^2\frac{d_F^{abcd}d_F^{abcd}}{N_A}\left(-\frac{704}{9} + \frac{512}{3}\zeta_3\right)
\end{aligned}
$$

Here ζ is the Riemann zeta-function ($\zeta_3 = 1.202\cdots$) and the colour factors for SU(N) are

$$T_F = \frac{1}{2}, \quad C_A = N, \quad C_F = \frac{N^2-1}{2N}, \quad \frac{d_A^{abcd}d_A^{abcd}}{N_A} = \frac{N^2(N^2+36)}{24},$$

$$\frac{d_F^{abcd}d_A^{abcd}}{N_A} = \frac{N(N^2+6)}{48}, \quad \frac{d_F^{abcd}d_F^{abcd}}{N_A} = \frac{N^4-6N^2+18}{96N^2}$$

Substitution of these colour factors for $N=3$ yields the following numerical results for QCD:

$$\begin{aligned}
\beta_0 &\approx 11 - 0.66667 n_f \\
\beta_1 &\approx 102 - 12.6667 n_f \\
\beta_2 &\approx 1428.50 - 279.611 n_f + 6.01852 n_f^2 \\
\beta_3 &\approx 29243.0 - 6946.30 n_f + 405.089 n_f^2 + 1.49931 n_f^3
\end{aligned}$$


$\beta_g = \frac{-g^3}{16\pi^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)$
$\frac{-g^5}{(16\pi^2)^2}\left(\frac{34}{3}N_c^2 + \cdots\cdot\right)$
$\frac{-g^7}{(16\pi^2)^3}\left(\frac{2857}{54}N_c^3 + \cdots\cdots\right)$
$\frac{-g^9}{(16\pi^2)^4}\left(\cdots\cdots\cdots\right)$

QCD Running Coupling

- We have $q^2 \frac{d\alpha_S}{dq^2} = \beta_{QCD}(\alpha_S)$
- Hence

$$\int_{\alpha_S(\mu^2)}^{\alpha_S(Q^2)} \frac{d\alpha_S}{\beta_{QCD}(\alpha_S)} = \int_{\mu^2}^{Q^2} \frac{dq^2}{q^2} = \ln \frac{Q^2}{\mu^2}$$

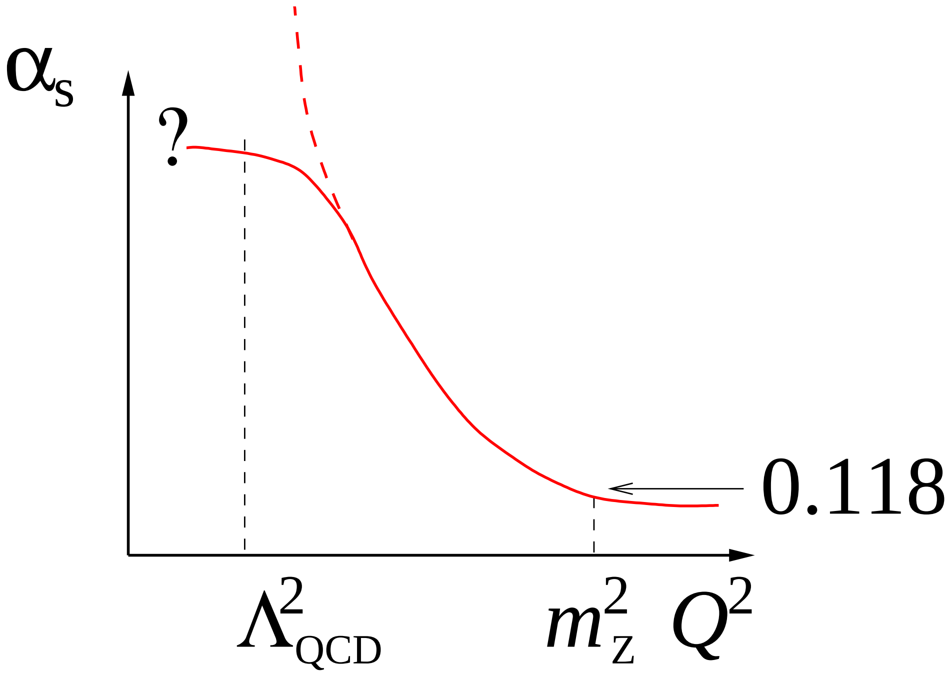
- In lowest order, $\beta_{QCD}(\alpha_S) = -\beta_0 \alpha_S^2$ and thus

$$\begin{aligned} \frac{1}{\alpha_S(Q^2)} - \frac{1}{\alpha_S(\mu^2)} &= \beta_0 \ln \frac{Q^2}{\mu^2} \\ \Rightarrow \quad \alpha_S(Q^2) &= \frac{\alpha_S(\mu^2)}{1 + \alpha_S(\mu^2)\beta_0 \ln(Q^2/\mu^2)} \end{aligned}$$

- We had to introduce a dimensionful parameter μ to specify the boundary conditions. Alternatively, we can define Λ_{QCD} (not to be confused with the cut-off Λ introduced earlier) as the scale at which the (perturbative) solution diverges . . .

$$\frac{1}{\alpha_S(Q^2)} - 0 = \beta_0 \ln \frac{Q^2}{\Lambda^2_{\text{QCD}}}$$

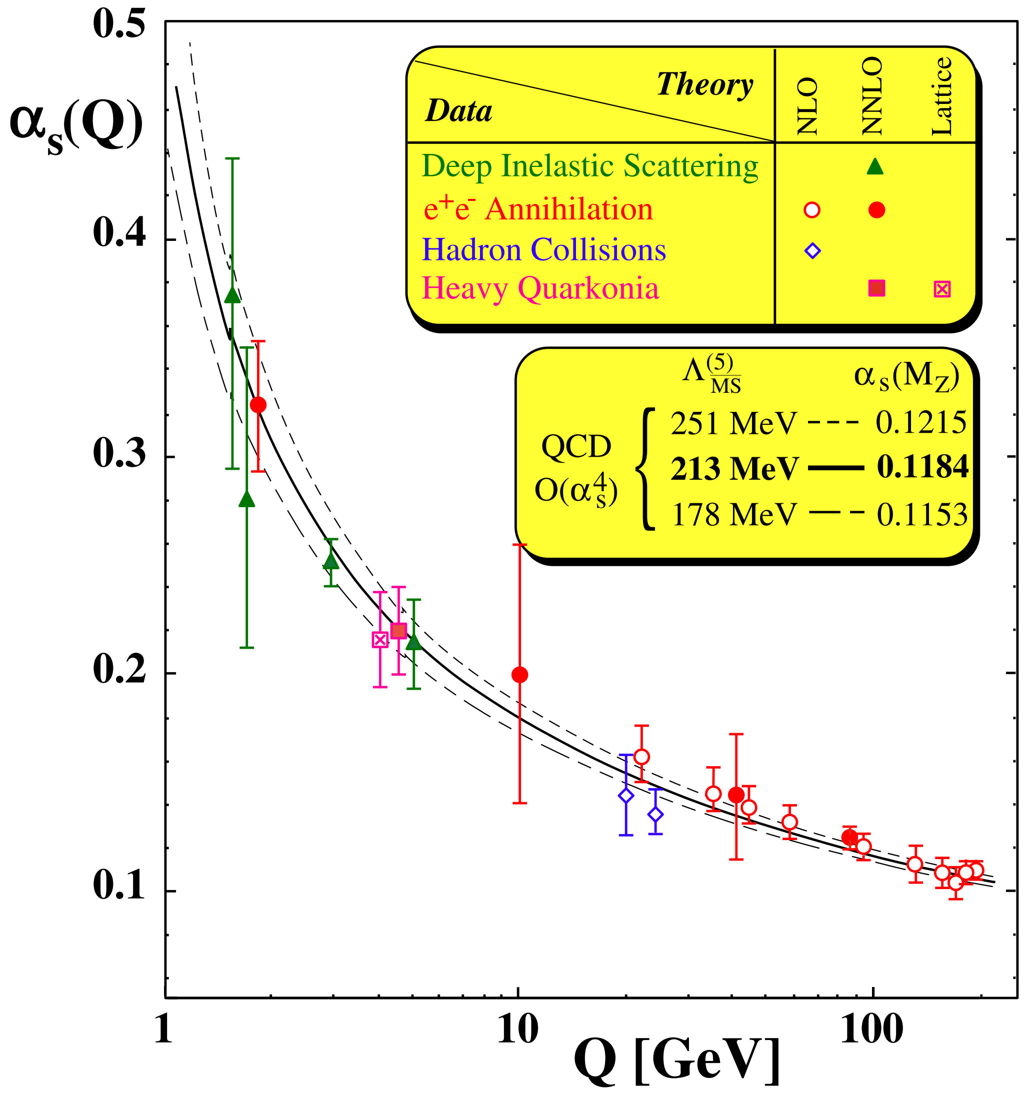
$$\Rightarrow \quad \alpha_S(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2_{\text{QCD}})}$$



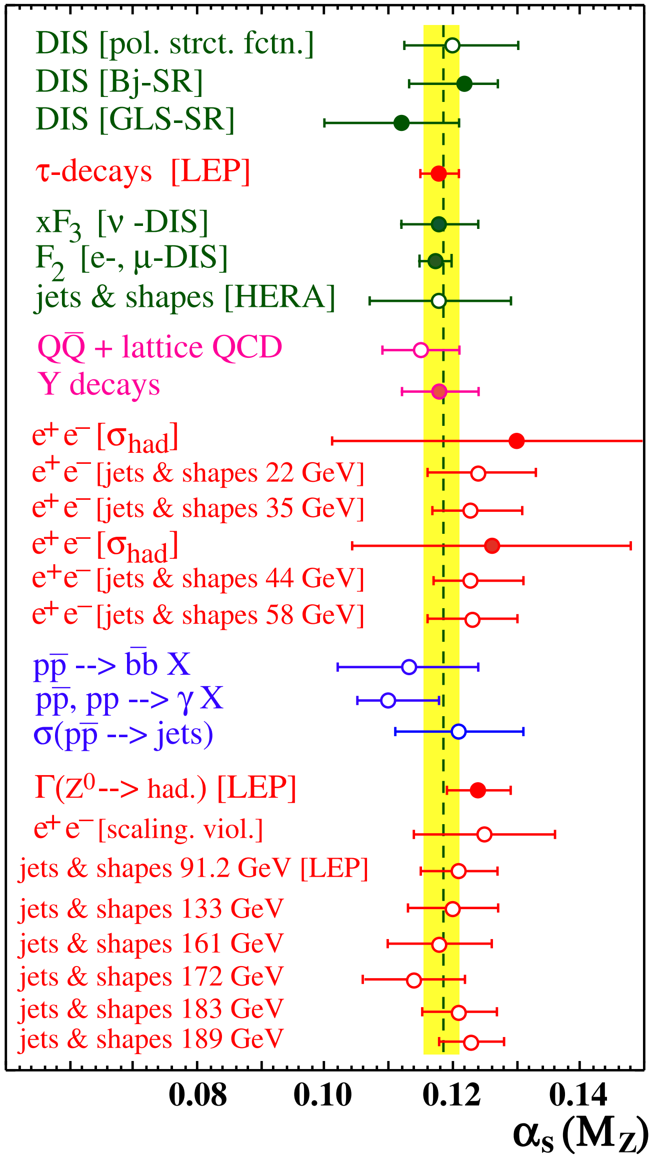
- Experimentally the fundamental scale of QCD is

$$\Lambda_{\text{QCD}} \simeq 210 \pm 40 \text{ MeV}$$

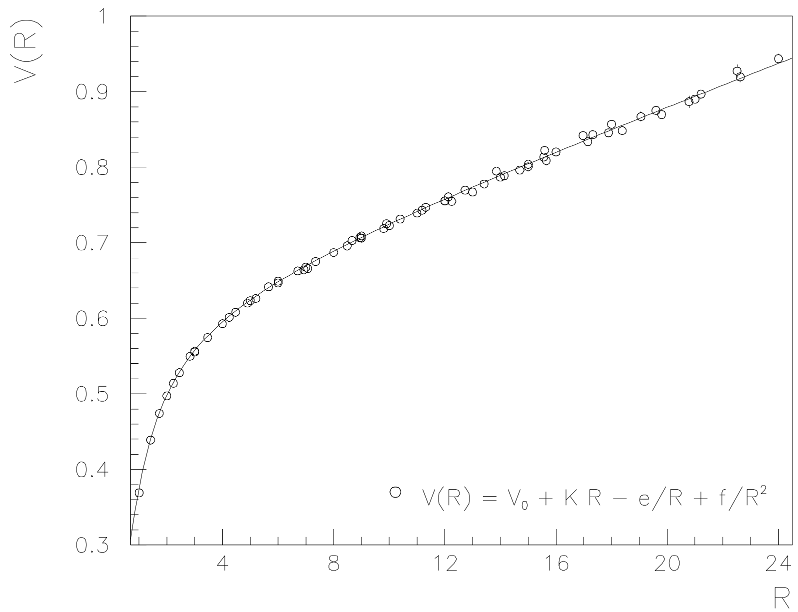
- Measurements at different scales show clear evidence that α_S does run:



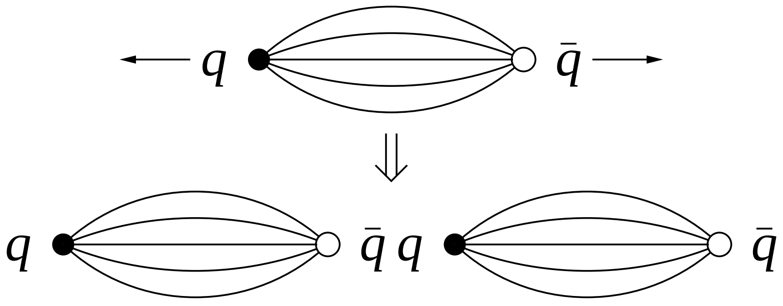
- Using the formula for the running of α_S, measurements at different scales can be expressed as measurements of $\alpha_S(M_Z^2)$:



- Non-perturbative (lattice) studies of QCD indicate that force between two colour charges (e.g. quarks) becomes constant at large distances, corresponding to a linear potential.



- It follows that quarks (and gluons) cannot be observed as isolated objects and exist only in colour-singlet bound states. Thus QCD explains quark confinement.



Grand Unification

- The complete Standard Model covariant derivative is

$$D^\mu = \partial^\mu + ig_{\mathsf{S}}\, \boldsymbol{t}\cdot\boldsymbol{\mathcal{A}}^\mu + ig\, \boldsymbol{I}\cdot\boldsymbol{W}^\mu + ig'\, Y B^\mu$$

  where extra terms represent the SU(2) and U(1) *electroweak* gauge field interactions.

- We would like to write this as

$$D^\mu = \partial^\mu + ig_{\mathsf{GUT}}\boldsymbol{T}\cdot\boldsymbol{X}^\mu$$

  where g_{GUT} is coupling of a Grand Unified Theory with gauge fields X^μ_α, and the gauge group G is a simple Lie group with generators T_α.
  - Definition: A *simple* Lie group is one with no *invariant subalgebras*, i.e. no A such that $X_\alpha \in A$ implies $[X_\alpha, X_\beta] \in A$ for all $X_\beta \in G$.
  - Theorem: G is simple iff

$$\mathrm{Tr}_R(T_\alpha T_\beta) = N_R \delta_{\alpha\beta}$$

  where Tr_R represents a sum over all states in a representation R of G and N_R is a number characteristic of the representation R.

- For $R =$ a Standard Model generation, e.g. u, d, e^- and ν_e, we have

$$\mathsf{Tr}_R(t_3 t_3) = \mathsf{Tr}_R(t_8 t_8) = \mathsf{Tr}_R(I_3 I_3) = 2 \; , \quad \mathsf{Tr}_R(Y^2) = \frac{10}{3}$$

- Hence for grand unification we require

$$g_{\mathsf{GUT}} = g_{\mathsf{S}} = g = \sqrt{\frac{5}{3}}\, g'$$

- However, this should hold at the GUT energy scale, not at present energies. . .

Running of Standard Model Couplings

- Setting $\alpha_3 = \frac{g_S^2}{4\pi}$, $\alpha_2 = \frac{g^2}{4\pi}$, $\alpha_1 = \frac{5}{3}\frac{g'^2}{4\pi}$, we have (ignoring Higgs contributions)

$$q^2 \frac{d\alpha_i}{dq^2} = -\frac{\alpha_i^2}{\pi}\left(\frac{11}{12}C_1 - \frac{1}{3}C_2\right)$$

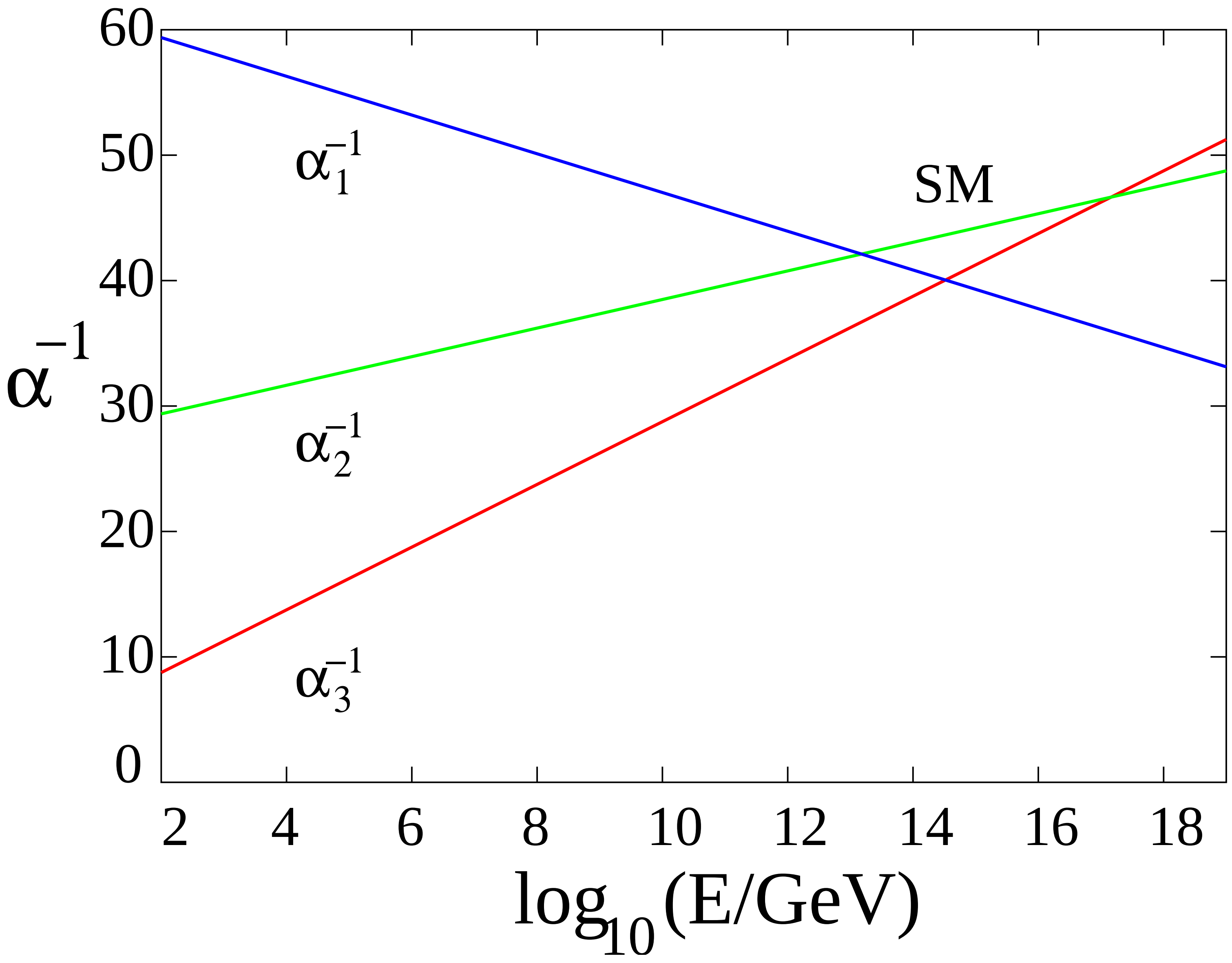
where

$$\begin{aligned} \mathrm{Tr}\,(T^A T^B) &= C_1 \delta_{AB} \quad \text{(gauge bosons)} \\ \mathrm{Tr}\,(t_a t_b) &= C_2 \delta_{ab} \quad \text{(fundamental fermions)} \end{aligned}$$

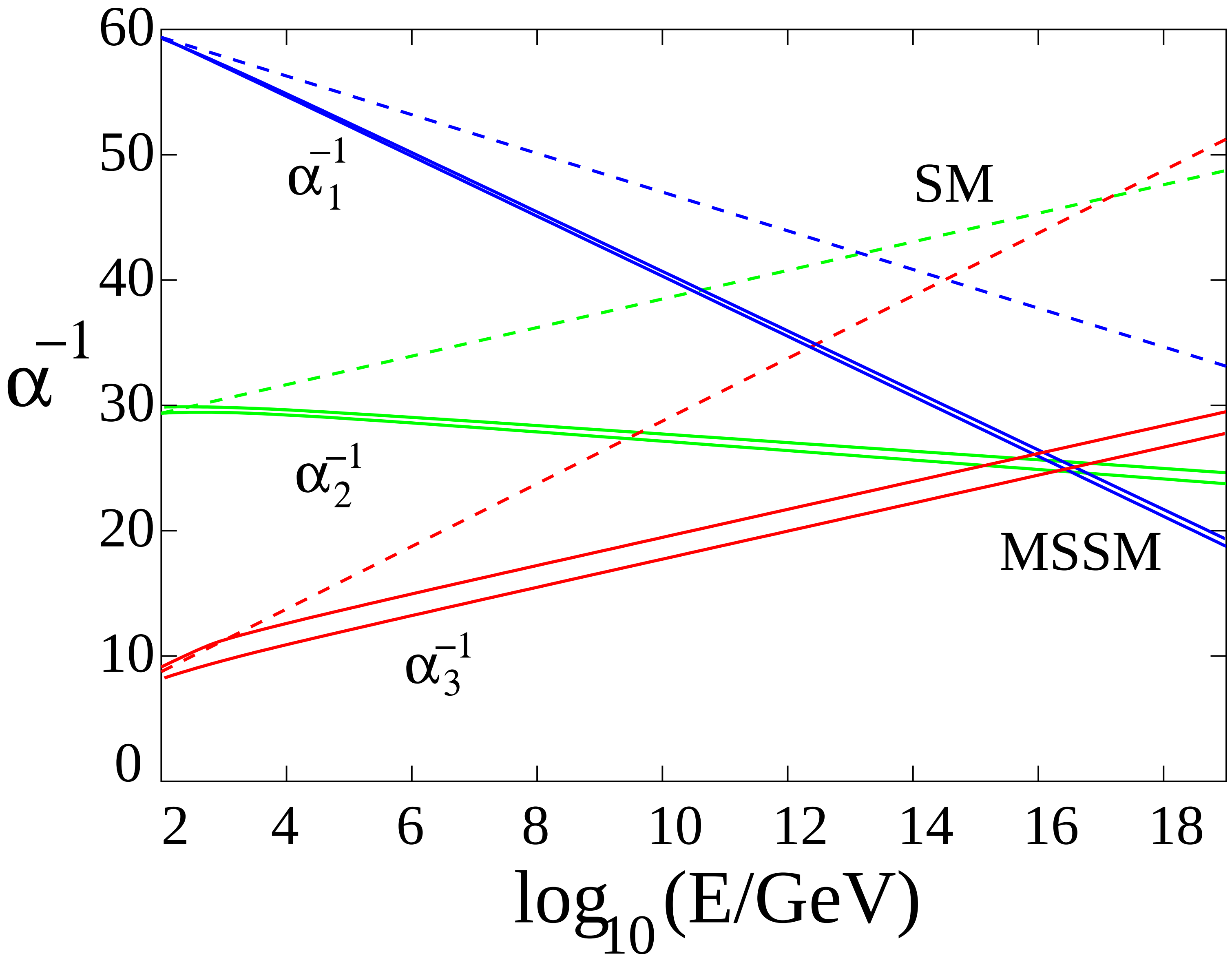
- For SU(N), $C_1 = N$, $C_2 = n_f/2$, and for n_g generations we therefore find (now including Higgs contributions)

$$\begin{aligned} q^2 \frac{d\alpha_3}{dq^2} &= -\frac{\alpha_3^2}{\pi}\left(\frac{11}{4} - \frac{1}{3}n_g\right) \\ q^2 \frac{d\alpha_2}{dq^2} &= -\frac{\alpha_2^2}{\pi}\left(\frac{11}{6} - \frac{1}{3}n_g - \frac{1}{24}\right) \\ q^2 \frac{d\alpha_1}{dq^2} &= -\frac{\alpha_1^2}{\pi}\left(-\frac{1}{3}n_g - \frac{1}{40}\right) \ . \end{aligned}$$

- Then running couplings almost meet at scale $\sim 10^{16}$ GeV.



- Adding supersymmetric partners for all Standard Model particles gives much better unification, provided superpartner masses are not more than ~ 1 TeV $\Rightarrow$ Large Hadron Collider



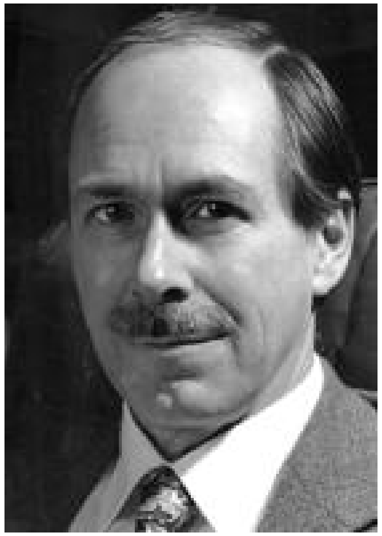
Conclusion: How to Win the Nobel Prize in Physics

- Discover or predict something important
- Understand its significance
- Publish first
- Be ≤ 3
- For theorists: be confirmed by experiment
- Live long enough

N.B. All are necessary, but not sufficient!

Thanks to Marek Karliner for many helpful suggestions & references

Iosif Khriplovich



Gerhard ’t Hooft