

QCD Phenomenology at High Energy

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CERN Academic Training Lectures

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Lecture 5: Matching Fixed Order Matrix Elements with Parton Showers

ME-PS Matching

- Two rather different objectives:
- Matching parton showers to **NLO** matrix elements, without double counting
 - MC@NLO
 - POWHEG
- Matching parton showers to **LO n-jet** matrix elements, minimizing jet resolution dependence
 - CKKW
 - Dipole
 - MLM Matching
 - Comparisons

MC@NLO

Recall simple one-dim. example from lecture 2:

$$|\mathcal{M}_{m+1}|^2 \equiv \frac{1}{x} \mathcal{M}(x)$$

x = gluon energy or two-parton invariant mass.

Divergences regularized by $d = 4 - 2\epsilon$ dimensions.

$$|\mathcal{M}_m^{\text{one-loop}}|^2 \equiv \frac{1}{\epsilon} \mathcal{V}$$

Cross section in d dimensions is:

$$\sigma = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_1^J(x) + \frac{1}{\epsilon} \mathcal{V} F_0^J$$

Infrared safety: $F_1^J(0) = F_0^J$

KLN cancellation theorem: $\mathcal{M}(0) = \mathcal{V}$

Subtraction Method

Exact identity:

$$\begin{aligned}\sigma^J &= \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_1^J(x) - \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V} F_0^J \\ &\quad + \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V} F_0^J + \frac{1}{\epsilon} \mathcal{V} F_0^J \\ &= \int_0^1 \frac{dx}{x} \left(\mathcal{M}(x) F_1^J(x) - \mathcal{V} F_0^J \right) + \mathcal{O}(1) \mathcal{V} F_0^J.\end{aligned}$$

➔ Two separate finite integrals.

Modified Subtraction

$$\sigma^J = \int_0^1 \frac{dx}{x} (\mathcal{M}(x) F_1^J(x) - \mathcal{V} F_0^J) + \mathcal{O}(1) \mathcal{V} F_0^J$$

Now add parton shower:

$F_{0,1}^J \Rightarrow$ result from showering after 0,1 emissions.

But shower adds $\mathcal{M}_{\text{MC}}/x$ to 1 emission. Must subtract this, and add to 0 emission (so that $F_{0,1}^{\text{tot}} = 1 \Rightarrow \sigma^{\text{tot}}$ fixed)

$$\begin{aligned} \sigma^J = \int_0^1 \frac{dx}{x} & (\{\mathcal{M}(x) - \mathcal{M}_{\text{MC}}(x)\} F_1^J(x) \\ & - \{\mathcal{V} - \mathcal{M}_{\text{MC}}(x)\} F_0^J) + \mathcal{O}(1) \mathcal{V} F_0^J \end{aligned}$$

MC good for soft and/or collinear $\Rightarrow \mathcal{M}_{\text{MC}}(0) = \mathcal{M}(0)$

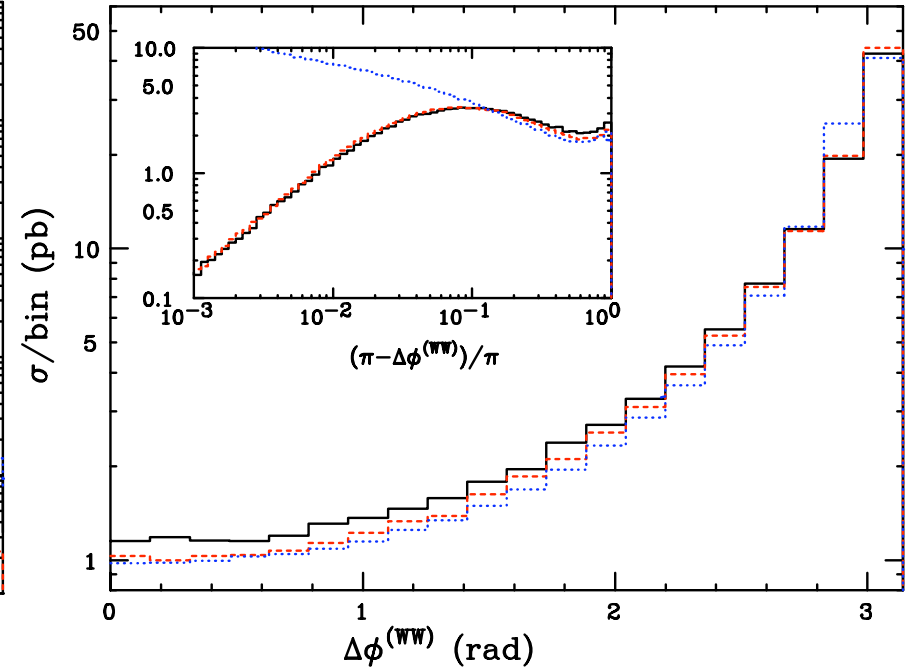
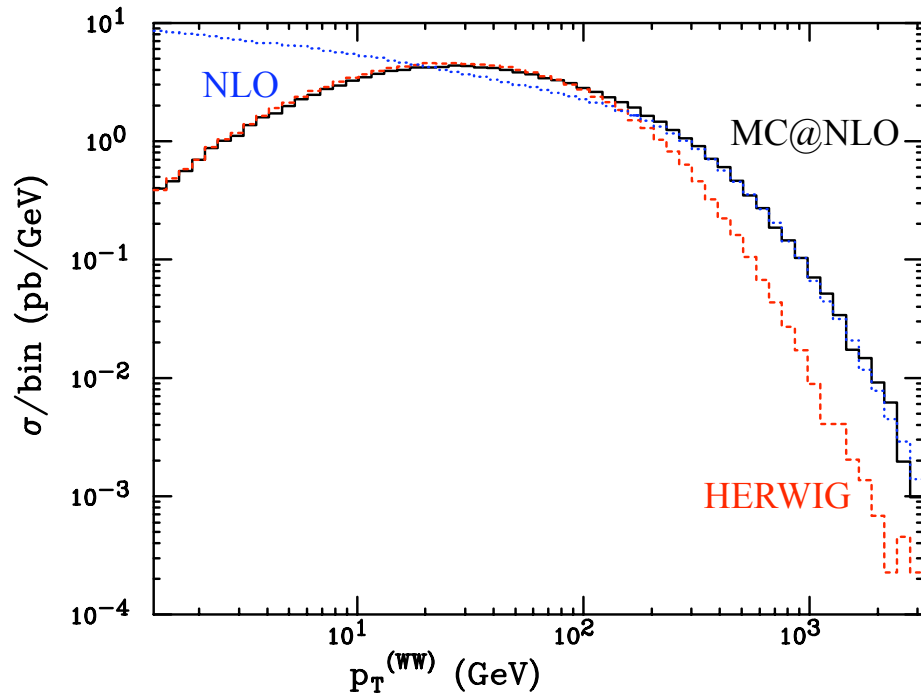
 0 & 1 emission contributions separately finite now!
(But some can be negative “counter-events”)

MC@NLO Processes

IPROC	IV	IL ₁	IL ₂	Spin	Process
-1350-IL				✓	$H_1 H_2 \rightarrow (Z/\gamma^* \rightarrow) l_{\text{IL}} \bar{l}_{\text{IL}} + X$
-1360-IL				✓	$H_1 H_2 \rightarrow (Z \rightarrow) l_{\text{IL}} \bar{l}_{\text{IL}} + X$
-1370-IL				✓	$H_1 H_2 \rightarrow (\gamma^* \rightarrow) l_{\text{IL}} \bar{l}_{\text{IL}} + X$
-1460-IL				✓	$H_1 H_2 \rightarrow (W^+ \rightarrow) l_{\text{IL}}^+ \nu_{\text{IL}} + X$
-1470-IL				✓	$H_1 H_2 \rightarrow (W^- \rightarrow) l_{\text{IL}}^- \bar{\nu}_{\text{IL}} + X$
-1396				×	$H_1 H_2 \rightarrow \gamma^*(\rightarrow \sum_i f_i \bar{f}_i) + X$
-1397				×	$H_1 H_2 \rightarrow Z^0 + X$
-1497				×	$H_1 H_2 \rightarrow W^+ + X$
-1498				×	$H_1 H_2 \rightarrow W^- + X$
-1600-ID					$H_1 H_2 \rightarrow H^0 + X$
-1705					$H_1 H_2 \rightarrow b\bar{b} + X$
-1706		7	7	×	$H_1 H_2 \rightarrow t\bar{t} + X$
-1706		i	j	✓	$H_1 H_2 \rightarrow (t \rightarrow) b l_i^+ \nu_i (\bar{t} \rightarrow) \bar{b} l_j^- \bar{\nu}_j + X$
-2000-IC		7		×	$H_1 H_2 \rightarrow t/\bar{t} + X$
-2000-IC		i		✓	$H_1 H_2 \rightarrow (t \rightarrow) b l_i^+ \nu_i / (\bar{t} \rightarrow) \bar{b} l_i^- \bar{\nu}_i + X$
-2001-IC		7		×	$H_1 H_2 \rightarrow \bar{t} + X$
-2001-IC		i		✓	$H_1 H_2 \rightarrow (\bar{t} \rightarrow) \bar{b} l_i^- \bar{\nu}_i + X$
-2004-IC		7		×	$H_1 H_2 \rightarrow t + X$
-2004-IC		i		✓	$H_1 H_2 \rightarrow (t \rightarrow) b l_i^+ \nu_i + X$
-2600-ID	1	7		×	$H_1 H_2 \rightarrow H^0 W^+ + X$
-2600-ID	1	i		✓	$H_1 H_2 \rightarrow H^0 (W^+ \rightarrow) l_i^+ \nu_i + X$
-2600-ID	-1	7		×	$H_1 H_2 \rightarrow H^0 W^- + X$
-2600-ID	-1	i		✓	$H_1 H_2 \rightarrow H^0 (W^- \rightarrow) l_i^- \bar{\nu}_i + X$
-2700-ID	0	7		×	$H_1 H_2 \rightarrow H^0 Z + X$
-2700-ID	0	i		✓	$H_1 H_2 \rightarrow H^0 (Z \rightarrow) l_i \bar{l}_i + X$
-2850		7	7	×	$H_1 H_2 \rightarrow W^+ W^- + X$
-2850		i	j	✓	$H_1 H_2 \rightarrow (W^+ \rightarrow) l_i^+ \nu_i (W^- \rightarrow) l_j^- \bar{\nu}_j + X$
-2860		7	7	×	$H_1 H_2 \rightarrow Z^0 Z^0 + X$
-2870		7	7	×	$H_1 H_2 \rightarrow W^+ Z^0 + X$
-2880		7	7	×	$H_1 H_2 \rightarrow W^- Z^0 + X$

MC@NLO Results

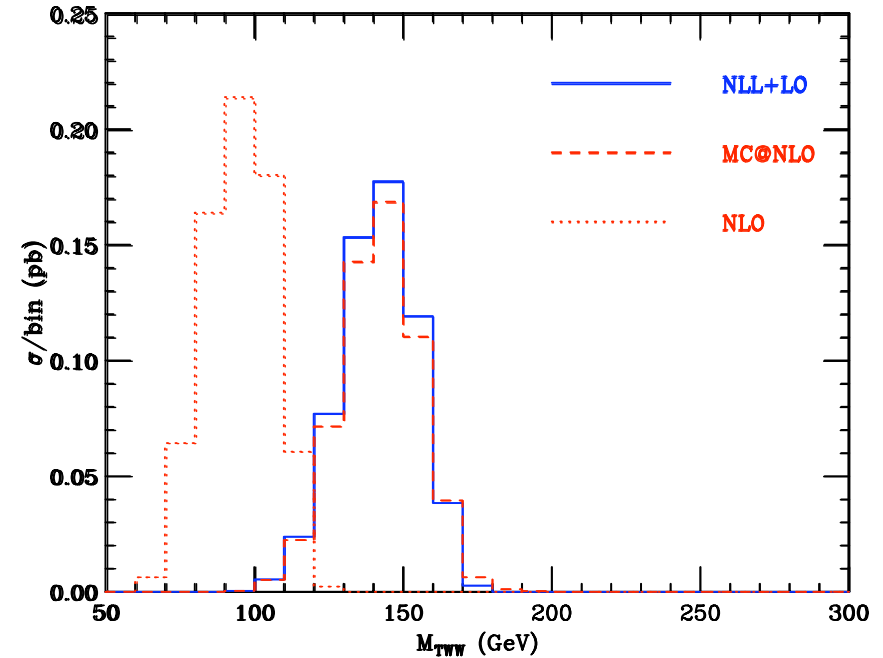
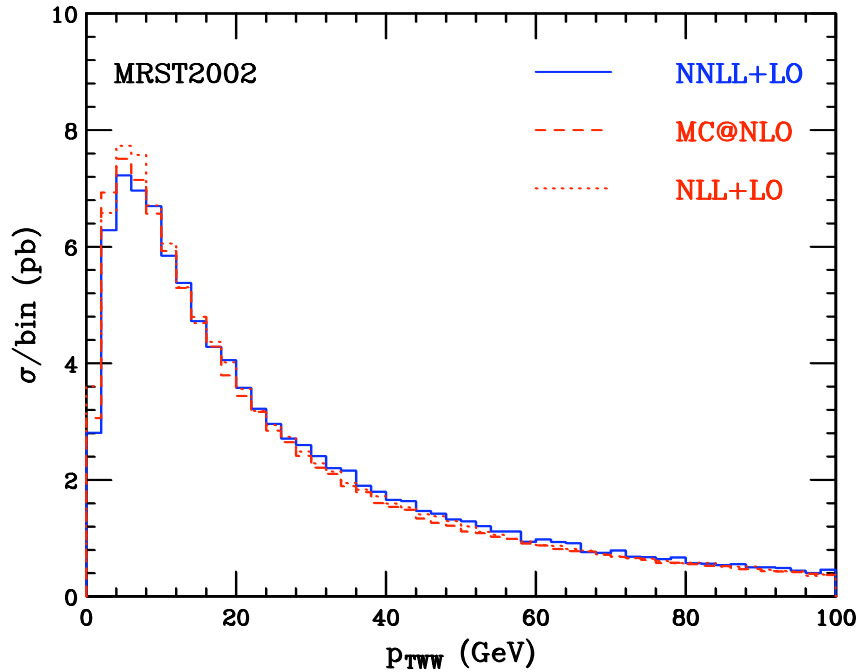
- WW production at LHC



- Interpolates between MC & NLO in $p_T^{(WW)}$
- Above both at $\Delta\phi^{(WW)} \simeq 0$

S Frixione & BW, JHEP 06(2002)029

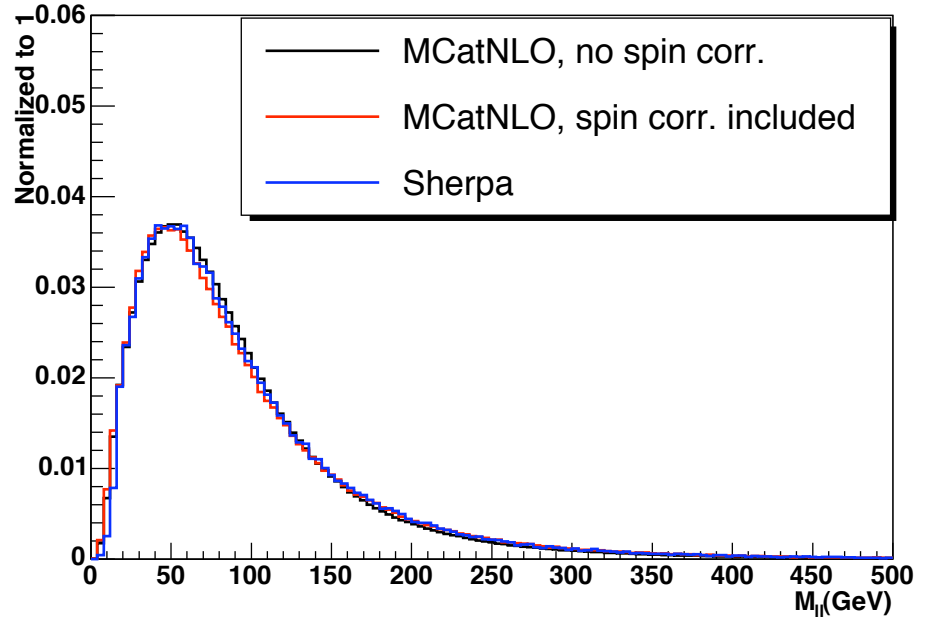
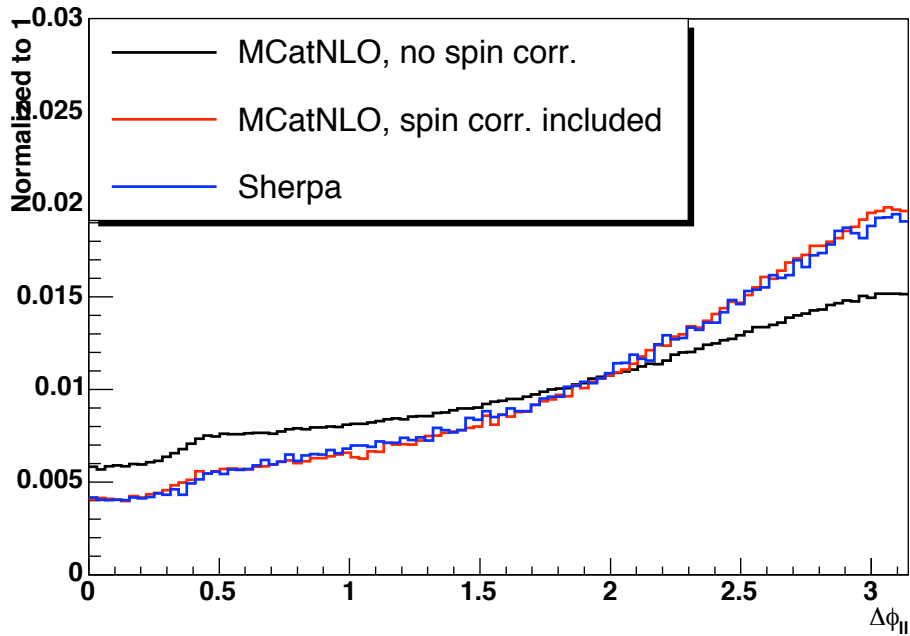
W^+W^- : MC@NLO vs Resummations



Plots from M. Grazzini JHEP 0601(2006)095

- ▶ Highly non-trivial test (of both computations) for *shapes* and *rates* !
- ▶ $M_{TWW} = \sqrt{(E_{Tu} + \cancel{E}_T)^2 - (\mathbf{p}_{Tu} + \cancel{\mathbf{p}}_T)^2}$ where $E_{Tu} = \sqrt{\mathbf{p}_{Tu}^2 + m_u^2}$ and $\cancel{E}_T \equiv \sqrt{\cancel{\mathbf{p}}_T^2 + m_l^2}$ (Rainwater & Zeppenfeld)
- ▶ Cuts involved in definition of M_{TWW} : $\Delta\phi_{l+l-} < \pi/4$, $M_{l+l-} > 35$ GeV, $p_{T\min}^{(l^+, l^-)} > 25$ GeV, $35 < p_{T\max}^{(l^+, l^-)} < 50$ GeV, $p_T^{WW} < 30$ GeV

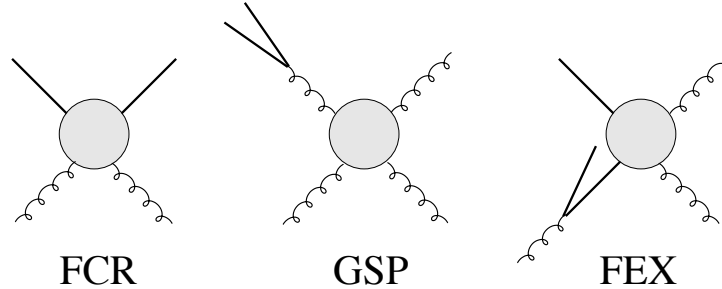
W^+W^- Spin Correlations



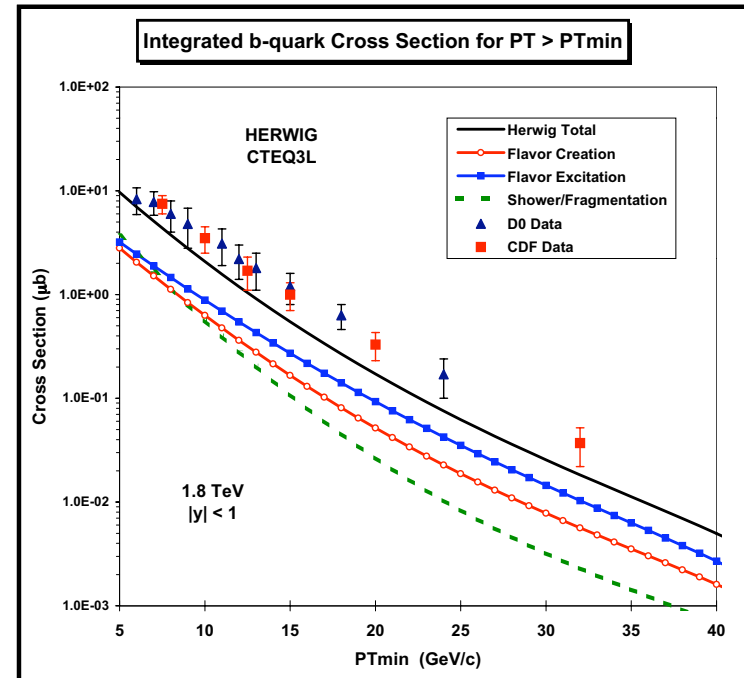
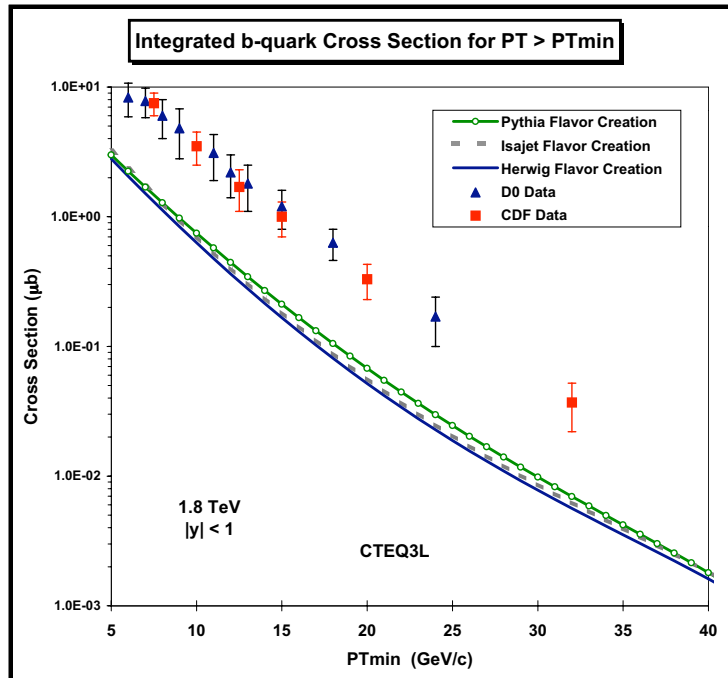
Plots from W. Quayle (preliminary)

b Production: PS MC vs MC@NLO

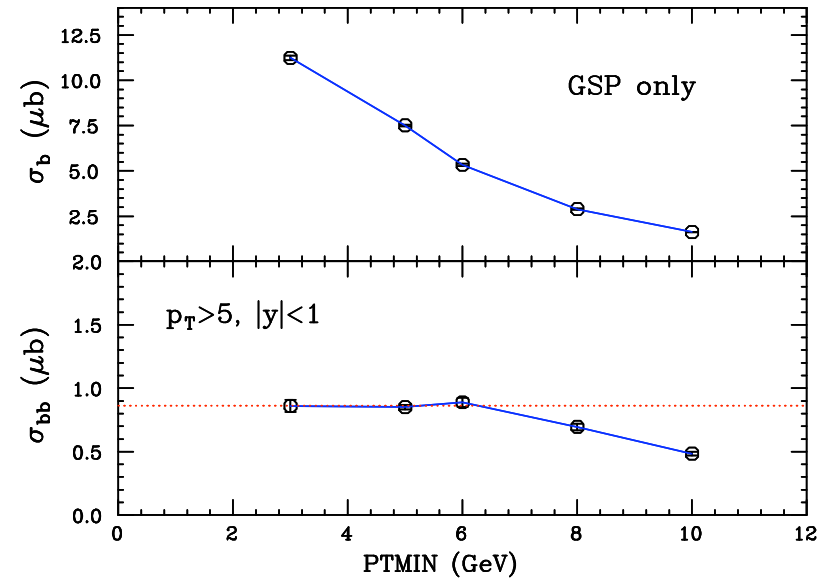
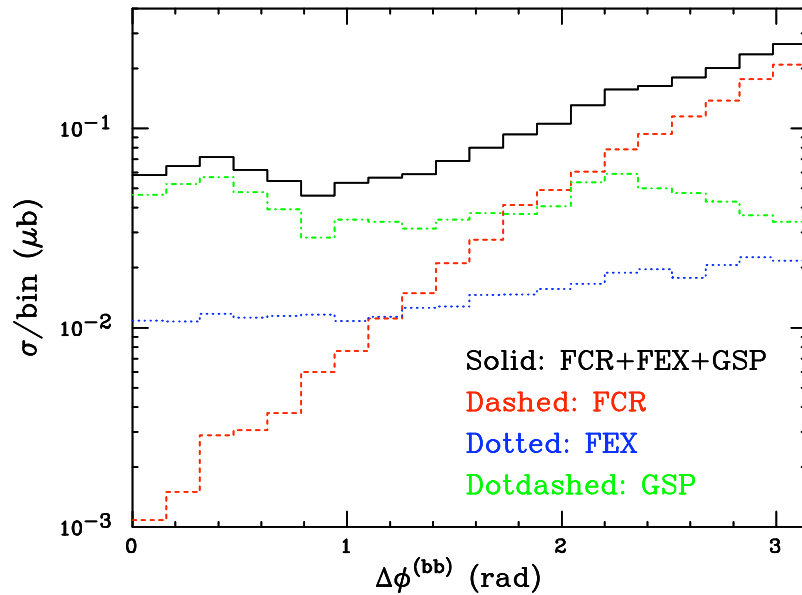
- In parton shower MC's, 3 classes of processes can contribute:



- All are needed to get close to data (RD Field, hep-ph/0201112):



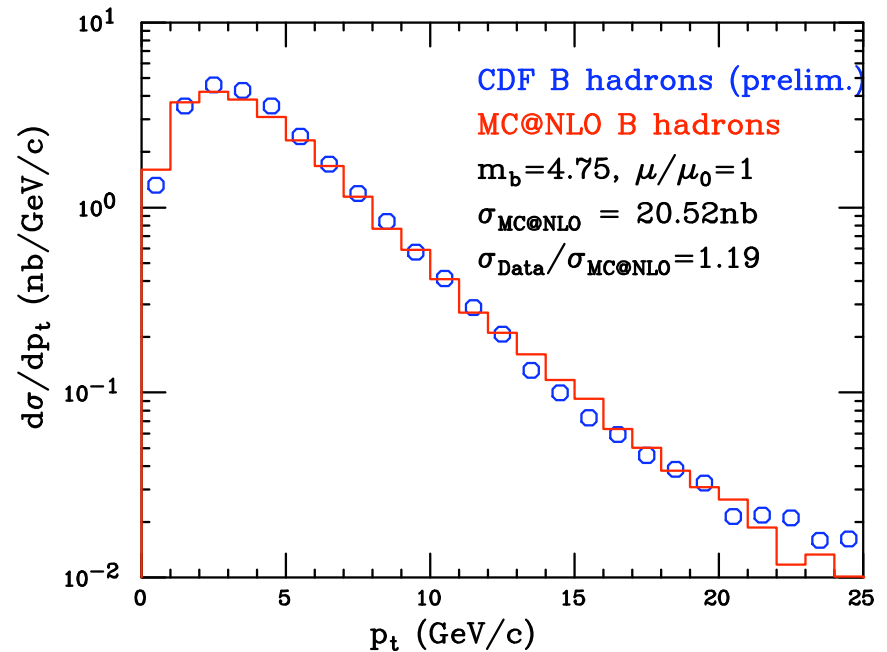
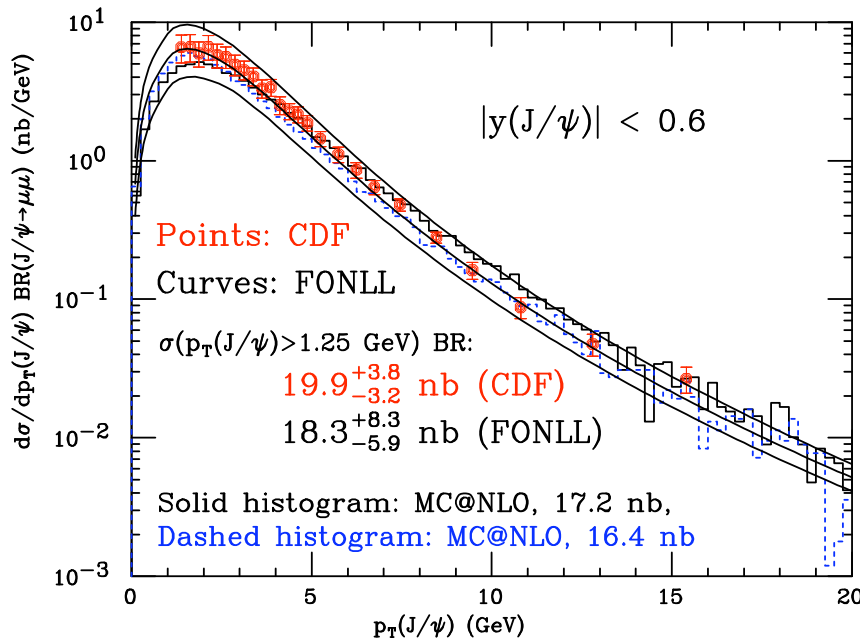
GSP and FEX contributions in HERWIG PS MC



- GSP, FEX and FCR are complementary and all must be generated
 - ❖ GSP cutoff (PT_{MIN}) sensitivity depends on cuts and observable
 - ❖ FEX sensitive to bottom PDF
 - ❖ GSP efficiency very poor, $\sim 10^{-4}$
- All these problems are avoided with MC@NLO!

MC@NLO: B Production at Tevatron

- $B \rightarrow J/\psi$ results from Tevatron Run II \Rightarrow B hadrons

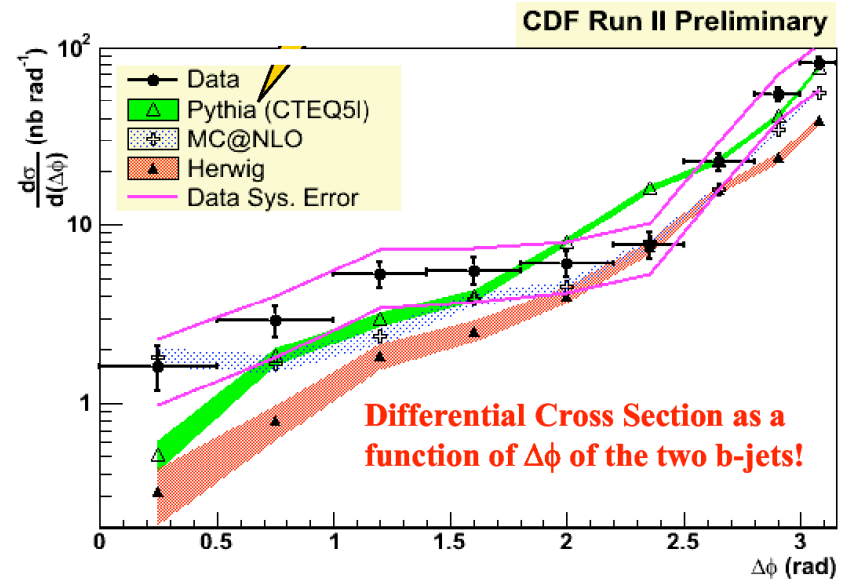
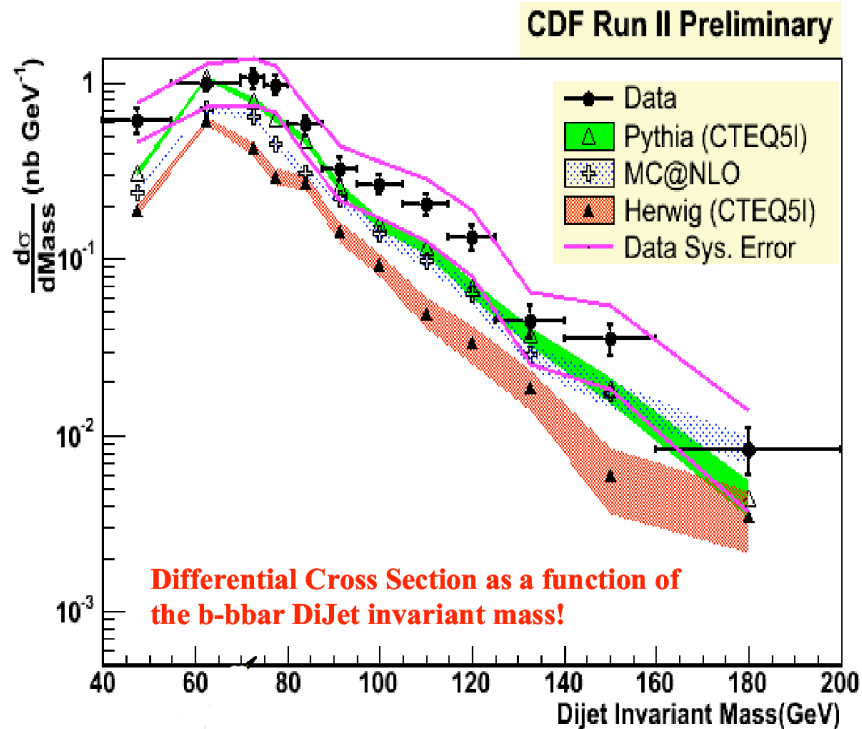


➔ Good agreement (and MC efficiency)

S Frixione, P Nason & BW, JHEP 0308(2003)007

M Cacciari et al., JHEP 0407(2004)033

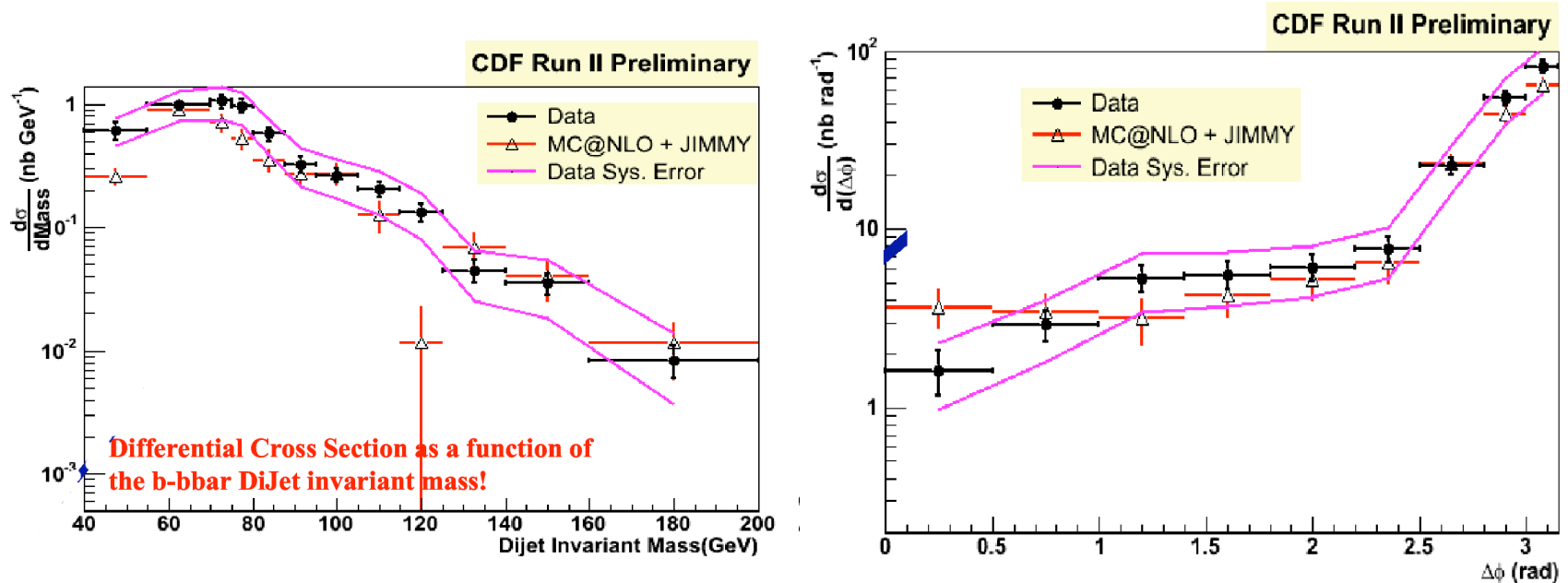
MC@NLO Di-b Jet Production



- ▶ These observables are very involved (b -jets at hadron level) and cannot be computed with analytical techniques;
- ▶ The underlying event in Pythia is fitted to data; default Herwig model (used in MC@NLO) does not fit data well (lack of MPI).

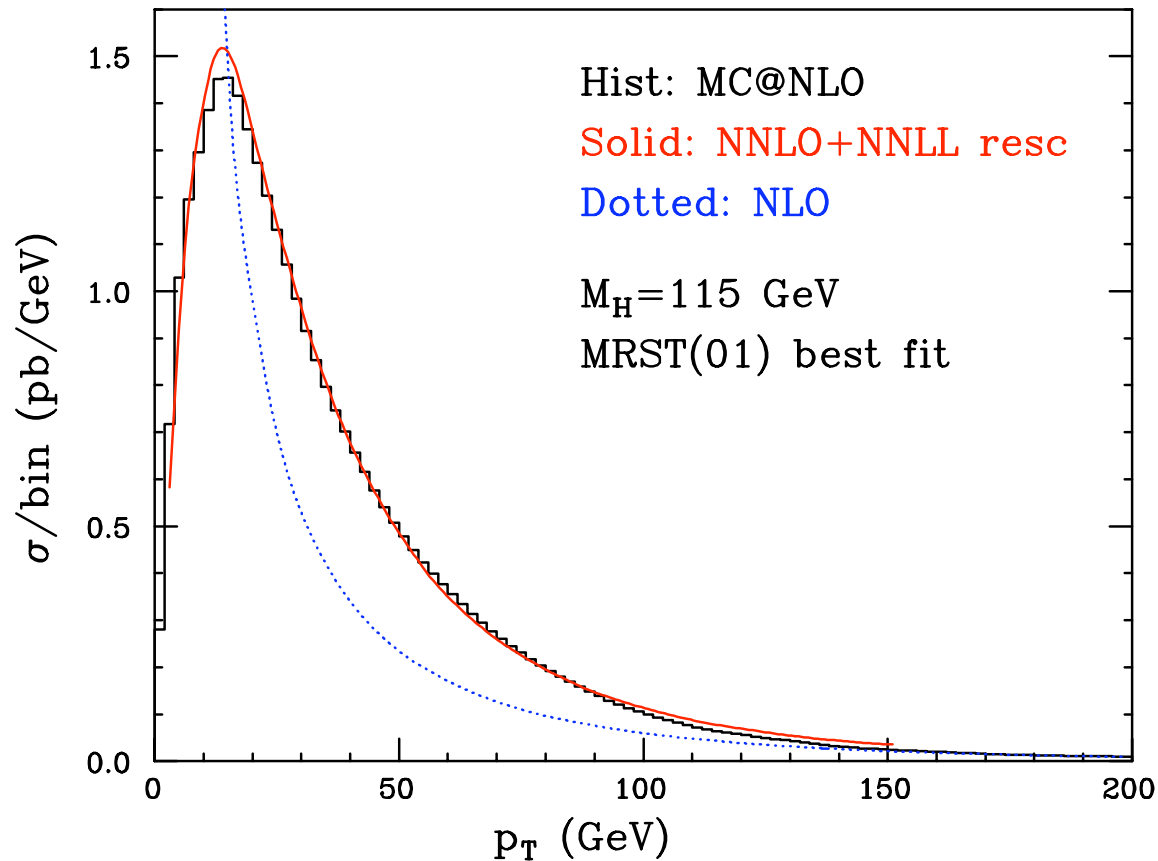
MC@NLO b-Jets: Improved Underlying Event

- ▶ The **JIMMY** underlying event model includes multiple parton interactions and interfaces to Herwig \Rightarrow interfaces to MC@NLO



- ▶ The importance of the underlying event shows the necessity of embedding precise computations in a Monte Carlo framework.

MC@NLO: Higgs Production at LHC



V Del Duca, S Frixione, C Oleari & BW, in prep.

➔ Good agreement with state-of-the-art resummation

POWHEG

Positive Weight Hardest Emission Generator

- Method to generate hardest emission first, with NLO accuracy, independent of PSEG
- Can be interfaced to any PSEG
- No negative weights
- Inaccuracies only affect next-to-hardest emission
- In principle, needs ‘truncated showers’

P Nason & G Ridolfi, JHEP08(2006)077

S Frixione, P Nason & G Ridolfi, arXiv:0707.3088

S Frixione, P Nason & C Oleari, arXiv:0709.2092

POWHEG

How it works (roughly)

In words: works like a standard Shower MC for the hardest radiation, with care to maintain higher accuracy.

Inclusive cross section \implies NLO inclusive cross section. Positive if $NL < LO$

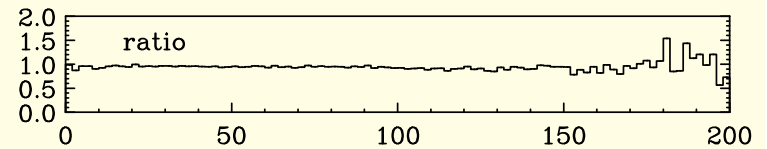
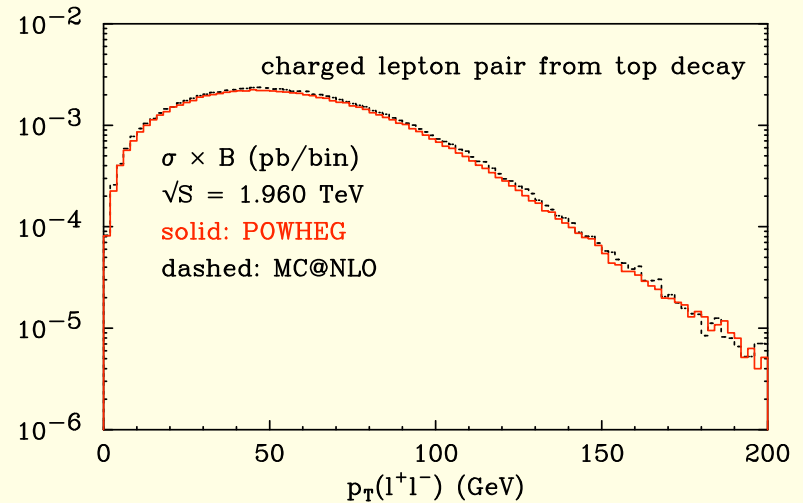
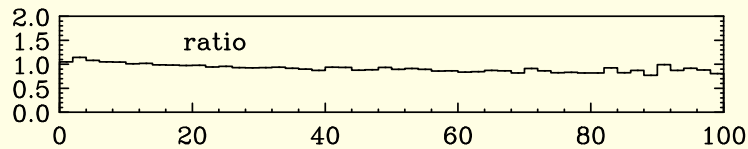
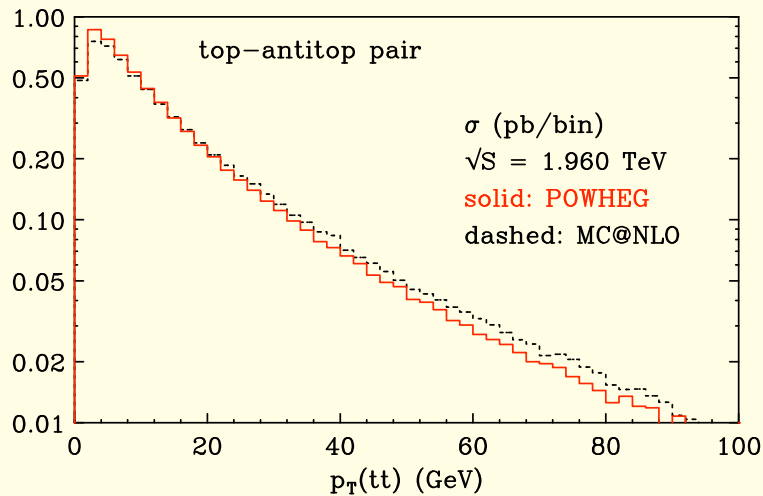
$$\begin{array}{l} \Phi_n = \text{Born variables} \\ \Phi_r = \text{radiation vars.} \end{array} \quad \bar{B}(\Phi_n) = B(\Phi_n) + \underbrace{\left[\overbrace{V(\Phi_n)}^{\text{INFINITE}} + \int \overbrace{R(\bar{\Phi}_n, \Phi_r)}^{\text{INFINITE}} d\Phi_r \right]}_{\text{FINITE!}}$$

Sudakov form factor for hardest emission built from exact NLO real emission

$$\Delta_t = \exp \left[- \underbrace{\int \theta(t_r - t) \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_r}_{\text{FINITE because of } \theta \text{ function}} \right]$$

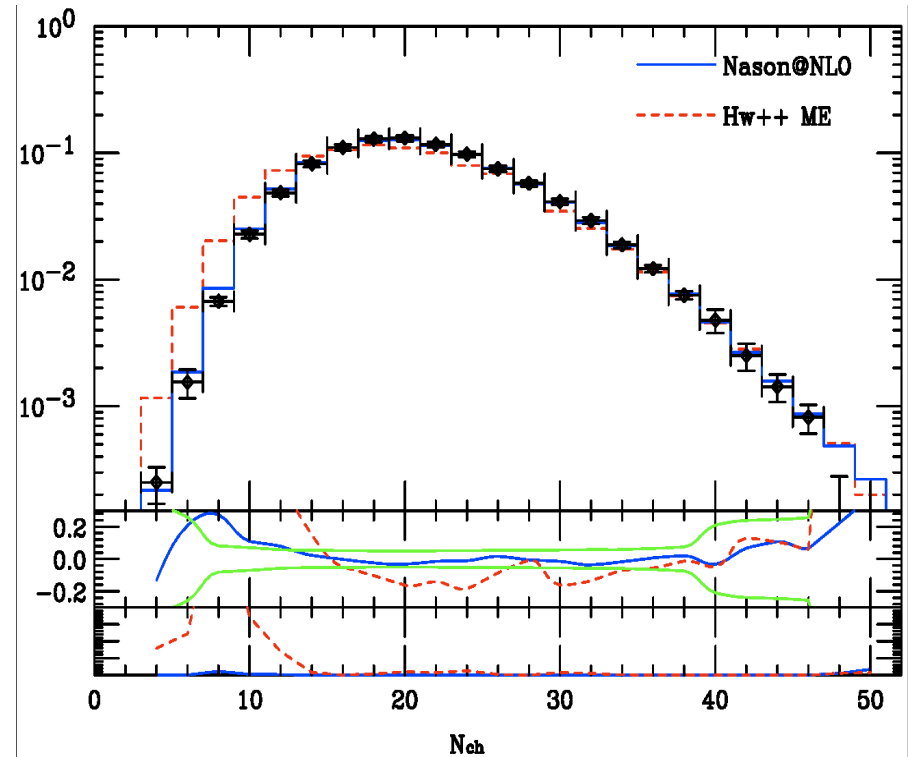
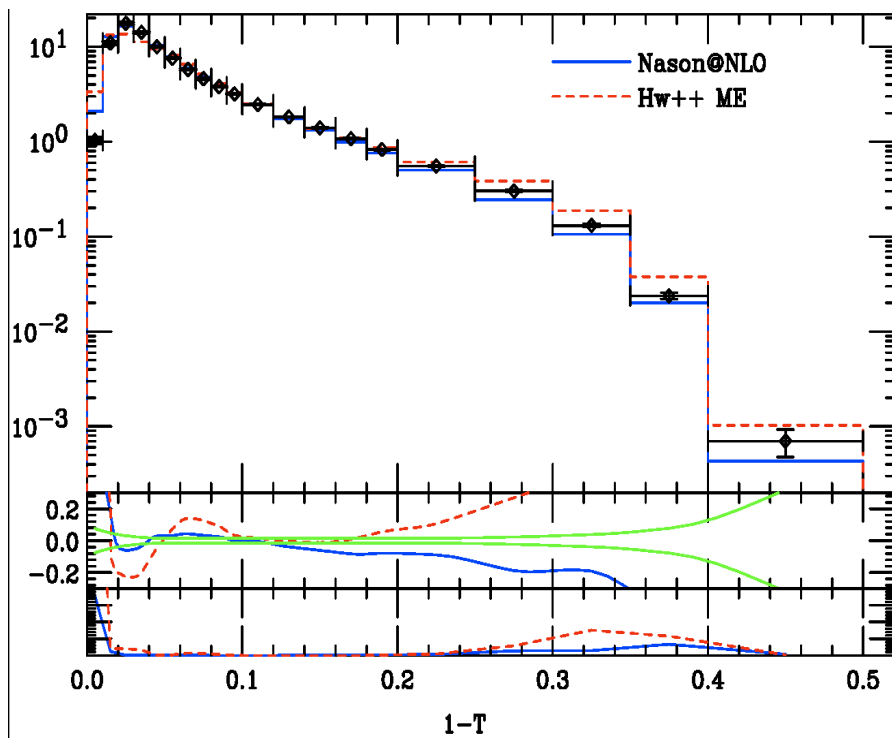
with $t_r = k_T(\Phi_n, \Phi_r)$, the transverse momentum for the radiation.

POWHEG and MC@NLO comparison: Top pair production



Good agreement for all observable considered
(differences can be ascribed to different treatment of higher order terms)

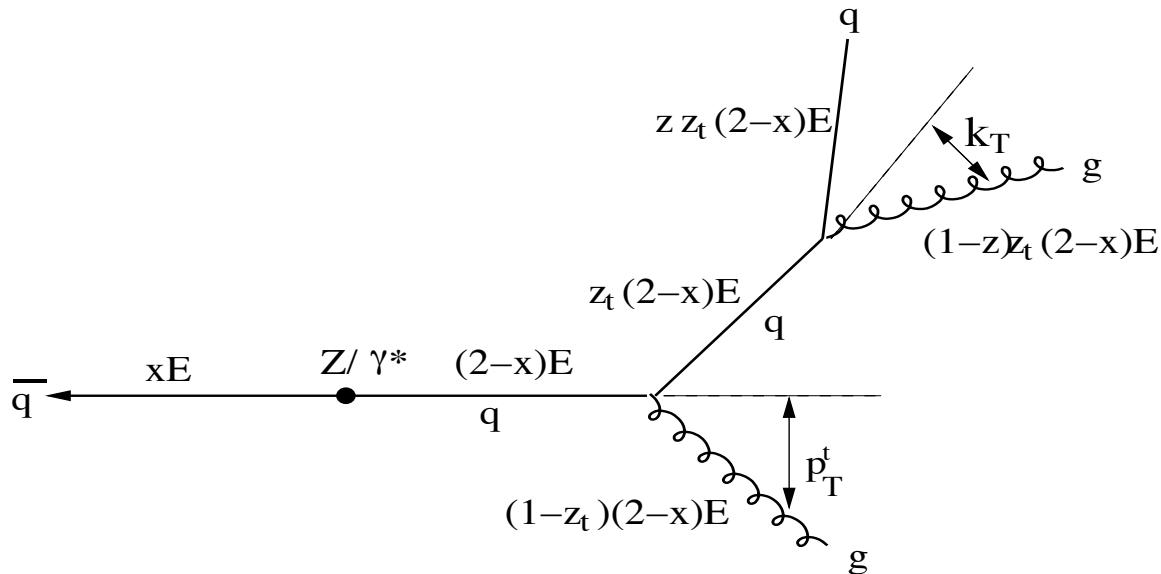
POWHEG for $e^+e^- \rightarrow \text{hadrons}$



O Latunde-Dada, S Gieseke, B Webber, JHEP02 (2007) 051, hep-ph/0612281

Truncated Shower

- In angular-ordered shower, hardest emission is not necessarily the first
- Need to add softer, wider-angle emissions
- Checked for up to one such emission in e^+e^-



Effect of truncated shower

Observable	Herwig++ ME	Nason@NLO	
		with truncated shower	w/o truncated shower
$1 - T$	36.52	9.03	9.81
Thrust Major	267.22	36.44	37.65
Thrust Minor	190.25	86.30	90.59
Oblateness	7.58	6.86	6.28
Sphericity	9.61	7.55	9.01
Aplanarity	8.70	22.96	25.33
Planarity	2.14	1.19	1.45
C Parameter	96.69	10.50	11.14
D Parameter	84.86	8.89	10.88
M_{high}	14.70	5.31	6.61
M_{low}	7.82	12.90	13.44
M_{diff}	5.11	1.89	2.09
B_{max}	39.50	11.42	12.17
B_{min}	45.96	35.2	36.16
B_{sum}	91.03	28.83	30.58
B_{diff}	8.94	1.40	1.14
N_{ch}	43.33	1.58	10.08
$\langle \chi^2 \rangle / \text{bin}$	56.47	16.96	18.49

Table 2: χ^2/bin for all observables we studied.



Small but beneficial effect

CKKW Matching

- Use Matrix Elements down to scale Q_1
- Use Parton Showers below Q_1
- Correct ME by **reweighting**
- Correct PS by **vetoing**
- Ensure that Q_1 cancels (to NLL)

S Catani, F Krauss, R Kuhn & BW, JHEP11 (2001) 063

Example: $e^+e^- \rightarrow \text{hadrons}$

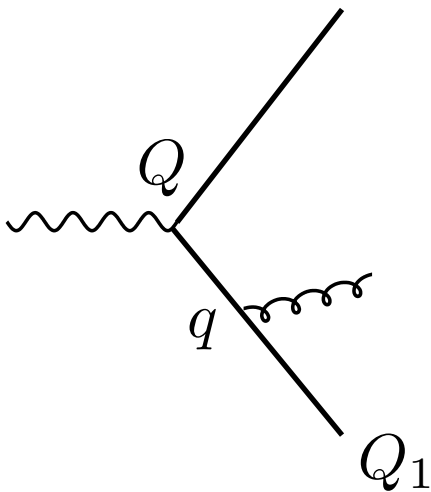
- 2- & 3-jet rates at scale Q_1 :

$$R_2(Q, Q_1) = [\Delta_q(Q, Q_1)]^2 ,$$

$$R_3(Q, Q_1) = 2\Delta_q(Q, Q_1) \int_{Q_1}^Q dq \frac{\Delta_q(Q, Q_1)}{\Delta_q(q, Q_1)} \Gamma_q(Q, q) \\ \times \Delta_q(q, Q_1) \Delta_g(q, Q_1)$$

$$= 2 [\Delta_q(Q, Q_1)]^2 \int_{Q_1}^Q dq \Gamma_q(Q, q) \Delta_g(q, Q_1)$$

$$\Gamma_q(Q, q) = \frac{2C_F}{\pi} \frac{\alpha_S(q)}{q} \left(\ln \frac{Q}{q} - \frac{3}{4} \right)$$

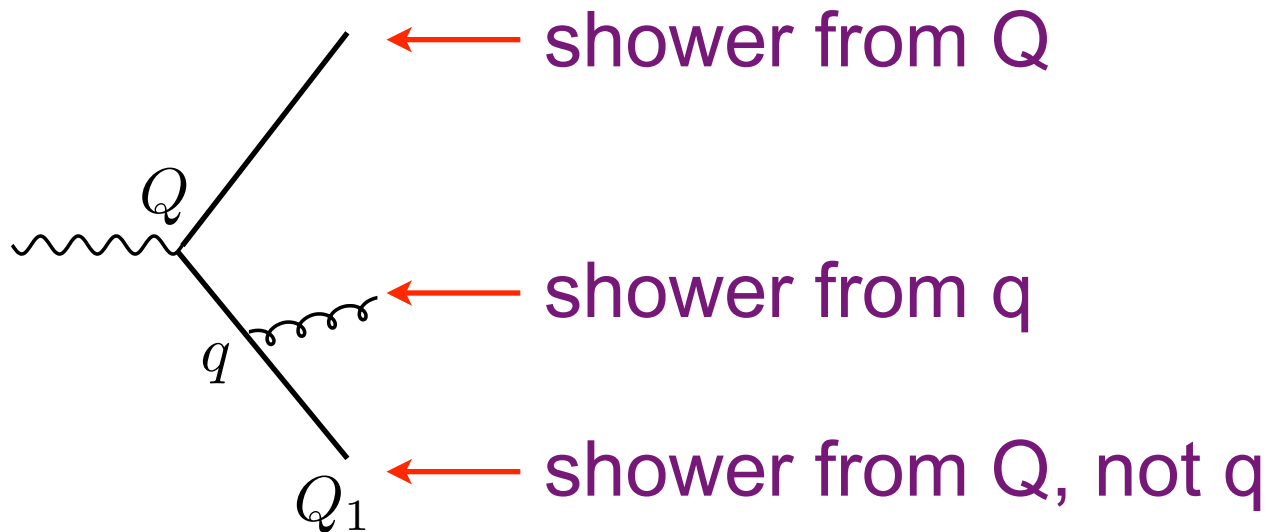


CKKW reweighting

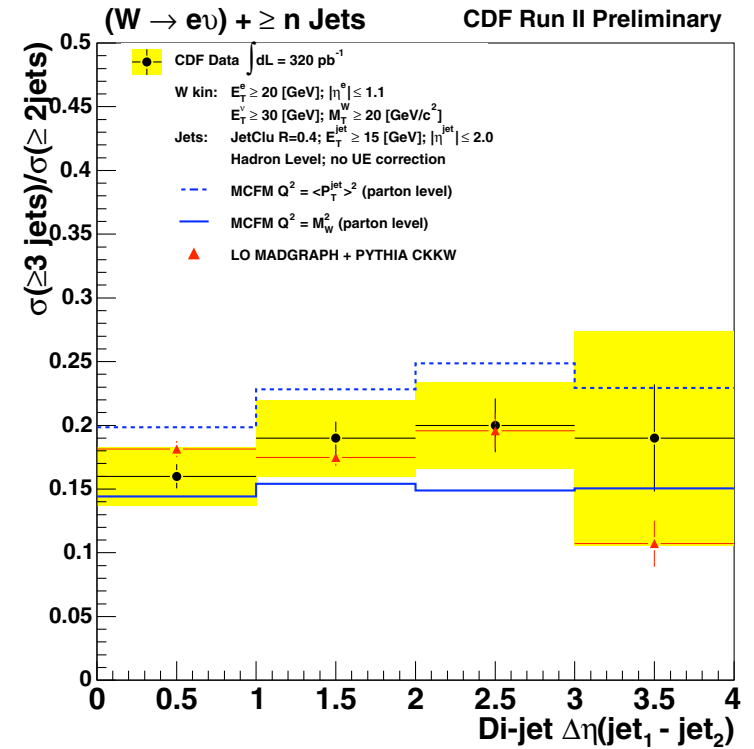
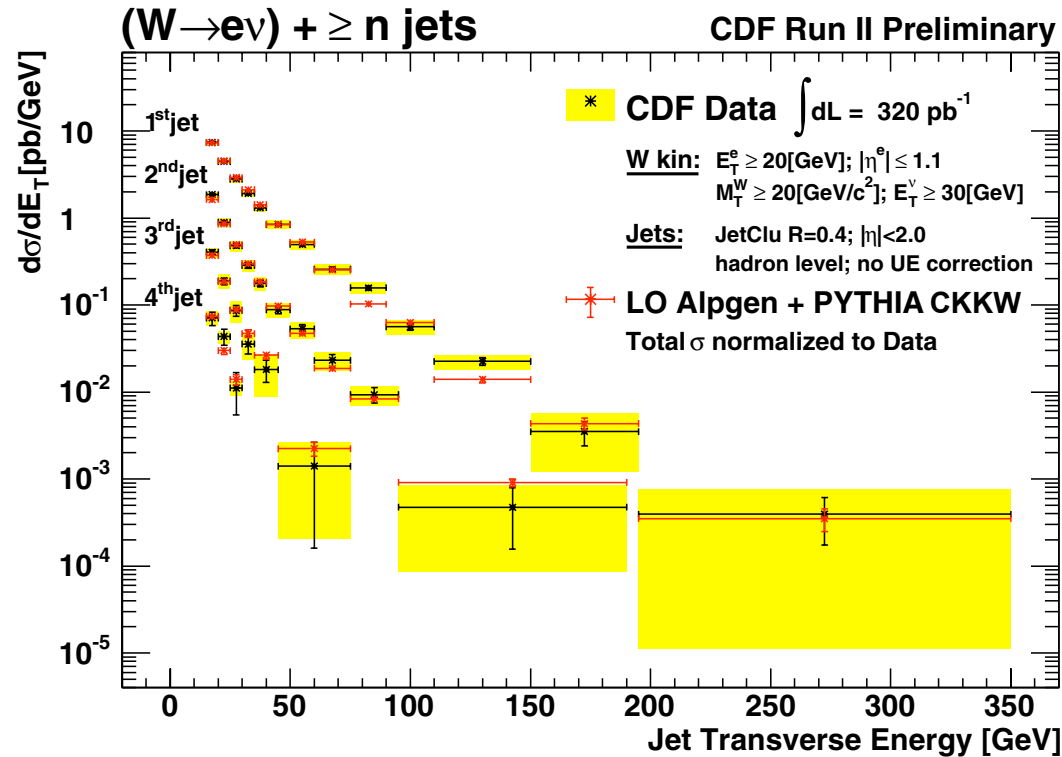
- Choose n according to $R_n(Q, Q_1)$ (LO)
 - use $[\alpha_S(Q_1)]^n$
- Use exact LO ME to generate n partons
- Construct “equivalent shower history”
 - preferably using k_T -type algorithm
- Weight vertex at scale q by $\alpha_S(q)/\alpha_S(Q_1) < 1$
- Weight parton of type i from Q_j to Q_k by
$$\Delta_i(Q_j, Q_1)/\Delta_i(Q_k, Q_1)$$

CKKW shower veto

- Shower n partons from “creation scales”
 - includes coherent soft emission
- Veto emissions at scales above Q_1
 - cancels leading (LL&NLL) Q_1 dependence



Comparisons with Tevatron data



from JM Campbell, JW Huston & WJ Stirling, Rept.Prog.Phys.70(2007)89

➔ M.E. + PYTHIA CKKW looks good

Dipole Matching

- Implemented in ARIADNE dipole MC
- Dipole cascade replaces parton shower
- Construct equivalent dipole history $\{p_{T_i}\}$
- Rejection replaces Sudakov weights
 - cascade from p_{T_i} , reject if $p_T > p_{T_{i+1}}$

L Lönnblad, JHEP05(2002)046

MLM Matching

- Use cone algorithm for jet definition:

$$R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$$

$$E_{Ti} > E_{Tmin}, R_{ij} > R_{min}$$

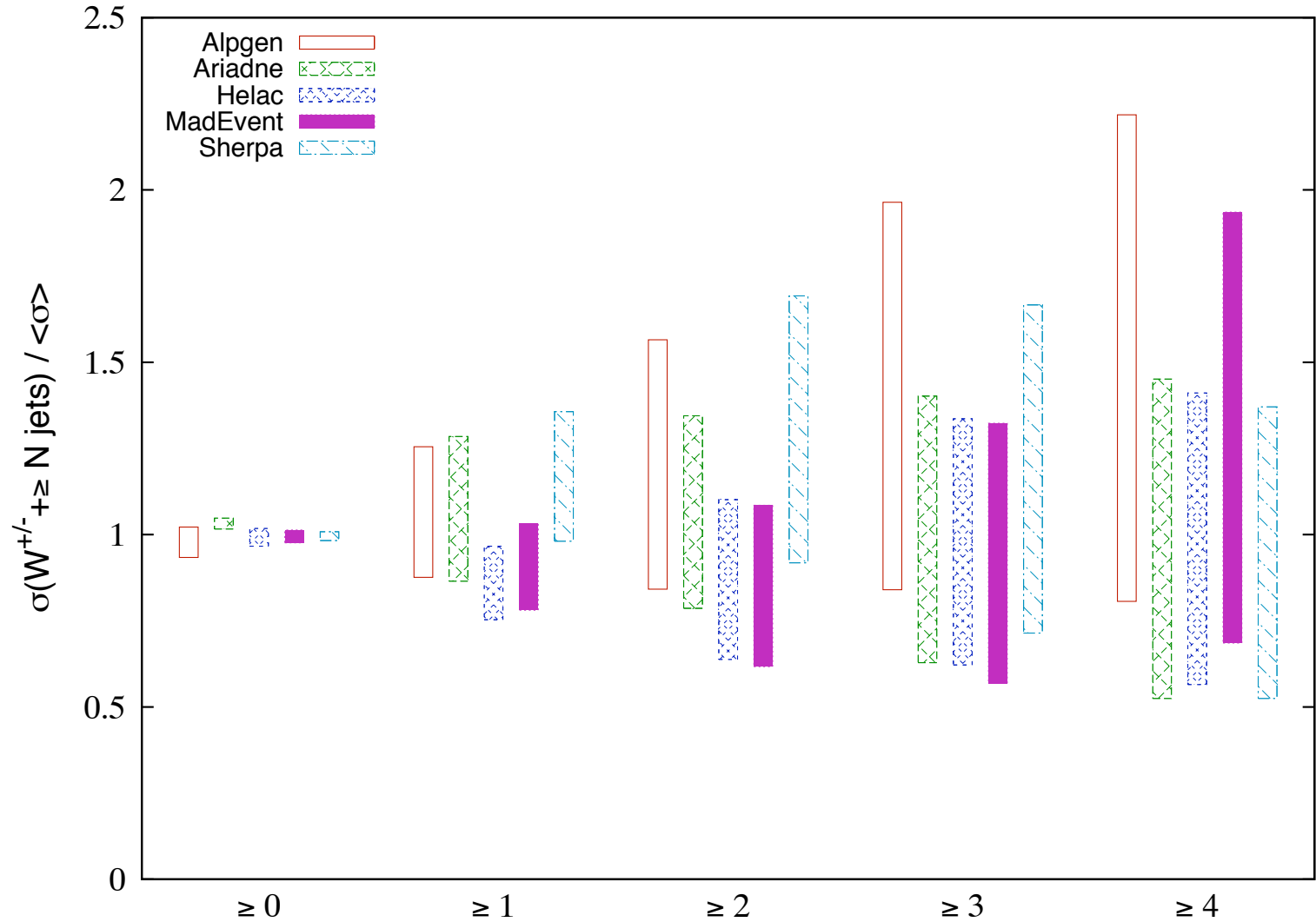
- Generate n-parton configurations with $E_{Ti} > E_{Tmin}, R_{ij} > R_{min}$ (no Sudakov weights)
- Generate showers (no vetos)
- Form jets using same jet definition
- Reject event if $n_{jets} \neq n_{partons}$

Comparisons

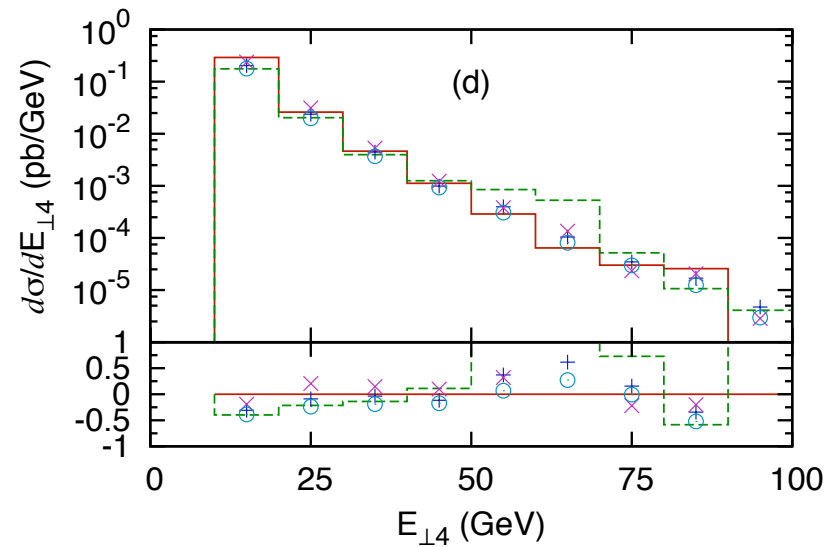
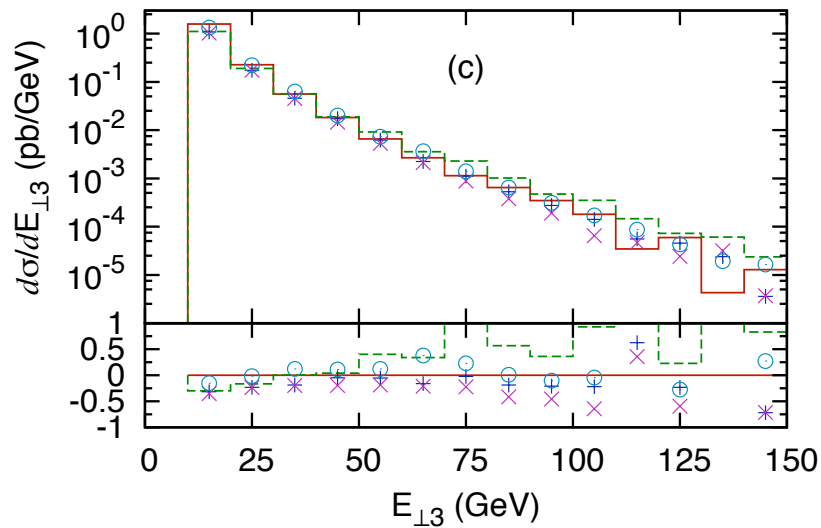
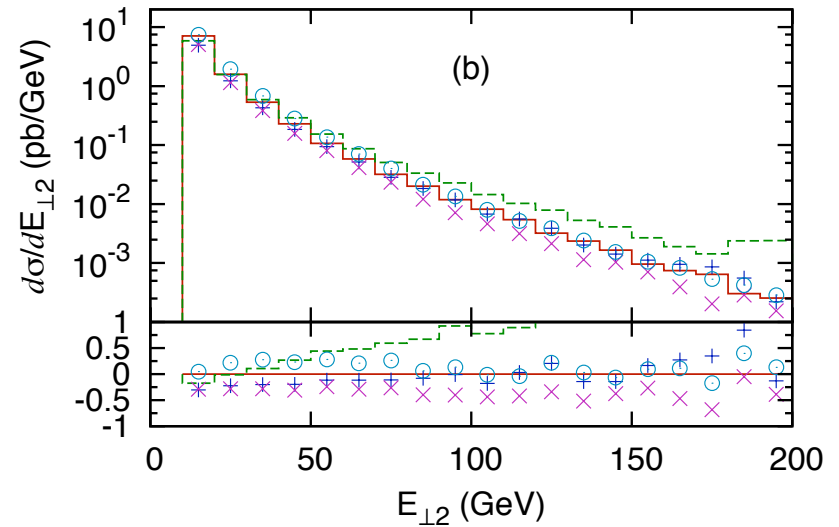
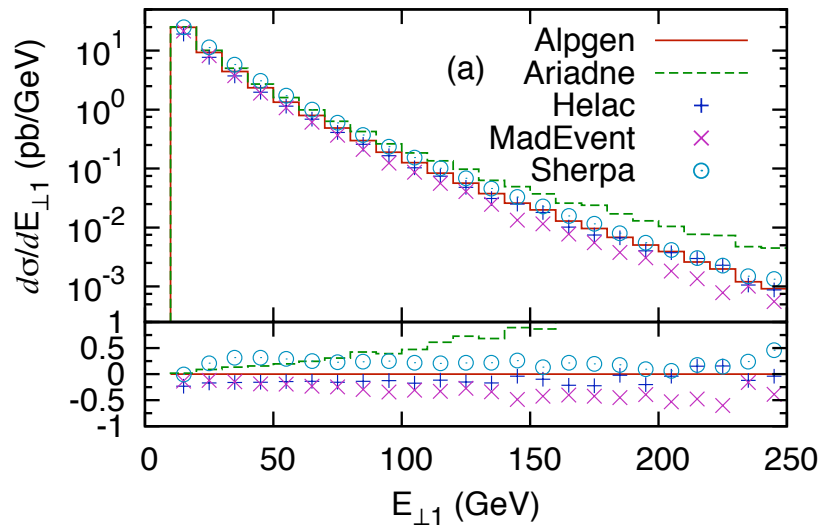
- ALPGEN: MLM matching
- ARIADNE: Dipole matching
- HELAC: MLM matching
- MadEvent: hybrid MLM/CKKW
- SHERPA: CKKW matching

J.Alwall et al., arXiv:0706.2569

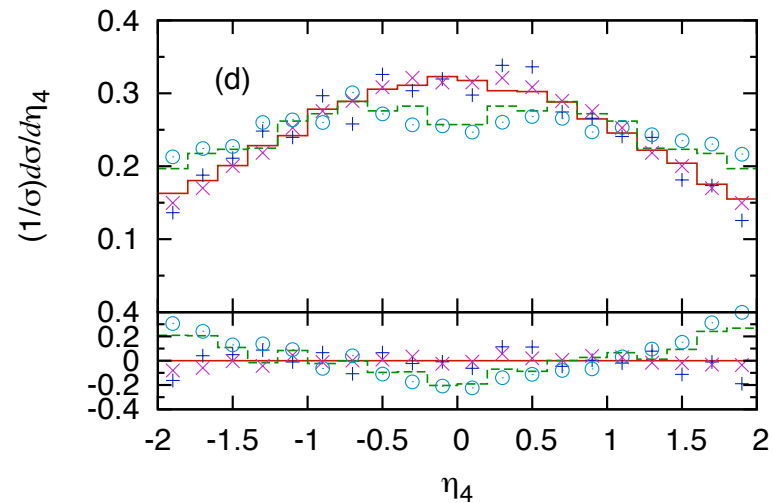
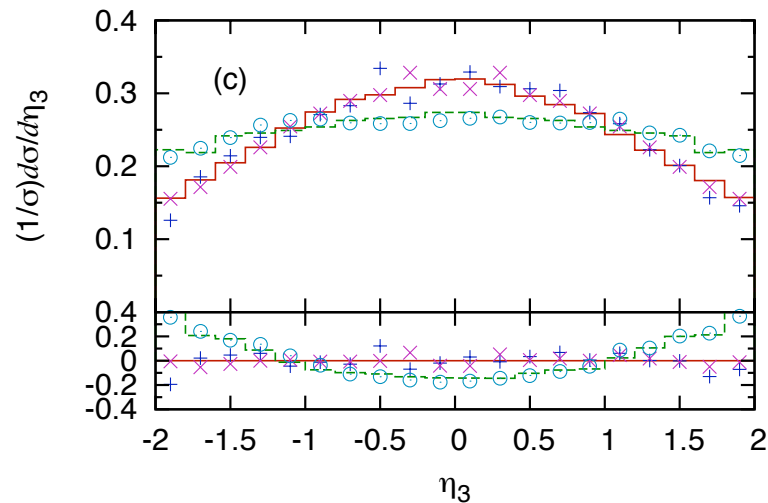
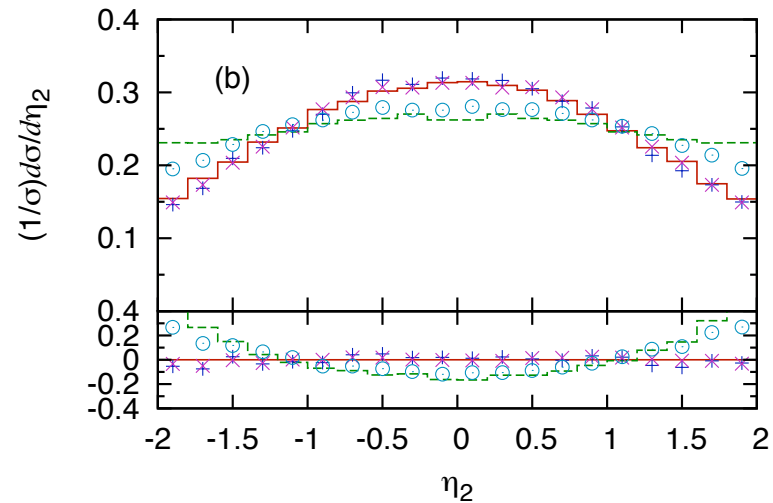
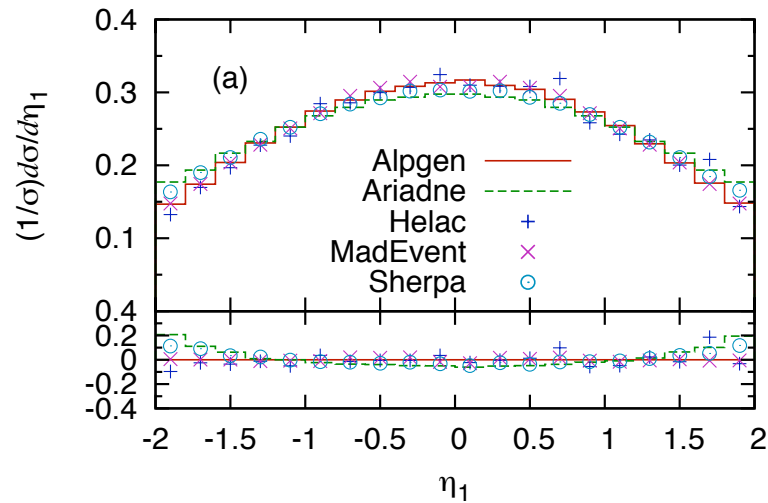
W + Multijets (Tevatron)



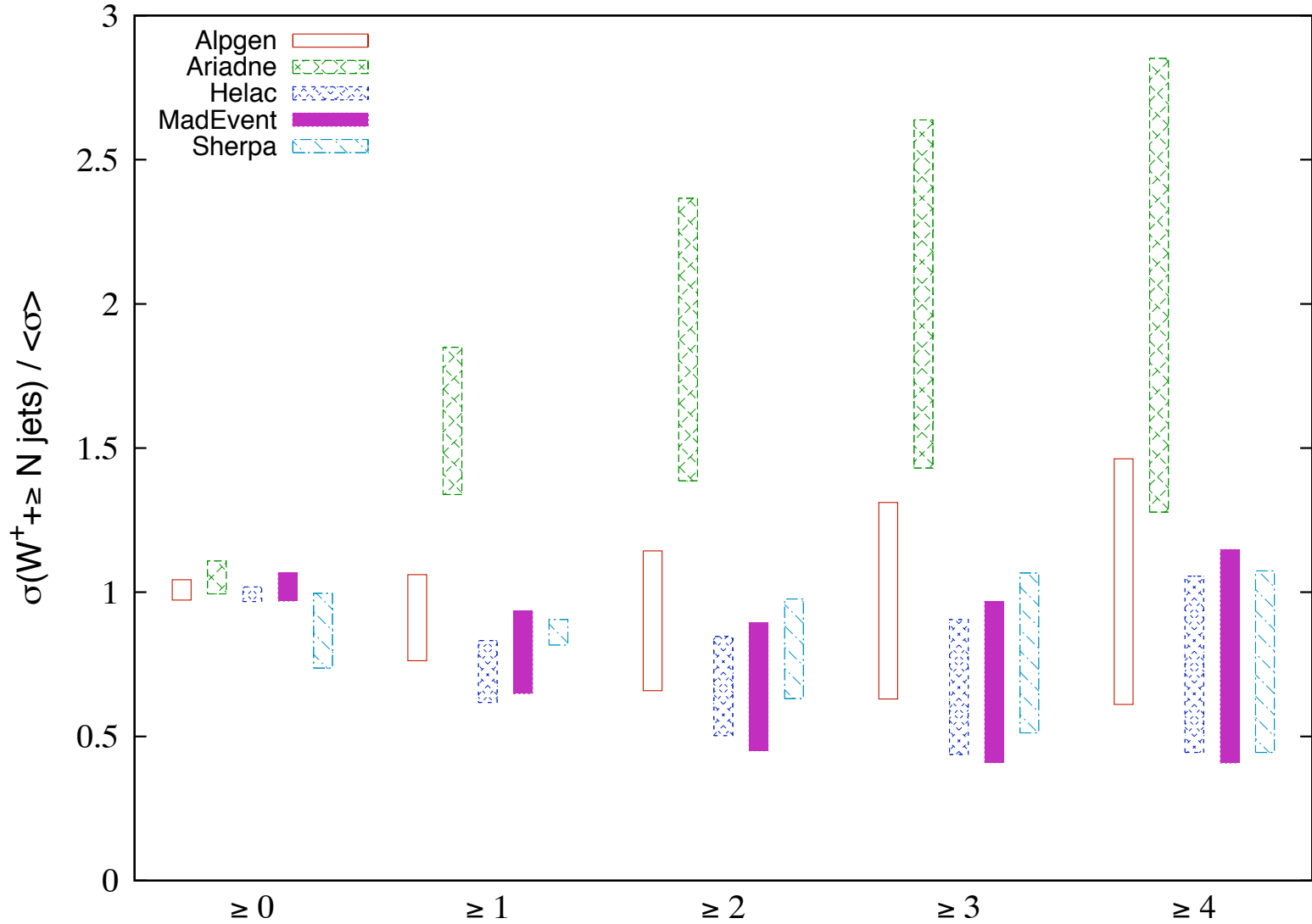
W + Multijets (Tevatron)



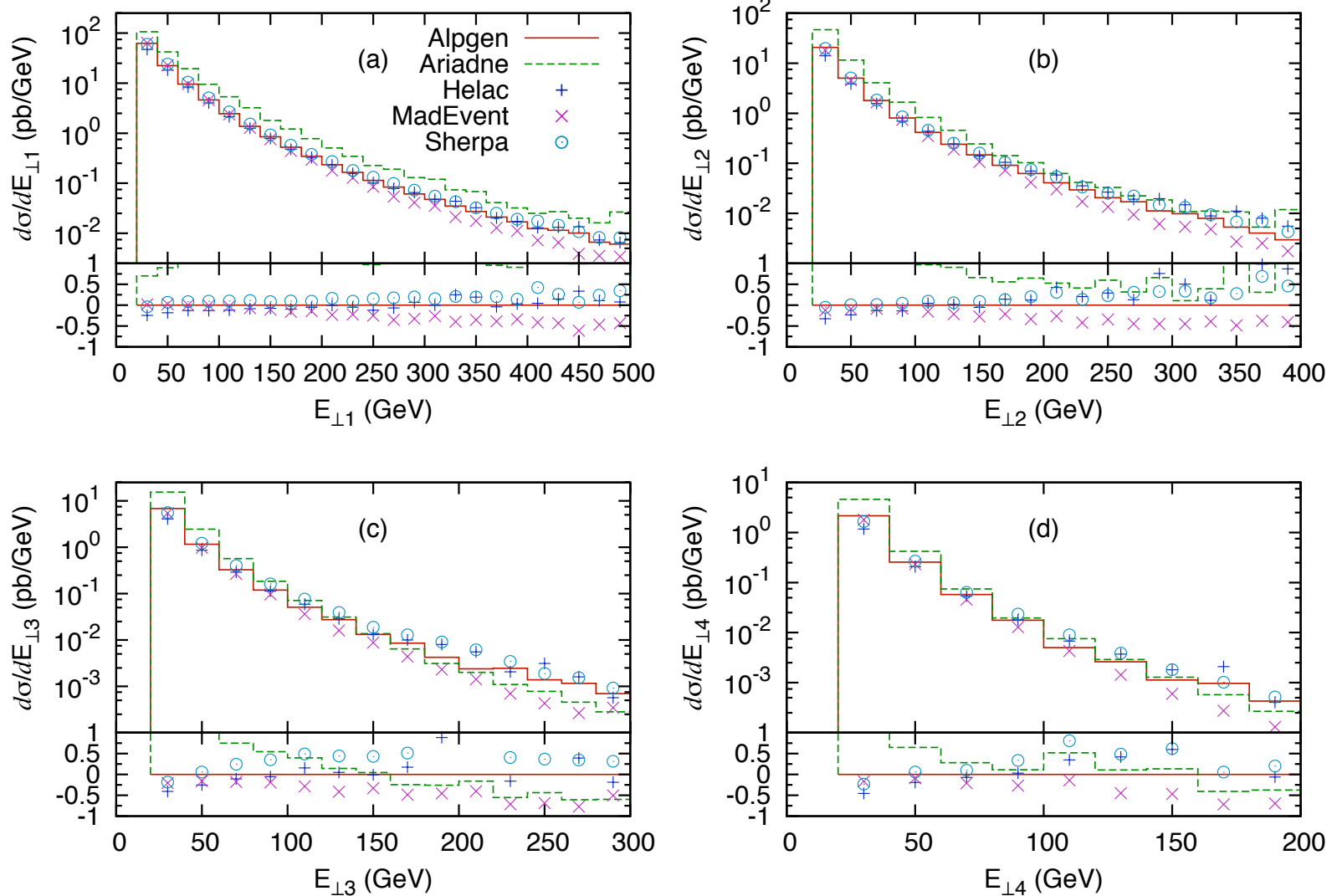
W + Multijets (Tevatron)



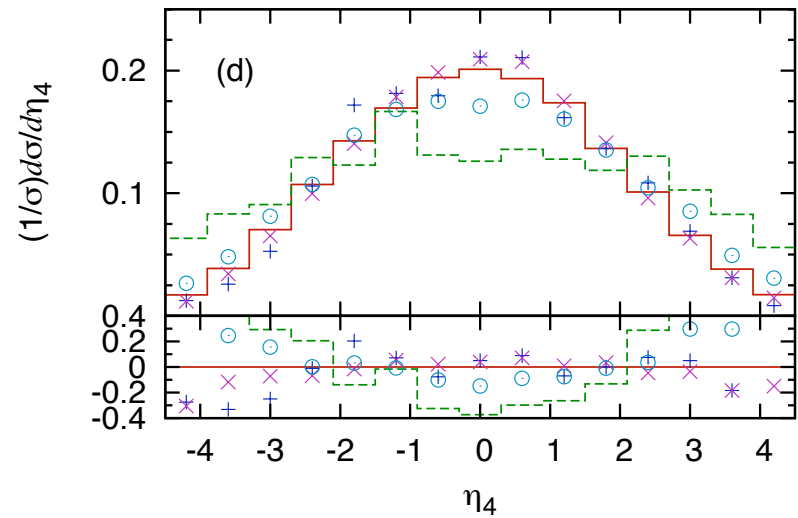
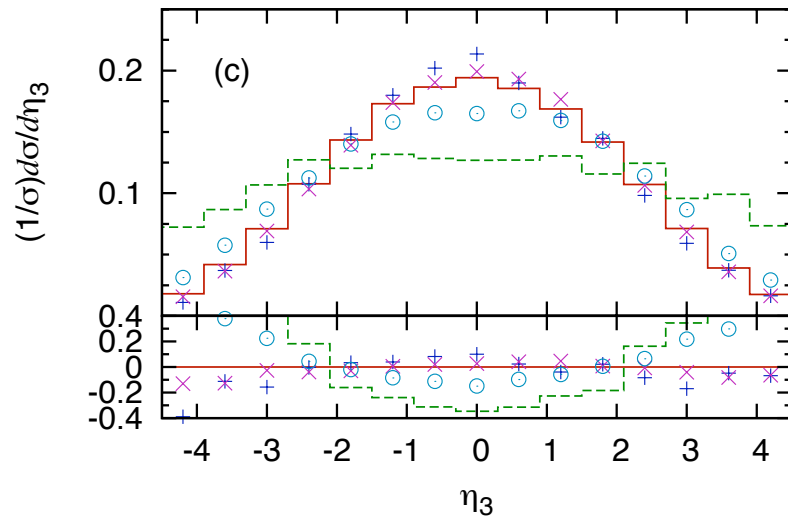
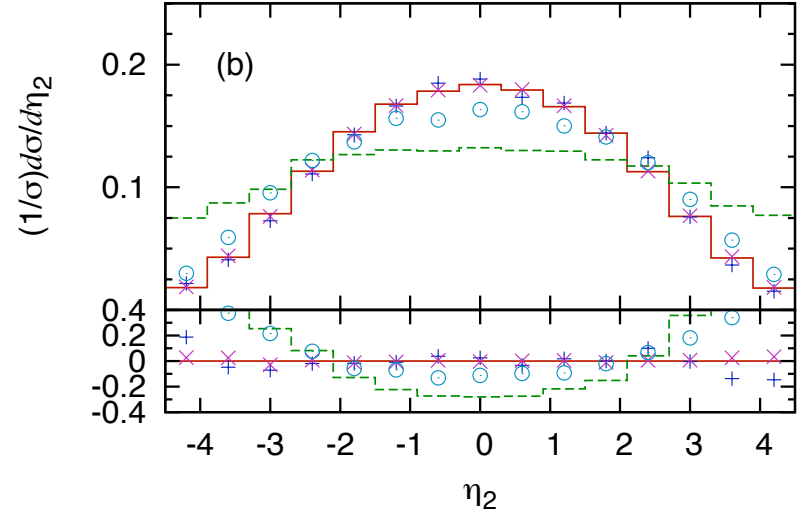
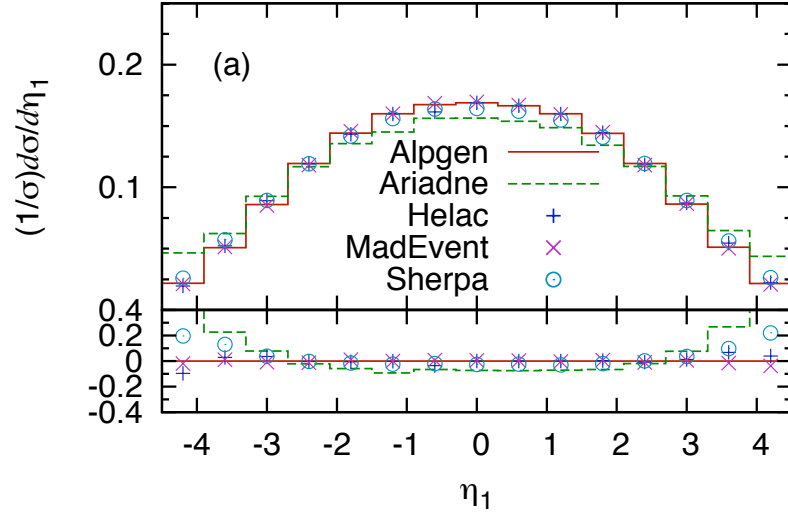
W + Multijets (LHC)



W + Multijets (LHC)



W + Multijets (LHC)



Summary of Lecture 5

- Matching Parton Showers to Matrix Elements comes in different forms:
 - matching to NLO for better precision
 - matching to LO for multijets
- MC@NLO is main scheme for NLO matching
 - newer POWHEG method looks promising
- Several options for LO multijets
 - reasonably consistent
 - spread indicates uncertainties (?)
- Field still very active
 - NLO matching for jets, spin correlations,...
 - building multijet matching into event generators