QCD Phenomenology at High Energy

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CERN Academic Training Lectures 2008

Lecture 4: Jet Fragmentation and Hadron-Hadron Processes

- Jet Fragmentation
  - Fragmentation functions
  - Small-x fragmentation
  - Average multiplicity
- Hadronization Models
  - General ideas
  - Cluster model
  - String model
- Hadron-Hadron Processes
  - Parton-parton luminosities
  - Lepton pair, jet and heavy quark production
  - Higgs boson production
- Survey of NLO Calculations for LHC

Jet Fragmentation

- Fragmentation functions $F_i^h(x,t)$ gives distribution of momentum fraction x for hadrons of type h in a jet initiated by a parton of type i, produced in a hard process at scale t.
- Like parton distributions in a hadron, $D_i^h(x,t)$, these are factorizable quantities, in which infrared divergences of PT can be factorized out and replaced by experimentally measured factor that contains all long-distance effects.
- In e^+e^- annihilation, for example, the hard process is $e^+e^- \longrightarrow q\bar{q}$ at scale equal to c.m. energy squared s; distribution of $x = 2p_h/\sqrt{s}$ is (for $s \ll M_Z^2$)

$$\frac{d\sigma}{dx} = 3\sigma_0 \sum_q Q_q^2 \left\{ F_q^h(x,s) + F_{\bar{q}}^h(x,s) \right\}$$

  where σ_0 is $e^+e^- \longrightarrow \mu^+\mu^-$ cross section.
- Fragmentation functions satisfy DGLAP evolution equation

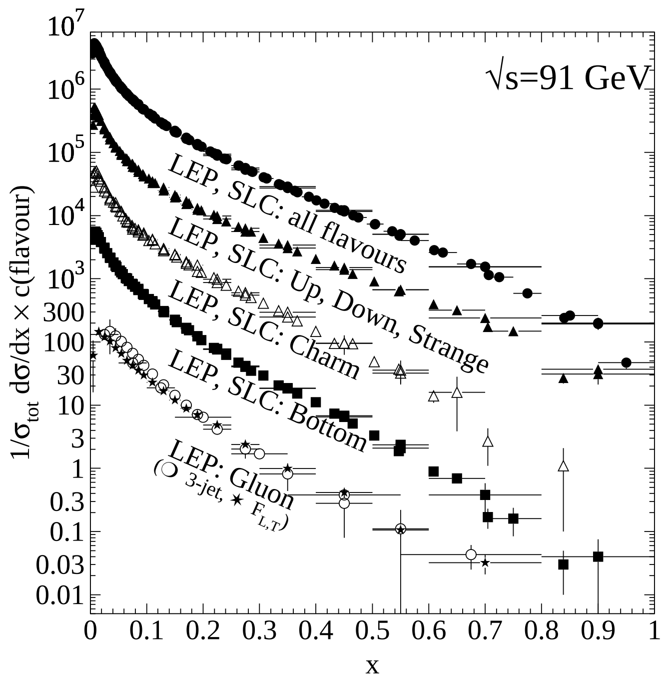
$$t\frac{\partial}{\partial t} F_i^h(x,t) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_S}{2\pi} P_{ji}(z,\alpha_S) F_j^h(x/z,t) \ .$$

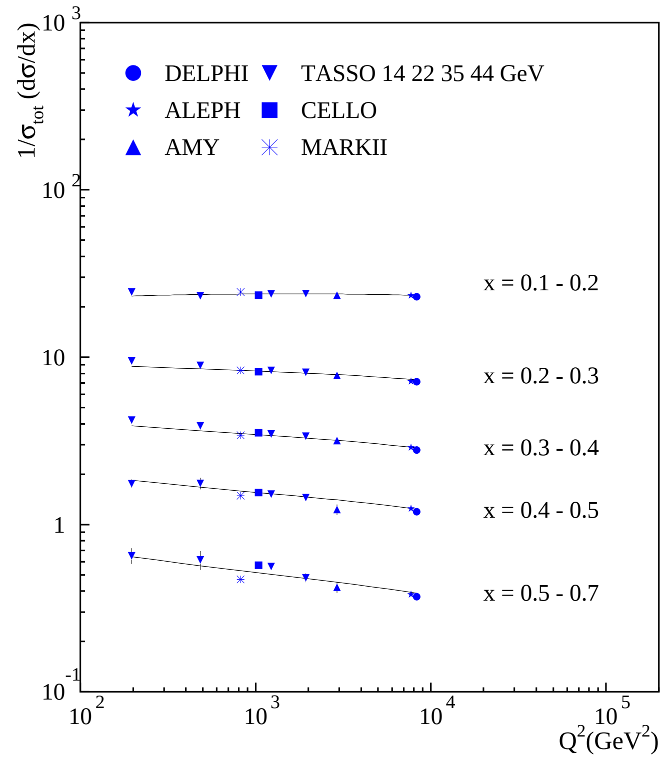
  Splitting functions P_{ji} have perturbative expansions of the form

$$P_{ji}(z,\alpha_S) = P_{ji}^{(0)}(z) + \frac{\alpha_S}{2\pi} P_{ji}^{(1)}(z) + \cdots$$

Leading terms $P_{ji}^{(0)}(z)$ were given earlier. Notice that splitting function is P_{ji} rather than P_{ij} since F_j^h represents fragmentation of final parton j.

- Solve DGLAP equation by taking moments as explained for DIS. As in that case, scaling violation is clearly seen.





Small-x fragmentation

- Evolution of fragmentation functions at small x sensitive to moments near $N = 1$. However, anomalous dimensions $\gamma_{gq}^{(0)}$, $\gamma_{gg}^{(0)}$ are not defined at $N = 1$: moment integrals for $N \leq 1$ are dominated by small z, where $P_{gi}(z)$ diverges due to soft gluon emission.
- At small z must take into account coherence effects. Recall evolution variable becomes $\tilde{t} = E^2[1 - \cos\theta]$, with angular ordering condition $\tilde{t}' < z^2\tilde{t}$. Thus, redefining t as $\tilde{t}$, evolution equation in integrated form is

$$\begin{aligned} F_i(x,t) &= F_i(x,t_0) \\ &+ \sum_j \int_x^1 \frac{dz}{z} \int_{t_0}^{z^2 t} \frac{dt'}{t'} \frac{\alpha_S}{2\pi} P_{ji}(z) F_j(x/z, t') \end{aligned}$$

or in differential form

$$t\frac{\partial}{\partial t} F_i(x,t) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_S}{2\pi} P_{ji}(z) F_j(x/z, z^2 t)\ .$$

- Only difference from DGLAP equation is z-dependent scale on the right-hand side — not important for most values of x but crucial at small x.
- For simplicity, consider first α_S fixed and neglect sum over j. Taking moments as usual,

$$t\frac{\partial}{\partial t} \tilde{F}(N,t) = \frac{\alpha_S}{2\pi} \int_x^1 dz\, z^{N-1} P(z) \tilde{F}(N, z^2 t)\ .$$

- Try solution of form $F(N,t) \propto t^{\gamma(N,\alpha_S)}$. Then anomalous dimension $\gamma(N,\alpha_S)$ must satisfy

$$\gamma(N,\alpha_S) = \frac{\alpha_S}{2\pi}\int_0^1 z^{N-1+2\gamma(N,\alpha_S)} P(z)\ .$$

- For $N-1$ not small, we can neglect $2\gamma(N,\alpha_S)$ in exponent and obtain usual formula for anomalous dimension. For $N \simeq 1$, $z \longrightarrow 0$ region dominates, where $P_{gg}(z) \simeq 2C_A/z$. Hence

$$\begin{aligned}
\gamma_{gg}(N,\alpha_S) &= \frac{C_A\alpha_S}{\pi}\frac{1}{N-1+2\gamma_{gg}(N,\alpha_S)} \\
&= \frac{1}{4}\left[\sqrt{(N-1)^2+\frac{8C_A\alpha_S}{\pi}}-(N-1)\right] \\
&= \sqrt{\frac{C_A\alpha_S}{2\pi}}-\frac{1}{4}(N-1)+\frac{1}{32}\sqrt{\frac{2\pi}{C_A\alpha_S}}(N-1)^2+\cdots
\end{aligned}$$

- To take account of running α_S, write

$$\tilde{F}(N,t) \sim \exp\left[\int^t \gamma_{gg}(N,\alpha_S)\frac{dt'}{t'}\right]\ ,$$

and note that $\gamma_{gg}(N, \alpha_S)$ should be $\gamma_{gg}(N, \alpha_S(t'))$. Use

$$\int^t \gamma_{gg}(N, \alpha_S(t'))\frac{dt'}{t'} = \int^{\alpha_S(t)} \frac{\gamma_{gg}(N, \alpha_S)}{\beta(\alpha_S)}\, d\alpha_S\ ,$$

where $\beta(\alpha_S) = -b\alpha_S^2 + \cdots$, to find

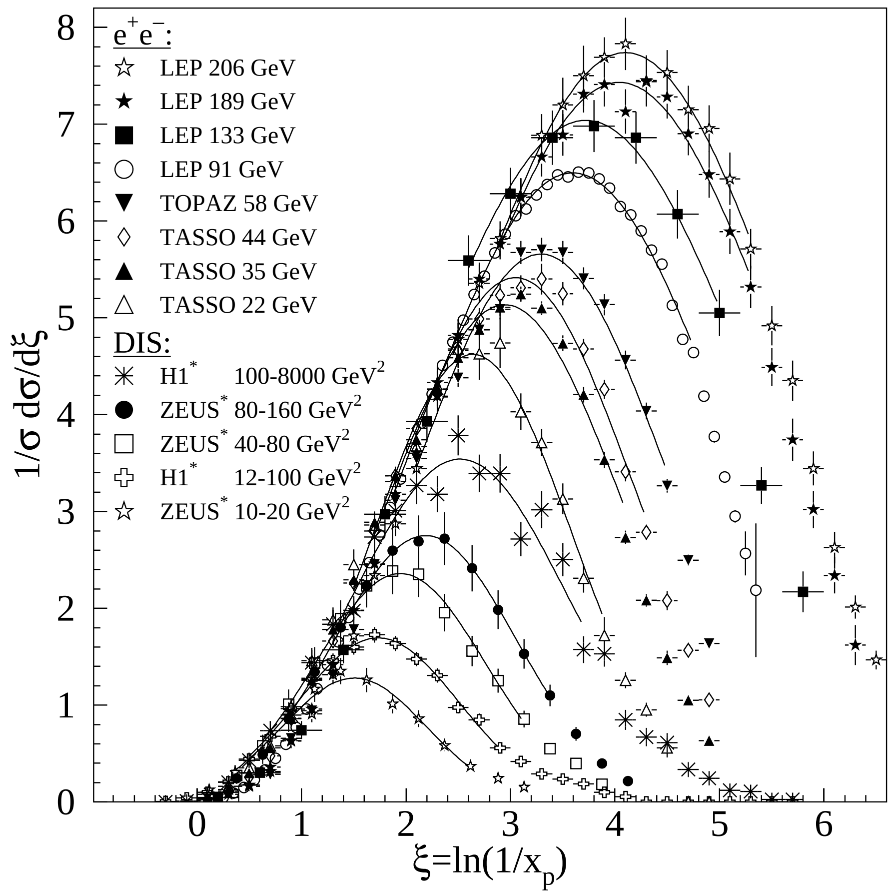
$$\begin{aligned} \tilde{F}(N,t) \quad &\sim \quad \exp\left[\frac{1}{b}\sqrt{\frac{2C_A}{\pi\alpha_S}} - \frac{1}{4b\alpha_S}(N-1)\right. \\ &+ \quad \left.\frac{1}{48b}\sqrt{\frac{2\pi}{C_A\alpha_S^3}}(N-1)^2 + \cdots\right]_{\alpha_S=\alpha_S(t)} \quad . \end{aligned}$$

- In e^+e^- annihilation, scale $t \sim s$ and behaviour of $\tilde{F}(N,s)$ near $N = 1$ determines form of small-x fragmentation functions. Keeping terms up to $(N-1)^2$ in exponent gives Gaussian function of N which transforms into Gaussian function of $\xi \equiv \ln(1/x)$:

$$xF(x,s) \propto \exp\left[-\frac{1}{2\sigma^2}(\xi - \xi_p)^2\right]\ ,$$

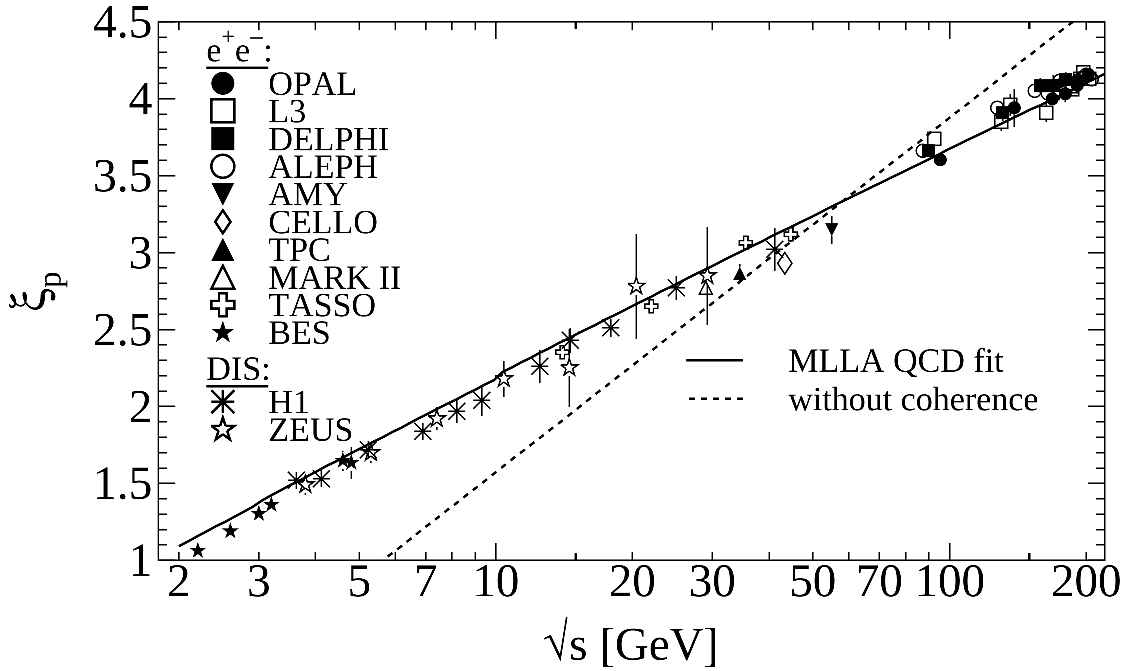
- Width of distribution

$$\sigma = \left(\frac{1}{24b} \sqrt{\frac{2\pi}{C_A \alpha_S^3(s)}} \right)^{\frac{1}{2}} \propto (\ln s)^{\frac{3}{4}} \; .$$

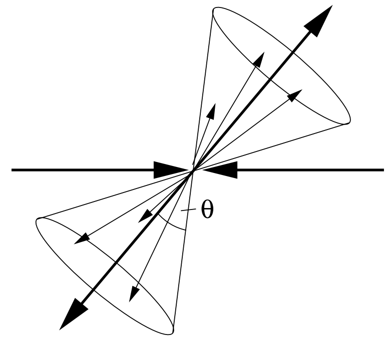


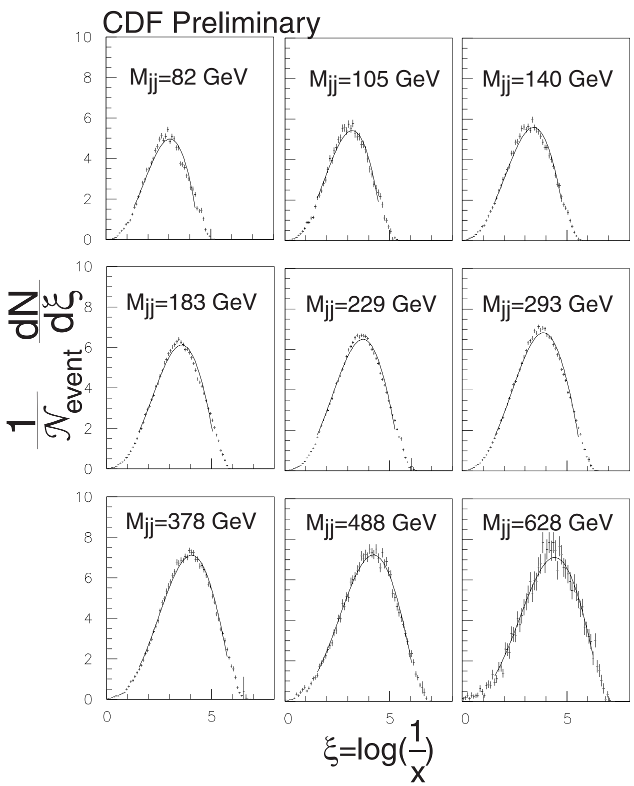
- Peak position

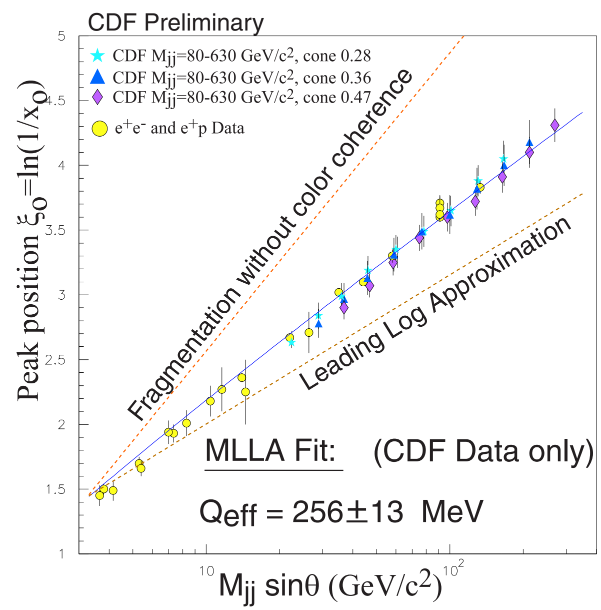
$$\xi_p = \frac{1}{4b\alpha_S(s)} \sim \frac{1}{4}\ln s$$



- Energy-dependence of the peak position ξ_p tests suppression of hadron production at small x due to soft gluon coherence. Decrease at very small x is expected on kinematical grounds, but this would occur at particle energies proportional to their masses, i.e. at $x \propto m/\sqrt{s}$, giving $\xi_p \sim \frac{1}{2}\ln s$. Thus purely kinematic suppression would give ξ_p increasing twice as fast.
- In $p\bar{p} \longrightarrow$ dijets, $\sqrt{s}$ is replaced by $M_{JJ}\sin\theta$ where M_{JJ} is dijet mass and θ is jet cone angle.


θ


CDF Preliminary
Mjj=82 GeV
Mjj=105 GeV
Mjj=140 GeV
Mjj=183 GeV
Mjj=229 GeV
Mjj=293 GeV
Mjj=378 GeV
Mjj=488 GeV
Mjj=628 GeV
1/Nevent dN/dξ
ξ=log(1/x)


CDF Preliminary
CDF Mjj=80-630 GeV/c2, cone 0.28
CDF Mjj=80-630 GeV/c2, cone 0.36
CDF Mjj=80-630 GeV/c2, cone 0.47
e+e- and e+p Data
Fragmentation without color coherence
Leading Log Approximation
MLLA Fit: (CDF Data only)
Qeff = 256±13 MeV
Peak position ξ0=ln(1/x0)
Mjj sinθ (GeV/c2)

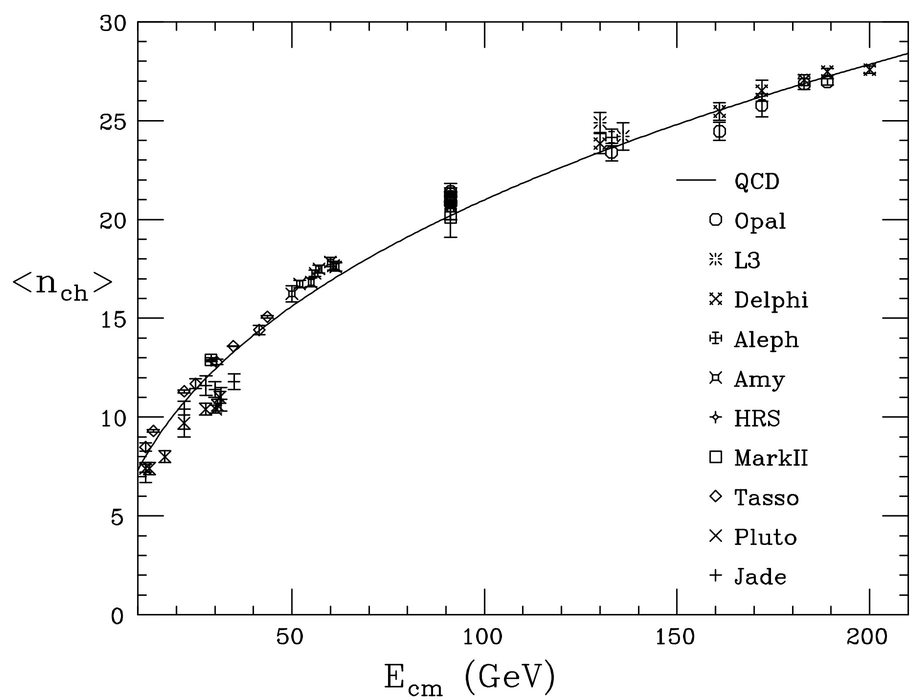
Average Multiplicity

- Mean number of hadrons is $N = 1$ moment of fragmentation function:

$$\langle n(s)\rangle = \int_0^1 dx\, F(x,s) = \tilde{F}(1,s)$$

$$\sim \exp\frac{1}{b}\sqrt{\frac{2C_A}{\pi\alpha_S(s)}} \sim \exp\sqrt{\frac{2C_A}{\pi b}\ln\left(\frac{s}{\Lambda^2}\right)}$$

(plus NLL corrections) in good agreement with data.

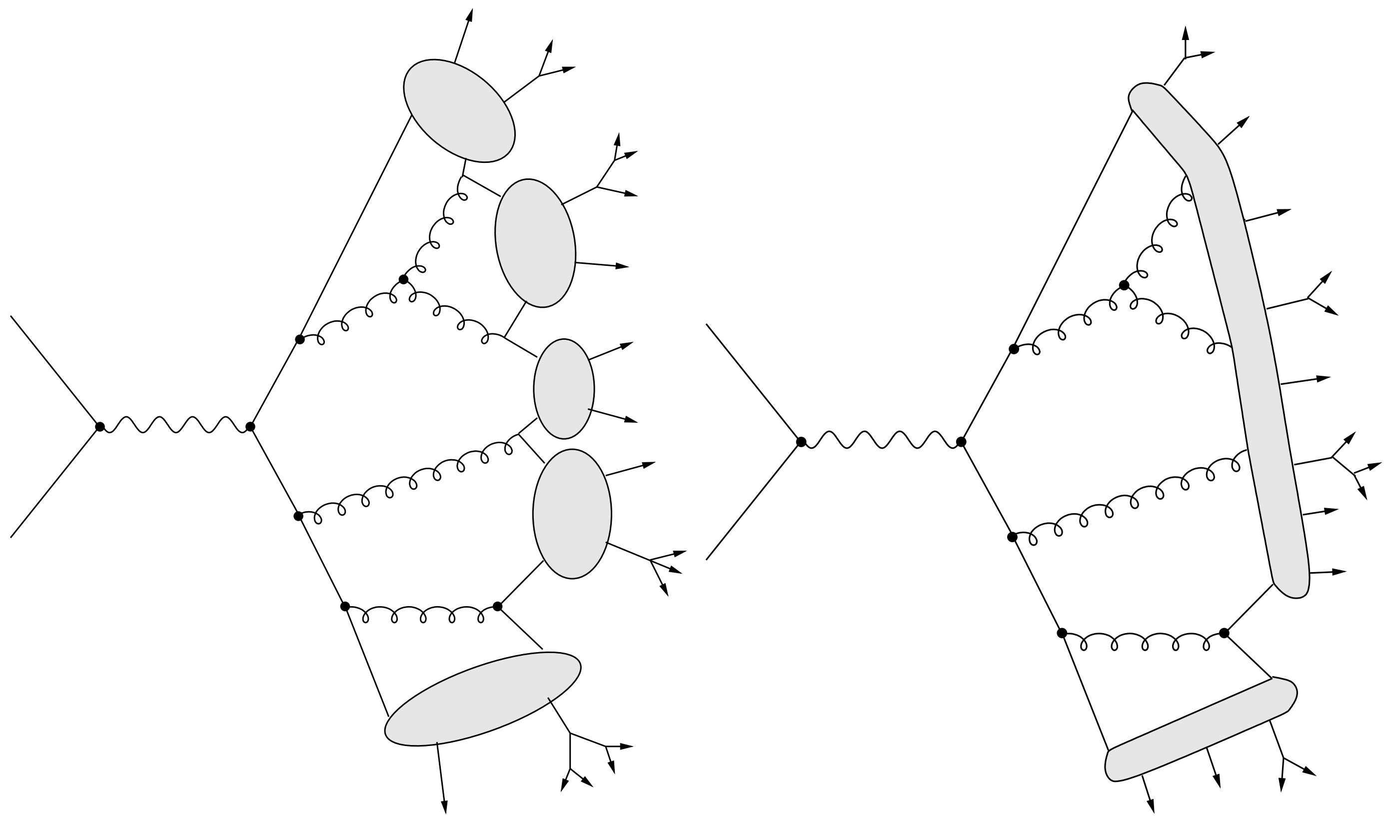


General ideas

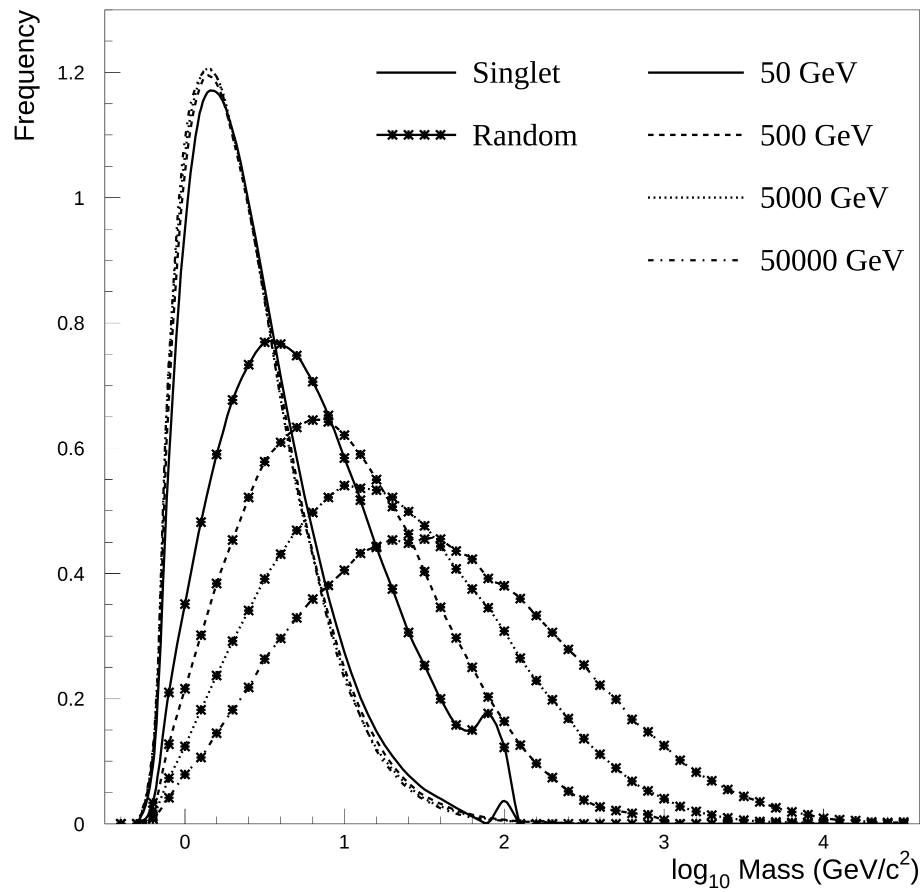
- Local parton-hadron duality
  - Hadronization is long-distance process, involving small momentum transfers. Hence hadron-level flow of energy-momentum, flavour should follow parton level.
  - Results on spectra and multiplicities support this.
- Universal low-scale α_S
  - PT works well down to very low scales, $Q \sim 1$ GeV.
  - Assume $\alpha_S(Q)$ defined (non-perturbatively) for all Q.
  - Good description of heavy quark spectra, event shapes.

Specific models

- General ideas do not describe hadron formation. Main current models are cluster and string.

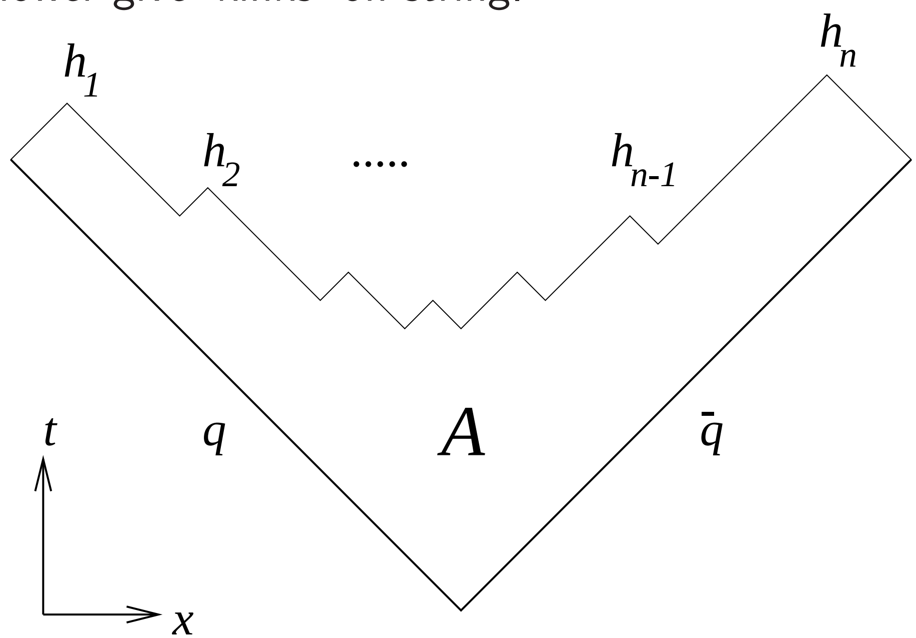


- Cluster (HERWIG)
  - Non-perturbative $g \longrightarrow q\bar{q}$ splitting after parton shower.
  - Colour singlet $q\bar{q}$ clusters have lower mass due to preconfinement property of parton shower.



  - Clusters decay according to 2-hadron density of states.
  - Few parameters: natural p_T and heavy particle suppression
  - Problems with massive clusters, baryons, heavy quarks

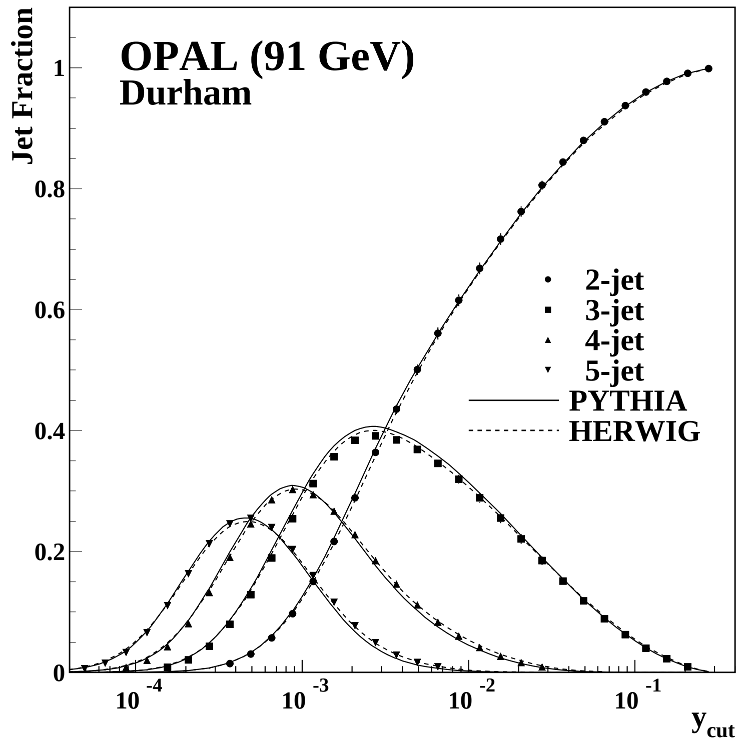
- String (PYTHIA)
  - Uses string dynamics: colour string stretched between initial $q\bar{q}$ breaks up into hadrons via $q\bar{q}$ pair production.
  - String gives linear confinement potential, area law for matrix elements.
  - Gluons produced in shower give 'kinks' on string.

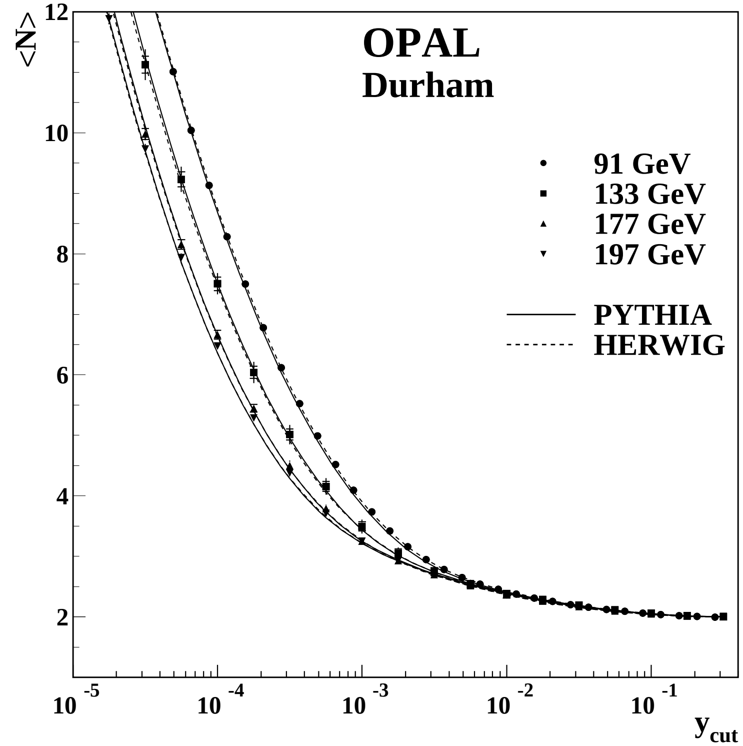


$$|M(q\bar{q} \to h_1 \cdots h_n)|^2 \propto e^{-bA}$$

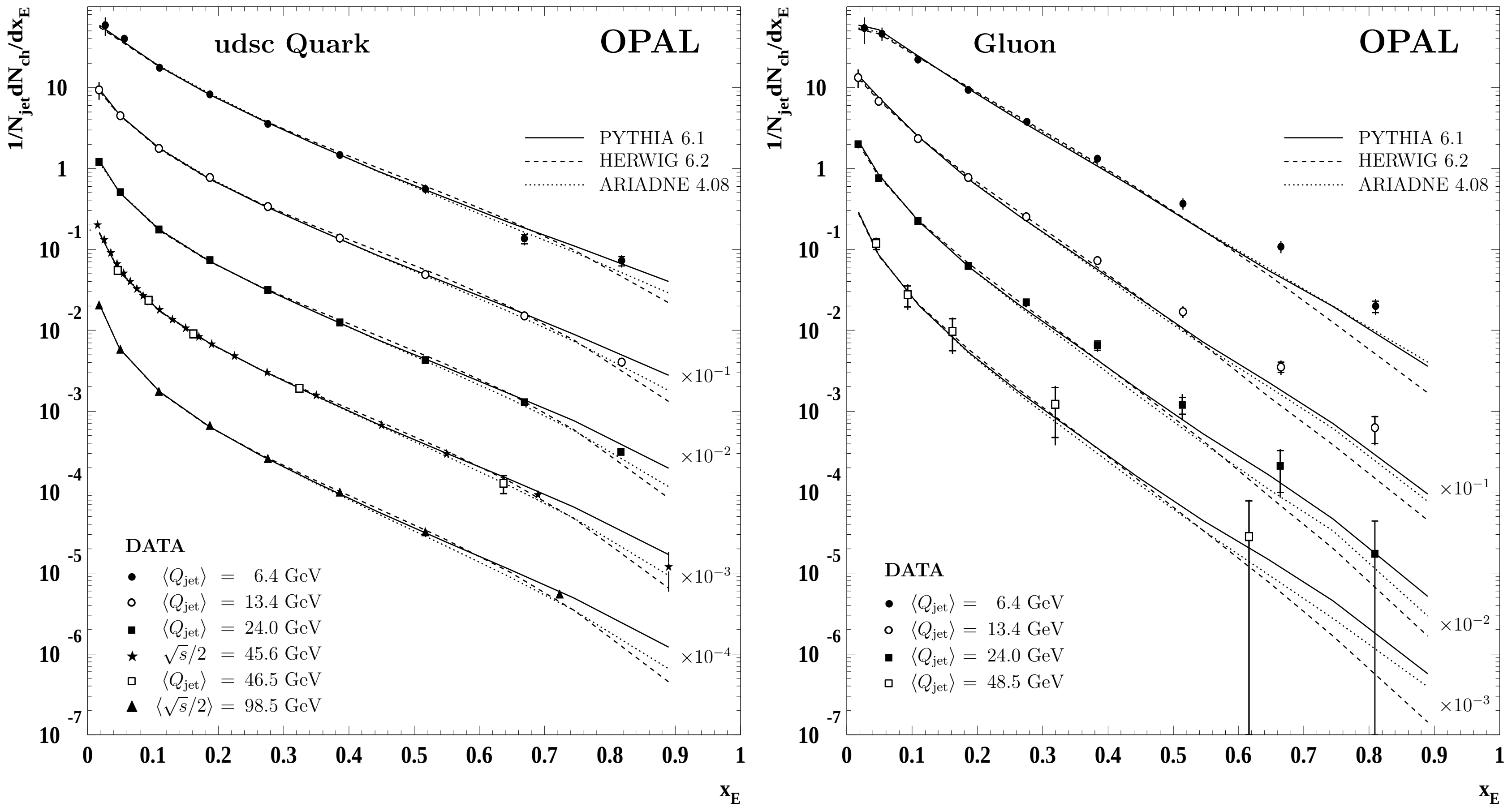
  - Extra parameters for p_T and heavy particle suppression.
  - Some problems with baryons.
- Both models describe e^+e^- data well . . .

- Jet rates and mean number of jets



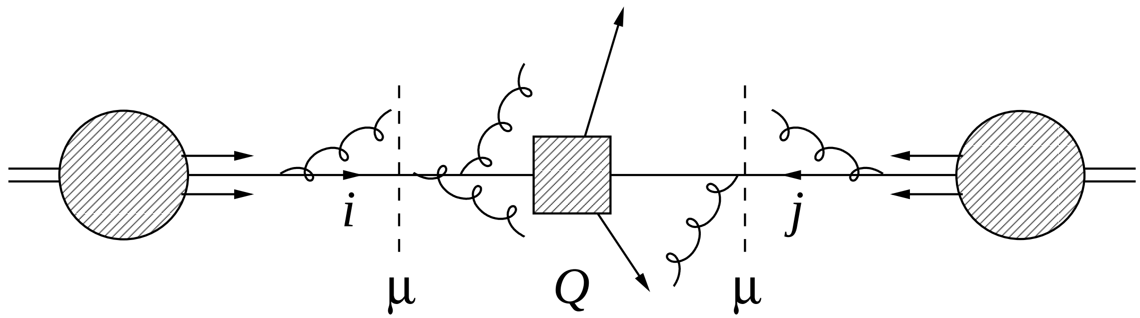


Light quark and gluon fragmentation functions



Hadron-Hadron Processes

- In hard hadron-hadron scattering, constituent partons from each incoming hadron interact at short distance (large momentum transfer Q^2).



- For hadron momenta P_1, P_2 ($S = 2P_1 \cdot P_2$), form of cross section is

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2 D_i(x_1, \mu) D_j(x_2, \mu) \hat{\sigma}_{ij}(\hat{s} = x_1 x_2 S, \alpha_S(\mu), Q/\mu)$$

where μ is factorization scale and $\hat{\sigma}_{ij}$ is subprocess cross section for parton types i, j.

  - Factorization scale is in principle arbitrary: it affects only what we call part of subprocess or part of initial-state evolution (parton shower).
  - Rapidity of subprocess c.m. frame $p^\mu = p_1^\mu + p_2^\mu$:

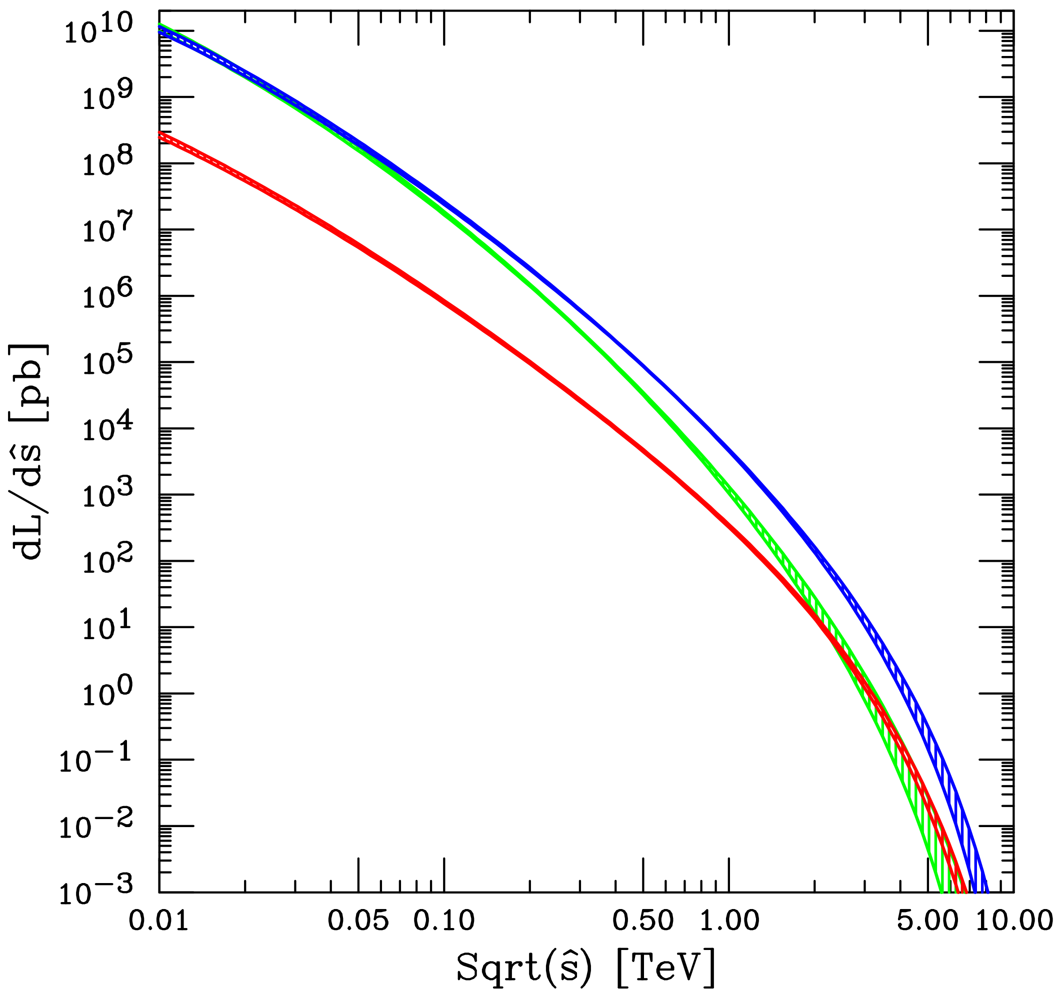
$$y \equiv \tfrac{1}{2} \ln \left[(p^0 + p_3)/(p^0 - p_3) \right] = \tfrac{1}{2} \ln (x_1/x_2)$$

  - Unlike e^+e^- or ep, we may have interaction between spectator partons, leading to *soft underlying event* and/or *multiple hard scattering*.

Parton-Parton Luminosities

- Useful to define the differential parton-parton luminosity $dL_{ij}/d\hat{s}\,dy$ and its integral $dL_{ij}/d\hat{s}$:
$$\frac{dL_{ij}}{d\hat{s}\,dy} = \frac{1}{S}\frac{1}{1+\delta_{ij}}\left[D_i(x_1,\mu)D_j(x_2,\mu) + (1 \leftrightarrow 2)\right] .$$
Factor with Kronecker delta avoids double-counting when partons are identical.
- We have $d\hat{s}\,dy = S\,dx_1\,dx_2$ and hence
$$\begin{aligned} \sigma &= \sum_{i,j}\int d\hat{s}\,dy\left(\frac{dL_{ij}}{d\hat{s}\,dy}\right)\hat{\sigma}_{ij}(\hat{s}) \\ &= \sum_{i,j}\int d\hat{s}\left(\frac{dL_{ij}}{d\hat{s}}\right)\hat{\sigma}_{ij}(\hat{s}) \end{aligned}$$
- This can be used to estimate the production rate for subprocesses at LHC.

- Figure shows parton-parton luminosities at $\sqrt{s} = 14\,\mathrm{TeV}$ for various parton combinations, calculated using the CTEQ6.1 parton distribution functions and scale $\mu = \sqrt{\hat{s}}$. Widths of curves estimate PDF uncertainties.



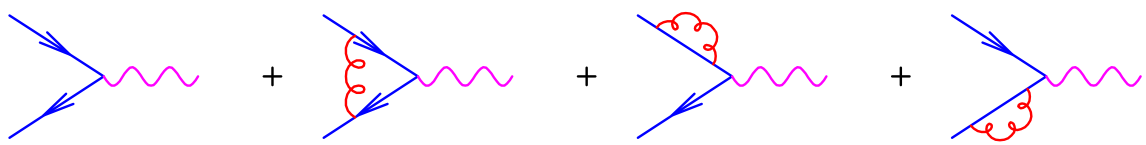
Green $= gg$, Blue $= gq + g\bar{q} + qg + \bar{q}g$, Red $= q\bar{q} + \bar{q}q$ $(q = d + u + s + c + b)$.

Lepton Pair Production

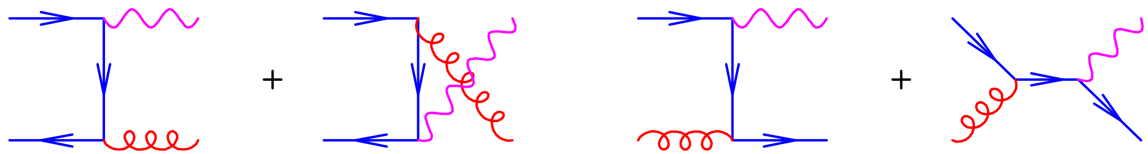
- Inverse of $e^+e^- \longrightarrow q\bar{q}$ is Drell-Yan process. At $\mathcal{O}(\alpha_S^0)$, mass distribution of lepton pair is given by

$$\frac{d\hat{\sigma}}{dM^2}(q\bar{q} \to \gamma^* \to l^+l^-) = \frac{4\pi\alpha^2}{\hat{s}}\frac{1}{3}Q_q^2\,\delta(M^2 - \hat{s})$$

❖ Factor of $1/3 = 1/N$ instead of $3 = N$ because of *average* over colours of incoming q.

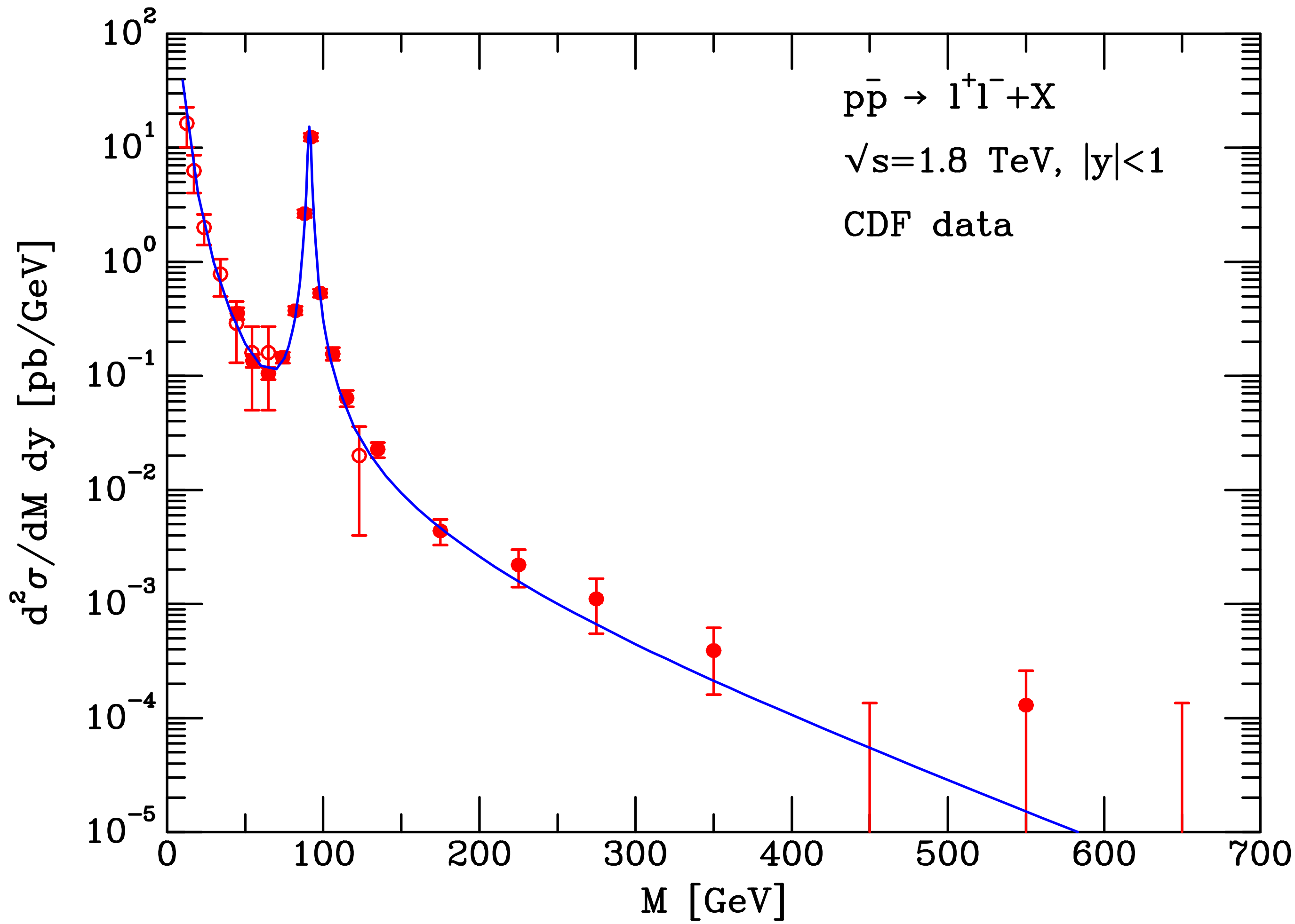


(a)



(b) (c)

❖ In higher orders *vertex corrections* (a) have $M^2 = \hat{s}$,
gluon emission (b) and *QCD Compton* (c) diagrams give $M^2 < \hat{s}$.

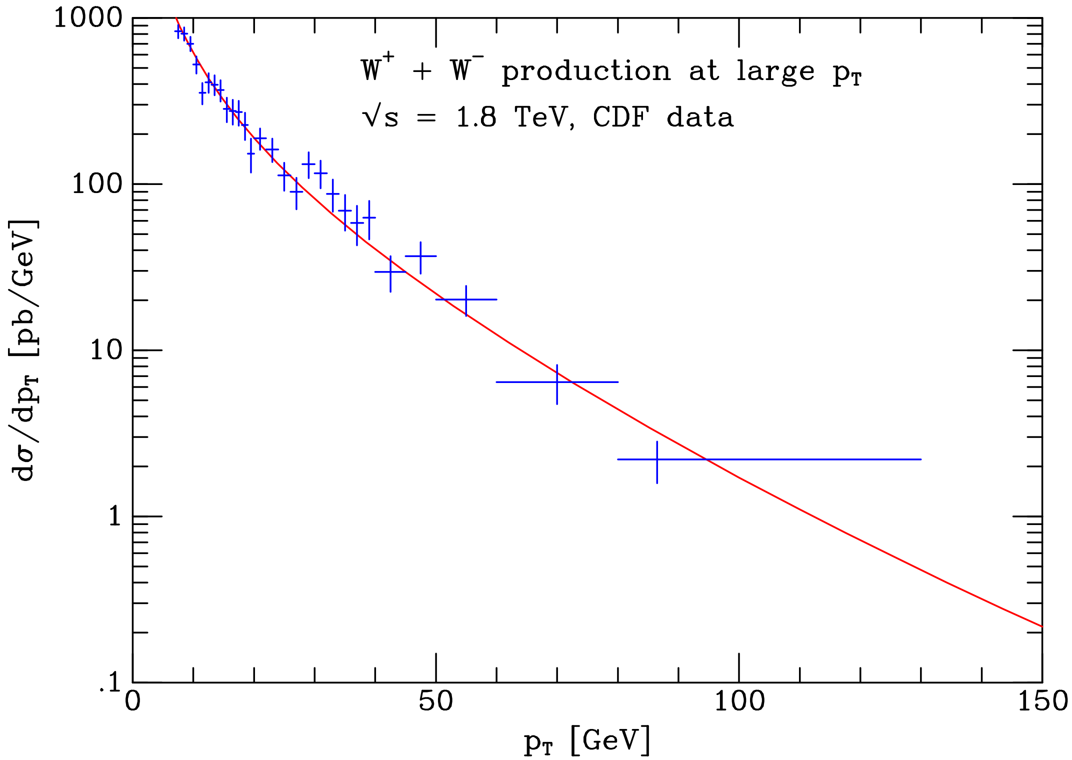


$p\bar{p} \to l^+l^- + X$
$\sqrt{s}=1.8$ TeV, $|y|<1$
CDF data
$d^2\sigma/dM\,dy$ [pb/GeV]
M [GeV]

- $W^{\pm}$ boson production is similar, except sensitive to different parton distributions, e.g.

$$u\bar{d} \to W^{+} \to l^{+}\nu_{l}$$

- Transverse momentum of lepton pair, p_T measures net transverse momentum of colliding partons plus any *intrinsic* p_T:

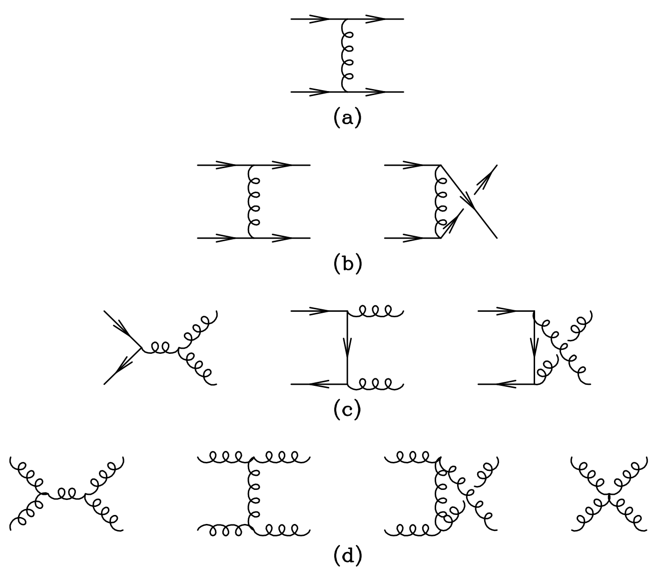


Jet Production

- Lowest-order subprocess for purely hadronic jet production is $2 \longrightarrow 2$ scattering $p_1 + p_2 \longrightarrow p_3 + p_4$

$$\frac{d\hat{\sigma}}{d\Phi_{34}} \equiv \frac{E_3 E_4 d^6\hat{\sigma}}{d^3\boldsymbol{p}_3 d^3\boldsymbol{p}_4}$$
$$= \frac{1}{32\pi^2\hat{s}}\overline{\sum}|\mathcal{M}|^2\,\delta^4(p_1+p_2-p_3-p_4)\ .$$

- Many processes even at $\mathcal{O}(\alpha_S^2)$:



- Single-jet inclusive cross section obtained by integrating over one outgoing momentum:

$$\frac{E d^3\hat{\sigma}}{d^3\boldsymbol{p}} = \frac{d^3\hat{\sigma}}{d^2\boldsymbol{p}_T dy} \longrightarrow \frac{1}{2\pi E_T}\frac{d^3\hat{\sigma}}{dE_T\, d\eta}$$
$$= \frac{1}{16\pi^2\hat{s}}\overline{\sum}|\mathcal{M}|^2\,\delta(\hat{s}+\hat{t}+\hat{u})$$

where (neglecting jet mass)

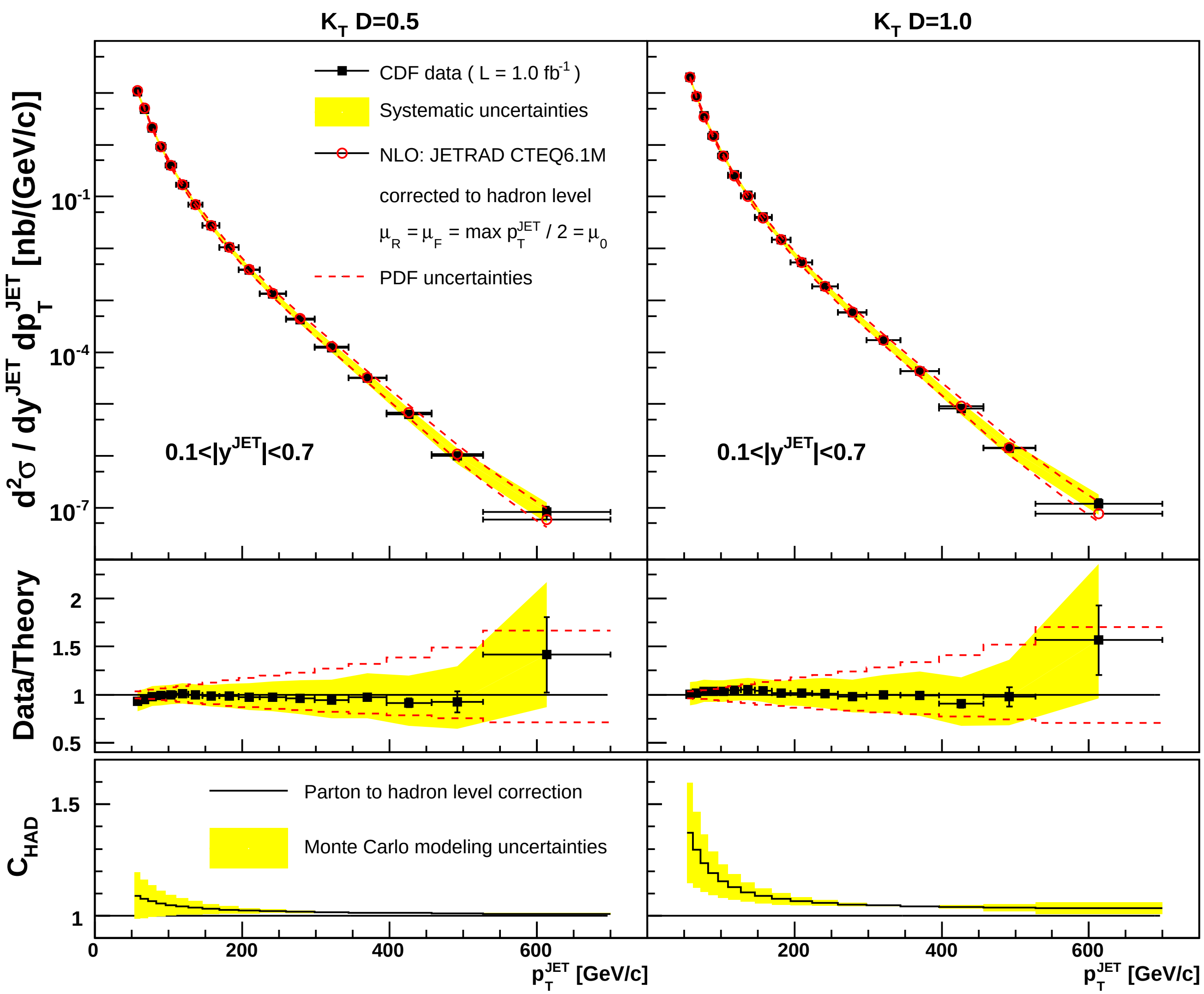
$$E_T \equiv E\sin\theta = |\boldsymbol{p}_T|\ , \quad \eta \equiv -\ln\tan(\theta/2) = y\ .$$

- Jets can be defined by a modified version of k_T algorithm discussed for e^+e^- in Lecture 2:
  - For each final-state momentum p_i and each pair of final-state momenta p_i, p_j, define

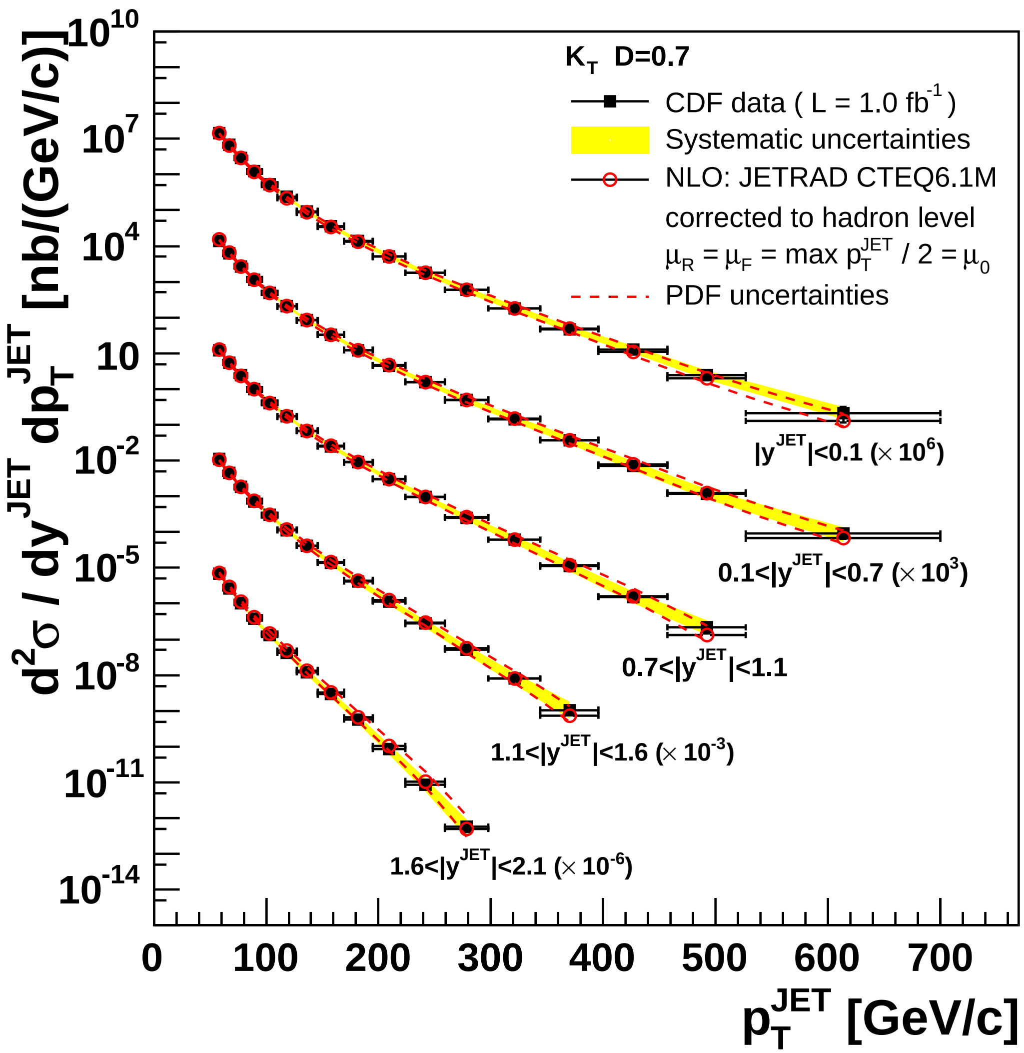
    $$k_{Ti} = E_{Ti}\ , \quad k_{Tij} = \min\{E_{Ti}, E_{Tj}\}\Delta R_{ij}/D$$

    where $\Delta R_{ij} = \sqrt{(\eta_i-\eta_j)^2+(\phi_i-\phi_j)^2}$ and $D =$ dimensionless parameter for angular size of jets ($D = 0.5 - 1.0$)
  - If k_{TI} is the smallest in the list of $\{k_{Ti}, k_{Tij}\}$, define I as a jet and remove from list.
  - If k_{TIJ} is the smallest, combine I, J into one object K with $p_K = p_I + p_J$.
  - Repeat until list is empty.
- Use η rather than θ for invariance under longitudinal boosts: $x_1 \to ax_1$, $x_2 \to x_2/a$ gives $\eta_i \to \eta_i + \ln a$, so $\eta_i - \eta_j$ is invariant.

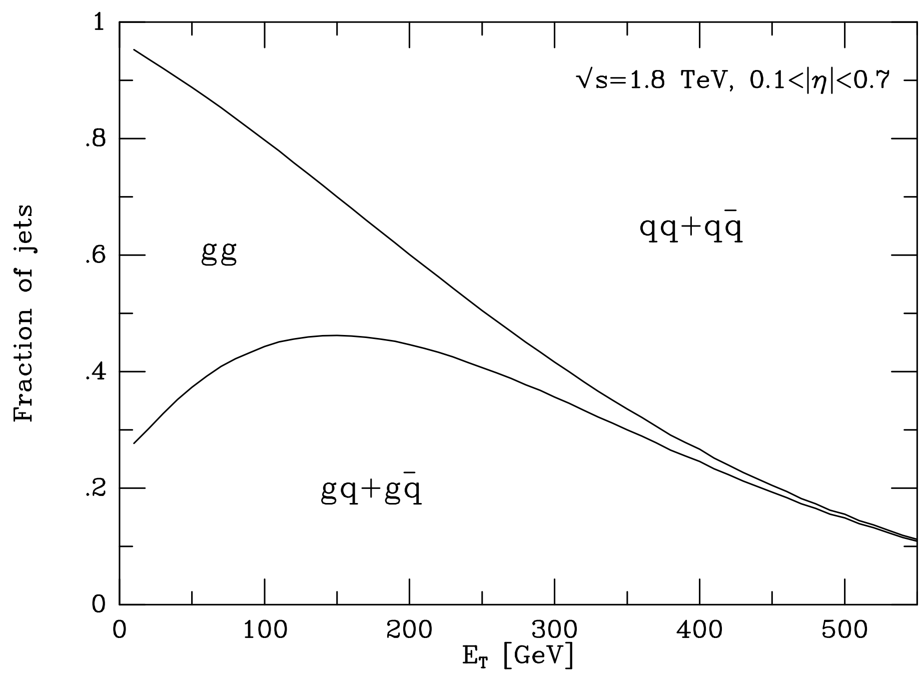
- NLO predictions and data agree very well:



- Rapidity dependence:



- Contribution of different parton combinations determined by subprocess cross sections and parton distributions.

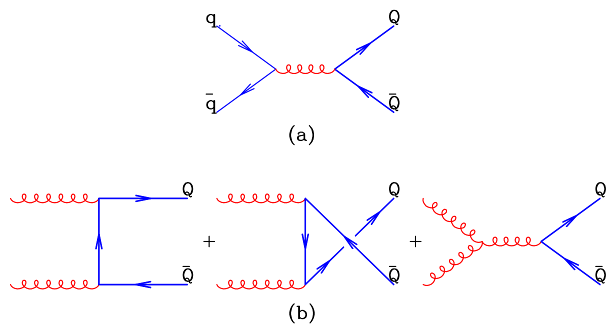


- Quarks dominate at large E_T since this selects large $x_{1,2}$:

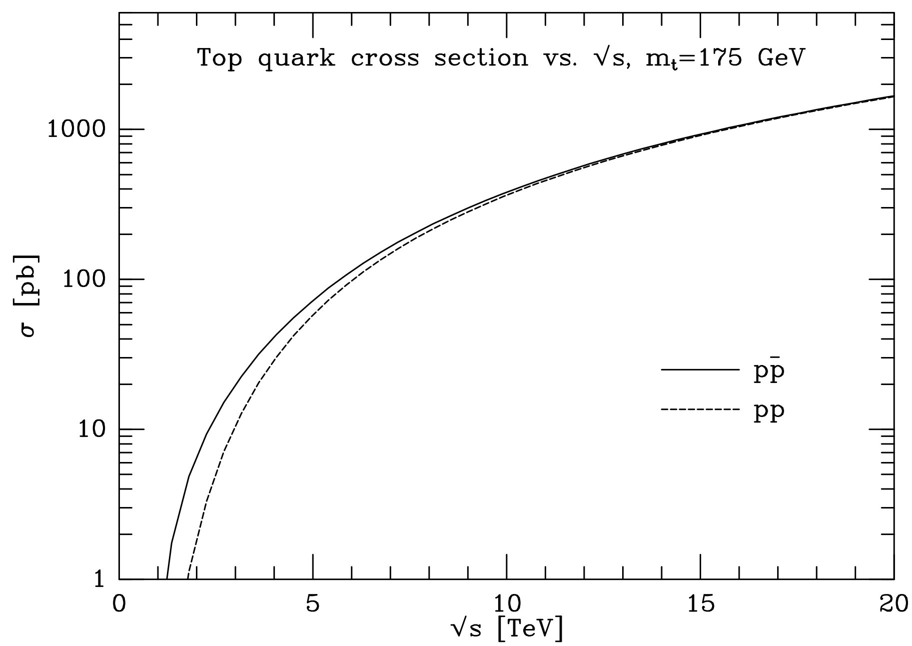
$$\hat{s} = x_1 x_2 S > 4E_T^2$$

Heavy Quark Production

- Lowest-order subprocesses for heavy quark production are (a) light quark-antiquark annihilation (10% at LHC) and (b) gluon-gluon fusion (90% at LHC)

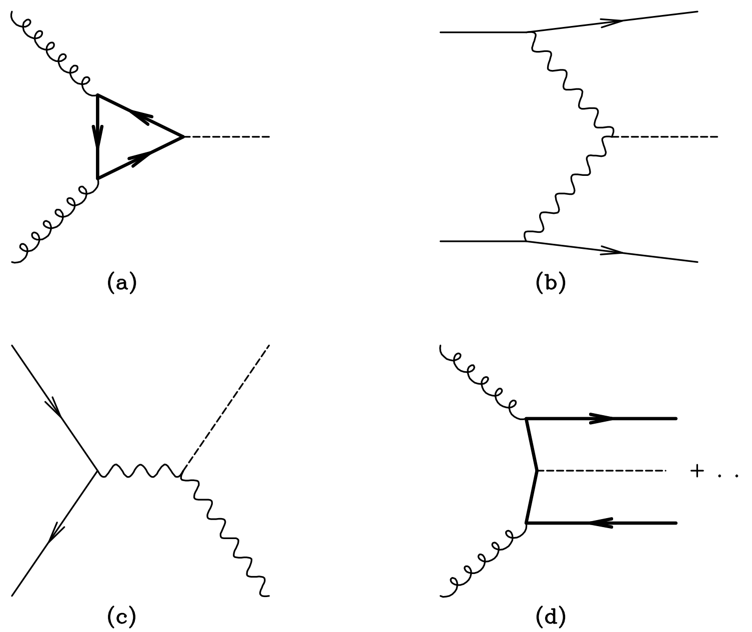


- NLO top quark cross section = 840±30(scale)±20(pdf) pb at LHC

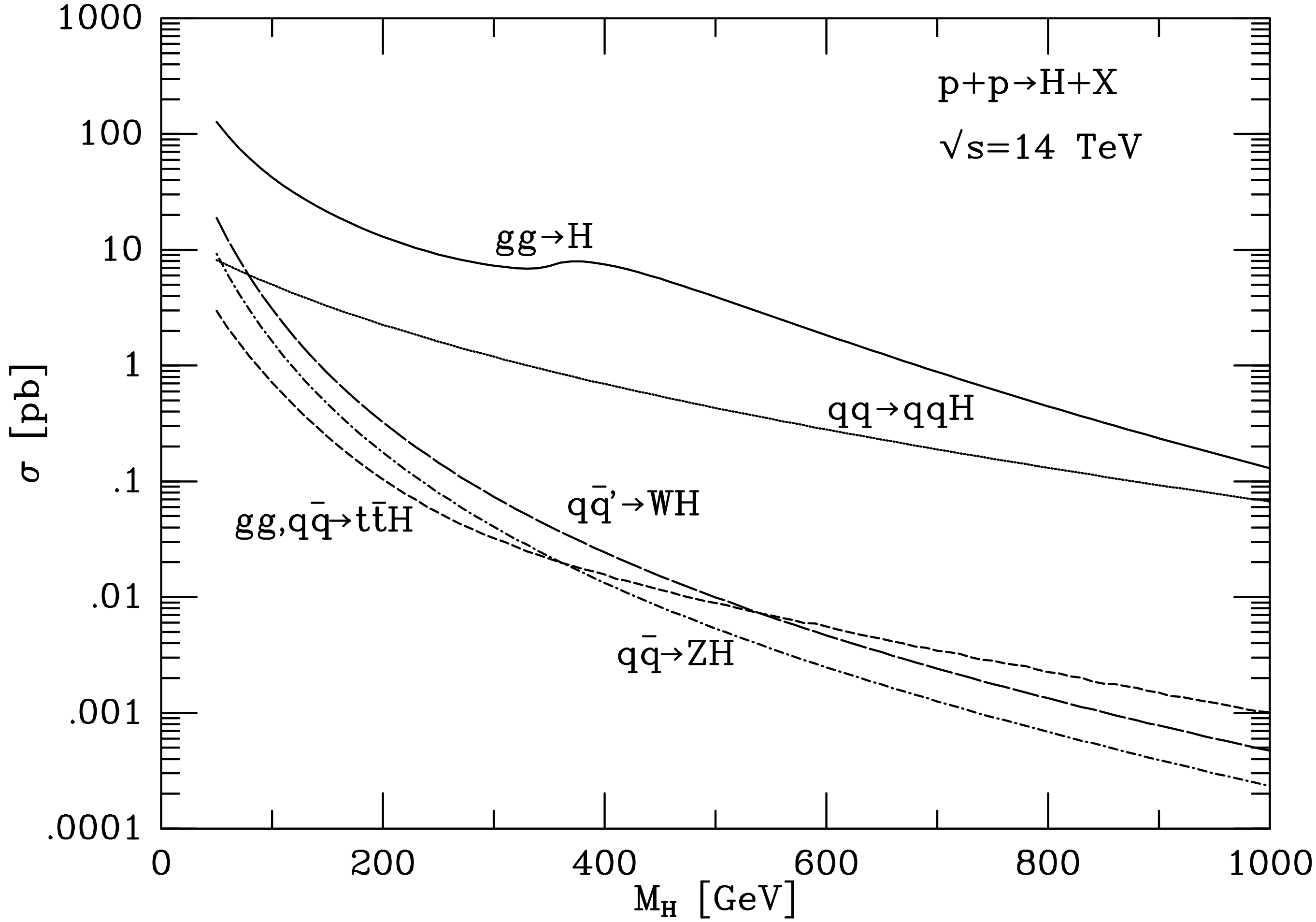


Standard Model Higgs Boson Production

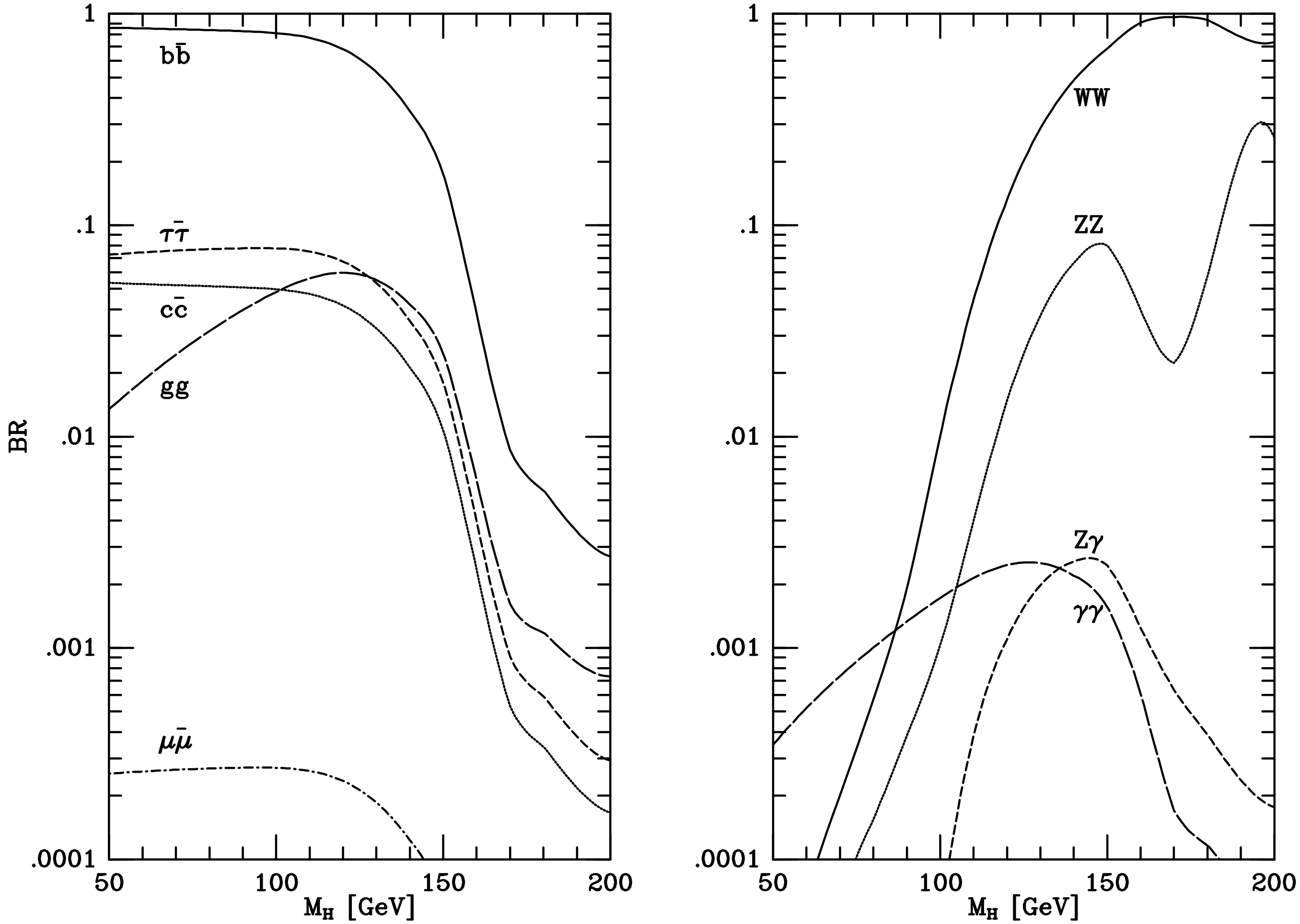
- Lowest-order subprocesses for Higgs boson production at hadron colliders:
  (a) Gluon-gluon fusion (via top loop)
  (b) Vector boson fusion
  (c) Associated production with W, Z boson
  (d) Associated production with $t\bar{t}$.



- NLO Higgs cross sections



- Discovery decay channels depend on Higgs mass



Status of NLO Calculations for LHC

- $2 \longrightarrow 2$ parton processes — all available, e.g. in MCFM (CaEl*)
- $2 \longrightarrow 3$ parton processes

Final State	Authors*	Comments
3 jets	KuSiTr,BerDixKo,GiKi,Na	Public code available
$V+$ 2 jets	ElCa,CaGlMi	Public code available
$V\,b\,\bar{b}$	ElCa	Massless b quarks
$V\,b\,\bar{b}$	ReFeWa	Massive b quarks
$H+$ 2 jets	FiOlZep	Vector boson fusion
$H+$ 2 jets	CaElZa	Gluon fusion
$V\,V+$ 2 jets	JaOlZep	Vector boson fusion
$\gamma\gamma$ jet	deFKu,DelMalNaTr,BiGuMah	
$t\bar{t}H$, $b\bar{b}H$	ReDaWaOr,BeeDitKrPlSpZer	
$t\bar{t}$ jet	DitUwWe	
$H\,H\,H$	PlRa,BiKarKauRu	
$W\,W$ jet	DiKalUw	
$Z\,Z\,Z$	LaMePe	

*Beenakker,Bern,Binoth,Campbell,Dawson,deFlorian,DelDuca,Dittmaier,Dixon,Ellis,FebresCordero,Figy, Giele,Glover,Guillet,Jager,Kallweit,Karg,Kauer,Kilgore,Kramer,Kosower,Kunszt,Lazopoulos,Mahmoudi, Maltoni,Melnikov,Miller,Nagy,Oleari,Orr,Petriello,Plehn,Plumper,Rauch,Reina,Ruckl,Signer,Spira, Troscsanyi,Uwer,Wackeroth,Weinzierl,Zanderighi,Zeppenfeld,Zerwas

- Les Houches 2005 wish list of "feasible" NLO calculations

Final State	Relevance	Progress?
$V\,V$ jet	$t\bar{t}H$, new physics	$VV=\gamma\gamma, WW$
$V\,V\,V$	SUSY trilepton	ZZZ
$V\,V\,b\bar{b}$	VBF$\rightarrow H \rightarrow VV$, $t\bar{t}H$, new physics	
$V\,V+2$ jets	VBF$\rightarrow H \rightarrow VV$	
$V+3$ jets	various new physics signatures	
$t\bar{t}+2$ jets	$t\bar{t}H$	
$t\bar{t}\,b\bar{b}$	$t\bar{t}H$	

- Les Houches 2007: WWW, $b\bar{b}b\bar{b}$, 4 jets added.

Summary of Lecture 4

- Jet fragmentation functions also obey DGLAP evolution equations.
  - Scaling violation seen in e^+e^-.
  - Small-x fragmentation sensitive to coherence effects.
  - Gaussian peak in $\ln(1/x)$, peak position shows coherence.
  - Average hadron multiplicity predicted.
- Hadronization models needed for simulation of full final states.
  - General ideas describe spectra and event shapes.
    - → Local parton-hadron duality.
    - → Universal low-scale α_S.
  - Specific models needed for hadron distributions.
    - → String model (PYTHIA).
    - → Cluster model (HERWIG).
- In hadron-hadron processes, factorization permits cross section calculations.
  - Parton-parton luminosities important: uncertainties $\sim 10 - 20\%$.
  - Lepton-pair, jet, top and Higgs production reliably predicted (NLO or NNLO).
  - All $2 \to 2$ and many $2 \to 3$ subprocesses predicted to NLO.