Monte Carlo Methods in Particle Physics

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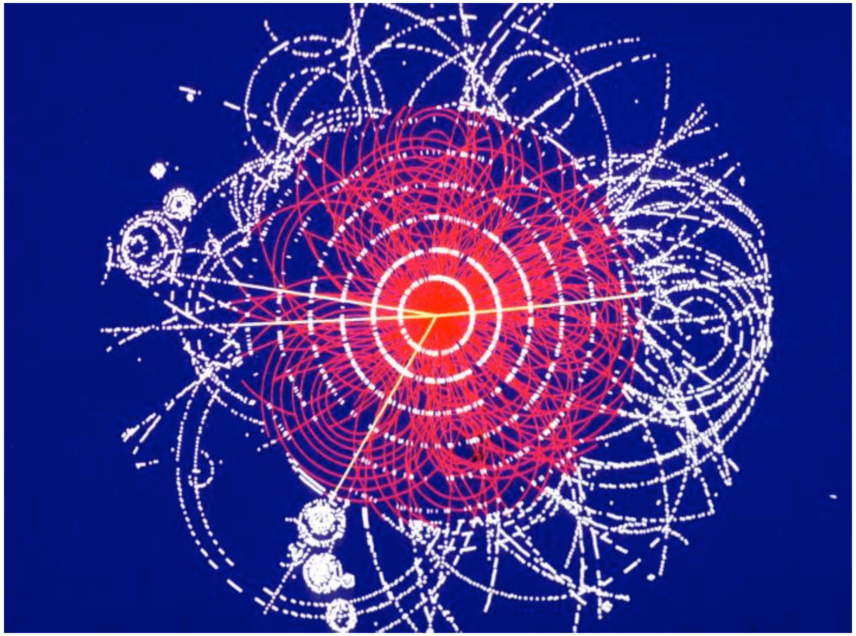
Monte Carlo Event Generation

- Basic Principles
- Event Generation
- Parton Showers
- Hadronization
- Underlying Event
- Event Generator Survey
- Matching to Fixed Order
- Beyond Standard Model



ME-PS Matching

- Two rather different objectives:
- Matching parton showers to NLO matrix elements, without double counting
  - MC@NLO
  - POWHEG
- Matching parton showers to LO n-jet matrix elements, minimizing jet resolution dependence
  - CKKW
  - Dipole
  - MLM Matching
  - Comparisons

MC@NLO

Recall simple one-dim. example from lecture 1:

$$|\mathcal{M}_{m+1}|^2 \equiv \frac{1}{x}\mathcal{M}(x)$$

x = gluon energy or two-parton invariant mass.

Divergences regularized by $d = 4 - 2\epsilon$ dimensions.

$$|\mathcal{M}_m^{\text{one-loop}}|^2 \equiv \frac{1}{\epsilon}\mathcal{V}$$

Cross section in d dimensions is:

$$\sigma = \int_0^1 \frac{dx}{x^{1+\epsilon}}\,\mathcal{M}(x)\;F_1^J(x) + \frac{1}{\epsilon}\mathcal{V}\;F_0^J$$

Infrared safety: $F_1^J(0) = F_0^J$

KLN cancellation theorem: $\mathcal{M}(0) = \mathcal{V}$

Subtraction Method

Exact identity:

$$
\begin{aligned}
\sigma^J = & \int_0^1 \frac{dx}{x^{1+\epsilon}}\,\mathcal{M}(x)\;F_1^J(x) - \int_0^1 \frac{dx}{x^{1+\epsilon}}\,\mathcal{V}\;F_0^J \\
& \qquad + \int_0^1 \frac{dx}{x^{1+\epsilon}}\,\mathcal{V}\;F_0^J + \frac{1}{\epsilon}\mathcal{V}\;F_0^J \\
= & \int_0^1 \frac{dx}{x}\left(\mathcal{M}(x)\;F_1^J(x) - \mathcal{V}\;F_0^J\right) + \mathcal{O}(1)\;\mathcal{V}\;F_0^J\,.
\end{aligned}
$$

→ Two separate finite integrals.

Modified Subtraction

$$\sigma^J = \int_0^1 \frac{dx}{x} \left(\mathcal{M}(x)\, F_1^J(x) - \mathcal{V}\, F_0^J \right) + \mathcal{O}(1)\, \mathcal{V}\, F_0^J$$

Now add parton shower:

$F_{0,1}^J \;\Rightarrow$ result from showering after 0,1 emissions.

But shower adds $\mathcal{M}_{\mathrm{MC}}/x$ to 1 emission. Must subtract this, and add to 0 emission (so that $F_{0,1}^{\mathrm{tot}} = 1 \;\Rightarrow \sigma^{\mathrm{tot}}$ fixed)

$$\begin{aligned}\sigma^J \quad &= \quad \int_0^1 \frac{dx}{x} \Big(\{\mathcal{M}(x) - \mathcal{M}_{\mathrm{MC}}(x)\}\; F_1^J(x) \\ &\qquad - \{\mathcal{V} - \mathcal{M}_{\mathrm{MC}}(x)\}\; F_0^J \Big) + \mathcal{O}(1)\, \mathcal{V}\, F_0^J\end{aligned}$$

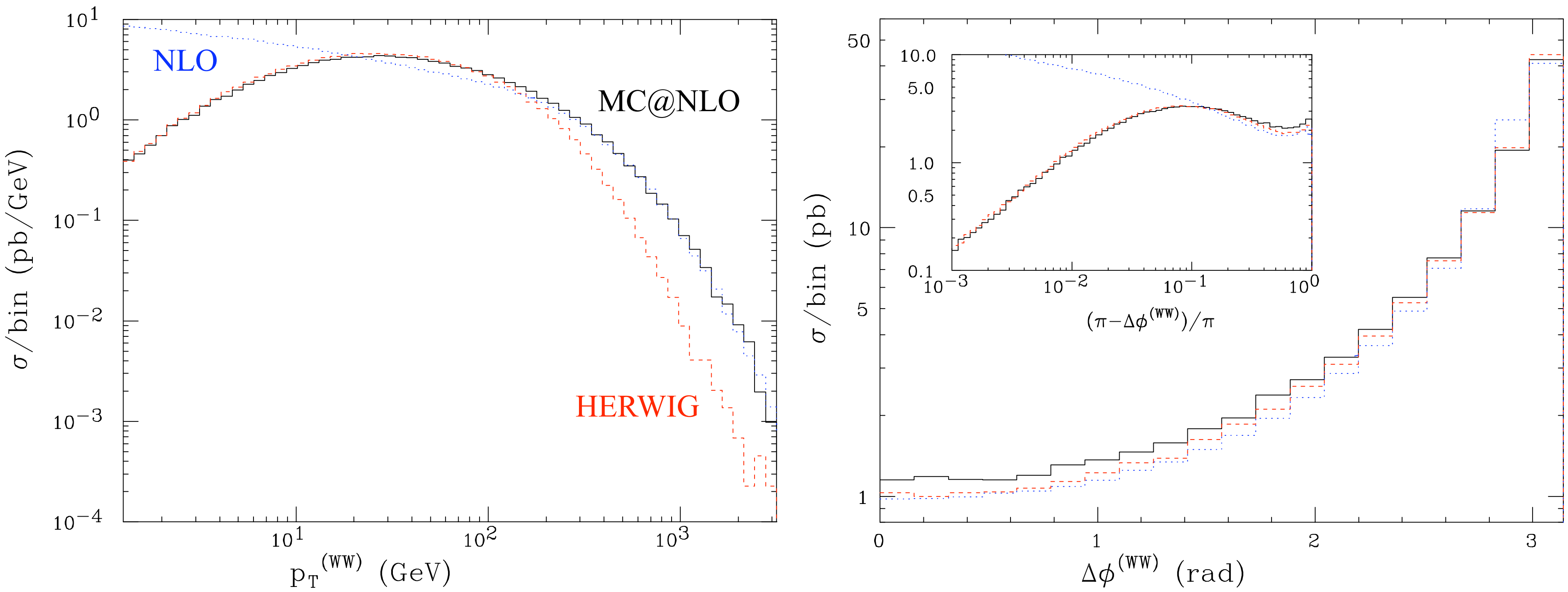
MC good for soft and/or collinear $\Rightarrow \; \mathcal{M}_{\mathrm{MC}}(0) = \mathcal{M}(0)$

➡ 0 & 1 emission contributions separately finite now!

(But some can be negative “counter-events”)

MC@NLO Results

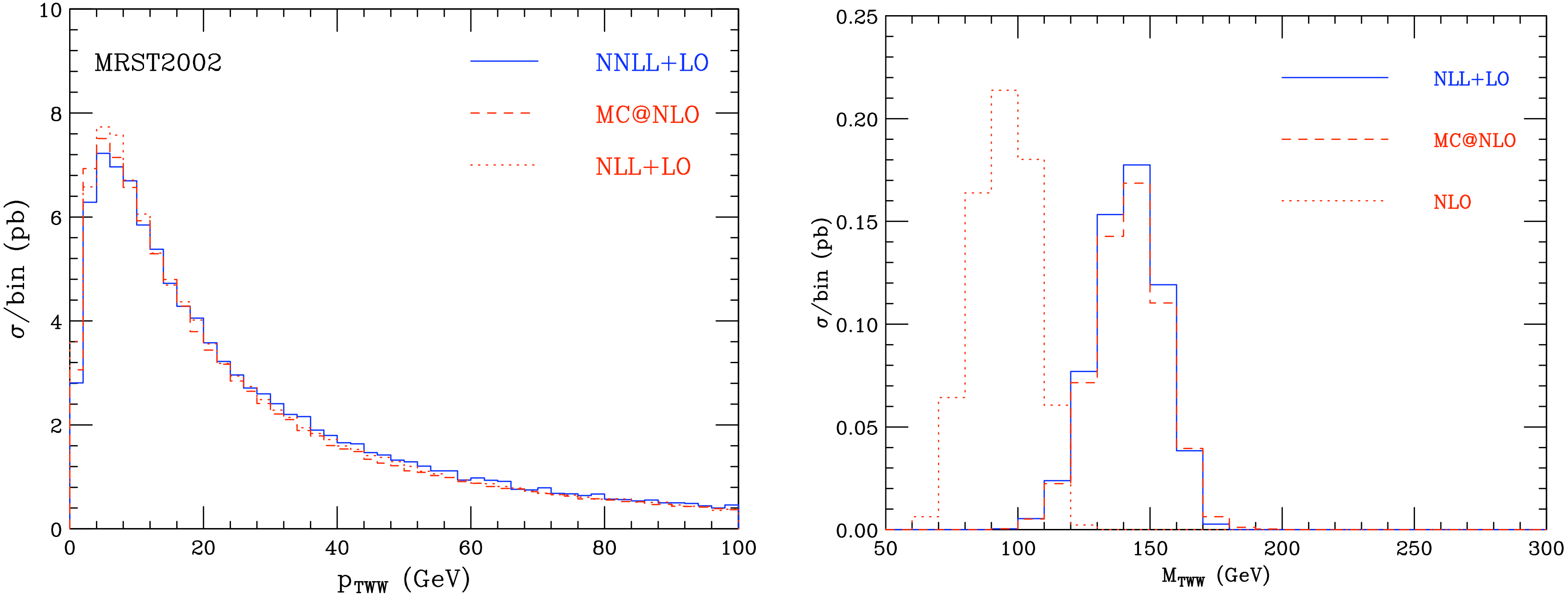
- WW production at LHC



- Interpolates between MC & NLO in $p_T^{(WW)}$
- Above both at $\Delta\phi^{(WW)} \simeq 0$

S Frixione & BW, JHEP 06(2002)029

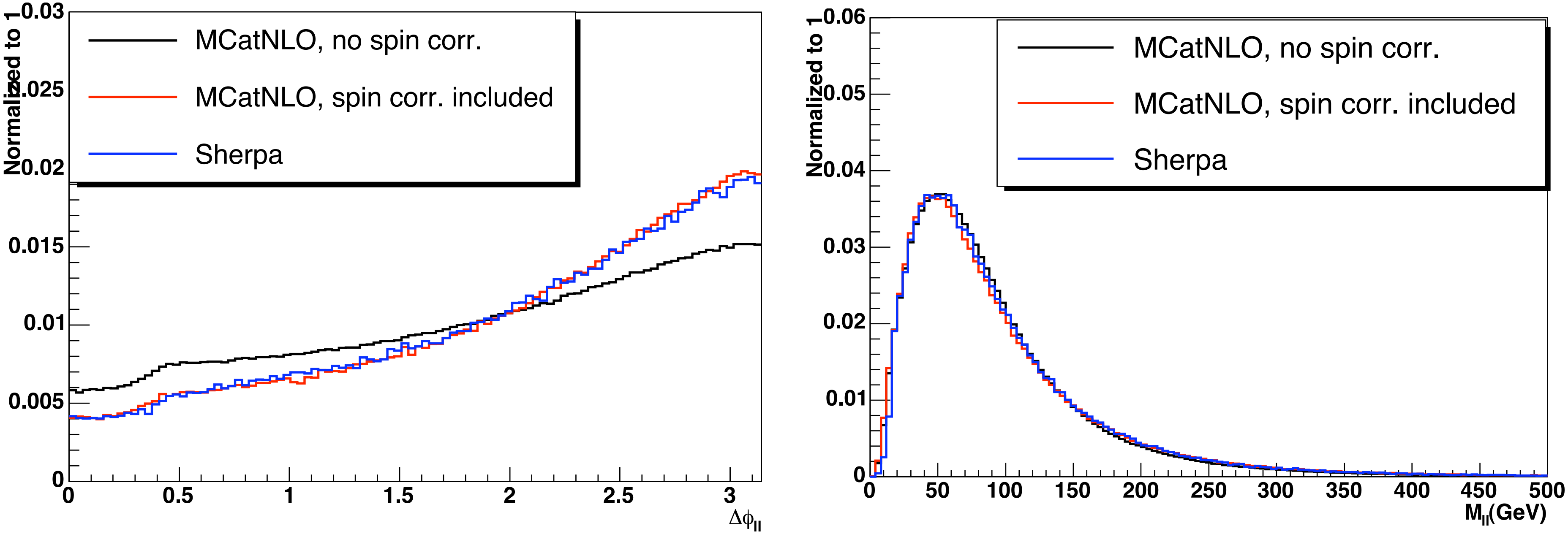
W^+W^-: MC@NLO vs Resummations



Plots from M. Grazzini JHEP 0601(2006)095

- Highly non-trivial test (of both computations) for *shapes* and *rates* !
- $M_{\rm TWW} = \sqrt{(E_{Tll} + \not{E}_T)^2 - (\mathbf{p}_{Tll} + \not{\mathbf{p}}_T)^2}$ where $E_{Tll} = \sqrt{\mathbf{p}_{Tll}^2 + m_{ll}^2}$ and $\not{E}_T \equiv \sqrt{\not{\mathbf{p}}_T^2 + m_{ll}^2}$ (Rainwater & Zeppenfeld)
- Cuts involved in definition of $M_{\rm TWW}$: $\Delta\phi_{l^+l^-} < \pi/4$, $M_{l^+l^-} > 35$ GeV, $p_{\rm Tmin}^{(l^+,l^-)} > 25$ GeV, $35 < p_{\rm Tmax}^{(l^+,l^-)} < 50$ GeV, $p_{\rm T}^{\rm WW} < 30$ GeV

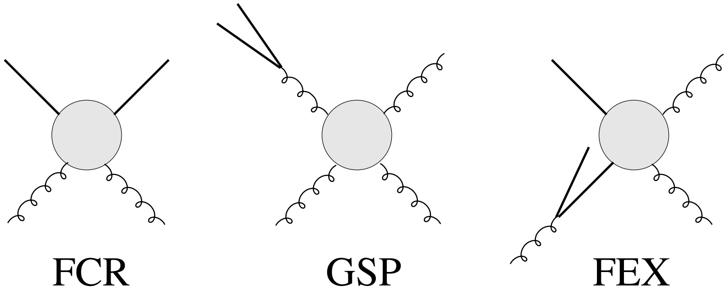
W^+W^- Spin Correlations



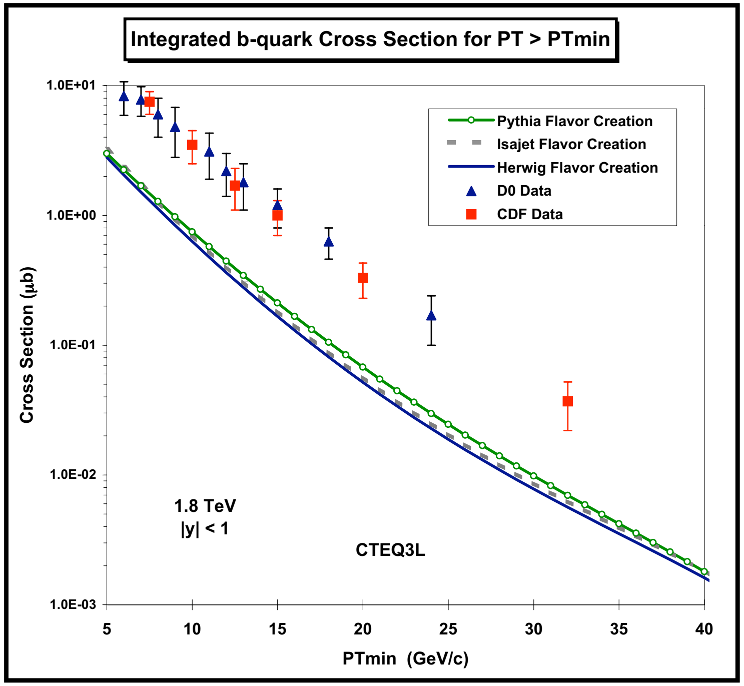
Plots from W. Quayle (preliminary)

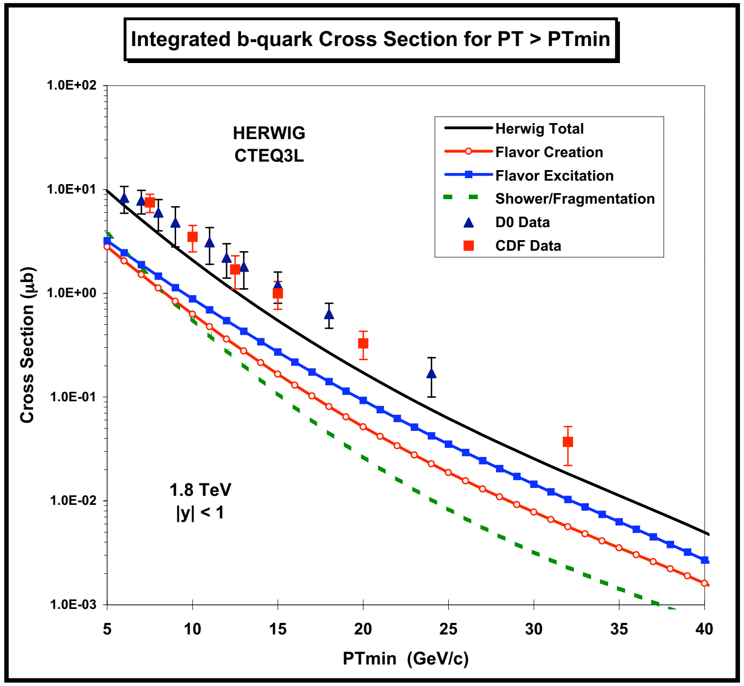
b Production: PS MC vs MC@NLO

- In parton shower MC's, 3 classes of processes can contribute:

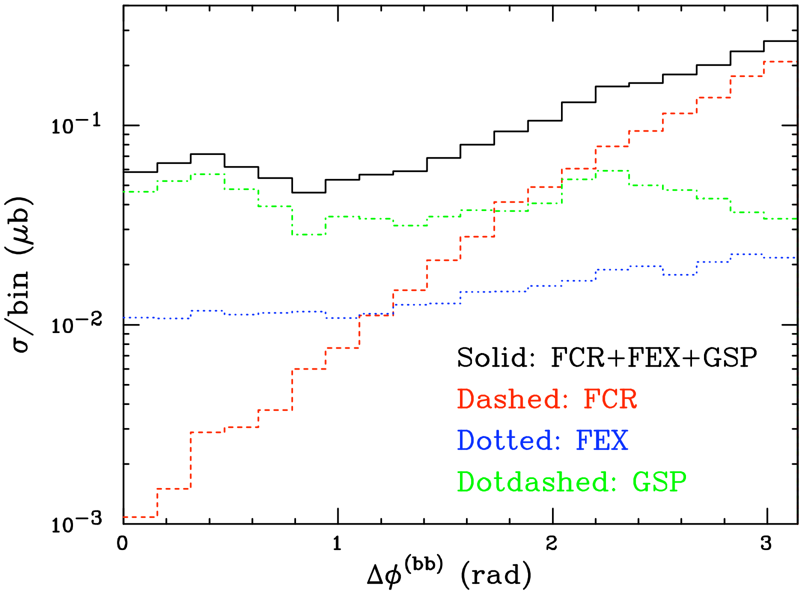


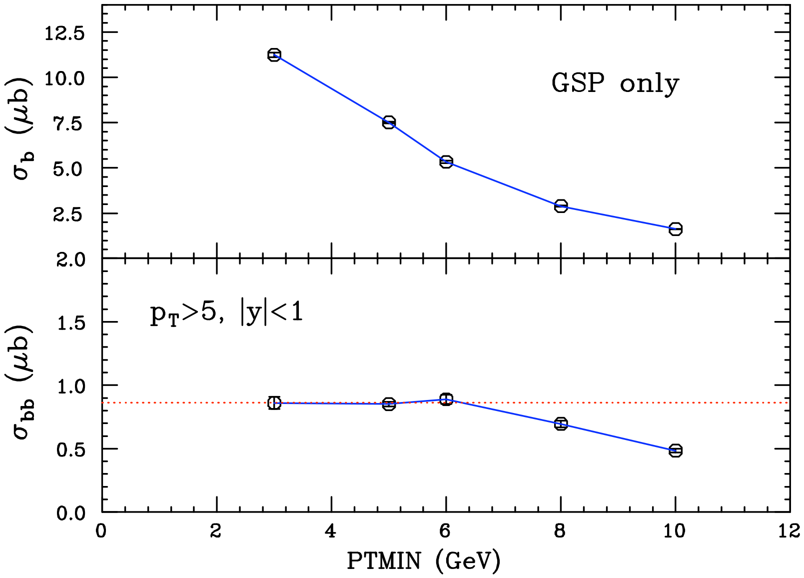
- All are needed to get close to data (RD Field, hep-ph/0201112):





GSP and FEX contributions in HERWIG PS MC

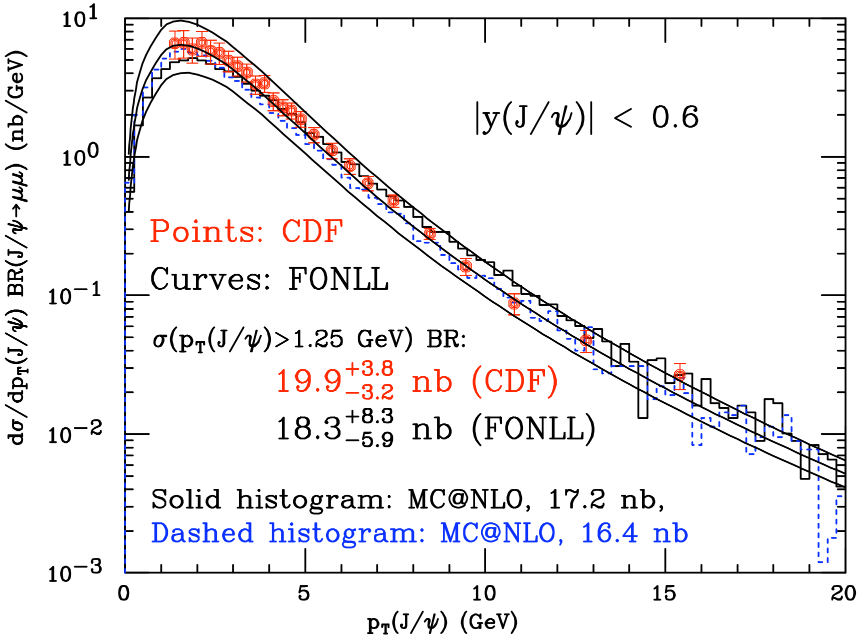


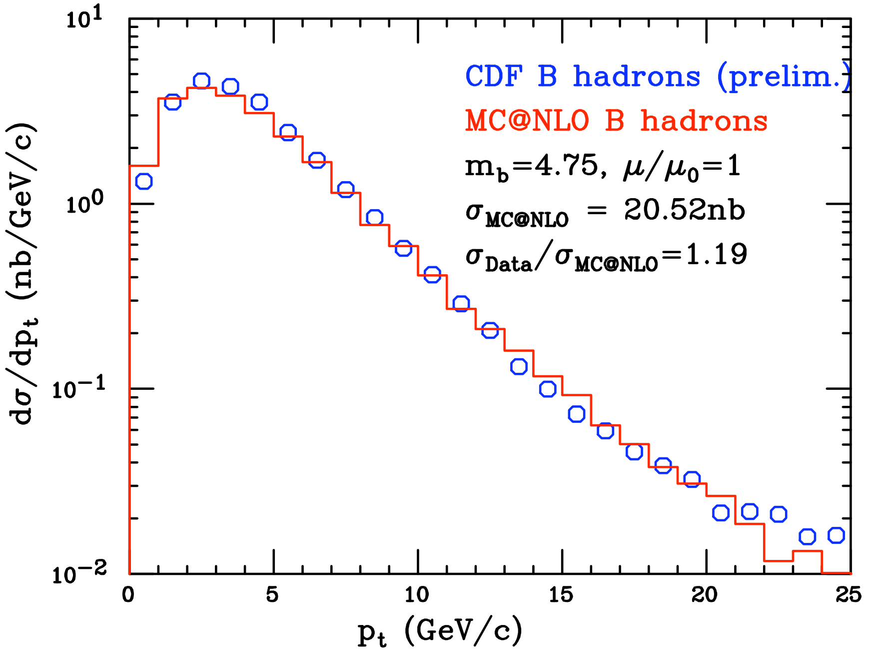


- GSP, FEX and FCR are complementary and all must be generated
  - ❖ GSP cutoff (PTMIN) sensitivity depends on cuts and observable
  - ❖ FEX sensitive to bottom PDF
  - ❖ GSP efficiency very poor, $\sim 10^{-4}$
- All these problems are avoided with MC@NLO!

MC@NLO: B Production at Tevatron

- $B \rightarrow J/\psi$ results from Tevatron Run II $\Rightarrow$ B hadrons



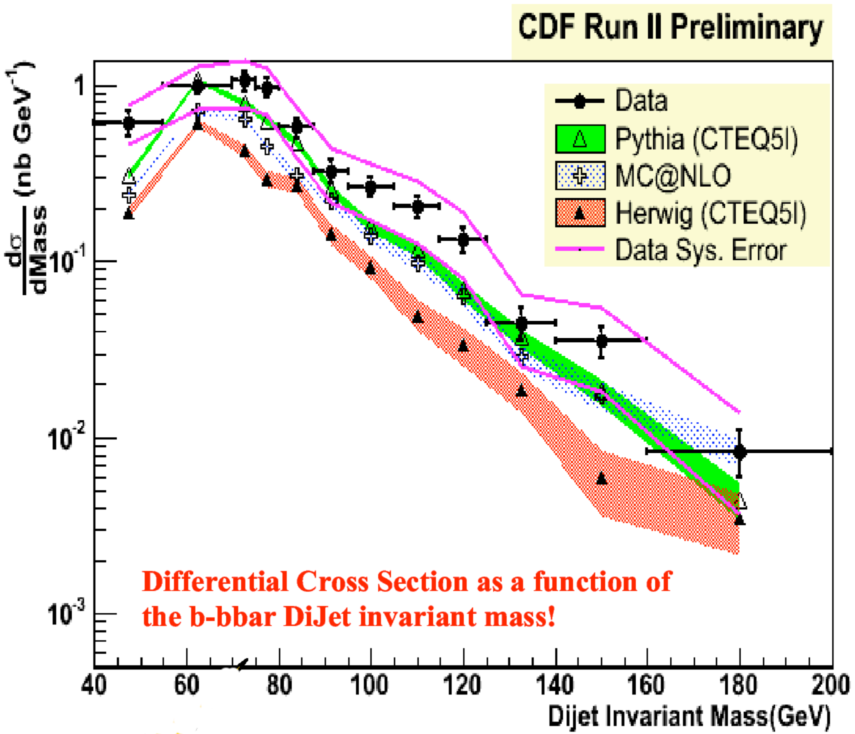


➡ Good agreement (and MC efficiency)

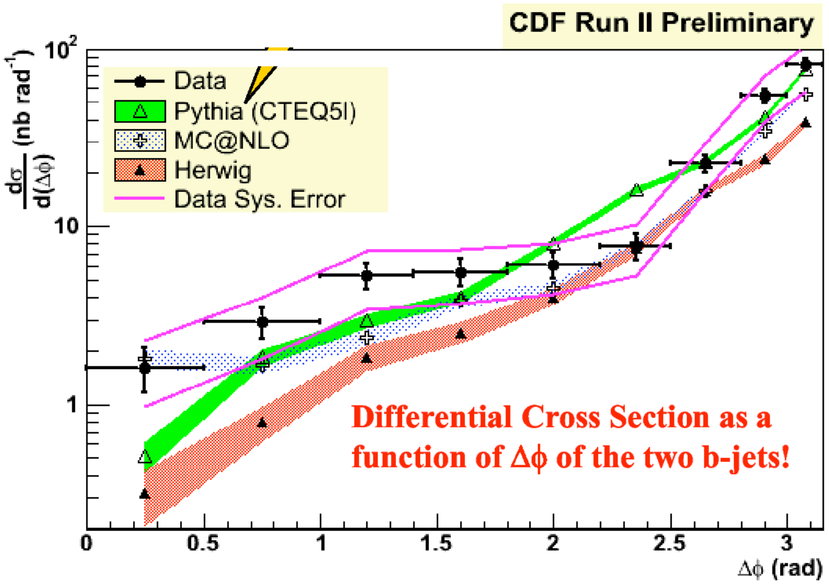
S Frixione, P Nason & BW, JHEP 0308(2003)007

M Cacciari et al., JHEP 0407(2004)033

MC@NLO Di-b Jet Production



Differential Cross Section as a function of the b-bbar DiJet invariant mass!

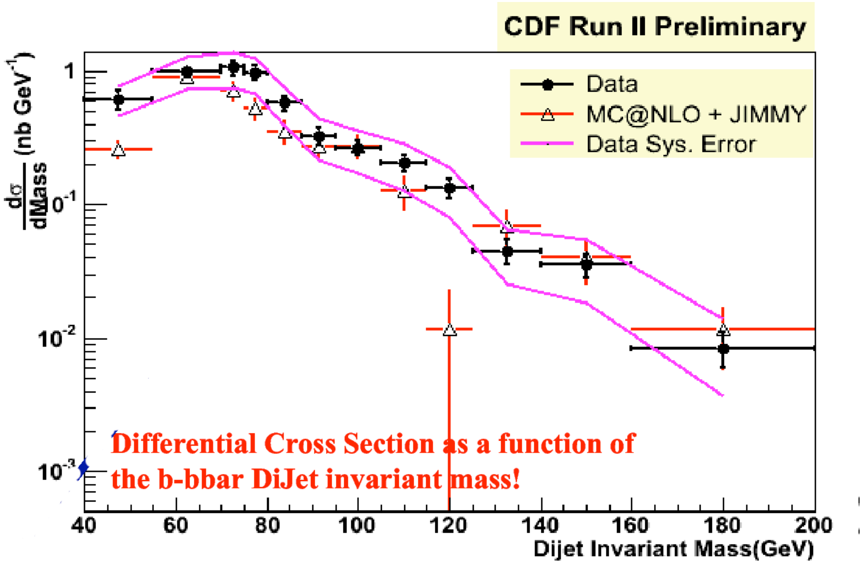


Differential Cross Section as a function of $\Delta\phi$ of the two b-jets!

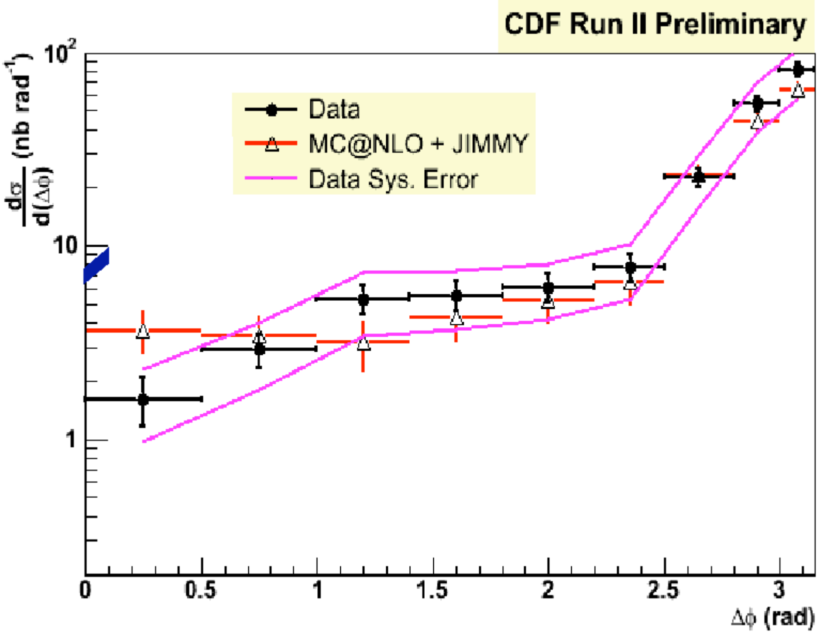
- These observables are very involved (b-jets at hadron level) and cannot be computed with analytical techniques;
- The underlying event in Pythia is fitted to data; default Herwig model (used in MC@NLO) does not fit data well (lack of MPI).

MC@NLO b-Jets: Improved Underlying Event

- The JIMMY underlying event model includes multiple parton interactions and interfaces to Herwig ⇒ interfaces to MC@NLO

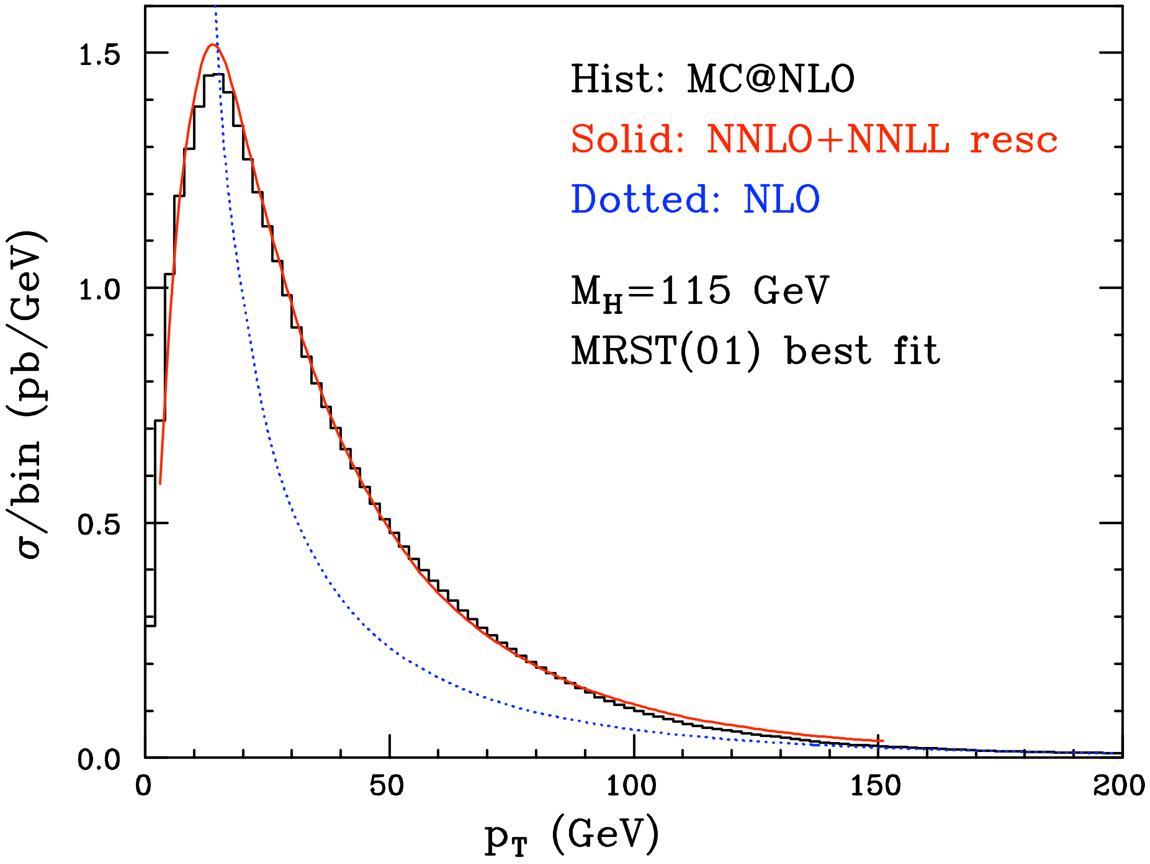


Differential Cross Section as a function of the b-bbar DiJet invariant mass!



- The importance of the underlying event shows the necessity of embedding precise computations in a Monte Carlo framework.

MC@NLO: Higgs Production at LHC



V Del Duca, S Frixione, C Oleari & BW, in prep.

➡ Good agreement with state-of-the-art resummation

POWHEG

Positive Weight Hardest Emission Generator

- Method to generate hardest emission first, with NLO accuracy, independent of PSEG
- Can be interfaced to any PSEG
- No negative weights
- Inaccuracies only affect next-to-hardest emission
- In principle, needs ‘truncated showers’

P Nason & G Ridolfi, JHEP08(2006)077

S Frixione, P Nason & G Ridolfi, arXiv:0707.3088

S Frixione, P Nason & C Oleari, arXiv:0709.2092

POWHEG

How it works (roughly)

In words: works like a standard Shower MC for the hardest radiation, with care to maintain higher accuracy.

Inclusive cross section $\Longrightarrow$ NLO inclusive cross section. Positive if $\mathrm{NL} < \mathrm{LO}$

$\Phi_n = \text{Born variables}$
$\Phi_r = \text{radiation vars.}$

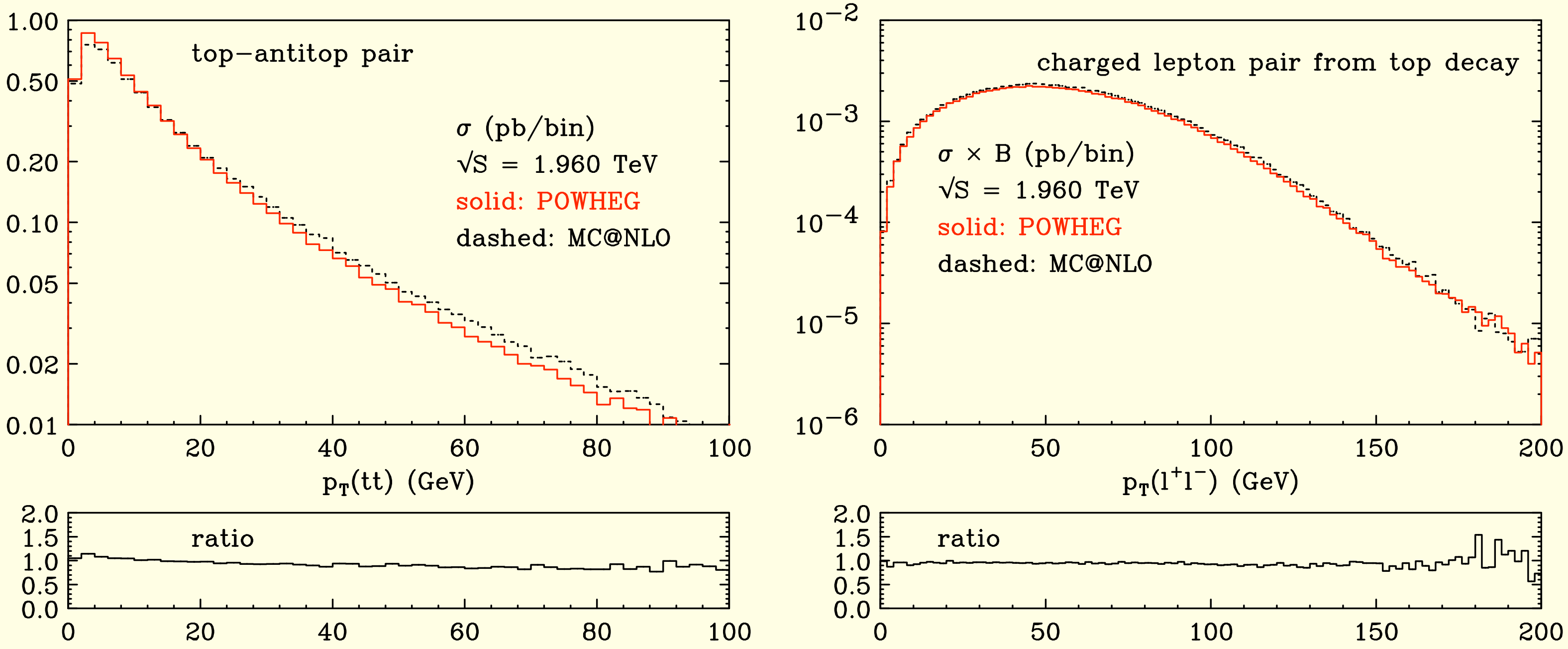
$$\bar{B}(\Phi_n) = B(\Phi_n) + \underbrace{\left[\overbrace{V(\Phi_n)}^{\text{INFINITE}} + \overbrace{\int R(\bar{\Phi}_n, \Phi_r)\, d\Phi_r}^{\text{INFINITE}} \right]}_{\text{FINITE!}}$$

Sudakov form factor for hardest emission built from exact NLO real emission

$$\Delta_t = \exp\left[\underbrace{-\int \theta(t_r - t) \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_r}_{\text{FINITE because of } \theta \text{ function}} \right]$$

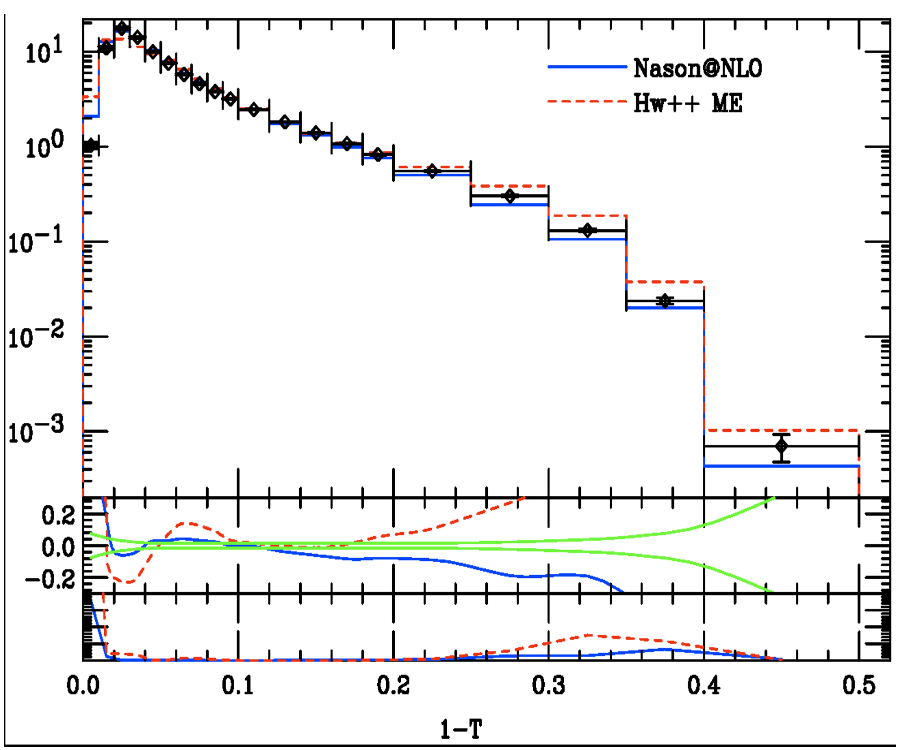
with $t_r = k_T(\Phi_n, \Phi_r)$, the transverse momentum for the radiation.

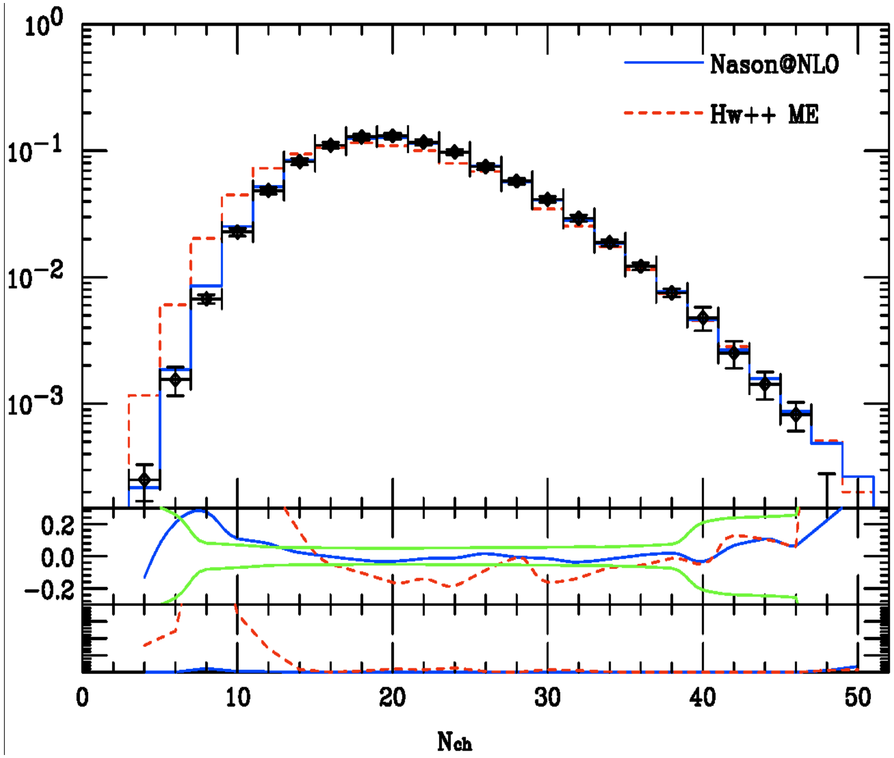
POWHEG and MC@NLO comparison: Top pair production



Good agreement for all observable considered
(differences can be ascribed to different treatment of higher order terms)

POWHEG for $e^+e^- \rightarrow$ hadrons

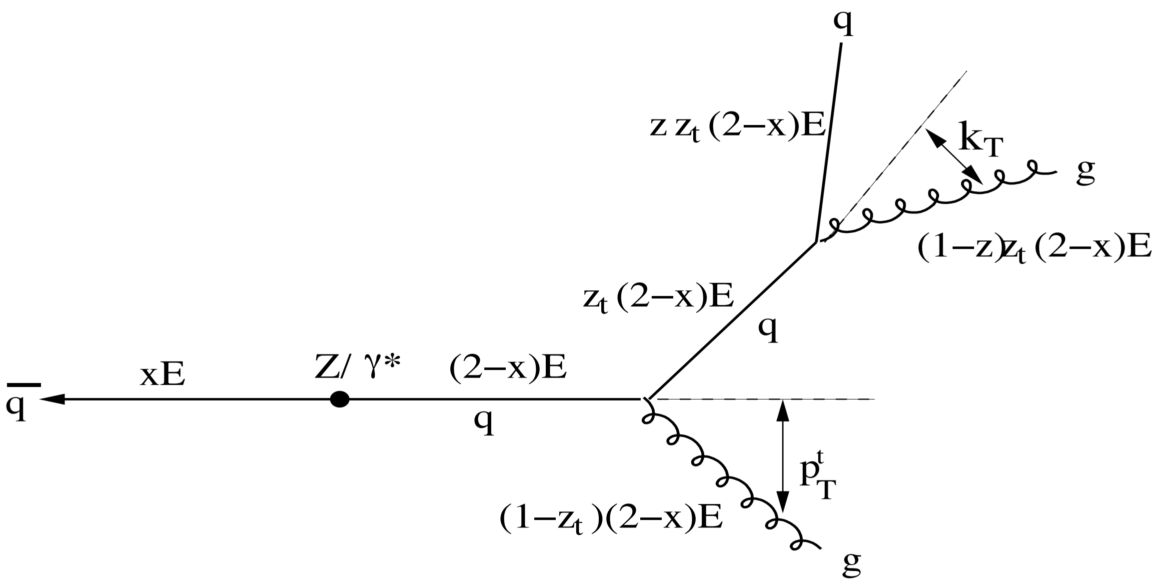




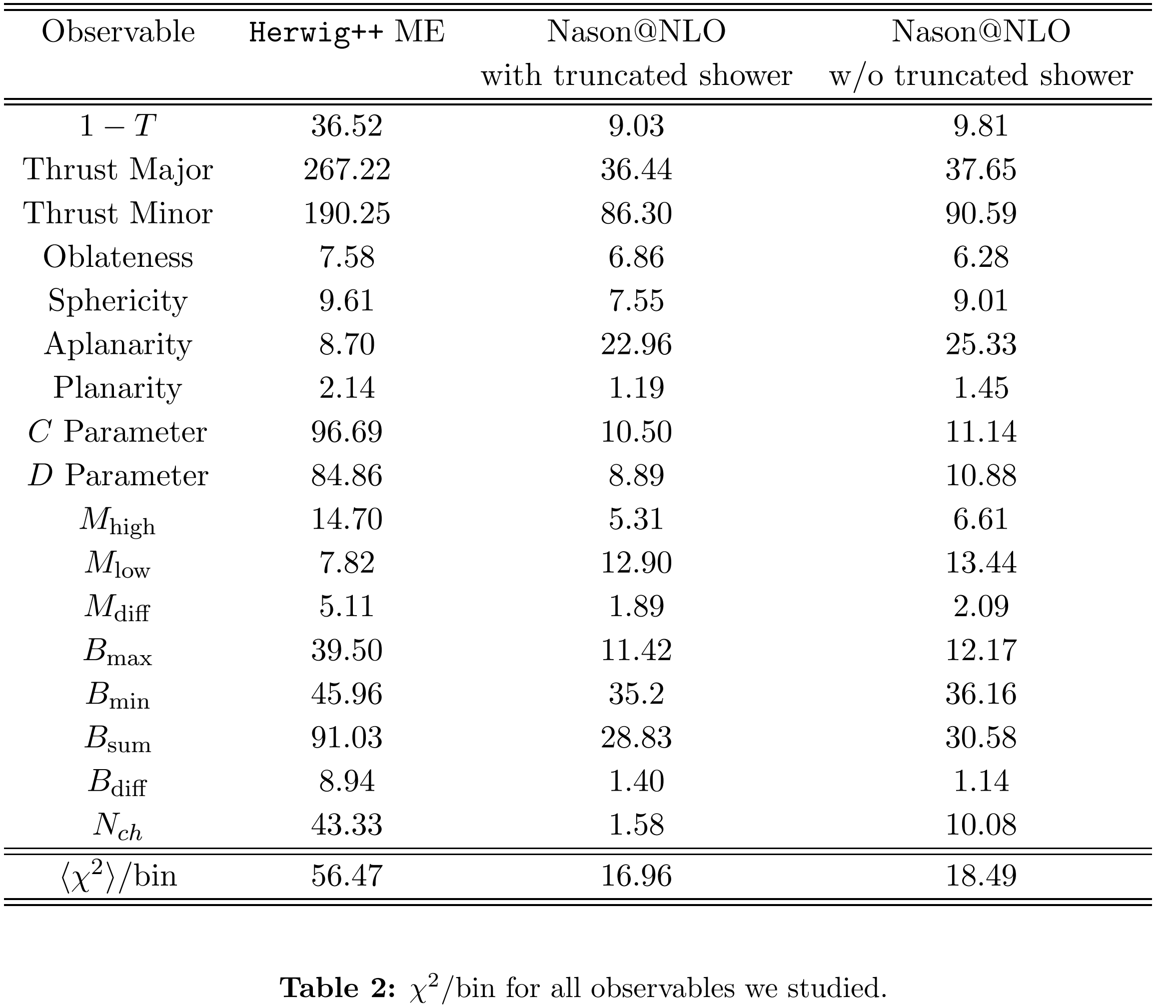
O Latunde-Dada, S Gieseke, B Webber, JHEP02 (2007) 051, hep-ph/0612281

Truncated Shower

- In angular-ordered shower, hardest emission is not necessarily the first
- Need to add softer, wider-angle emissions
- Checked for up to one such emission in e^+e^-



Effect of truncated shower



Observable	`Herwig++` ME	Nason@NLO with truncated shower	Nason@NLO w/o truncated shower
$1-T$	36.52	9.03	9.81
Thrust Major	267.22	36.44	37.65
Thrust Minor	190.25	86.30	90.59
Oblateness	7.58	6.86	6.28
Sphericity	9.61	7.55	9.01
Aplanarity	8.70	22.96	25.33
Planarity	2.14	1.19	1.45
C Parameter	96.69	10.50	11.14
D Parameter	84.86	8.89	10.88
M_{high}	14.70	5.31	6.61
M_{low}	7.82	12.90	13.44
M_{diff}	5.11	1.89	2.09
B_{max}	39.50	11.42	12.17
B_{min}	45.96	35.2	36.16
B_{sum}	91.03	28.83	30.58
B_{diff}	8.94	1.40	1.14
N_{ch}	43.33	1.58	10.08
$\langle\chi^2\rangle$/bin	56.47	16.96	18.49

Table 2: χ^2/bin for all observables we studied.

Small but beneficial effect

CKKW Matching

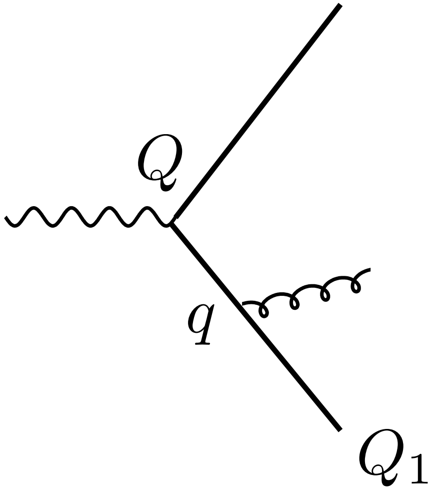
- Use Matrix Elements down to scale Q_1
- Use Parton Showers below Q_1
- Correct ME by reweighting
- Correct PS by vetoing
- Ensure that Q_1 cancels (to NLL)

S Catani, F Krauss, R Kuhn & BW, JHEP11 (2001) 063

Example: $e^+e^- \rightarrow$ hadrons

- 2- & 3-jet rates at scale Q_1:

$$
\begin{aligned}
R_2(Q,Q_1) &= \left[\Delta_q(Q,Q_1)\right]^2 \;, \\
R_3(Q,Q_1) &= 2\Delta_q(Q,Q_1)\int_{Q_1}^{Q} dq\,\frac{\Delta_q(Q,Q_1)}{\Delta_q(q,Q_1)}\Gamma_q(Q,q) \\
&\quad \times\Delta_q(q,Q_1)\Delta_g(q,Q_1) \\
&= 2\left[\Delta_q(Q,Q_1)\right]^2\int_{Q_1}^{Q} dq\,\Gamma_q(Q,q)\Delta_g(q,Q_1)
\end{aligned}
$$



$$
\Gamma_q(Q,q) = \frac{2C_F}{\pi}\frac{\alpha_S(q)}{q}\left(\ln\frac{Q}{q}-\frac{3}{4}\right)
$$

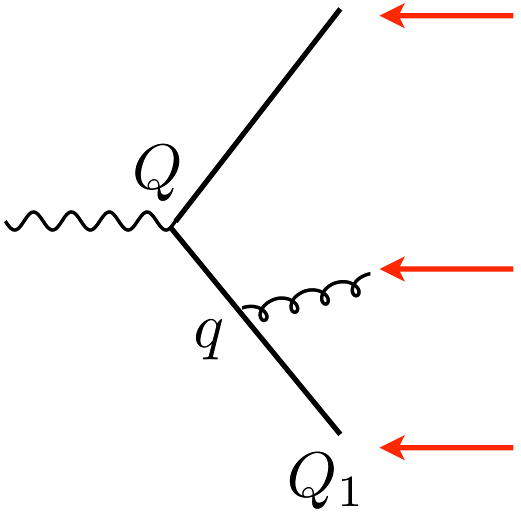
CKKW reweighting

- Choose n according to $R_n(Q,Q_1)$ (LO)
  - – use $[\alpha_S(Q_1)]^n$
- Use exact LO ME to generate n partons
- Construct "equivalent shower history"
  - – preferably using k_T-type algorithm
- Weight vertex at scale q by $\alpha_S(q)/\alpha_S(Q_1) < 1$
- Weight parton of type i from Q_j to Q_k by

$$\Delta_i(Q_j,Q_1)/\Delta_i(Q_k,Q_1)$$

CKKW shower veto

- Shower n partons from “creation scales”
  - includes coherent soft emission
- Veto emissions at scales above Q_1
  - cancels leading (LL&NLL) Q_1 dependence

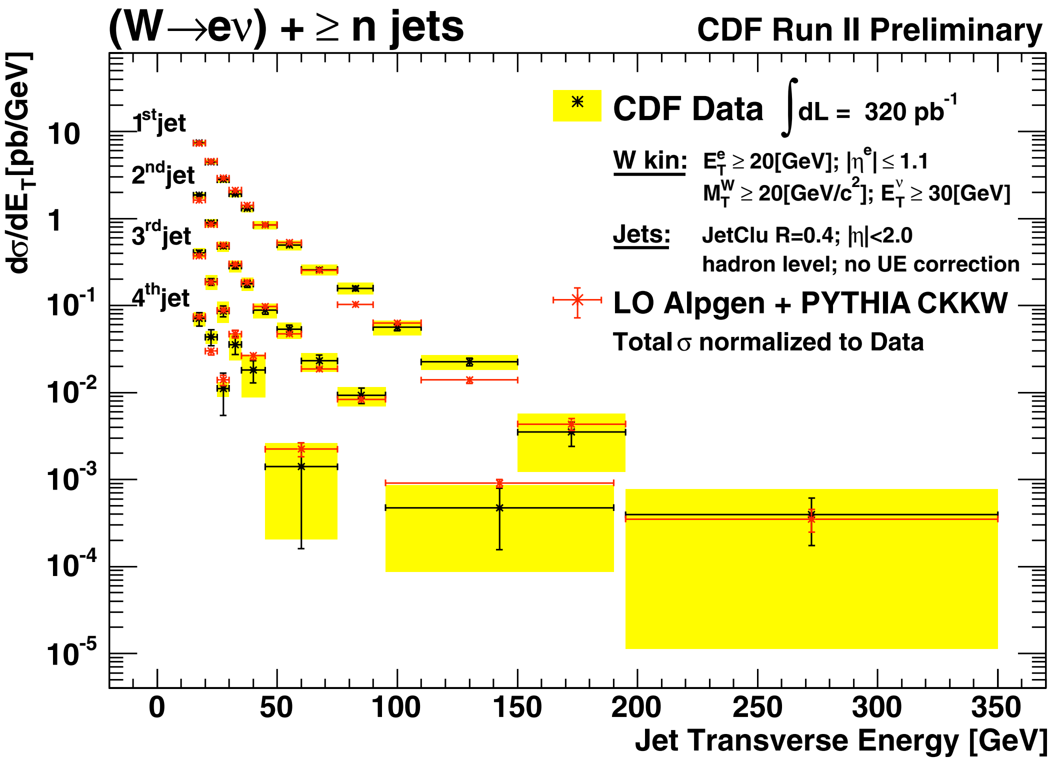


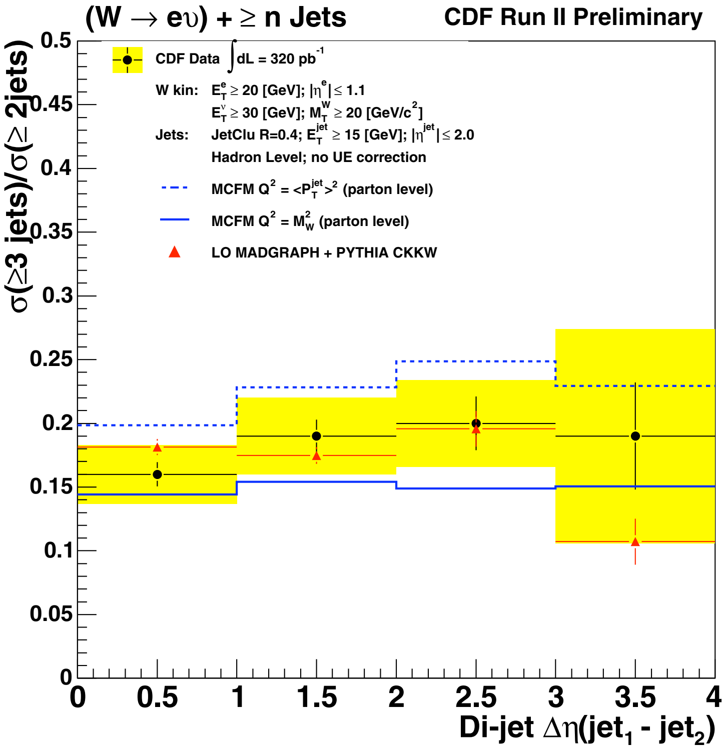
shower from Q

shower from q

shower from Q, not q

Comparisons with Tevatron data





from JM Campbell, JW Huston & WJ Stirling, Rept.Prog.Phys.70(2007)89

➡ M.E. + PYTHIA CKKW looks good

Dipole Matching

- Implemented in ARIADNE dipole MC
- Dipole cascade replaces parton shower
- Construct equivalent dipole history $\{p_{Ti}\}$
- Rejection replaces Sudakov weights
  - cascade from p_{Ti}, reject if $p_T > p_{Ti+1}$

L Lönnblad, JHEP05(2002)046

MLM Matching

- Use cone algorithm for jet definition:

$$R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$$

$$E_{Ti} > E_{Tmin}, \; R_{ij} > R_{min}$$

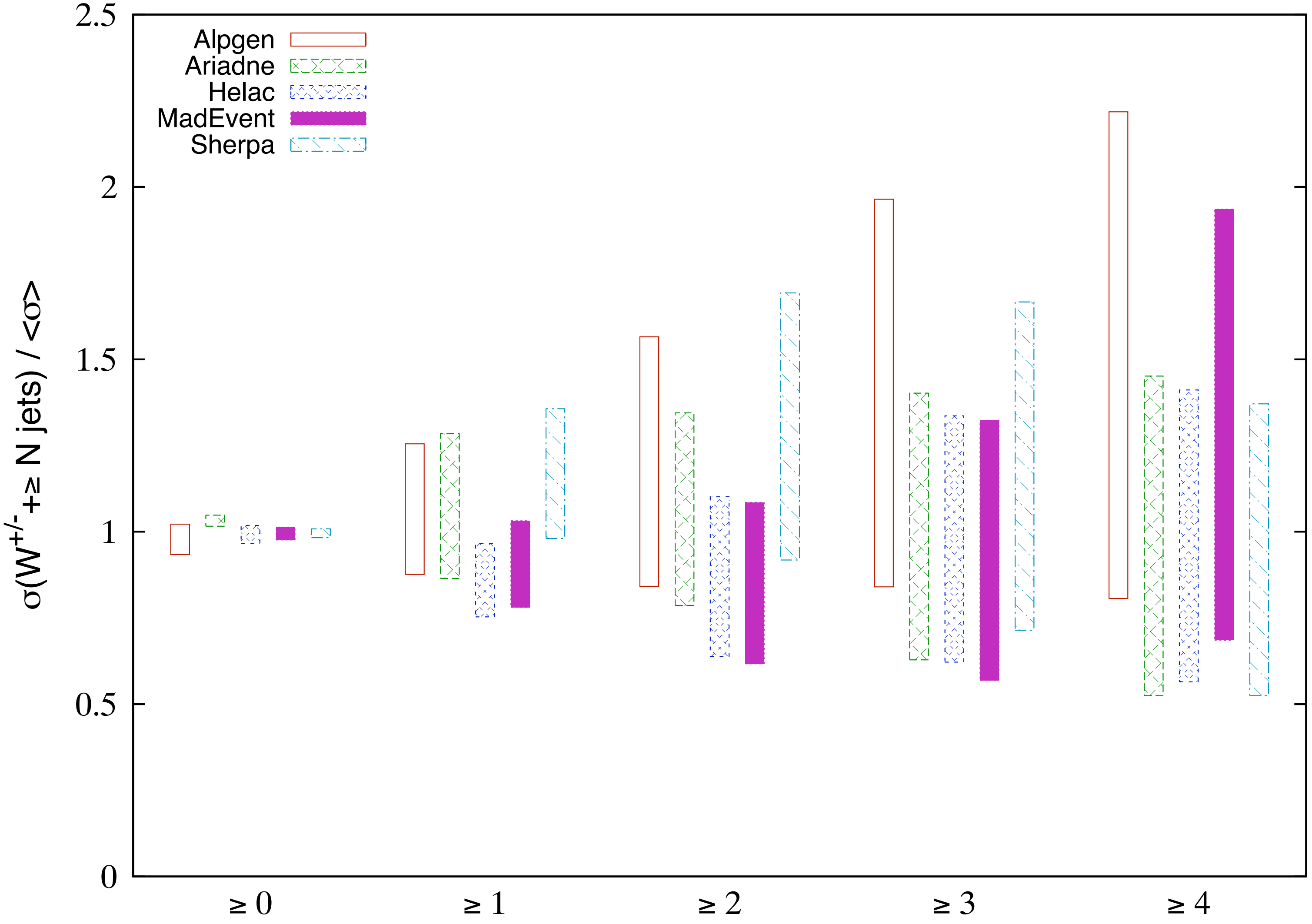
- Generate n-parton configurations with $E_{Ti} > E_{Tmin}, \; R_{ij} > R_{min}$ (no Sudakov weights)
- Generate showers (no vetos)
- Form jets using same jet definition
- Reject event if $n_{\text{jets}} \neq n_{\text{partons}}$

Comparisons

- ALPGEN: MLM matching
- ARIADNE: Dipole matching
- HELAC: MLM matching
- MadEvent: hybrid MLM/CKKW
- SHERPA: CKKW matching

J. Alwall el al., arXiv:0706.2569

W + Multijets (Tevatron)



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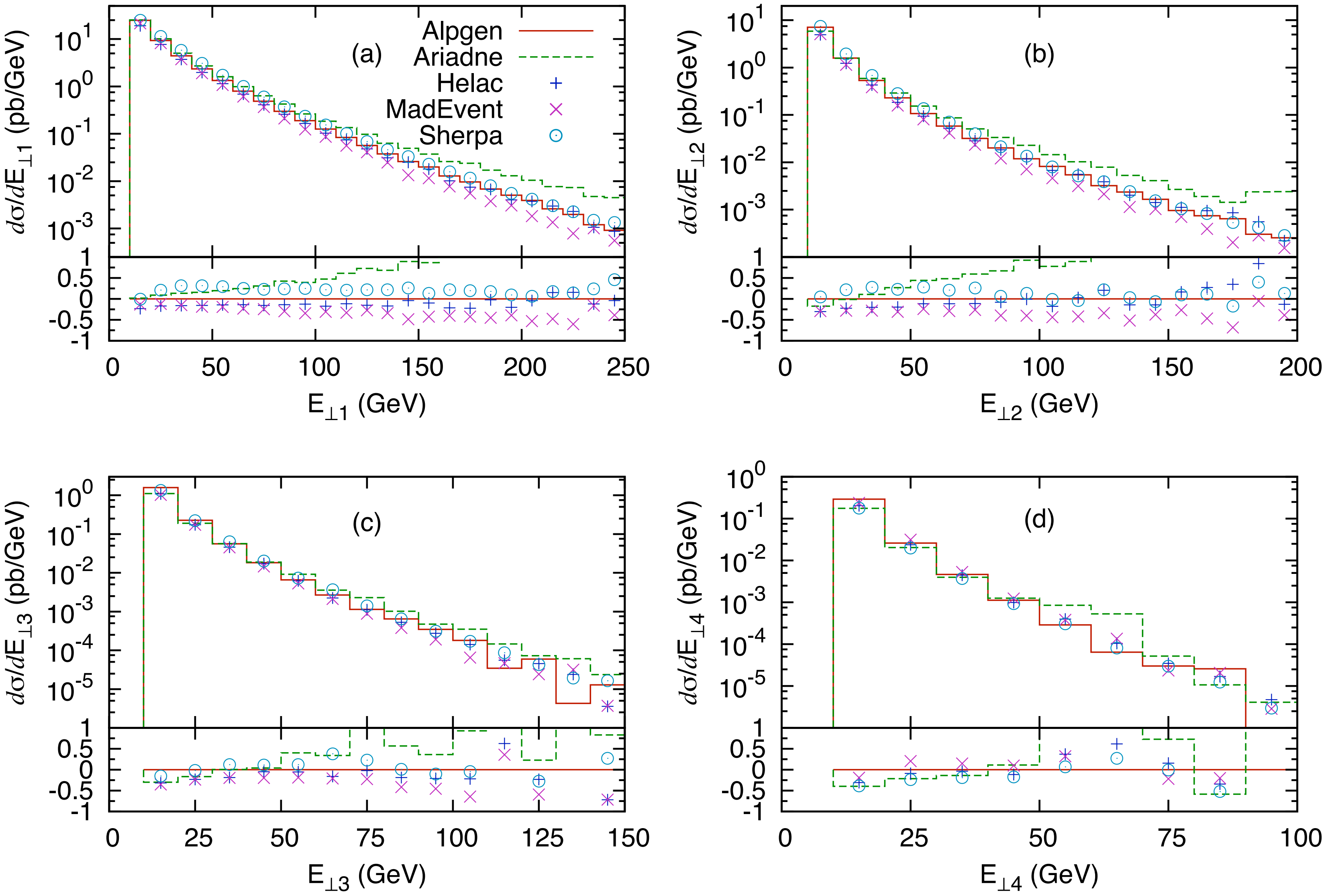

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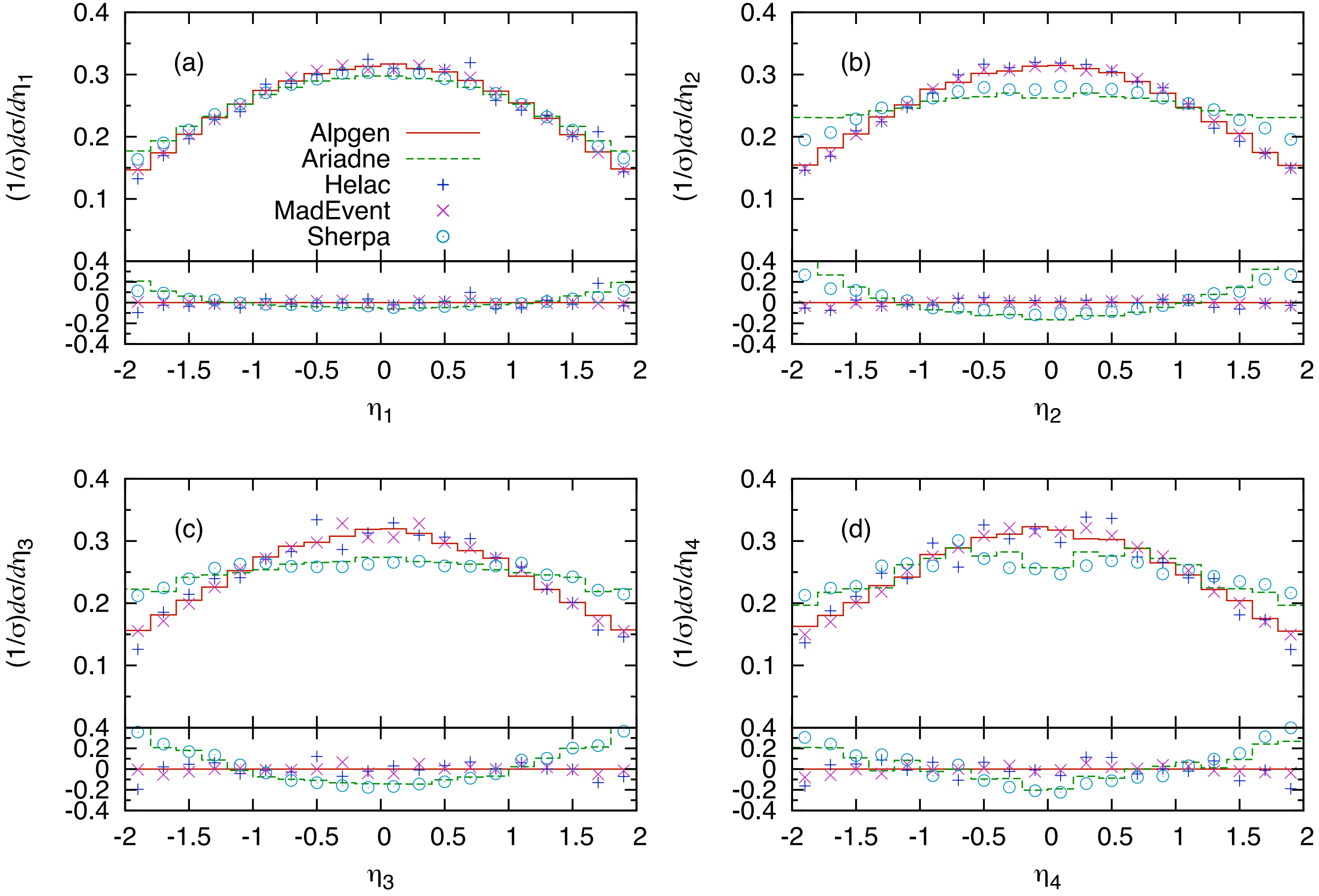
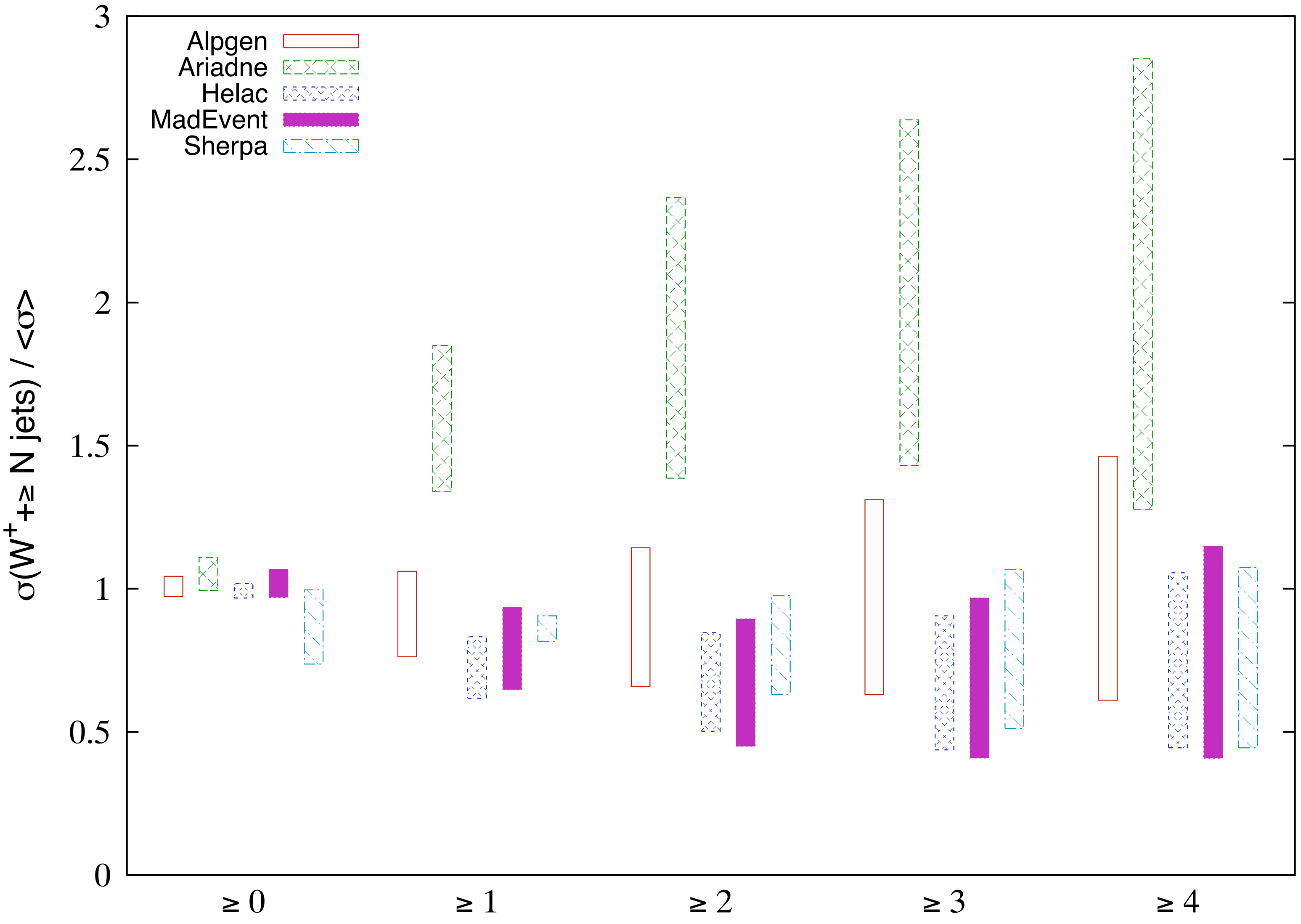
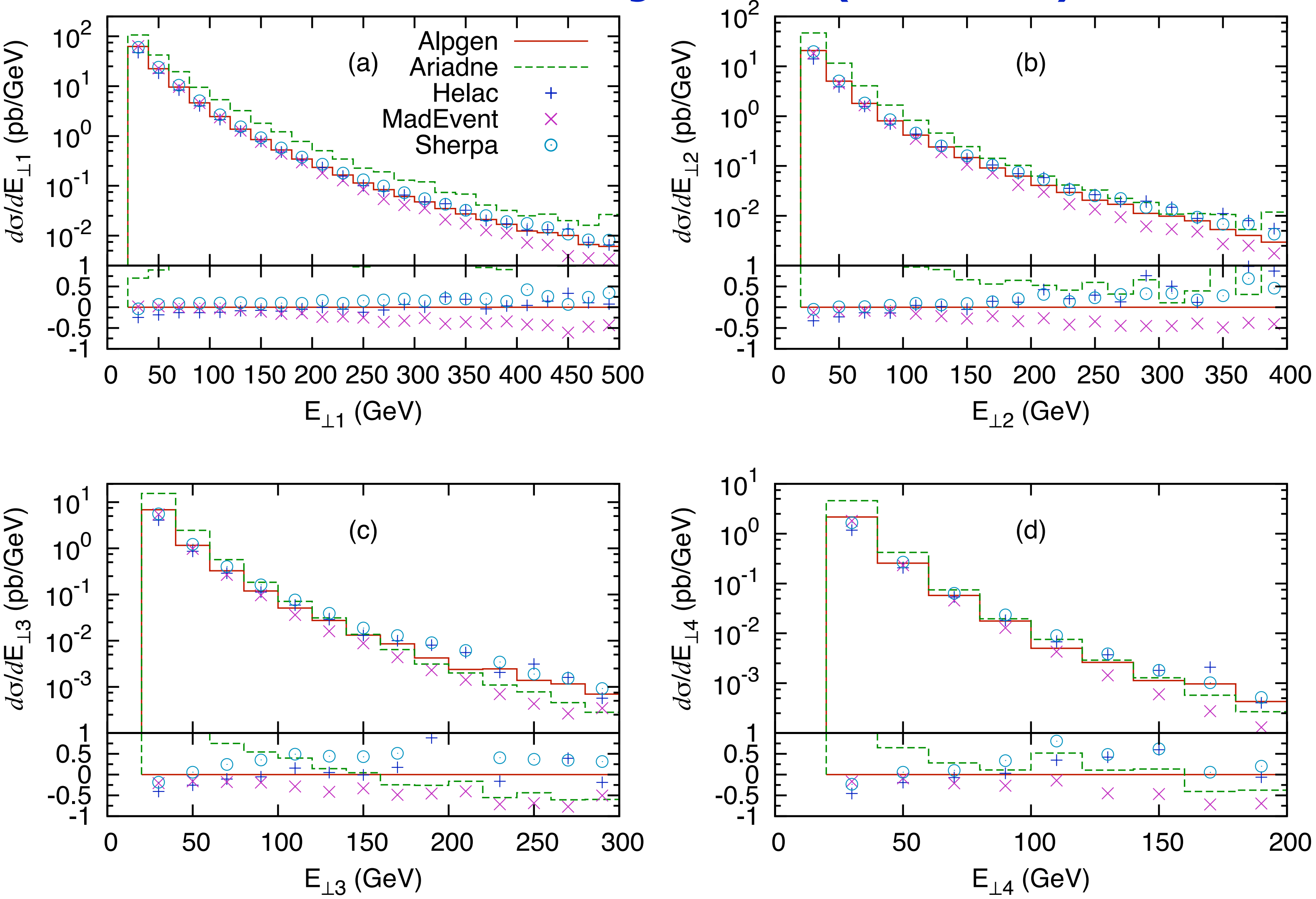


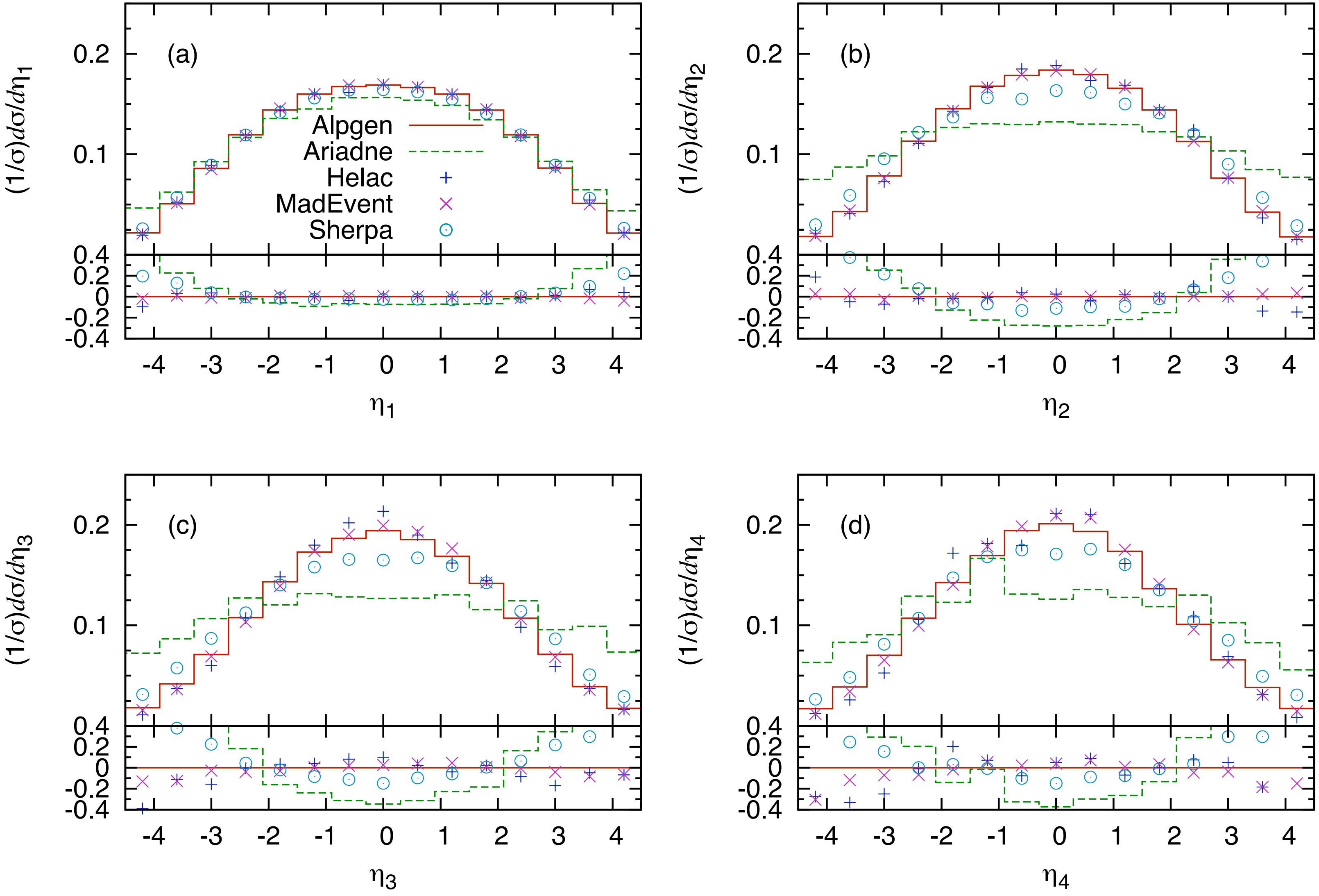
W + Multijets (LHC)



W + Multijets (LHC)



W + Multijets (LHC)



Summary

- Matching Parton Showers to Matrix Elements comes in different forms:
  - matching to NLO for better precision
  - matching to LO for multijets
- MC@NLO is main scheme for NLO matching
  - newer POWHEG method looks promising
- Several options for LO multijets
  - reasonably consistent
  - spread indicates uncertainties (?)
- Field still very active
  - NLO matching for jets, spin correlations,...
  - building multijet matching into OO generators