Monte Carlo Methods in Particle Physics

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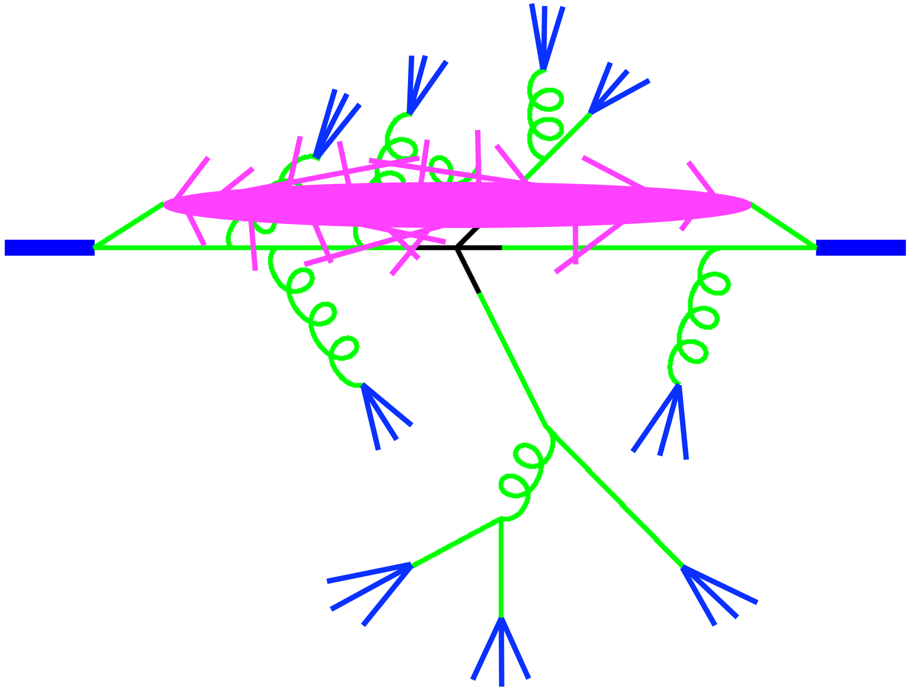
Structure of LHC Events

1. Hard process
2. Parton shower
3. Hadronization
4. Underlying event



Lecture 2: Parton Showers

QED: accelerated charges radiate.

QCD identical: accelerated colours radiate.

gluons also charged.

→ cascade of partons.

= parton shower.

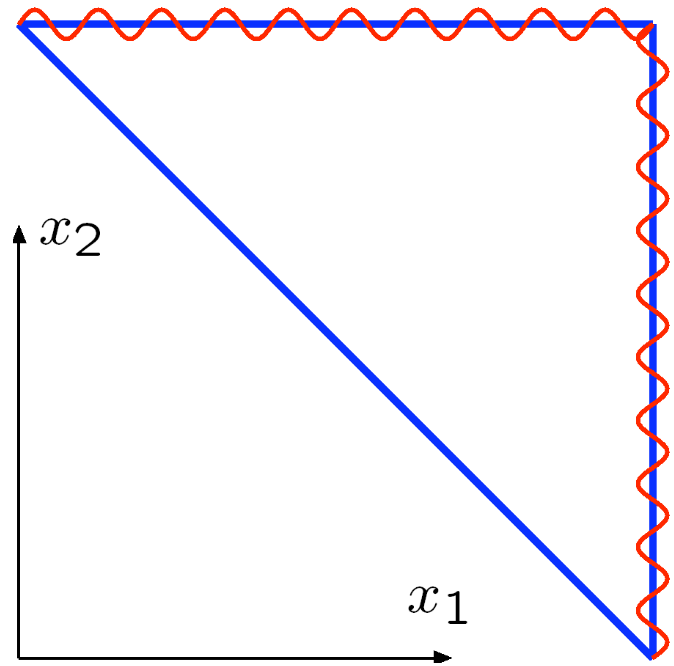
1. e^+e^- annihilation to jets.
2. Universality of collinear emission.
3. Sudakov form factors.
4. Universality of soft emission.
5. Angular ordering.
6. Initial-state radiation.
7. Hard scattering.
8. The Colour Dipole Model.

e^+e^- annihilation to jets

Three-jet cross section:

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 \, C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

singular as $x_{1,2} \to 1$



Rewrite in terms of quark-gluon opening angle θ and gluon energy fraction x_3 :

$$\frac{d\sigma}{d\cos\theta \, dx_3} = \sigma_0 \, C_F \frac{\alpha_s}{2\pi} \left\{ \frac{2}{\sin^2\theta} \frac{1+(1-x_3)^2}{x_3} - x_3 \right\}$$

Singular as $\sin\theta \to 0$ and $x_3 \to 0$.

can separate into two independent jets:

$$
\begin{aligned}
\frac{2\, d\cos\theta}{\sin^2\theta} &= \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\theta}{1+\cos\theta} \\
&= \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\bar\theta}{1-\cos\bar\theta} \\
&\approx \frac{d\theta^2}{\theta^2} + \frac{d\bar\theta^2}{\bar\theta^2}
\end{aligned}
$$

jets evolve independently

$$
d\sigma = \sigma_0 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2}\, dz \frac{1+(1-z)^2}{z}
$$

Exactly same form for anything $\propto \theta^2$

eg transverse momentum: $k_\perp^2 = z^2(1-z)^2\,\theta^2\,E^2$

invariant mass: $q^2 = z(1-z)\,\theta^2\,E^2$

$$
\frac{d\theta^2}{\theta^2} = \frac{dk_\perp^2}{k_\perp^2} = \frac{dq^2}{q^2}
$$

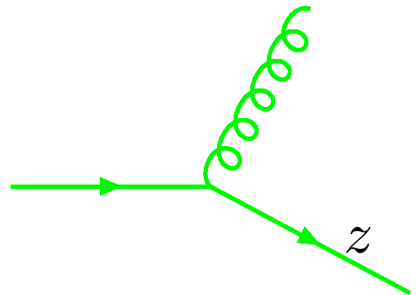
Collinear Limit

Universal:

$$d\sigma = \sigma_0 \, \frac{\alpha_s}{2\pi} \, \frac{d\theta^2}{\theta^2} \, dz \, P(z,\phi) d\phi$$

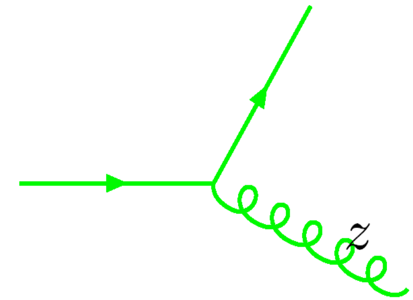
$$P(z,\phi) =$$

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi splitting kernel: dependent on flavour and spin



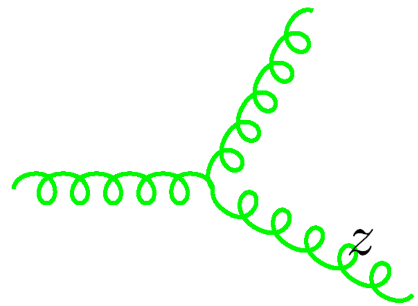
$q \to qg$

$C_F \dfrac{1+z^2}{1-z}$



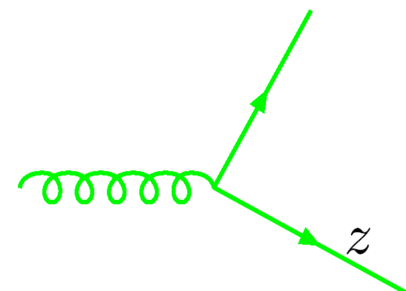
$q \to gq$

$C_F \dfrac{1+(1-z)^2}{z}$



$g \to gg$

$C_A \dfrac{z^4+1+(1-z)^4}{z(1-z)}$



$g \to q\bar{q}$

$T_R \left(z^2 + (1-z)^2\right)$

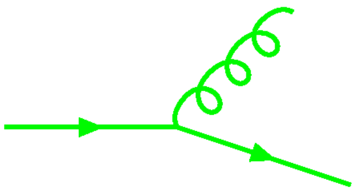
Resolvable partons

What is a parton?

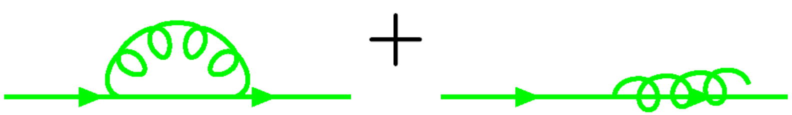
Collinear parton pair $\longleftrightarrow$ single parton

Introduce resolution criterion, eg $k_\perp > Q_0$.

Virtual corrections must be combined with unresolvable real emission



Resolvable emission: Finite



Virtual + Unresolvable emission: Finite

Unitarity: P(resolved) + P(unresolved) = 1

Sudakov form factor

Probability(emission between q^2 and $q^2 + dq^2$)

$$d\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz\ P(z) \equiv \frac{dq^2}{q^2}\bar{P}(q^2).$$

Define probability(no emission between Q^2 and q^2) to be $\Delta(Q^2, q^2)$. Gives evolution equation

$$-\frac{d\Delta(Q^2,q^2)}{dq^2} = \Delta(Q^2,q^2)\frac{d\mathcal{P}}{dq^2}$$

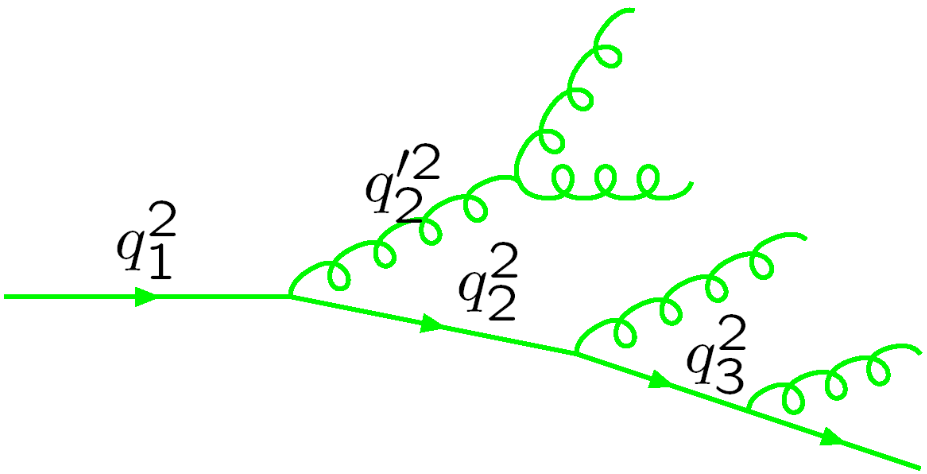
$$\Rightarrow \Delta(Q^2,q^2) = \exp - \int_{q^2}^{Q^2}\frac{dk^2}{k^2}\bar{P}(k^2).$$

$\Delta(Q^2, Q_0^2) \equiv \Delta(Q^2)$ Sudakov form factor factor

=Probability(emitting no resolvable radiation)

$$\Delta_q(Q^2) \sim \exp -C_F\frac{\alpha_s}{2\pi}\log^2\frac{Q^2}{Q_0^2}$$

Multiple emission



$$q_1^2 > q_2^2 > q_3^2 > \ldots$$
$$q_1^2 > q_2'^2 \ldots$$

But initial condition? $q_1^2 < ???$

Process dependent

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Monte Carlo implementation

Can generate branching according to

$$d\mathcal{P} = \frac{dq^2}{q^2}\bar{P}(q^2)\,\Delta(Q^2,q^2)$$

By choosing $0 < \rho < 1$ uniformly:

If $\rho < \Delta(Q^2)$ no resolvable radiation, evolution stops.

Otherwise, solve $\rho = \Delta(Q^2,q^2)$

for q^2 = emission scale

Considerable freedom:

Evolution scale: $q^2/k_\perp^2/\theta^2$?

z: Energy? Light-cone momentum?

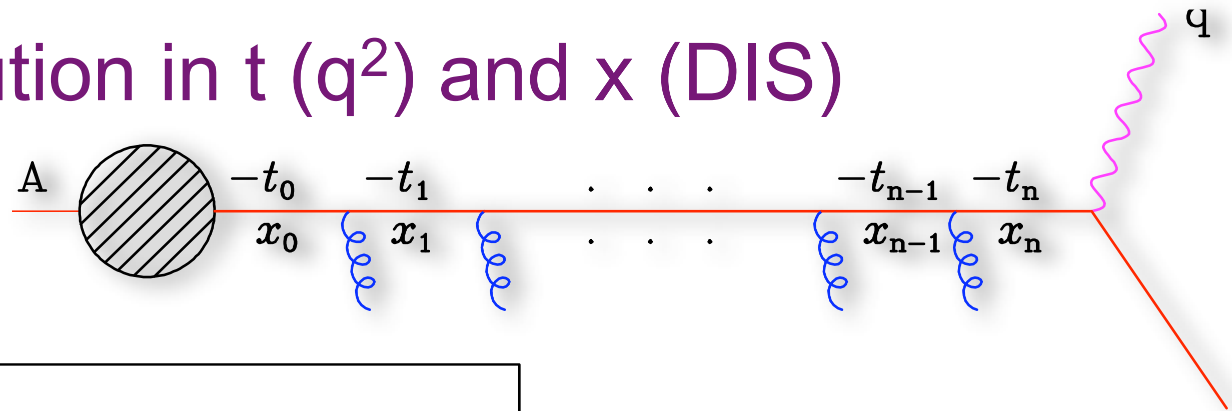
Massless partons become massive. How?

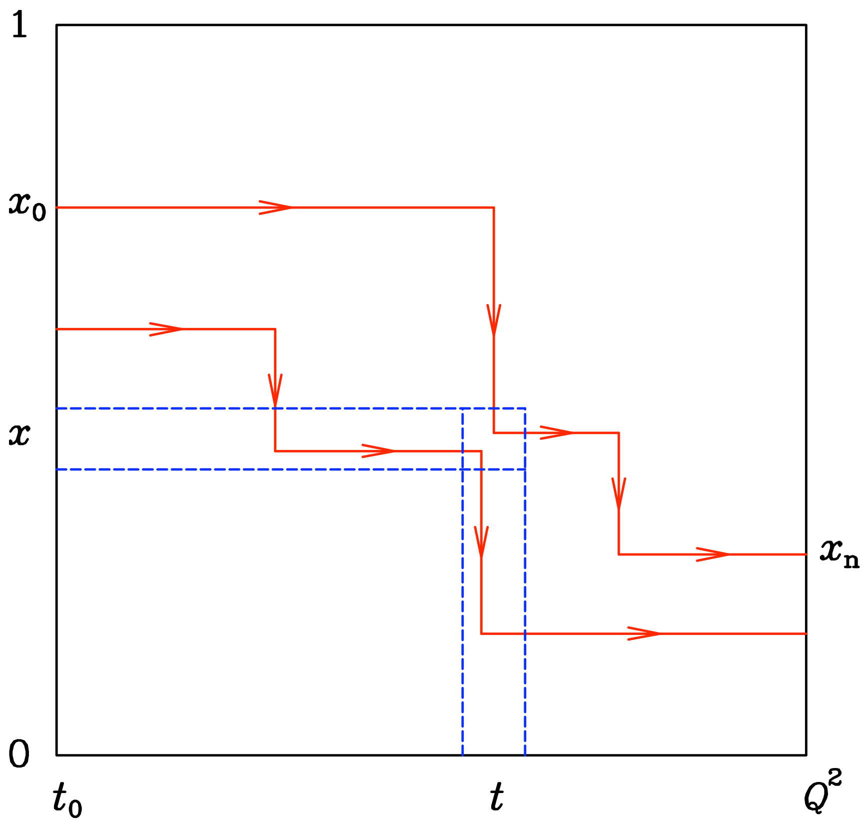
Upper limit for q^2?

$\Bigg\}$ Equivalent at this stage, but can be very important numerically

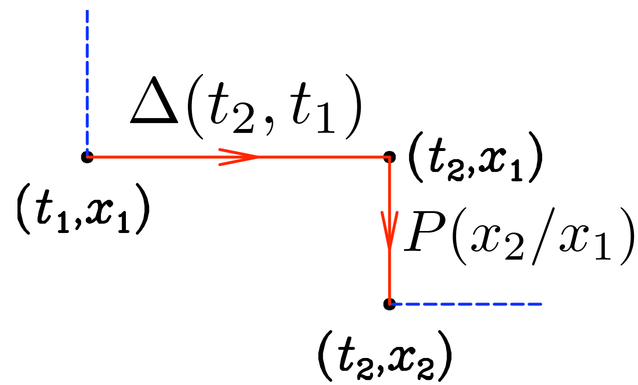
Parton Shower

- Evolution in t (q^2) and x (DIS)





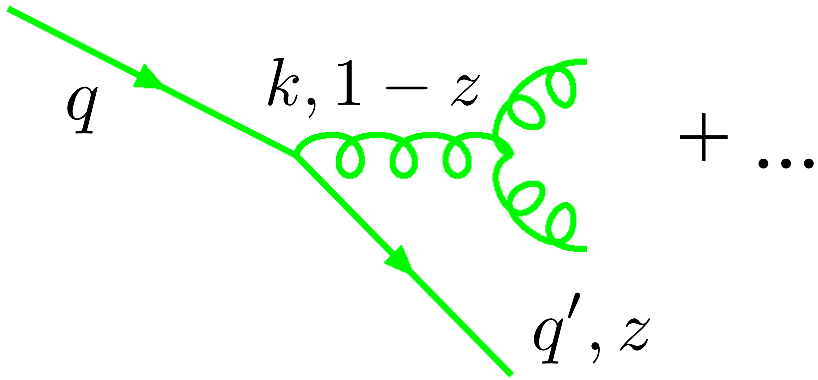
Basic 2-step:



e^+e^-: same formula, opposite direction!

Running coupling

Effect of summing up higher orders:



Scale is set by maximum virtuality of emitted gluon

$$k^2_{\rm max} = (1-z)q^2$$

Similarly in $g \to gg'$, scale is set by

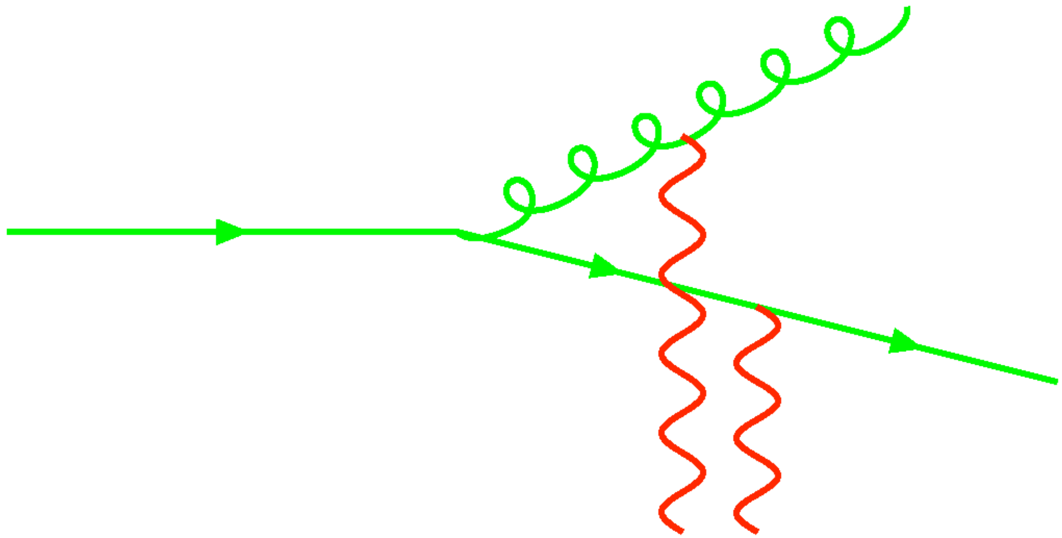
$$\min\{k^2_{\rm max}, k'^2_{\rm max}\} = \min\{z, (1-z)\}q^2 \simeq z(1-z)q^2 \equiv k_T^2$$

Scale change absorbed by replacing $\alpha_{\rm S}(q^2)$ by $\alpha_{\rm S}(k_T^2)$

➡ Faster parton multiplication

Soft limit

Also universal. But at amplitude level…



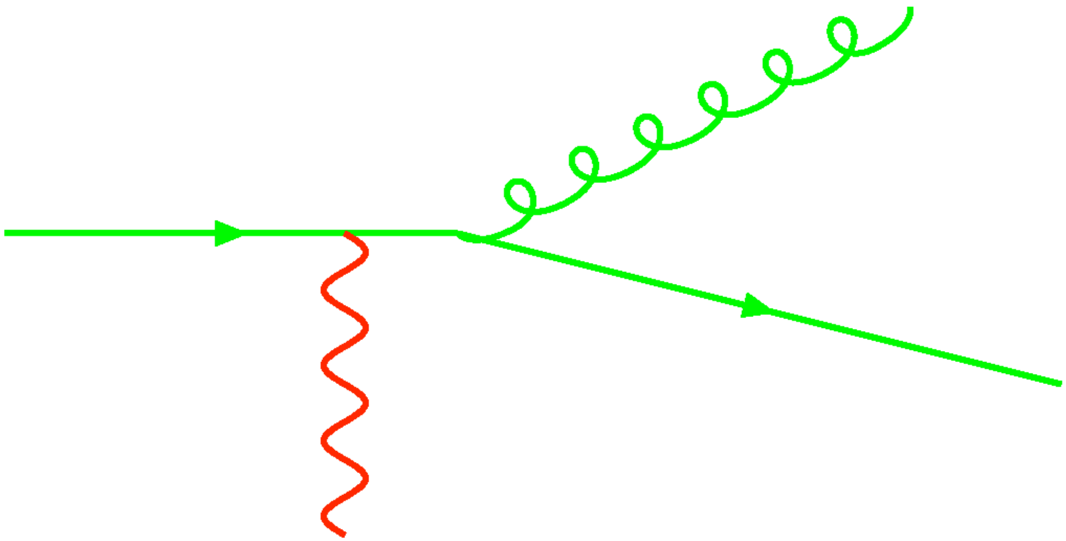
soft gluon comes from everywhere in event.

→ Quantum interference.

Spoils independent evolution picture?

Angular Ordering

NO:



outside angular ordered cones, soft gluons sum coherently: only see colour charge of whole jet.

Soft gluon effects fully incorporated by using θ^2 as evolution variable: angular ordering

First gluon not necessarily hardest!

Soft Gluon Emission

- Propagator factor for emission from external line, energy E, mass m

$$\frac{1}{(p\pm q)^2 - m^2} = \frac{\pm 1}{2p\cdot q} = \frac{\pm 1}{2\omega E(1 - v\cos\theta)}$$

Including numerator, get universal eikonal factor in soft limit

$$F_{\rm soft} = \frac{p\cdot\varepsilon}{p\cdot q}$$

No enhancement for emission from internal lines

$$(p+q)^2 - m^2 \to p^2 - m^2 \neq 0 \text{ as } \omega \to 0$$

Enhancement factor in amplitude for each external line implies cross section enhancement is sum over all *pairs* of external lines:

$$d\sigma_{n+1} = d\sigma_n \frac{d\omega}{\omega}\frac{d\Omega}{2\pi}\frac{\alpha_s}{2\pi}\sum_{i,j} C_{ij} W_{ij}$$

where $d\Omega$ is element of solid angle for emitted gluon, C_{ij} is a colour factor, and radiation function W_{ij} is given by

$$W_{ij} = \frac{\omega^2 p_i \cdot p_j}{p_i \cdot q \, p_j \cdot q} = \frac{1 - v_i v_j \cos\theta_{ij}}{(1 - v_i \cos\theta_{iq})(1 - v_j \cos\theta_{jq})}$$

Colour-weighted sum of radiation functions $C_{ij} W_{ij}$ is antenna pattern of hard process.

Angular Ordering

Radiation function can be separated into two parts containing collinear singularities along lines i and j. Consider for simplicity massless particles, $v_{i,j} = 1$. Then $W_{ij} = W_{ij}^i + W_{ij}^j$ where

$$W_{ij}^i = \frac{1}{2}\left(W_{ij} + \frac{1}{1-\cos\theta_{iq}} - \frac{1}{1-\cos\theta_{jq}}\right) .$$

This function has the remarkable property of angular ordering. Write angular integration in polar coordinates w.r.t. direction of i, $d\Omega = d\cos\theta_{iq}\, d\phi_{iq}$

Performing azimuthal integration, we find

$$\int_0^{2\pi} \frac{d\phi_{iq}}{2\pi} W_{ij}^i = \frac{1}{1-\cos\theta_{iq}} \quad \text{if } \theta_{iq} < \theta_{ij}, \text{ otherwise } 0.$$

To prove angular ordering property, write $1-\cos\theta_{jq} = a - b\cos\phi_{iq}$ where $a = 1-\cos\theta_{ij}\cos\theta_{iq}$, $b = \sin\theta_{ij}\sin\theta_{iq}$. Defining $z = \exp(i\phi_{iq})$, we have

$$I^i_{ij} \equiv \int_0^{2\pi}\frac{d\phi_{iq}}{2\pi}\frac{1}{1-\cos\theta_{jq}} = \frac{1}{i\pi b}\oint\frac{dz}{(z-z_+)(z-z_-)}$$

where z-integration contour is the unit circle and $z_\pm = a/b \pm \sqrt{a^2/b^2-1}$. Now only pole at $z = z_-$ can lie inside unit circle, so

$$I^i_{ij} = \sqrt{\frac{1}{a^2-b^2}} = \frac{1}{|\cos\theta_{iq}-\cos\theta_{ij}|}$$

Hence

$$\begin{aligned}\int_0^{2\pi}\frac{d\phi_{iq}}{2\pi}W^i_{ij} &= \frac{1}{2(1-\cos\theta_{iq})}[1+(\cos\theta_{iq}-\cos\theta_{ij})I^i_{ij}] \\ &= \frac{1}{1-\cos\theta_{iq}} \quad \text{if } \theta_{iq}<\theta_{ij},\ \text{otherwise } 0.\end{aligned}$$

Coherent Branching

Angular ordering provides basis for coherent parton branching formalism, which includes leading soft gluon enhancements to all orders. In place of virtual mass-squared variable t in earlier treatment, use angular variable

$$\zeta = \frac{p_b \cdot p_c}{E_b\, E_c} \simeq 1 - \cos\theta$$

as evolution variable for branching $a \to bc$, and impose angular ordering $\zeta' < \zeta$ for successive branchings. Iterative formula for n-parton emission becomes

$$d\sigma_{n+1} = d\sigma_n \frac{d\zeta}{\zeta} dz \frac{\alpha_s}{2\pi}$$

In place of virtual mass-squared cutoff, we must use angular cutoff for coherent branching. This is to some extent arbitrary, depending on how we classify emission as unresolvable. Simplest choice is $\zeta_0 = t_0/E^2$.

With this cutoff, the most convenient definition of evolution variable is not ζ itself but rather $\tilde{t} = E^2\zeta \geq t_0$
Angular ordering condition $\zeta_b, \zeta_c < \zeta_a$ for timelike branching $a \to bc$ becomes $\tilde{t}_b < z^2\tilde{t}\ , \quad \tilde{t}_c < (1-z)^2\tilde{t}$
where $\tilde{t} = \tilde{t}_a$ and $z = E_b/E_a$. Thus cutoff on z becomes

$$\sqrt{t_0/\tilde{t}} < z < 1 - \sqrt{t_0/\tilde{t}}$$

Neglecting masses of b & c, virtual mass-squared of a and transverse momentum of branching are

$$t = z(1-z)\tilde{t}\ , \quad p_t^2 = z^2(1-z)^2\tilde{t}$$

Thus for coherent branching Sudakov form factor of quark becomes

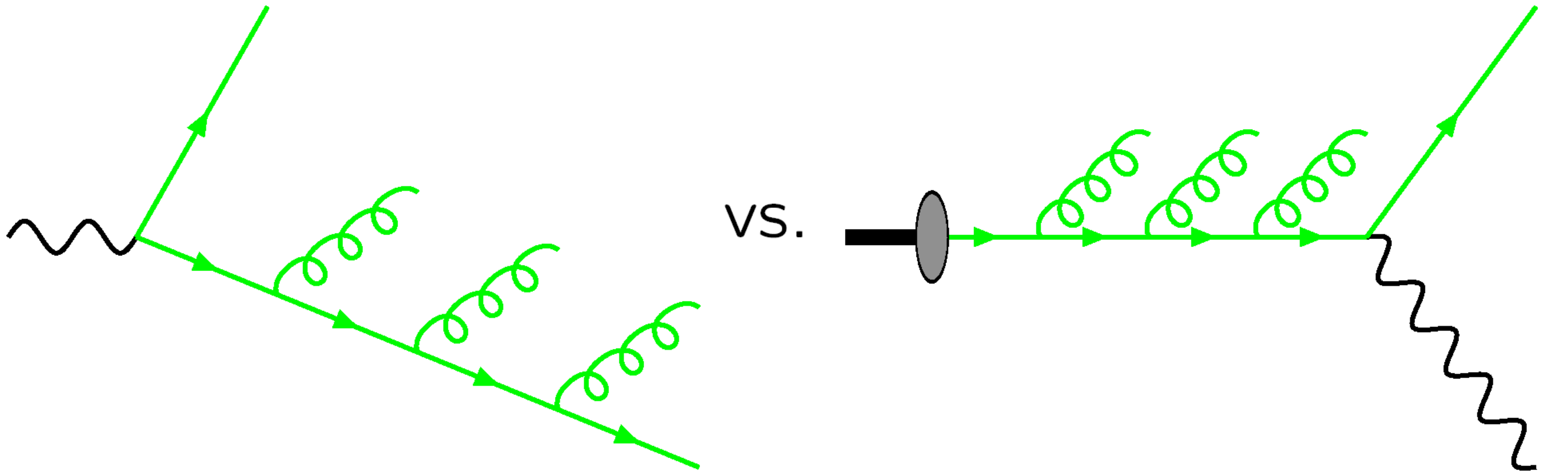
$$\tilde{\Delta}_q(\tilde{t}) = \exp\left[-\int_{4t_0}^{\tilde{t}} \frac{dt'}{t'} \int_{\sqrt{t_0/t'}}^{1-\sqrt{t_0/t'}} \frac{dz}{2\pi} \alpha_s(z^2(1-z)^2 t')\hat{P}_{qq}(z)\right]$$

This falls more slowly than without coherence, due to suppression of soft gluon emission by angular ordering.

Initial state radiation

In principle identical to final state (for not too small x)

In practice different because both ends of evolution fixed:



Use approach based on evolution equations…

Backward Evolution

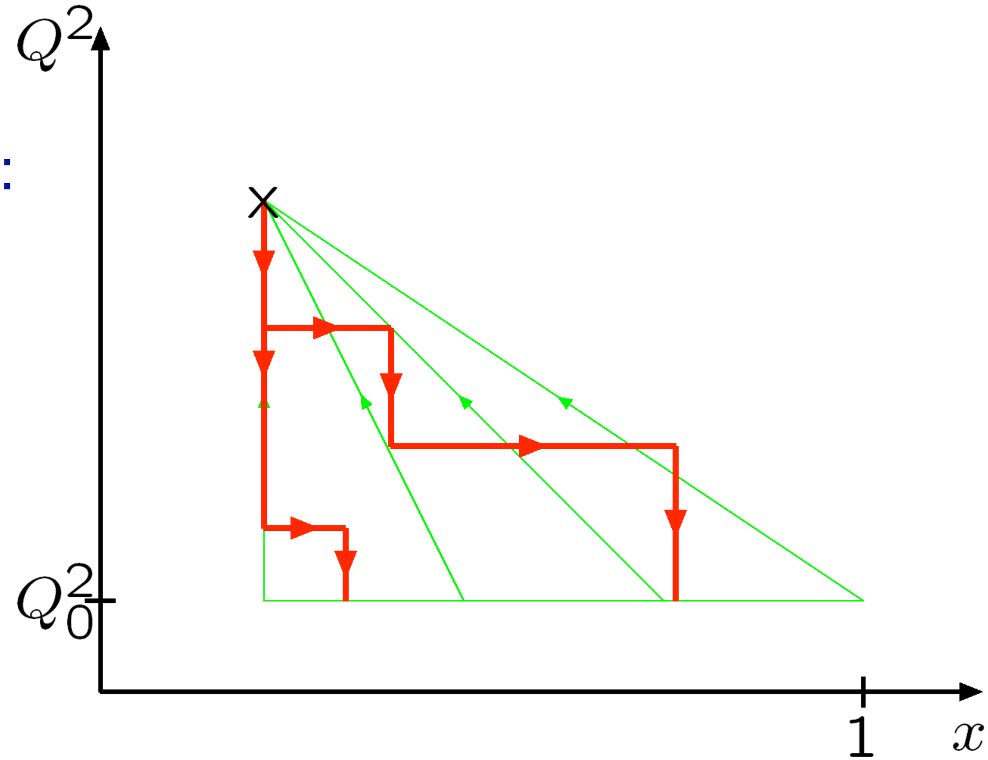
DGLAP evolution: pdfs at (x, Q^2) as function of pdfs at $(> x, Q_0^2)$:

Evolution paths sum over all possible events.

Formulate as backward evolution: start from hard scattering and work down in q^2, up in x towards incoming hadron.

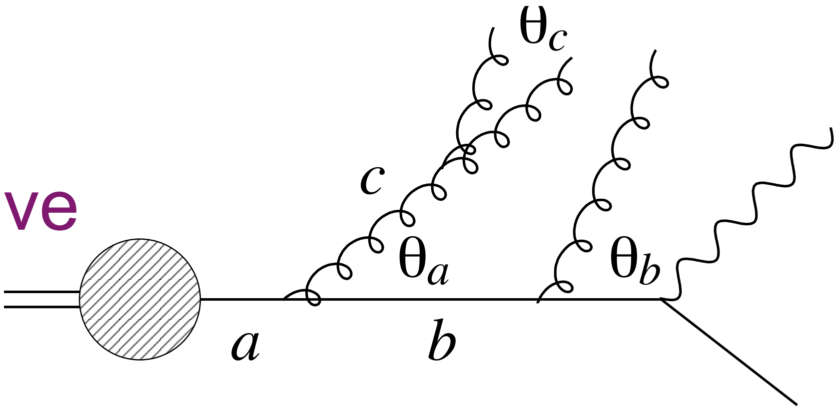
Algorithm identical to final state with $\Delta_i(Q^2, q^2)$ replaced by

$$\Delta_i(Q^2, q^2)/f_i(x, q^2).$$



Note that for initial-state (spacelike) branching $a \to bc$ (a incoming, b spacelike), angular ordering condition is

$$\theta_b > \theta_a > \theta_c$$



and so for $z = E_b/E_a$ we now have

$$\tilde{t}_b > z^2\tilde{t}_a \ , \qquad \tilde{t}_c < (1-z)^2\tilde{t}_a$$

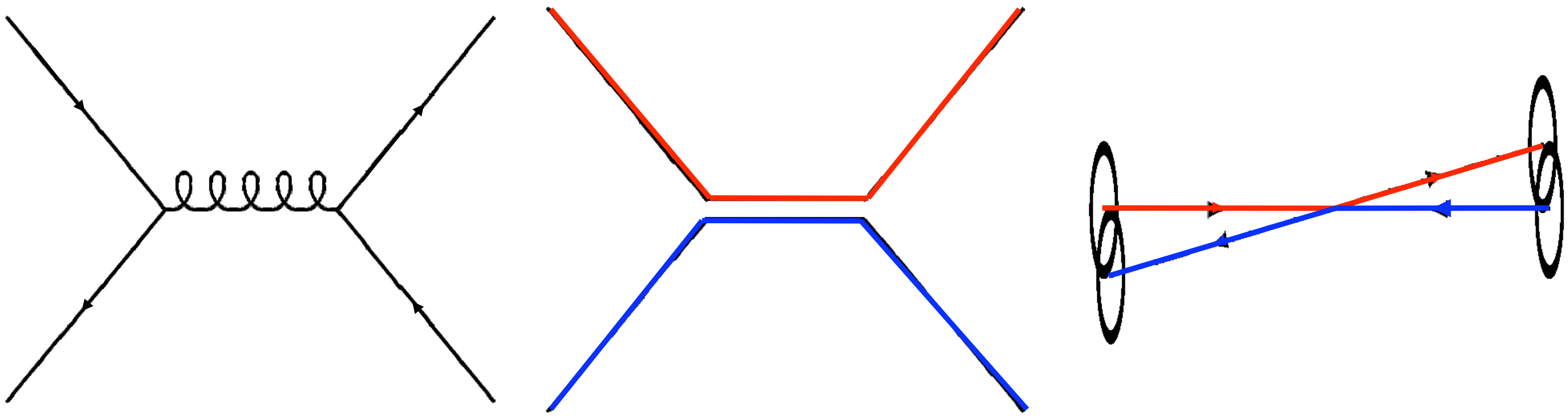
Thus we can have either $\tilde{t}_b > \tilde{t}_a$ or $\tilde{t}_a > \tilde{t}_b$, especially at small z

➡ Spacelike branching becomes disordered at small x.

Hard Scattering

Sets up initial conditions for parton showers.

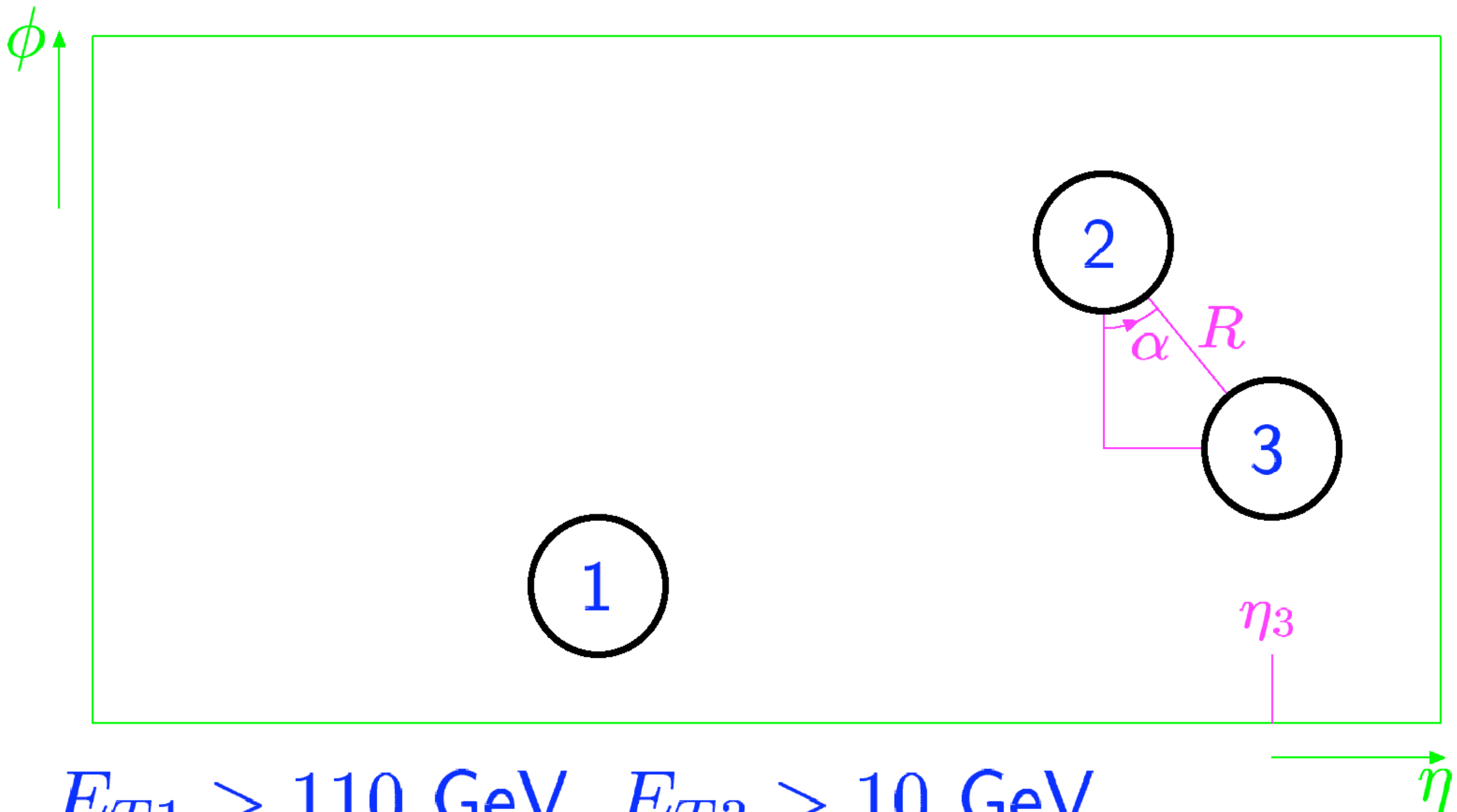
Colour coherence important here too.



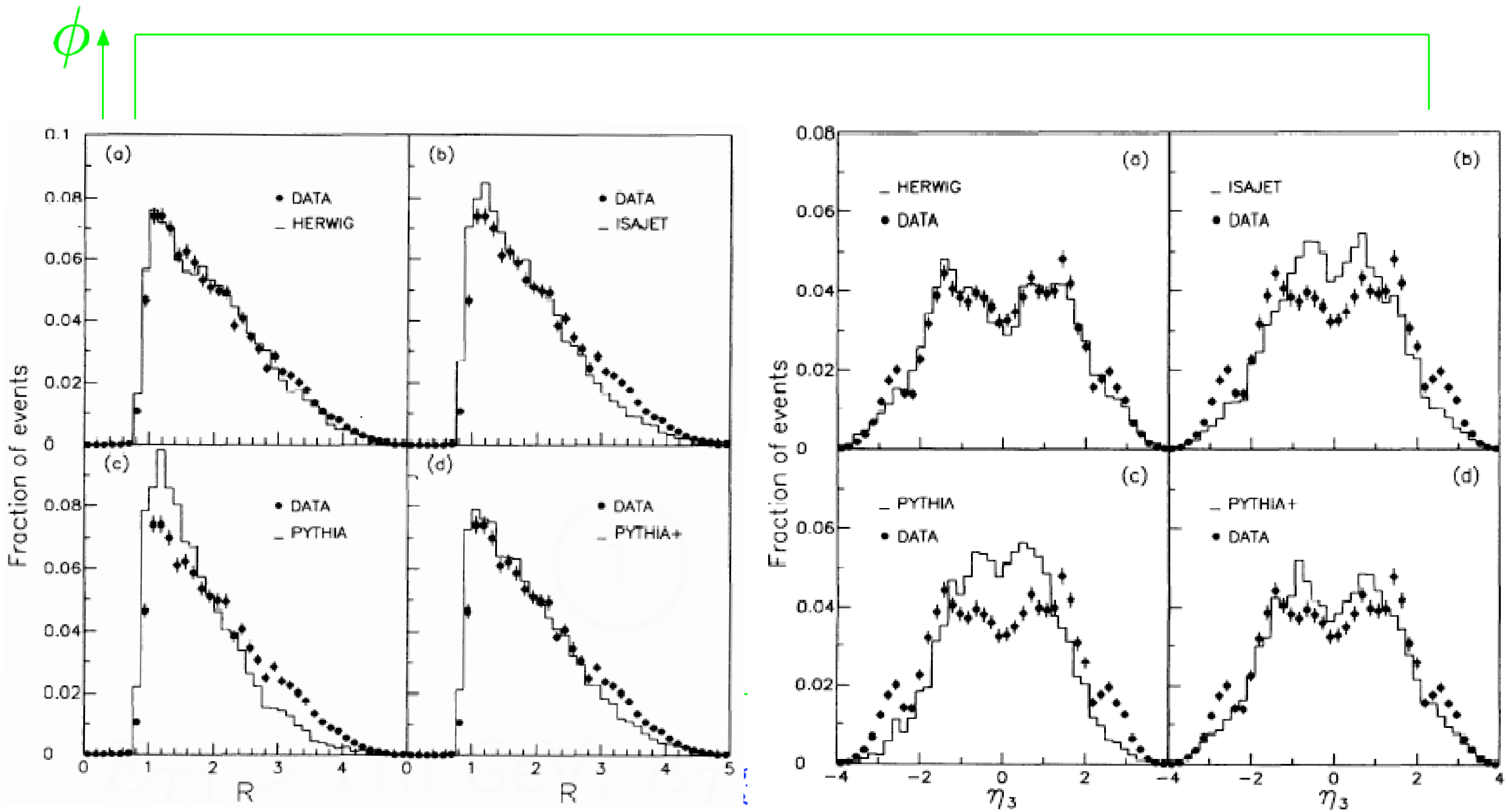
Emission from each parton confined to cone stretching to its colour partner

Essential to fit Tevatron data…

Three-jet correlations (CDF)



$E_{T1} > 110$ GeV, $E_{T3} > 10$ GeV.



Distributions of third-hardest jet in multi-jet events

HERWIG has complete treatment of colour coherence, PYTHIA+ has partial

The Colour Dipole Model

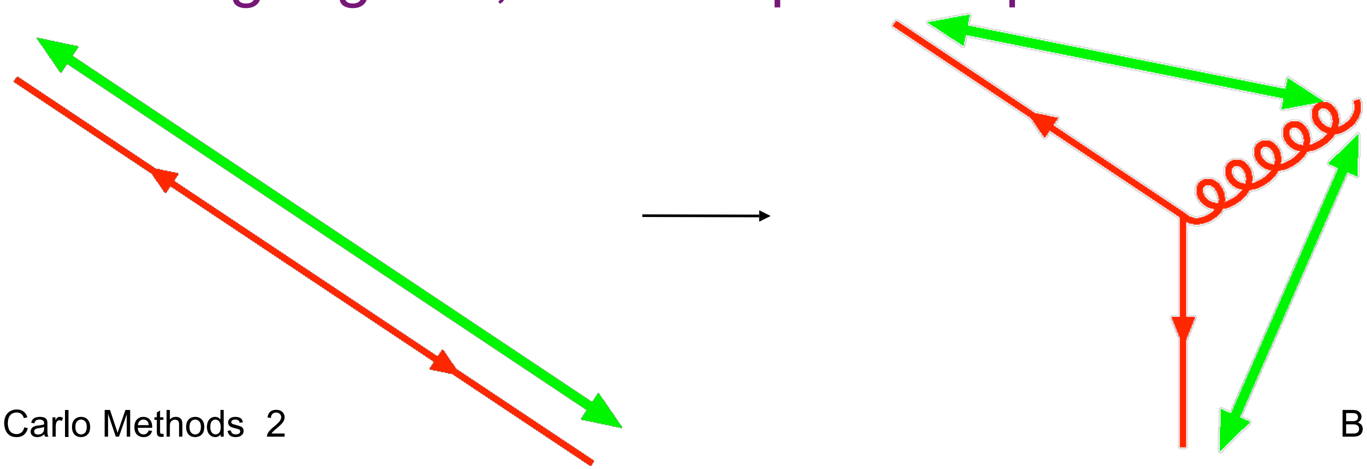
Conventional parton showers: start from collinear limit, modify to incorporate soft gluon coherence

Colour Dipole Model: start from soft limit

Emission of soft gluons from colour-anticolour dipole universal (and classical):

$$d\sigma \approx \sigma_0 \, \tfrac{1}{2} C_A \frac{\alpha_s(k_\perp)}{2\pi} \frac{dk_\perp^2}{k_\perp^2} \, dy, \qquad y = \mathrm{rapidity} = \log\tan\theta/2$$

After emitting a gluon, colour dipole is split:

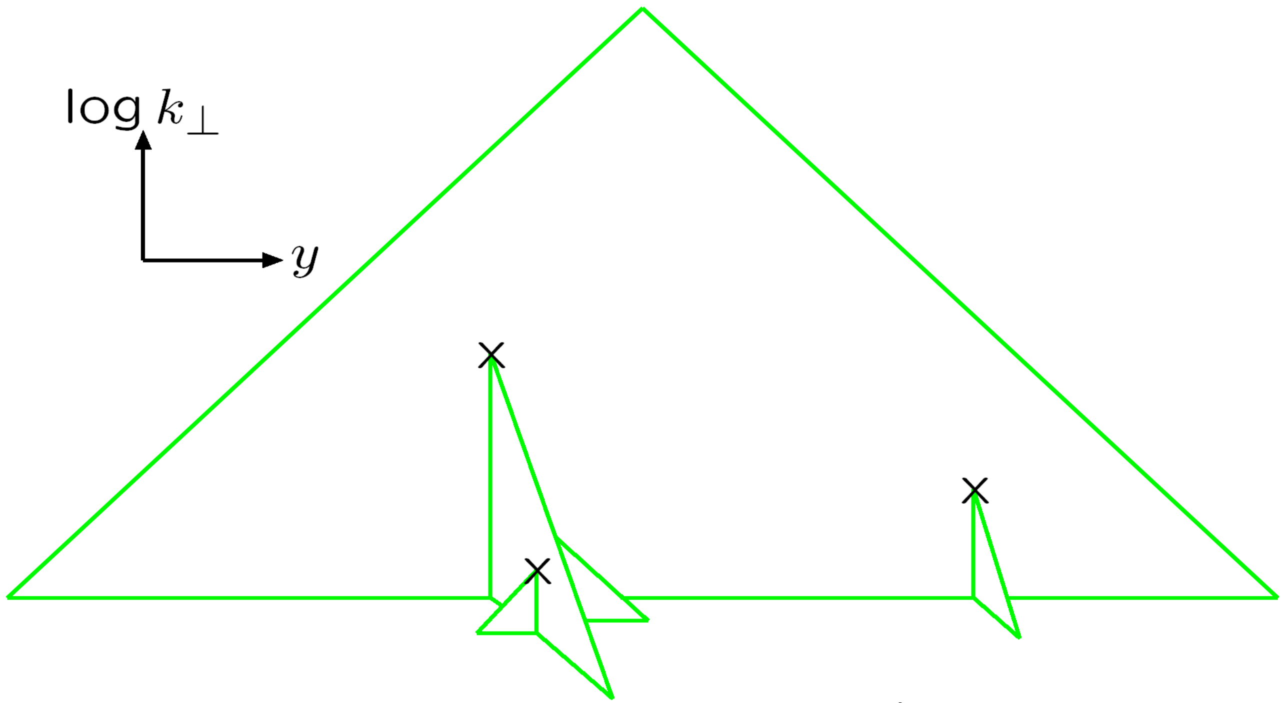


Subsequent dipoles continue to cascade

c.f. parton shower: one parton → two

CDM: one dipole → two = two partons → three

Represented in 'origami diagram':



Similar to angular-ordered parton shower for e^+e^- annihilation

Summary

- Accelerated colour charges radiate gluons.

  Gluons are also charged → cascade.

- Probabilistic language derived from factorization theorems of full gauge theory.

  Colour coherence → angular ordering.

- Modern parton shower models are very sophisticated implementations of perturbative QCD, but would be useless without hadronization models…