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Resummed C-Parameter Distribution in e^+e^- Annihilation*

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Abstract

We give perturbative predictions for the distribution of the C-parameter event shape variable in e^+e^- annihilation, including resummation of large logarithms in the two-jet (small-C) region, matched to next-to-leading order results. We also estimate the leading non-perturbative power correction and make a preliminary comparison with experimental data.

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1 Introduction

The study of event shape distributions in e^+e^- annihilation has proved to be a valuable testing-ground for QCD. For an infrared-safe shape variable, i.e. one that is insensitive to the splitting of a final-state momentum into collinear momenta and to the emission of soft momenta, the distribution can be calculated order-by-order in perturbation theory. However, one finds that higher-order contributions are enhanced by large logarithmic factors near the two-jet region, where most of the data reside. Hence for a detailed quantitative understanding one should resum as many of these terms as possible to all orders. This has been achieved for a limited number of shape variables, namely those shown to satisfy an *exponentiation* property, to be specified in more detail later. These variables include the thrust [1], heavy jet mass [2], jet broadening [3], energy-energy correlation [4] and differential two-jet rate [5].

In the present paper we show that the C-parameter [6] shape variable also has the required property of exponentiation, so that we are able to carry out the resummation of large logarithms to the same accuracy as for those mentioned above. The exponentiation of soft gluon contributions to QCD matrix elements is a general phenomenon; the critical property for event shapes is that the corresponding phase space should factorize with sufficient accuracy to maintain exponentiation of at least the leading and next-to-leading logarithms. In the case of the C-parameter, we are able to show this by exploiting a simple connection between the C-parameter and the thrust in the dominant phase-space regions, namely those where C is small.

In Sect. 2 we review the definition and kinematics of the C-parameter, and the relationship between it and the thrust in the small-C region. Sect. 3 presents the fixed-order predictions for the C-parameter distribution up to order α_S^2, together with the large logarithmic terms that appear at small C. The resummation of these terms, to next-to-leading logarithmic accuracy, is performed in Sect. 4 using the connection with the thrust.

In order to describe the C-parameter distribution over the widest possible range, one should match the resummed results to those at fixed order outside the two-jet region. A suitable matching procedure, chosen from those proposed in Ref. [1], is outlined in Sect. 5.

A further improvement in the prediction can be made by estimating non-perturbative effects, which have been found to be substantial for many shape variables at present energies. This is discussed in Sect. 6, where we argue that the effects on the C-parameter distribution can again be related to those for the thrust, via their connection in the soft region.

Finally in Sect. 7 we make a preliminary comparison with experimental data and discuss the results.

2 Kinematics of the C-parameter

The C-parameter was initially defined as [6]

$$C = 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1) \tag{1}$$

where λ_α ($0 \le \lambda_\alpha \le 1$, $\sum_\alpha \lambda_\alpha = 1$) are the eigenvalues of the linearized momentum tensor

$$\Theta^{\alpha\beta} = \frac{\sum_i \boldsymbol{p}_i^\alpha \boldsymbol{p}_i^\beta / |\boldsymbol{p}_i|}{\sum_j |\boldsymbol{p}_j|} \,, \tag{2}$$

the sums being over all final-state particles. Neglecting particle masses, we may express C in terms of invariants as

$$C = 3 - \frac{3}{2}\sum_{i,j} \frac{(p_i \cdot p_j)^2}{(p_i \cdot Q)(p_j \cdot Q)} \tag{3}$$

where Q^μ is the total four-momentum

$$Q^\mu = \sum_i p_i^\mu \,. \tag{4}$$

Introducing the c.m. energy fractions $x_i = 2(p_i \cdot Q)/Q^2$, we can write Eq. (3) as

$$C = \tfrac{3}{8}\sum_{i,j} x_i x_j \sin^2\theta_{ij} \,. \tag{5}$$

The kinematic range is $0 \le C \le 1$, with $C = 0$ for a perfectly two-jet-like final state and 1 for an isotropic and acoplanar distribution of final-state momenta. In fact the maximal value $C = 1$ can only be achieved when there are four or more final-state particles. Planar events fill up the kinematic region $C \le \frac{3}{4}$. For three particles, the maximum value $C = \frac{3}{4}$ corresponds to the symmetric configuration $x_1 = x_2 = x_3 = \frac{2}{3}$.

2.1 C-parameter and thrust at small C

Let us consider the definition (1) of the C-parameter and assume the ordering of eigenvalues $\lambda_1 \ge \lambda_2 \ge \lambda_3$. It follows that in the small-C limit the maximum eigenvalue approaches unity, $\lambda_1 \to 1$, whilst λ_2 and λ_3 are parametrically small, $\lambda_2 \sim \lambda_3 \sim \mathcal{O}(1-\lambda_1)$. Using the normalization condition $\Sigma_\alpha \lambda_\alpha = 1$, we thus obtain

$$C = 3\{(1-\lambda_1) - \tfrac{1}{2}[(1-\lambda_1)^2 + \lambda_2^2 + \lambda_3^2]\} \simeq 3(1-\lambda_1)[1 + \mathcal{O}(1-\lambda_1)] \,, \tag{6}$$

or, equivalently,

$$C = 3(1-\lambda_1)[1 + \mathcal{O}(C)] \,, \qquad (C \to 0) \,. \tag{7}$$

We can now relate the eigenvalue λ_1 to the thrust. The definition of the thrust T is [7]

$$T = \mathrm{Max}\frac{\Sigma_i |\boldsymbol{p}_i \cdot \boldsymbol{n}|}{\Sigma_j |\boldsymbol{p}_j|} \,, \tag{8}$$

where the maximum is with respect to the unit 3-vector $\boldsymbol{n}$. At the maximum $\boldsymbol{n} = \boldsymbol{n}_T$, the thrust axis. Denoting by $\boldsymbol{n}_1$ the eigenvector of the linearized momentum tensor (2) corresponding to the maximum eigenvalue λ_1, by definition we have

$$\lambda_1 = \frac{1}{\Sigma_j |\boldsymbol{p}_j|} \Sigma_i \frac{|\boldsymbol{p}_i \cdot \boldsymbol{n}_1|^2}{|\boldsymbol{p}_i|} \leq \frac{\Sigma_i |\boldsymbol{p}_i \cdot \boldsymbol{n}_1|}{\Sigma_j |\boldsymbol{p}_j|} \leq T \quad , \tag{9}$$

where the last inequality comes from Eq. (8). From Eq. (7) we thus obtain

$$C \geq 3(1-T)[1 + \mathcal{O}(C)] \; , \qquad (C \to 0) \; . \tag{10}$$

Equation (10) implies that in the small-C limit the thrust approaches unity. The opposite is also true. In order to show this, let us consider the separation of the final state into two hemispheres S_+ and S_- by the plane orthogonal to the thrust axis $\boldsymbol{n}_T$. We can write Eq. (5) as

$$C = \frac{3}{8} \left(\Sigma'_{i,j} x_i x_j \sin^2\theta_{ij} + \Sigma''_{i,j} x_i x_j \sin^2\theta_{ij} \right) \quad , \tag{11}$$

where $\sum'_{i,j}$ ($\sum''_{i,j}$) denotes the sum over all final-state pairs belonging to the same (opposite) hemisphere(s). Then, applying the identity $\sin^2\theta_{ij} = 2(1-\cos\theta_{ij}) - (1-\cos\theta_{ij})^2$ ($\sin^2\theta_{ij} = 2(1+\cos\theta_{ij}) - (1+\cos\theta_{ij})^2$) whenever the particles i and j belong to the same (opposite) hemisphere(s) and using energy-momentum conservation, it is straightforward to recast Eq. (11) in the form

$$C = 3\,[\,2(w_+ + w_-) - (w_+ - w_-)^2] - \Delta_T \quad , \tag{12}$$

where w_+ and w_- are the rescaled invariant masses-squared of the two hemispheres,

$$w_\pm = \frac{1}{Q^2} \left(\sum_{i \in S_\pm} p_i \right)^2 \quad , \tag{13}$$

and Δ_T is defined by

$$\Delta_T = \frac{3}{8} \left(\Sigma'_{i,j} x_i x_j (1-\cos\theta_{ij})^2 + \Sigma''_{i,j} x_i x_j (1+\cos\theta_{ij})^2 \right) \quad . \tag{14}$$

Note that Δ_T is positive definite and that the expression in the square bracket in Eq. (12) is exactly equal to $1 - T^2$ [1]. Therefore from Eq. (12) we obtain

$$C = 3(1-T^2) - \Delta_T \leq 3(1-T^2) \leq 6(1-T) \; . \tag{15}$$

Thus, combining the inequalities (10) and (15), we see that for sufficiently small C (i.e., neglecting corrections of order C^2) we have

$$3(1-T) \leq C \leq 6(1-T) \; . \tag{16}$$

3 The C-parameter in perturbation theory

The C-parameter is manifestly insensitive to the splitting of a final-state momentum into collinear momenta ($\theta_{ij} = 0$) and to the emission of soft momenta ($x_i = 0$). It follows that the C-parameter distribution can be computed in QCD perturbation theory as a power series expansion in the strong coupling α_S. For $C \neq 0$, the predicted distribution has the general form

$$\frac{1}{\sigma_0}\frac{d\sigma}{dC} = \bar{\alpha}_S A(C) + \bar{\alpha}_S^2 B(C) + \mathcal{O}(\alpha_S^3)\ , \tag{17}$$

where $\bar{\alpha}_S = \alpha_S/2\pi$ and we normalize to the Born cross section σ_0, as was done in Refs. [8,9]. The first-order distribution $A(C)$ is shown by the dashed curve in Fig. 1. Notice that it diverges as $C \to 0$; in fact at small C one finds

$$A(C) = 4C_F \frac{1}{C}\left[\ln\left(\frac{6}{C}\right) - \frac{3}{4}\right] + \mathcal{O}(\ln C)\ . \tag{18}$$

At larger values of C, $A(C)$ smoothly approaches a finite value at the three-parton upper limit $C = \frac{3}{4}$:

$$A(\tfrac{3}{4}) = \frac{256}{243}\pi\sqrt{3}C_F = 7.6433\ . \tag{19}$$

It will be useful to define the fraction of events with C-parameter values less than C:

$$R(C) = \int_0^C \frac{dC}{\sigma_t}\frac{d\sigma}{dC} = 1 + \bar{\alpha}_S R_1(C) + \bar{\alpha}_S^2 R_2(C) + \mathcal{O}(\alpha_S^3) \tag{20}$$

where σ_t is the total cross section. Then since $R(C_{\max}) = 1$ and $\sigma_t/\sigma_0 = 1 + \frac{3}{2}C_F\bar{\alpha}_S + \mathcal{O}(\alpha_S^2)$, we have

$$\begin{aligned} R_1(C) &= -\int_C^{\frac{3}{4}} A(C)dC \\ R_2(C) &= -\int_C^1 B(C)dC + \tfrac{3}{2}C_F\int_C^{\frac{3}{4}} A(C)dC\ . \end{aligned} \tag{21}$$

Introducing

$$L \equiv \ln\left(\frac{6}{C}\right) \tag{22}$$

we find as $C \to 0$

$$R_1(C) = -2C_F(L^2 - \tfrac{3}{2}L - \tfrac{1}{3}\pi^2 + \tfrac{5}{4}) + \mathcal{O}(C\ln C)\ . \tag{23}$$

Notice that in first order we obtain up to two large logarithmic factors L at small C, corresponding to the emission of one collinear and/or soft gluon. Note also that by rescaling C by a factor of 6 in the definition (22) we absorb all logarithms in Eq. (23) into L. In Sect. 4 we shall see that this is a consequence of the relationship (15) between the C-parameter and the thrust at small C.

Upon adding the second-order contribution $B(C)$, we obtain the solid curve in Fig. 1, which exhibits a number of new features [8]: the behaviour at $C \to 0$ is modified, in fact

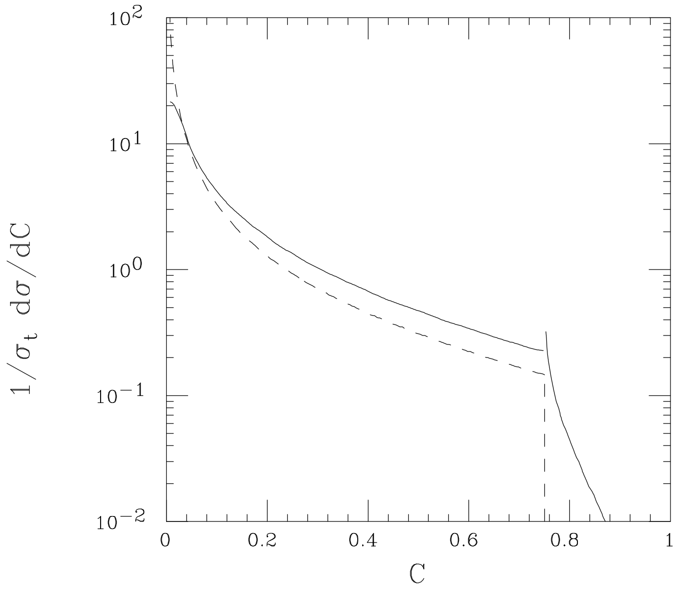

Figure 1: Fixed-order predictions of the C-parameter distribution for $\alpha_S = 0.12$. Dashed: $\mathcal{O}(\alpha_S)$. Solid: $\mathcal{O}(\alpha_S^2)$.

becoming strongly negative divergent; the distribution remains finite as $C \to \frac{3}{4}^-$ but there is a new divergence as $C \to \frac{3}{4}^+$; and finally the distribution vanishes smoothly as $C \to 1$.

The new features at large C arise from the abrupt change in the $\mathcal{O}(\alpha_S)$ contribution $A(C)$ at $C = \frac{3}{4}$, see Eq. (19). They have been discussed in Ref. [10] and the resummation of large terms of the type $\ln(C - \frac{3}{4})$ in this region will be considered in a separate paper [11]. In the present paper we concentrate on terms that diverge as $C \to 0$, and the matching of the resummed and fixed-order predictions in the region $0 < C < \frac{3}{4}$, where the latter remains smooth.

In the region of small C, the emission of two collinear and/or soft gluons can yield up to four powers of $L = \ln(6/C)$. Thus, writing the logarithmic dependence explicitly, the second-order contribution R_2 to the event fraction $R(C)$ defined in Eq. (20) has the form

$$R_2(C) = \sum_{m=0}^{4} R_{2m} L^m + D_2(C) \, , \tag{24}$$

where $D_2 \to 0$ as $C \to 0$. We have computed the coefficients R_{24}, R_{23}, R_{22} analytically, and find that their values, together with Eq. (23), are consistent with the following exponentiating form for $R(C)$:

$$R(C) = \left(1 + \sum_{n=1}^{\infty} C_n \bar{\alpha}_S^n\right) \exp\left(\sum_{n=1}^{\infty} \sum_{m=1}^{n+1} G_{nm} \bar{\alpha}_S^n L^m\right) + \sum_{n=1}^{\infty} \bar{\alpha}_S^n D_n(C) \, . \tag{25}$$

Here C_n and G_{nm} are C-independent coefficients and the remainder functions $D_n(C)$ vanish as $C \to 0$.

In the next section we argue that the exponentiation formula (25) is actually valid to all orders in perturbation theory. Therefore it is convenient to express the coefficients in

C_1	$=$	$+C_F(4\pi^2 - 15)/6$
G_{11}	$=$	$+3C_F$
G_{12}	$=$	$-2C_F$
G_{22}	$=$	$-C_F[48\pi^2 C_F + (169 - 12\pi^2)C_A - 22N_f]/36$
G_{23}	$=$	$-C_F(11C_A - 2N_f)/3$

Table 1: Coefficients C_1 and G_{nm} in Eq. (25).

Eq. (24) in terms of those in Eq. (25), as follows

$$\begin{aligned} R_{20} &= C_2 \, , \quad R_{21} = G_{21} + C_1 G_{11} \, , \quad R_{22} = G_{22} + \tfrac{1}{2} G_{11}^2 + C_1 G_{12} \, , \\ R_{23} &= G_{23} + G_{12} G_{11} \, , \quad R_{24} = \tfrac{1}{2} G_{12}^2 \, . \end{aligned} \tag{26}$$

The coefficients which determine R_{2m} for $m \geq 2$ are as given in Table 1.

The coefficients C_2 and G_{21} which enter into R_{20} and R_{21} are not known analytically but can be fitted to numerical data from the Monte Carlo matrix element evaluation program EVENT [9]. From Eqs. (21) and (25) we have, for $N_f = 5$,

$$\begin{aligned} -\int_C^1 dC B(C) &= R_2(C) + \tfrac{3}{2} C_F R_1(C) \qquad (27) \\ &\underset{C \to 0}{\sim} 3.556 L^4 - 20.889 L^3 - 36.778 L^2 + (G_{21} + 29.758) L + C_2 + 10.879 \, . \end{aligned}$$

By fitting we find

$$C_2 = 76.5 \pm 2.9 \, , \quad G_{21} = 63.4 \pm 6.0 \, . \tag{28}$$

Then Eq. (27) gives a good description of the matrix element, as illustrated in Fig. 2. We checked that similar results are obtained with the program EVENT2 [12].

4 Resummation for $C \to 0$

As explained in the Introduction, in order to carry out the resummation of the $\log C$ contributions for $C \to 0$, we exploit the connection between the C-parameter and the thrust in the two-jet region, discussed in Sec. 2. However, to derive the relevant relation between C and $1 - T$, we cannot simply rely on the kinematics but rather we have to consider QCD dynamics.

As discussed in detail in Ref. [1], the structure of the soft and collinear singularities in the multiparton QCD matrix elements can be described in terms of a branching process. In the large-T region and to next-to-leading logarithmic accuracy, this process takes place in an

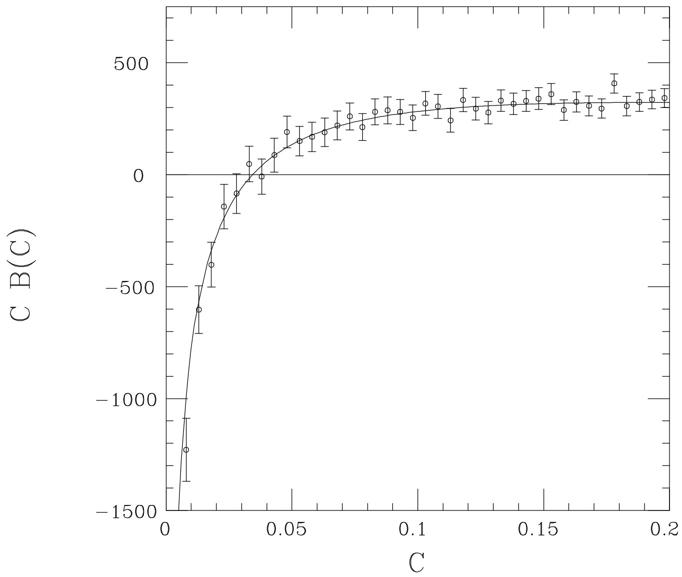

Figure 2: Second-order prediction of the C-parameter distribution for $C \to 0$. Points: EVENT Monte Carlo. Curve: Eqs. (27,28).

angular-ordered region around the thrust axis. Because of this angular ordering (enforced by QCD dynamics), we can introduce in Eq. (11) the approximation $\sin^2\theta_{ij} \simeq 2(1-\cos\theta_{ij})$ ($\sin^2\theta_{ij} \simeq 2(1+\cos\theta_{ij})$) in the terms appearing in $\sum'_{i,j}$ ($\sum''_{i,j}$). Equivalently, we can say that when $T \to 1$ the phase-space region contributing to Δ_T in Eq. (15) is subdominant with respect to that contributing to $3(1-T^2) \simeq 6(1-T)$. It follows that to *next-to-leading logarithmic* accuracy at small C we have

$$C \simeq 6(1-T)\;. \tag{29}$$

In Ref. [1] the perturbative expression for $R_T(\tau)$, the fraction of events with thrust larger than $T = 1-\tau$, was written in the following form:

$$R_T(\tau = 1-T) = K_T(\alpha_S)\Sigma_T(\tau,\alpha_S) + D_T(\tau,\alpha_S)\;. \tag{30}$$

By definition the functions $K_T(\alpha_S)$, $D_T(\tau,\alpha_S)$ and $\Sigma_T(\tau,\alpha_S)$ are power series expansions in α_S whose coefficients are respectively constant in τ, vanishing for $\tau \to 0$ and polynomials in $L = \ln 1/\tau$. The perturbative contributions to $R_T(\tau)$ which are logarithmically enhanced in the two-jet region $\tau = 1-T \to 0$ are thus embedded in Σ_T. At small τ it becomes important to resum the series of large logarithms in Σ_T.

By naïve counting of logarithms R_T, and hence Σ_T, contain terms of the type $\alpha_S^n L^m$ with $m \le 2n$. However, by explicitly performing the resummation of the leading logarithms (those with $n < m \le 2n$), it was shown [1] that they *exponentiate*. The word exponentiation means that the terms $\alpha_S^n L^m$ with $m > n+1$ are absent from $\ln R_T$ ($\ln \Sigma_T$), although they do appear in R_T itself.

The result in Eq. (29) allows us to obtain the leading and next-to-leading $\log C$ contributions to the C-parameter distribution simply by replacing $\ln(1/\tau)$ by $L = \ln(6/C)$ in the corresponding formula for the thrust distribution. It follows that the function $R(C)$

defined in Eq. (20) has the same form as Eq. (30), that is,

$$R(C) = K(\alpha_S)\Sigma(C,\alpha_S) + D(C,\alpha_S)\ , \tag{31}$$

with

$$\begin{aligned} \ln\Sigma(C,\alpha_S) &= \sum_{n=1}^{\infty}\sum_{m=1}^{n+1} G_{nm}\bar{\alpha}_S^n L^m \\ &= L\, g_1(\alpha_S L) + g_2(\alpha_S L) + \alpha_S\, g_3(\alpha_S L) + \cdots\ . \end{aligned} \tag{32}$$

The function g_1 resums all the leading logarithmic contributions $\alpha_S^n L^{n+1}$, while g_2 contains the next-to-leading logarithmic terms $\alpha_S^n L^n$, and g_3 etc. represent the remaining subdominant corrections $\alpha_S^n L^m$ with $0 < m < n$. All the functions g_i vanish at $L = 0$ since they resum terms with $m > 0$.

Because of the result in Eq. (29), the functions g_1 and g_2 in Eq. (32) are the same as those for the thrust distribution, given in Ref. [1], provided we identify L with $\ln(6/C)$. This relation between $\ln\Sigma(C)$ and $\ln\Sigma_T(\tau)$ breaks down at subdominant orders, i.e. the functions g_i for $i > 2$ are different in the two cases. Similarly the non-logarithmic coefficient functions K and K_T and the remainder functions D and D_T are different, as may be seen by comparing the fixed-order results in Sec. 3 with the corresponding ones in Ref. [1].

5 Matching with fixed order

To combine the fixed-order and resummed predictions of the previous two sections without any double counting, we may adopt any of the matching procedures proposed in Ref. [1]. In the so-called log-R matching scheme, for example, one writes the next-to-leading order expression (20) in the equivalent form

$$\ln R(C) = \bar{\alpha}_S R_1(C) + \bar{\alpha}_S^2\{R_2(C) - \tfrac{1}{2}[R_1(C)]^2\} + \mathcal{O}(\alpha_S^3) \tag{33}$$

and then replaces the computed logarithmic terms by the corresponding resummed expressions, to obtain

$$\begin{aligned} \ln R(C) &= L\, g_1(\alpha_S L) + g_2(\alpha_S L) + \bar{\alpha}_S\{R_1(C) - G_{11}L - G_{12}L^2\} \\ &\quad + \bar{\alpha}_S^2\{R_2(C) - \tfrac{1}{2}[R_1(C)]^2 - G_{22}L^2 - G_{23}L^3\}\ . \end{aligned} \tag{34}$$

An advantage of this scheme is that, since all terms are exponentiated, it is not necessary to separate out the coefficient and remainder functions K and D in Eq. (31) explicitly. A potential disadvantage is that the resummed terms may dominate over the fixed-order ones far from the two-jet region, if the latter become very small there. This should not be a severe problem for the C-parameter distribution, since the first-order prediction does not vanish at the three-parton boundary, Eq. (19). Thus we expect the log-R prescription (34) to be satisfactory at high energies, as long as matching is limited to the region $C < \frac{3}{4}$. Note, however, that the sensitivity to the matching procedure can increase at low energies, and therefore other schemes should also be tried.

6 Non-perturbative power corrections

Even at high energies ($Q \gtrsim M_Z$), it has been found that substantial non-perturbative 'hadronization corrections' need to be applied to perturbative predictions concerning event shape variables. Traditionally these have been estimated [13,14] using Monte Carlo hadronization models [15,16]. Recently, an alternative 'renormalon' or 'dispersive' method for estimating them has been developed [17-25]. According to this approach, the dominant non-perturbative correction to the thrust distribution, at thrust values that are not too close to the kinematic boundaries, should correspond to a simple shift in the resummed perturbative prediction, of the form [24]

$$R_T(\tau) = R_T^{\rm pert}(\tau - 4A_1/Q)\;, \tag{35}$$

where $R_T^{\rm pert}$ is the function called R_T in Eq. (30) and A_1 is a non-perturbative parameter to be determined experimentally.

Because of the close connection (16) between the C-parameter and thrust, the same type of result holds for the C-parameter distribution. Furthermore, the assumption that the leading non-perturbative effect is associated with a universal effective strong coupling at low scales [23] leads to a relationship between the shifts in the two distributions. Consider the emission of a single soft gluon at angle θ with energy fraction x. We have from Eq. (5) a contribution to C of $\frac{3}{2}x\sin^2\theta$, whereas the contribution to τ is $\frac{1}{2}x\min\{1-\cos\theta, 1+\cos\theta\}$. As expected, the ratio is between 3 (at $\theta = \pi/2$) and 6 (in the collinear regions, $\theta = 0, \pi$). Integrating at a fixed small value of the gluon transverse momentum k_t, we obtain

$$\left\{ \begin{array}{c} \delta C \\ \delta\tau \end{array} \right\} = 2C_F \frac{\alpha_S(k_t)}{\pi} \int_0^1 \frac{dx}{x} \int_0^\pi \frac{d\theta}{\sin\theta} \delta(k_t - \tfrac{1}{2}xQ\sin\theta) \left\{ \begin{array}{c} \frac{3}{2}x\sin^2\theta \\ \frac{1}{2}x\min\{1-\cos\theta, 1+\cos\theta\} \end{array} \right\}\;, \tag{36}$$

giving [17]

$$\frac{\delta C}{\delta\tau} = \frac{3\pi}{2}\;. \tag{37}$$

This is smaller than the factor of 6 in Eq. (29), because the non-perturbative effect comes from the whole soft region of gluon emission, whereas the perturbative logarithmic enhancement comes from the collinear region. As in the case of the thrust distribution, the soft-gluon contribution exponentiates and, taking into account Eq. (37), we have

$$R(C) = R^{\rm pert}(C - 6\pi A_1/Q)\;. \tag{38}$$

In Ref. [22] it was pointed out that higher-loop effects can alter the coefficients of the power corrections to event shapes predicted by the dispersive approach. However, recent studies of two-loop contributions [26] have shown that Eqs. (35) and (38) remain valid, provided that the relationship between the non-perturbative constant A_1 and the effective coupling at low scales is renormalized by a common overall 'Milan factor' $\mathcal{M} \simeq 1.8$.

7 Comparison with data

At present the experimental data on the C-parameter distribution are limited to high energies, $Q = M_Z$ [14,27] and above [28]. Figure 3 shows the LEP1 data [27] together with

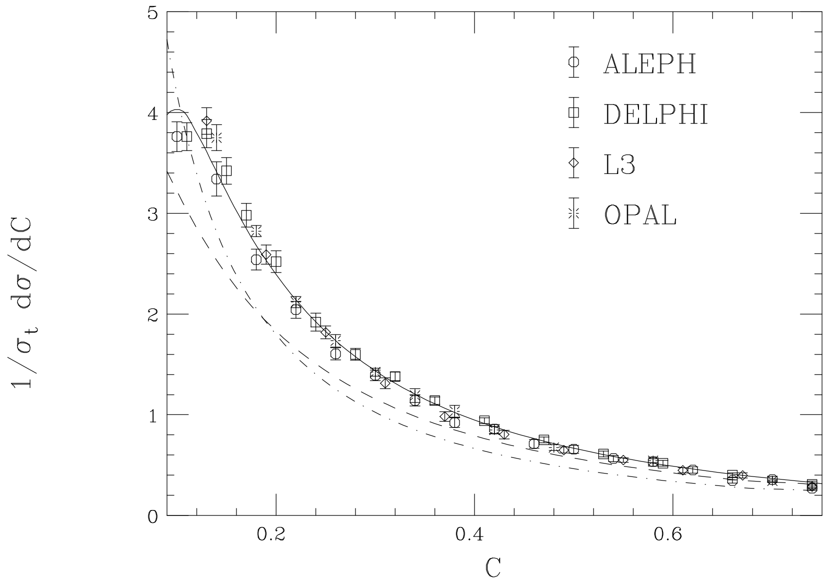

Figure 3: The C-parameter distribution at $Q = M_Z$. Solid curve: Eq. (38) with $A_1 = 0.24$ GeV. Dashed: $A_1 = 0$. Dot-dashed: $\mathcal{O}(\alpha_S^2)$.

various theoretical calculations, all assuming $\alpha_S(M_Z) = 0.118$. The solid curve represents Eq. (38) with the best-fit value of $A_1 = 0.24$ GeV, which gives very good agreement throughout the region $0.09 < C < 0.75$. For the function $R^{\rm pert}$ the resummed expression with log-R matching, Eq. (34), was used. The dashed curve shows the effect of neglecting the non-perturbative shift A_1, while the dot-dashed curve shows the fixed-order prediction (17), with neither resummation nor non-perturbative correction.

The corresponding value of A_1 obtained from fitting the thrust distribution [24] is $A_1 = 0.22$ GeV. Thus the results obtained from the C-parameter and thrust data are consistent within 10%. Taking into account the Milan enhancement factor $\mathcal{M}$ in the coefficient of the power correction [26], these results correspond to a value $\bar{\alpha}_0 \simeq 0.35$ for the non-perturbative parameter $\bar{\alpha}_0$ (the mean value of the effective strong coupling at scales below 2 GeV) introduced in Refs. [20,24].

The fitted values of α_S and A_1 are not well constrained separately by the existing C-parameter data, owing to the lack of measurements of this quantity at lower energies. A comprehensive analysis over a wide range of energies, similar to those presented in Refs. [29-31] for other shape variables, would therefore be very useful.

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