Cavendish-HEP-96/1
hep-ph/9604388

Power Corrections and Renormalons in Deep Inelastic Structure Functions*

M. Dasgupta and B.R. Webber
Cavendish Laboratory, University of Cambridge,
Madingley Road, Cambridge CB3 0HE, U.K.

Abstract

We study the power corrections (infrared renormalon contributions) to the coefficient functions for non-singlet deep inelastic structure functions due to gluon vacuum polarization insertions in one-loop graphs. Remarkably, for all the structure functions F_1, F_2, F_3 and g_1, there are only two such contributions, corresponding to $1/Q^2$ and $1/Q^4$ power corrections. We compute their dependence on Bjorken x. The results could be used to model the dominant higher-twist contributions.

Cavendish–HEP–96/1
April 1996

*Research supported in part by the U.K. Particle Physics and Astronomy Research Council and by the EC Programme "Human Capital and Mobility", Network "Physics at High Energy Colliders", contract CHRX-CT93-0357 (DG 12 COMA).

1 Introduction

A number of recent papers have discussed power corrections to non-singlet deep inelastic structure functions and sum rules, using either the language of infrared renormalons [1–4] or a dispersive approach based on assumed analyticity properties of the running coupling [5]. The contributions computed correspond to the sum of vacuum polarization insertions ('renormalon chains' [6]) on the gluon line in one-loop corrections to the coefficient function. Phenomenologically, it appears [4,5] that the corrections computed in this way provide a good guide to the form of the higher-twist contributions observed experimentally. This may be understandable in terms of the notion of a universal infrared-finite effective coupling [5].

In the present paper we point out that, within the framework of the approaches mentioned, there are only two power-behaved contributions, corresponding to $1/Q^2$ and $1/Q^4$ corrections. This appears to be a special feature of deep inelastic structure functions. It is not the case for e^+e^- fragmentation functions, for example, where the same graphs generate an infinite set of $1/Q^{2p}$ corrections. Of course, higher power corrections might also be generated by inserting renormalon chains in multi-loop graphs. Nevertheless, to the extent that the calculations based on one-loop graphs are phenomenologically successful, one may regard higher-power contributions as correction terms in the deep inelastic case.

We present our calculation using the dispersive approach introduced in Ref. [5]. This makes it simple to compute the power-behaved contributions and their dependence on Bjorken x. As mentioned above, the results could be useful as a model for the dominant higher-twist contributions. They are expressed in terms of two non-perturbative parameters, which could be determined experimentally by fitting deep inelastic data at moderate Q^2.

2 Dispersive method

The method of Ref. [5] starts from a (formal) dispersion relation for the QCD running coupling $\alpha_s(k^2)$ of the form

$$\alpha_s(k^2) = -\int_0^\infty \frac{d\mu^2}{\mu^2+k^2}\,\rho_s(\mu^2)\,, \qquad \rho_s(\mu^2) = -\frac{1}{2\pi i}\,\mathrm{Disc}\left\{\alpha_s(-\mu^2)\right\}\,. \tag{2.1}$$

Introducing the effective coupling $\alpha_{\rm eff}(\mu^2)$, defined in terms of the 'spectral function' $\rho_s(\mu^2)$ by

$$\rho_s(\mu^2) = \frac{d}{d\ln\mu^2}\alpha_{\rm eff}(\mu^2)\,, \tag{2.2}$$

it follows that

$$\alpha_s(k^2) = k^2\int_0^\infty \frac{d\mu^2}{(\mu^2+k^2)^2}\alpha_{\rm eff}(\mu^2)\,. \tag{2.3}$$

In the perturbative domain $\alpha_s \ll 1$ the standard and effective couplings are approximately the same:

$$\alpha_{\rm eff}(\mu^2) = \alpha_s(\mu^2) - \frac{\pi^2}{6}\frac{d^2\alpha_s}{d\ln^2\mu^2} + \ldots \;=\; \alpha_s \;+\; \mathcal{O}(\alpha_s^3)\,. \tag{2.4}$$

Thus one may regard $\alpha_{\rm eff}$ defined by (2.2) as an effective measure of QCD interaction strength, extending the physical perturbative coupling down to the non-perturbative domain.

Next we write the strong coupling in the form

$$\alpha_s(k^2) \;=\; \alpha_s^{\rm PT}(k^2) + \delta\alpha_s(k^2)\,, \tag{2.5}$$

where $\alpha_s^{\rm PT}$ is the perturbative coupling and $\delta\alpha_s$ is a modification to the effective interaction strength at small momentum scales, responsible for non-perturbative effects. The corresponding "effective coupling modification" $\delta\alpha_{\rm eff}$, which generates the non-perturbative interaction strength $\delta\alpha_s$, must satisfy a dispersion relation of the form (2.3):

$$\delta\alpha_s(k^2) = k^2 \int_0^\infty \frac{d\mu^2}{(\mu^2+k^2)^2}\,\delta\alpha_{\rm eff}(\mu^2)\,. \tag{2.6}$$

It follows that an arbitrary finite modification of the effective coupling at low scales would generally introduce power corrections of the form $1/k^{2p}$ into the ultraviolet behaviour of the running coupling α_s itself. As discussed in Ref. [5], such a modification would destroy the basis of the operator product expansion [7]. One must therefore require that at least the first few integer moments of the coupling modification should vanish:

$$\int_0^\infty \frac{d\mu^2}{\mu^2}\,\Big(\mu^2\Big)^p\,\delta\alpha_{\rm eff}(\mu^2) \;=\; 0\,; \quad p = 1,\ldots,p_{\rm max}\,. \tag{2.7}$$

The upper bound $p_{\rm max}$ could be set by instanton–anti-instanton contributions at $p_{\rm max} < \beta_0 \sim 9$.

The effect on some observable $\hat{F}$ of gluon vacuum polarization insertions in one-loop graphs is represented in terms of the spectral function $\rho_s(\mu^2)$ by a *characteristic function* $\mathcal{F}(\mu^2)$, as follows:

$$\hat{F} \;=\; \alpha_s(0)\mathcal{F}(0) + \int_0^\infty \frac{d\mu^2}{\mu^2}\rho_s(\mu^2)\cdot\mathcal{F}(\mu^2) \;=\; \int_0^\infty \frac{d\mu^2}{\mu^2}\rho_s(\mu^2)\cdot\Big[\,\mathcal{F}(\mu^2)-\mathcal{F}(0)\,\Big]\,, \tag{2.8}$$

where we have made use of the formal relation (2.1) to eliminate $\alpha_s(0)$. The characteristic function is obtained by computing the relevant graphs with a non-zero gluon mass μ [8,9]. Note that we do not intend to imply that the gluon has a real effective mass, but only that the dispersive representation (2.1) can be expressed in this way.

Introducing the effective coupling $\alpha_{\rm eff}(\mu^2)$ using Eq. (2.2) and integrating by parts, we can write

$$\hat{F}(x,Q^2) \;=\; \int_0^\infty \frac{d\mu^2}{\mu^2}\,\alpha_{\rm eff}(\mu^2)\cdot\dot{\mathcal{F}}(x,Q^2;\mu^2)\,, \qquad \dot{\mathcal{F}} \equiv -\frac{\partial\mathcal{F}}{\partial\ln\mu^2}\,. \tag{2.9}$$

Here we have taken $\hat{F}$ to represent a quark structure function, with dependence on the deep inelastic scattering variables x and Q^2. To obtain the corresponding (non-singlet) hadron structure function $F(x,Q^2)$, we have to convolute the quark structure function with the appropriate combination of quark distribution functions $q(x)$. As discussed in Ref. [5], one finds that the characteristic function $\mathcal{F}$ has a collinear divergent part, which generates the scale dependence of the quark distributions and the usual logarithmic scaling violations. The remaining part $\mathcal{F}^{\rm reg}$ generates the coefficient function C in the relation between the structure function and the quark distribution:

$$F(x,Q^2) = \int_x^1 \frac{dz}{z} C(z,Q^2)\,q(x/z,Q^2) \tag{2.10}$$

where

$$C(x,Q^2) \;=\; \int_0^\infty \frac{d\mu^2}{\mu^2}\,\alpha_{\rm eff}(\mu^2)\cdot\dot{\mathcal{F}}^{\rm reg}(x,Q^2;\mu^2)\,. \tag{2.11}$$

Since $\mathcal{F}$ depends only on dimensionless ratios, we may write

$$\mathcal{F}(x,Q^2;\mu^2) = \mathcal{F}(x,\epsilon)\;, \qquad \dot{\mathcal{F}} \equiv -\epsilon\frac{\partial}{\partial\epsilon}\mathcal{F}(x,\epsilon)\;, \qquad \epsilon \equiv \frac{\mu^2}{Q^2}\;. \tag{2.12}$$

The power-behaved contributions depend on the small-ϵ behaviour of $\mathcal{F}$, which is of the generic form

$$\mathcal{F}(x,\epsilon) = -P(x)\ln\epsilon + C_0(x) - C_2(x)\epsilon\ln\epsilon - \tfrac{1}{2}C_4(x)\epsilon^2\ln\epsilon + \cdots\;. \tag{2.13}$$

The dots indicate terms that are analytic and vanishing at $\epsilon = 0$. Thus according to the above definition the regular part is

$$\mathcal{F}^{\rm reg}(x,\epsilon) = C_0(x) - C_2(x)\epsilon\ln\epsilon - \tfrac{1}{2}C_4(x)\epsilon^2\ln\epsilon + \cdots\;, \tag{2.14}$$

and

$$\dot{\mathcal{F}}^{\rm reg}(x,\epsilon) = C_2(x)\epsilon\ln\epsilon + C_4(x)\epsilon^2\ln\epsilon + \cdots\;. \tag{2.15}$$

The constraint (2.7) means that only those terms in the small-ϵ behaviour of the characteristic function that are *non-analytic* at $\epsilon = 0$ will lead to power-behaved non-perturbative contributions [9]. According to Eq. (2.11), the corresponding contributions to the coefficient function will be of the form

$$C^{\rm NP}(x,Q^2) = \int_0^\infty \frac{d\mu^2}{\mu^2}\,\delta\alpha_{\rm eff}(\mu^2)\dot{\mathcal{F}}^{\rm reg}(x,\epsilon=\mu^2/Q^2)\,. \tag{2.16}$$

Thus from the small-ϵ behaviour (2.15) we find

$$C^{\rm NP}(x,Q^2) = C_2(x)\frac{A_2'}{Q^2} + C_4(x)\frac{A_4'}{Q^4}\;, \tag{2.17}$$

where, following Ref. [5], we have defined the log-moment integrals

$$A_{2p}' \;=\; \frac{C_F}{2\pi}\int_0^\infty \frac{d\mu^2}{\mu^2}\,\mu^{2p}\,\ln(\mu^2/\mu_0^2)\;\delta\alpha_{\rm eff}(\mu^2)\,. \tag{2.18}$$

Notice that since integer μ^2-moments of $\delta\alpha_{\rm eff}$ vanish, these quantities are independent of the scale μ_0^2. For convenience, we extract a universal factor of $C_F/2\pi$ from the characteristic function.

The interpretation of Eq. (2.17) in terms of a universal low-energy effective coupling is optional. More generally, we could interpret this expression as the ambiguity in the perturbative evaluation of the coefficient function, arising from the factorial divergence of the perturbation series generated by gluon vacuum polarization insertions. In this language, the two terms correspond to infrared renormalons, and the constants A_2' and A_4' are proportional to powers of the QCD scale Λ, the constants of proportionality depending on how we choose to resolve the renormalon ambiguity.

3 Calculations

In this section we apply the dispersive method to compute the power correction terms (2.17) for non-singlet structure functions. Recall that the object of central importance is the characteristic function $\mathcal{F}(\epsilon)$ for the emission of a gluon with mass-squared $\mu^2 = \epsilon Q^2$ at the hard scale Q^2. For power corrections, the relevant contributions are given by the non-analytic terms in the small-ϵ behaviour of the logarithmic derivative $\dot{\mathcal{F}}(\epsilon)$.

The characteristic function for the structure function F_2 (actually F_2/x) was given in Ref. [5]:

$$\mathcal{F}_2(x,\epsilon) = \mathcal{F}_2^{(r)}(x,\epsilon)\,\Theta(1-x-x\epsilon) + \mathcal{F}^{(v)}(\epsilon)\,\delta(1-x)\;. \tag{3.1}$$

The contribution from real gluon emission is

$$\begin{aligned}\mathcal{F}_2^{(r)}(x,\epsilon) = {} & \left[\frac{2(1-\epsilon)^2}{1-x} - (1+x) + 2(2+x+6x^2)\epsilon - 2(1+x+9x^3)\epsilon^2\right]\ln\left[\frac{(1-x\epsilon)(1-x)}{x^2\epsilon}\right] \\ & - \frac{3+14\epsilon-15\epsilon^2}{2(1-x)} + \frac{\epsilon}{(1-x)^2} + \frac{\epsilon^2}{2(1-x)^3} + \frac{x}{1-x\epsilon} \\ & + 1 + 3x + 6(1-x)(1+3x)\epsilon - (8+9x+18x^2)\epsilon^2\;.\end{aligned} \tag{3.2}$$

The virtual contribution is

$$\mathcal{F}^{(v)}(\epsilon) = 2(1-\epsilon)^2\left[\mathrm{Li}_2(\epsilon) + \ln\epsilon\ln(1-\epsilon) - \frac{1}{2}\ln^2\epsilon - \frac{\pi^2}{3}\right] - \frac{7}{2} - (3-2\epsilon)\ln\epsilon + 2\epsilon\,, \tag{3.3}$$

where

$$\mathrm{Li}_2(\epsilon) = -\int_0^\epsilon \frac{dt}{t}\ln(1-t)\,. \tag{3.4}$$

We note the relation

$$\mathcal{F}^{(v)}(\epsilon) = -\int_0^1 \mathcal{F}_2^{(r)}(x;\epsilon)\Theta(1-x-x\epsilon)\,dx\;, \tag{3.5}$$

which means that the Adler sum rule is satisfied identically, that is, it receives neither perturbative nor power corrections (see Ref. [10]).

Taking the small-ϵ limit, we obtain an expression of the form (2.13). The coefficient of $-\ln\epsilon$ in $\mathcal{F}_2^{(r)}$ is the quark splitting function $P(x) = (1+x^2)/(1-x)$, which is singular for $x\to 1$. The singularity is regularized by including the virtual contribution. As discussed above, this term produces the usual logarithmic scaling violation. The second term $C_0(x)$ is the perturbative coefficient function (in the gluon mass regularization scheme). The remaining terms are analytic at $\epsilon = 0$, except for two terms proportional to $\epsilon\ln\epsilon$ and $\epsilon^2\ln\epsilon$. Thus we obtain only two power corrections, of the form (2.17).

When taking the $\epsilon\to 0$ limit of eq. (3.2), we have to be careful with the functions that become singular in this limit. Defining '+', '++' and '+++' prescriptions such that, for any test function f,

$$\begin{aligned}\int_0^1 F(x)_+\,f(x)\,dx &= \int_0^1 F(x)\,[f(x)-f(1)]\,dx \\ \int_0^1 F(x)_{++}\,f(x)\,dx &= \int_0^1 F(x)\,[f(x)-f(1)+(1-x)f'(1)]\,dx \\ \int_0^1 F(x)_{+++}\,f(x)\,dx &= \int_0^1 F(x)\,[f(x)-f(1)+(1-x)f'(1)-\tfrac{1}{2}(1-x)^2f''(1)]\,dx\end{aligned} \tag{3.6}$$

and recalling that

$$\int_0^1 \delta^{(n)}(1-x)\,f(x)\,dx = f^{(n)}(1)\;, \tag{3.7}$$

we should replace the singular terms in Eq. (3.2) at small ϵ, up to terms of order ϵ^2, as follows:

$$\begin{aligned}
\frac{1}{1-x} &\to \frac{1}{(1-x)_+} + (\epsilon - \tfrac{1}{2}\epsilon^2 - \ln\epsilon)\,\delta(1-x) \\
&\quad + \epsilon(1-\epsilon)\,\delta'(1-x) - \tfrac{1}{4}\epsilon^2\,\delta''(1-x) \\
\frac{\ln(1-x)}{1-x} &\to \left(\frac{\ln(1-x)}{1-x}\right)_+ + (\epsilon\ln\epsilon - \tfrac{1}{2}\epsilon^2\ln\epsilon - \tfrac{1}{2}\ln^2\epsilon - \tfrac{1}{2}\epsilon^2)\,\delta(1-x) \\
&\quad + \epsilon(\ln\epsilon - \epsilon\ln\epsilon - 1)\,\delta'(1-x) - \tfrac{1}{4}\epsilon^2(\ln\epsilon - \tfrac{1}{2})\,\delta''(1-x) \\
\frac{\epsilon}{(1-x)^2} &\to \frac{\epsilon}{(1-x)^2_{++}} + \delta(1-x) + \epsilon(\ln\epsilon - \epsilon)\,\delta'(1-x) + \tfrac{1}{2}\epsilon(1-\epsilon)\,\delta''(1-x) \\
\frac{\epsilon^2}{(1-x)^3} &\to \frac{\epsilon^2}{(1-x)^3_{+++}} + (\tfrac{1}{2}+\epsilon)\,\delta(1-x) - \epsilon\,\delta'(1-x) - \tfrac{1}{2}\epsilon^2\ln\epsilon\,\delta''(1-x)\,.
\end{aligned} \tag{3.8}$$

Individual terms of the form $\epsilon^p \ln\epsilon$ with $p > 2$ are generated by the above replacements, but they cancel in the sum (3.1). The only remaining terms in $\mathcal{F}^{\rm reg}$ that are non-analytic as $\epsilon \to 0$ are those of the form $\epsilon\ln\epsilon$ and $\epsilon^2\ln\epsilon$.

The characteristic function for the structure function $2F_1$ is given by

$$\mathcal{F}_1(x,\epsilon) = \mathcal{F}_2(x,\epsilon) - \mathcal{F}_L(x,\epsilon) \tag{3.9}$$

where $\mathcal{F}_L$, the characteristic function for the longitudinal contribution F_L/x, is

$$\begin{aligned}
\mathcal{F}_L(x,\epsilon) &= 4(2-3x\epsilon)x^2\epsilon\ln\left[\frac{(1-x\epsilon)(1-x)}{x^2\epsilon}\right] - \frac{2\epsilon(2-\epsilon)}{1-x} + \frac{2\epsilon^2}{(1-x)^2} \\
&\quad + 2x + 4(1-x)(1+3x)\epsilon - 2(2+3x+6x^2)\epsilon^2\,.
\end{aligned} \tag{3.10}$$

This contribution introduces no new power corrections, as noted in Ref. [4].

The characteristic function for the parity-violating structure function F_3 was given in Ref. [5]:

$$\mathcal{F}_3(x,\epsilon) = \mathcal{F}_2(x,\epsilon) - \mathcal{F}_d(x,\epsilon) \tag{3.11}$$

where

$$\begin{aligned}
\mathcal{F}_d(x,\epsilon) &= 2(4+\epsilon-9x\epsilon)x^2\epsilon\ln\left[\frac{(1-x\epsilon)(1-x)}{x^2\epsilon}\right] - \frac{\epsilon(4-3\epsilon)}{1-x} + \frac{2\epsilon^2}{(1-x)^2} + \frac{2x}{1-x\epsilon} \\
&\quad + 1 - x + 2(2+5x-9x^2)\epsilon - (5+7x+18x^2)\epsilon^2\,.
\end{aligned} \tag{3.12}$$

Again, no power corrections beyond those in Eq. (2.17) are introduced by this contribution.

The power corrections to the coefficient function for the polarized structure function g_1 are the same as those for F_3, since their characteristic functions are identical to one-loop order.

4 Results

The coefficients in Eq. (2.17) for the power corrections to the coefficient function for F_2/x are found from Eqs. (3.1)-(3.8) to be

$$\begin{aligned}
C_2(x) &= -\frac{4}{(1-x)_+} + 2(2+x+6x^2) - 9\,\delta(1-x) - \delta'(1-x) \\
C_4(x) &= \frac{4}{(1-x)_+} - 4(1+x+9x^3) + 15\,\delta(1-x) + \frac{1}{2}\,\delta''(1-x)\,.
\end{aligned} \tag{4.1}$$

The corresponding expressions in moment space, defined by

$$\tilde{C}(N) = \int_0^1 x^{N-1}\, C(x)\, dx \tag{4.2}$$

are

$$\begin{aligned} \tilde{C}_2(N) &= -N - 8 + \frac{4}{N} + \frac{2}{N+1} + \frac{12}{N+2} + 4S_1 \\ \tilde{C}_4(N) &= \frac{1}{2}N^2 - \frac{3}{2}N + 16 - \frac{4}{N} - \frac{4}{N+1} - \frac{36}{N+3} - 4S_1\ , \end{aligned} \tag{4.3}$$

with

$$S_1 = \sum_{j=1}^{N-1} \frac{1}{j} = \psi(N) + \gamma_E = \ln N + \mathcal{O}(1/N)\,. \tag{4.4}$$

For $2F_1 = F_2/x - F_L/x$, the corresponding results are[†]

$$\begin{aligned} C_2(x) &= -\frac{4}{(1-x)_+} + 2(2 + x + 2x^2) - 5\,\delta(1-x) - \delta'(1-x) \\ C_4(x) &= \frac{4}{(1-x)_+} - 4(1 + x + 3x^3) + 11\,\delta(1-x) + 4\,\delta'(1-x) + \frac{1}{2}\,\delta''(1-x)\,, \end{aligned} \tag{4.5}$$

$$\begin{aligned} \tilde{C}_2(N) &= -N - 4 + \frac{4}{N} + \frac{2}{N+1} + \frac{4}{N+2} + 4S_1 \\ \tilde{C}_4(N) &= \frac{1}{2}N^2 + \frac{5}{2}N + 8 - \frac{4}{N} - \frac{4}{N+1} - \frac{12}{N+3} - 4S_1\ . \end{aligned} \tag{4.6}$$

For F_3 (and g_1),

$$\begin{aligned} C_2(x) &= -\frac{4}{(1-x)_+} + 2(2 + x + 2x^2) - 5\,\delta(1-x) - \delta'(1-x) \\ C_4(x) &= \frac{4}{(1-x)_+} - 4(1 + x + x^2) + 9\,\delta(1-x) + 4\,\delta'(1-x) + \frac{1}{2}\,\delta''(1-x)\,, \end{aligned} \tag{4.7}$$

$$\begin{aligned} \tilde{C}_2(N) &= -N - 4 + \frac{4}{N} + \frac{2}{N+1} + \frac{4}{N+2} + 4S_1 \\ \tilde{C}_4(N) &= \frac{1}{2}N^2 + \frac{5}{2}N + 6 - \frac{4}{N} - \frac{4}{N+1} - \frac{4}{N+2} - 4S_1\ . \end{aligned} \tag{4.8}$$

Note that the $1/Q^2$ coefficients $C_2(x)$ are the same for F_3 and $2F_1$, but the $1/Q^4$ coefficients $C_4(x)$ are slightly different.

5 Discussion

To illustrate the above results we examine the $1/Q^2$ and $1/Q^4$ contributions arising from the valence quark distributions in neutrino scattering. Defining $F = \frac{1}{2}(F^\nu + F^{\bar{\nu}})_V$ and $q = u_V + d_V$, where V indicates the valence contribution, we can write

$$\begin{aligned} F(x,Q^2) &\simeq q(x,Q^2)\left(1 + \frac{D_2(x,Q^2)}{Q^2} + \frac{D_4(x,Q^2)}{Q^4}\right) \\ D_{2p}(x,Q^2) &= \frac{A'_{2p}}{q(x,Q^2)} \int_x^1 \frac{dz}{z} C_{2p}(z)\, q(x/z,Q^2)\ , \end{aligned} \tag{5.1}$$

[†]Our results for F_L agree with those of Ref. [4].

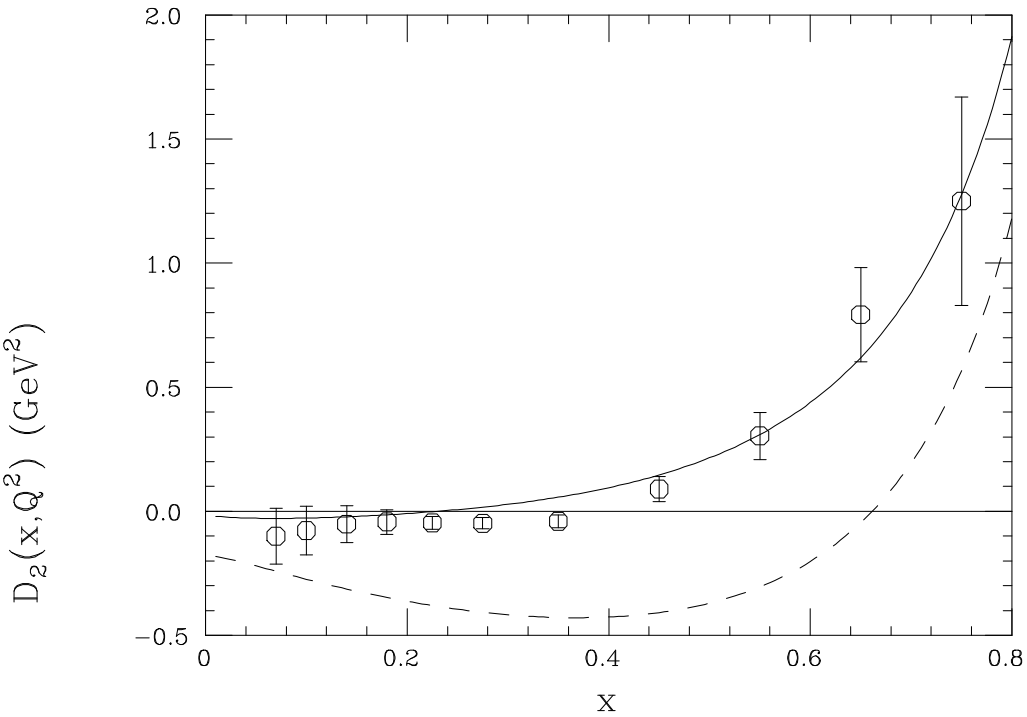

Figure 1: Coefficients of $1/Q^2$ contributions to F_2 (solid), and to F_1 and F_3 (dashed).

the coefficient functions C_{2p} being as given above for $F = F_2/x$, $2F_1$ and F_3. The valence quark distributions were taken from the MRSA parametrization [11]. Calculations were performed at $Q^2 = 10$ GeV^2, but the Q^2-dependence of the coefficients $D_{2p}(x, Q^2)$ is negligible.

Figure 1 shows the $1/Q^2$ coefficients $D_2(x, Q^2)$, assuming the value $A_2' = -0.2$ GeV^2 for the non-perturbative parameter defined by Eq. (2.18) with $p = 1$. We see that this gives remarkably good agreement with the data points for F_2, taken from Ref. [12].[‡] The generally negative value of the prediction for F_3 leads to a negative correction to the Gross–Llewellyn-Smith sum rule, in qualitative agreement with the predictions of Ref. [13], used in the test of the sum rule by the CCFR-NuTeV Collaboration [14].

Figure 2 shows corresponding predictions for the $1/Q^4$ coefficients $D_4(x, Q^2)$. Here the arbitrary choice $A_4' = (A_2')^2 = 0.04$ GeV^4 is made for illustration only. We see that the $1/Q^4$ contributions are peaked more sharply at high x, owing to the $\delta''(1-x)$ term in $C_4(x)$. This indicates that the power corrections are functions of $(1-x)Q^2$ rather than Q^2 at high x.

It would clearly be of interest to attempt a global fit to deep inelastic data at moderate Q^2, treating the quantities A_2' and A_4' as free parameters. For this purpose, a program to compute the predicted power corrections is available from the authors.

References

1. D.J. Broadhurst and A.L. Kataev, *Phys.Lett.* 315B (1993) 179.

2. X.-D. Ji, *Nucl.Phys.* B448 (1995) 51.

3. C.N. Lovett-Turner and C.J. Maxwell, *Nucl.Phys.* B452 (1995) 188.

[‡]The data are for charged leptons but the predicted coefficients $D_{2p}(x, Q^2)$ for neutral and charged lepton scattering are essentially identical.

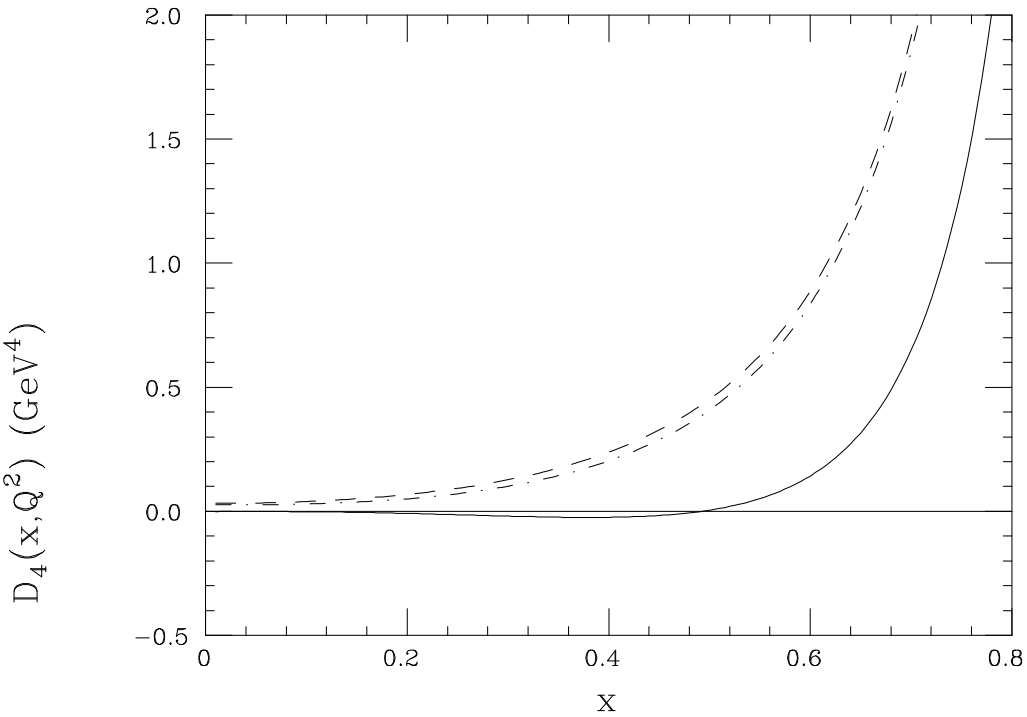

Figure 2: Coefficients of $1/Q^4$ contributions to F_2 (solid), F_1 (dashed) and F_3 (dot-dashed).

4. E. Stein, M. Meyer-Hermann, L. Mankiewicz and A. Schäfer, Frankfurt preprint UFTP 407/1996 [hep-ph/9601356].

5. Yu.L. Dokshitzer, G. Marchesini and B.R. Webber, CERN preprint CERN-TH/95-281 [hep-ph/9512336], to be published in Nucl. Phys. B.

6. For reviews and classic references see V.I. Zakharov, *Nucl.Phys.* B385 (1992) 452 and A.H. Mueller, in *QCD 20 Years Later*, vol. 1 (World Scientific, Singapore, 1993).

7. M.A. Shifman, A.I. Vainstein and V.I. Zakharov, *Nucl.Phys.* B147 (1979) 385,448,519; *Vacuum Structure and QCD Sum Rules: Reprints*, ed. M.A. Shifman (North-Holland, 1992: Current Physics, Sources and Comments, v. 10).

8. B.R. Webber, *Phys.Lett.* 339B (1994) 148.

9. M. Beneke, V.M. Braun and V.I. Zakharov, *Phys.Rev.Lett.* 73 (1994) 3058;
P. Ball, M. Beneke and V.M. Braun, *Nucl.Phys.* B452 (1995) 563;
M. Beneke and V.M. Braun, *Nucl.Phys.* B454 (1995) 253.

10. G. Altarelli, R.K. Ellis and G. Martinelli, *Nucl.Phys.* B143 (1978) 521;
J. Kubar-André and F.E. Paige, *Phys.Rev.* D19 (1979) 221.

11. A.D. Martin, R.G. Roberts and W.J. Stirling, *Phys.Rev.* D50 (1994) 6734.

12. M. Virchaux and A. Milsztajn, *Phys.Lett.* 274B (1992) 221.

13. V.M. Braun and A.V. Kolesnichenko, *Nucl.Phys.* B283 (1987) 723;

14. D.A. Harris, CCFR-NuTeV Collaboration, FERMILAB-CONF-95-144, to appear in *Proc. 30th Rencontres de Moriond, Meribel les Allues, France, March 1995.*