

# Reconstructing particle masses from pairs of decay chains

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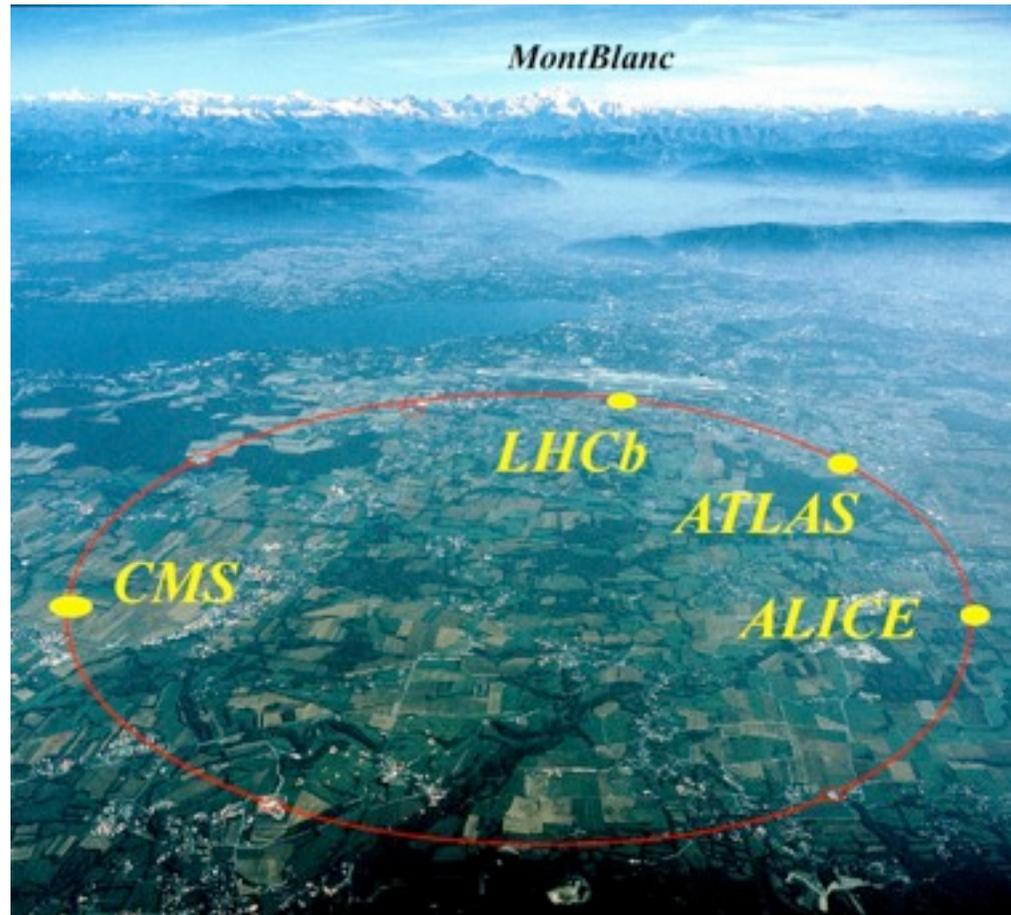
In collaboration with: Mihoko Nojiri and Bryan Webber

Based on: JHEP 1006:069 (2010)

## Outline

- Introduction
- A short revision of measurements of masses of new particles at the LHC
- A new method
- Summary

# LHC has started taking data



Search for new physics has already started.

Alves, Izaguirre, Wacker (arXiv:1008.0407)

ATLAS-CONF 2010-065

ATLAS-CONF 2010-066

# New physics at the LHC

- Dark Matter

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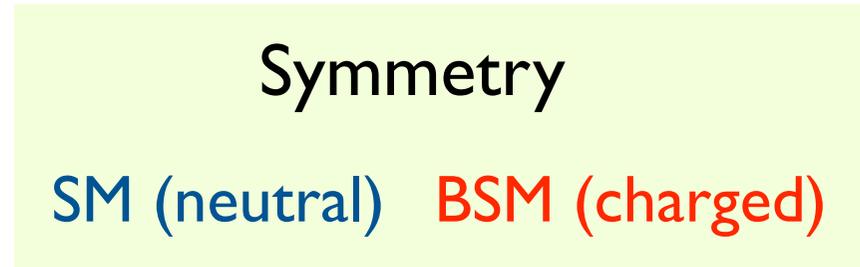


Symmetry

SM (neutral) BSM (charged)

# New physics at the LHC

- Dark Matter 
- Pair Production 

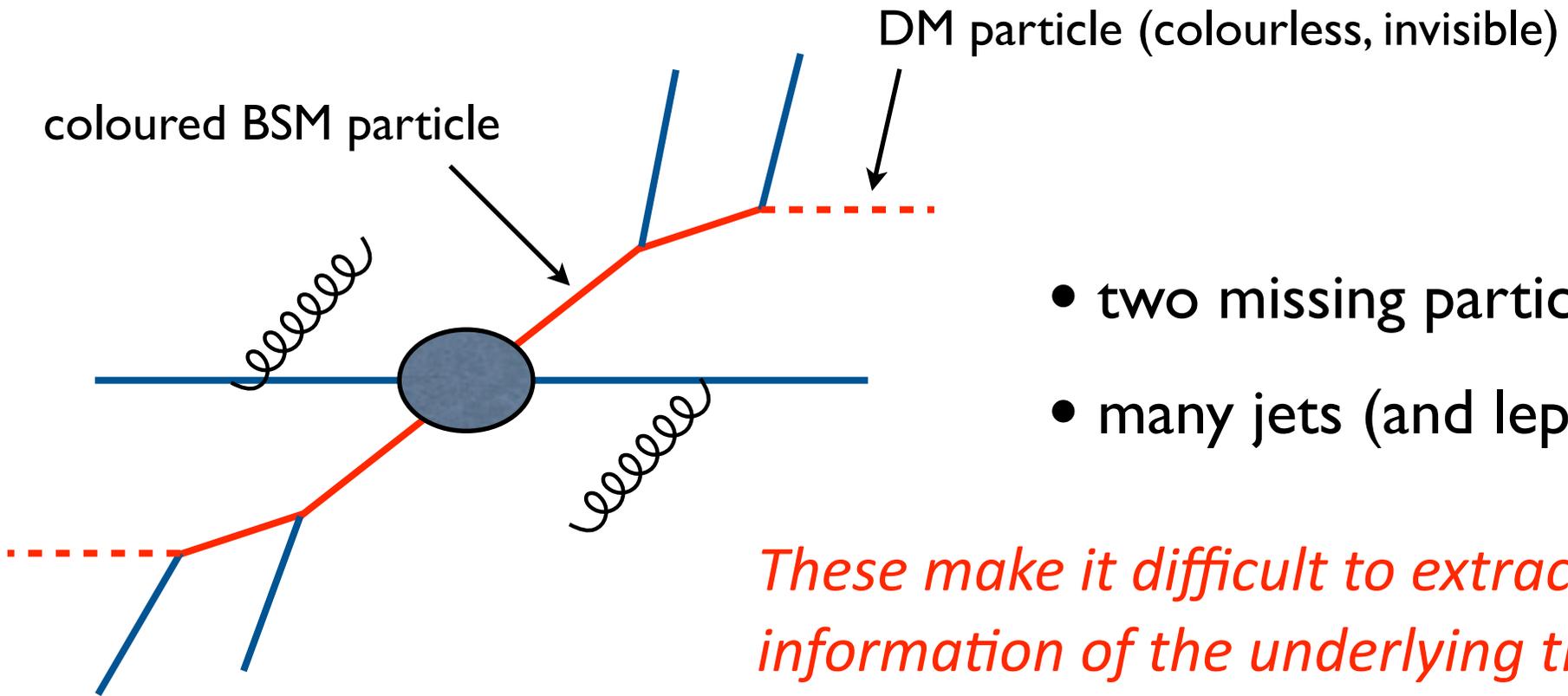


# New physics at the LHC

- Dark Matter
- Pair Production

Symmetry

SM (neutral)    BSM (charged)



- two missing particles
- many jets (and leptons)

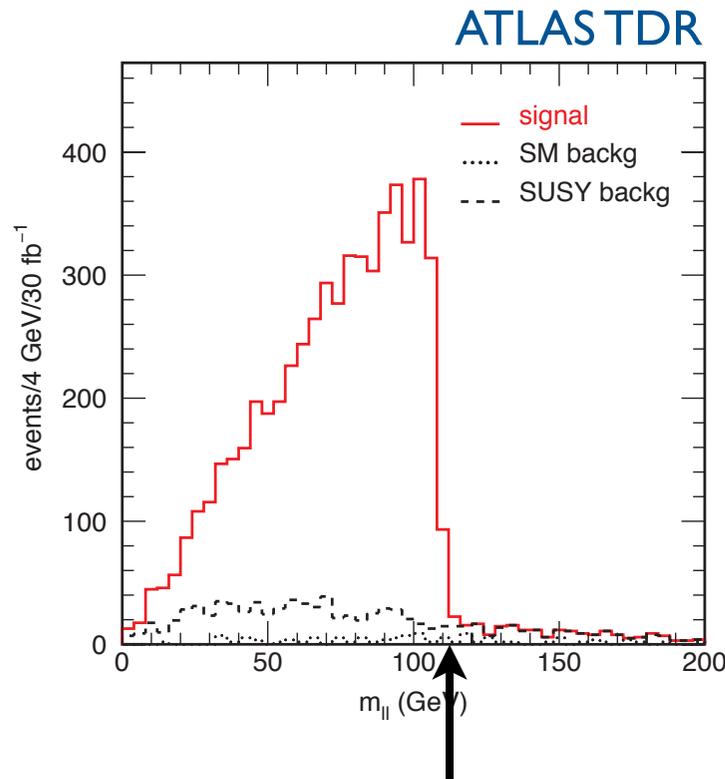
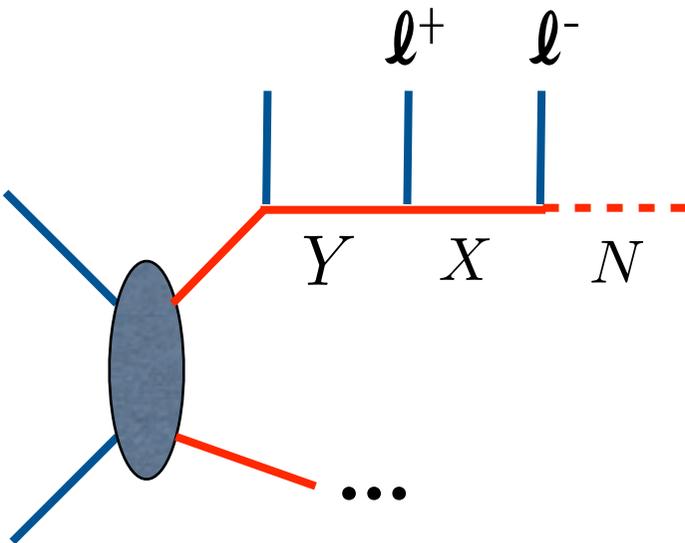
*These make it difficult to extract the information of the underlying theory.*

# Measuring Mass (1)

- Invariant Mass

$$M_{ll}^2 \equiv (p_{l^+}^\mu + p_{l^-}^\mu)^2$$

Hinchliffe, Paige, Shapiro, Soderqvist, Yao (hep-ph/9610544),  
 Bachacou, Hinchliffe, Paige (hep-ph/9907518),  
 Allanach, Lester, Parker, Webber (hep-ph/9907519),  
 ...



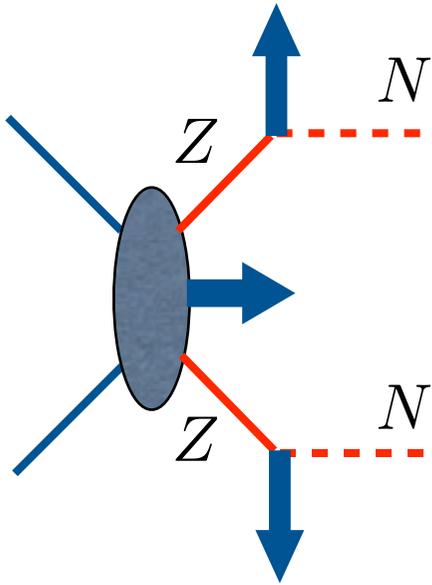
very accurate

error  $\lesssim 0.5\%$  ( $10\text{fb}^{-1}$ )

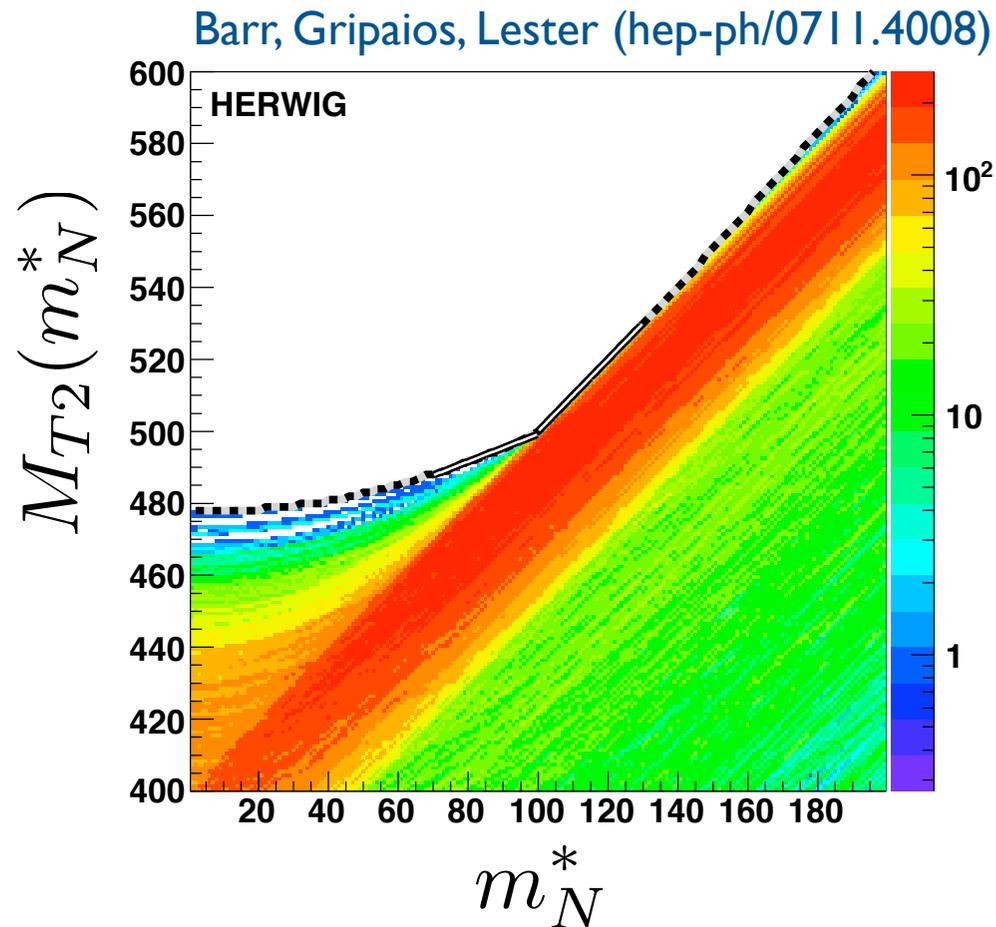
$$(M_{ll}^2)^{\text{max}} = \frac{1}{m_X^2} (m_Y^2 - m_X^2)(m_X^2 - m_N^2)$$

# Measuring Mass (2)

- Kinematical Variables ( $M_{T2}$ ,  $M_{CT}$ ,  $M_{2C}$ ,  $M_{CT2}$ , ...)



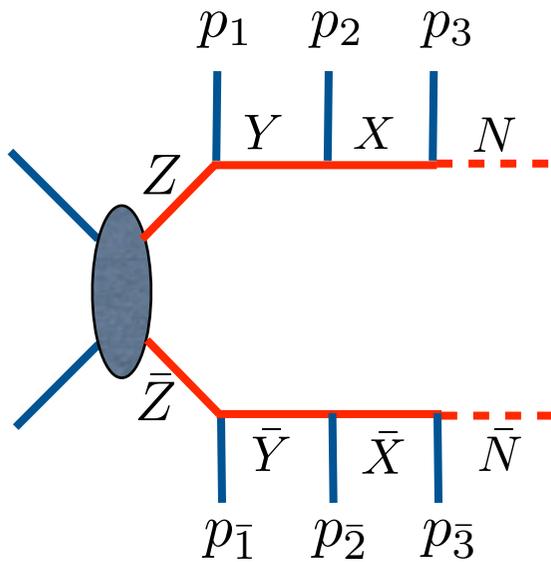
Lester, Summers (hep-ph/9906349),  
Tovey (arXiv:0802.2879),  
Barr, Ross, Serna (arXiv:0806.3224)  
W.S.Cho, J.E.Kim, J.H.Kim (arXiv:0912.2354)  
...



# Measuring Mass (3)

- Event reconstruction

Kawagoe, Nojiri, Polesello (hep-ph/0410160),  
Cheng, Engelhardt, Gunion, Han, McElrath (arXiv:0802.4290),  
Webber (arXiv:0907.5307),  
...

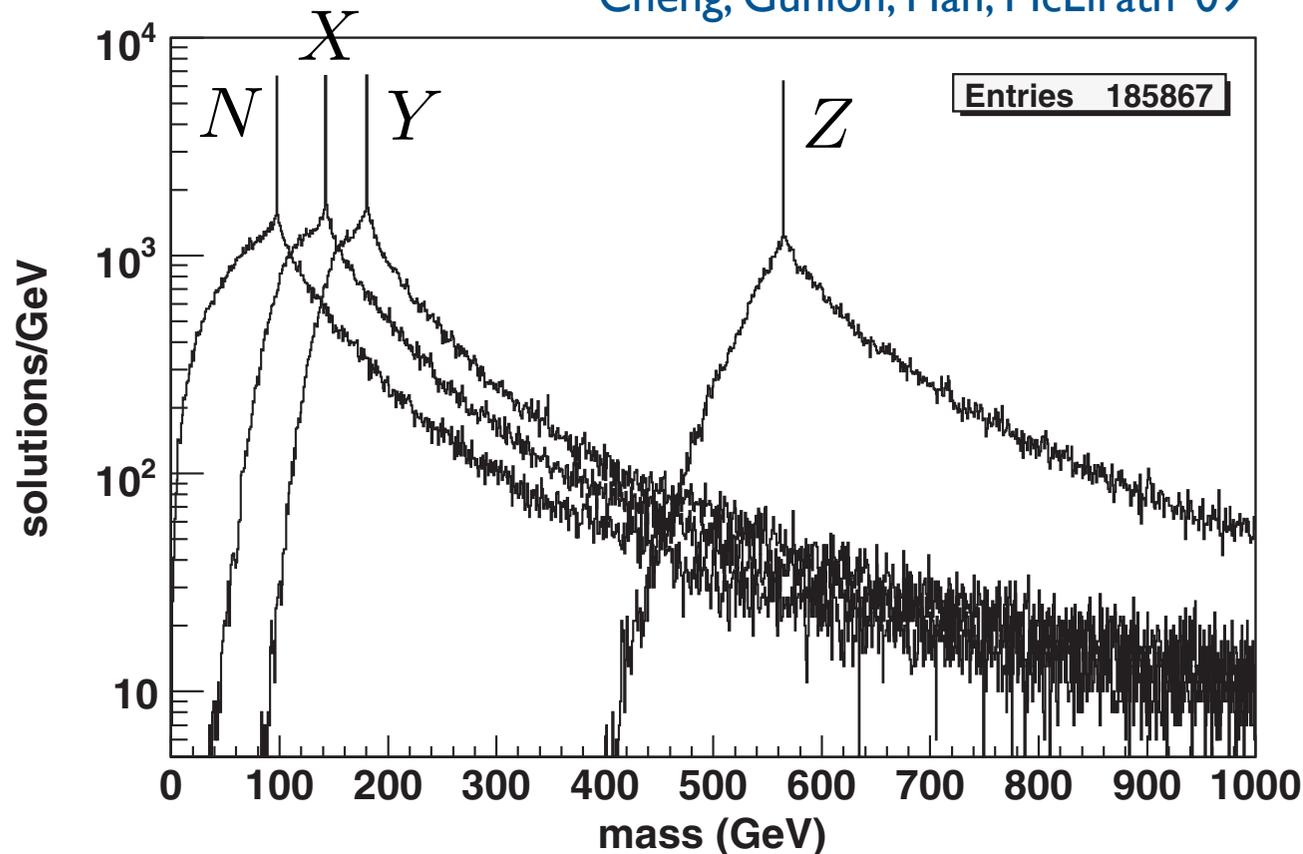


Two DMs' momenta  $P_N, P_{\bar{N}}$  can be determined by mass-shell conditions and missing momentum.

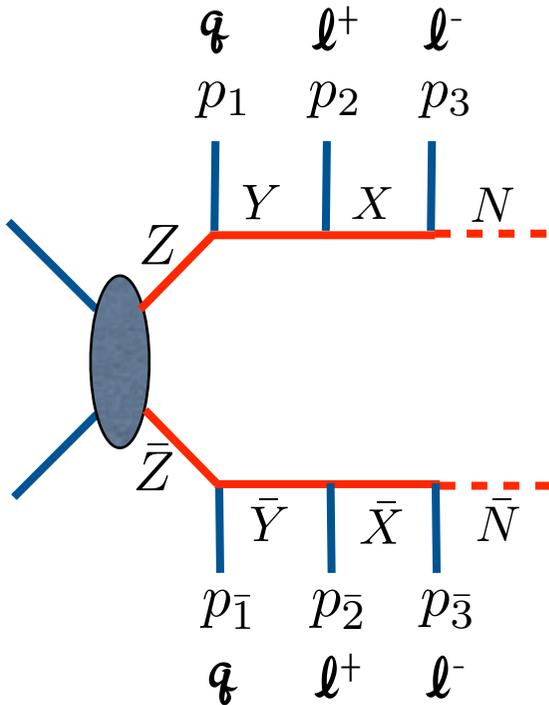
*All events contribute to determining masses.*

*All masses can be determined at the same time.*

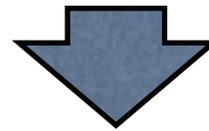
Cheng, Gunion, Han, McElrath '09



# Event reconstruction



$$\begin{aligned}
 & [\# \text{ of Constraints}] - [\# \text{ of Unknowns}] \\
 &= [(8 + 2)N] - [(8N + 4)]
 \end{aligned}$$

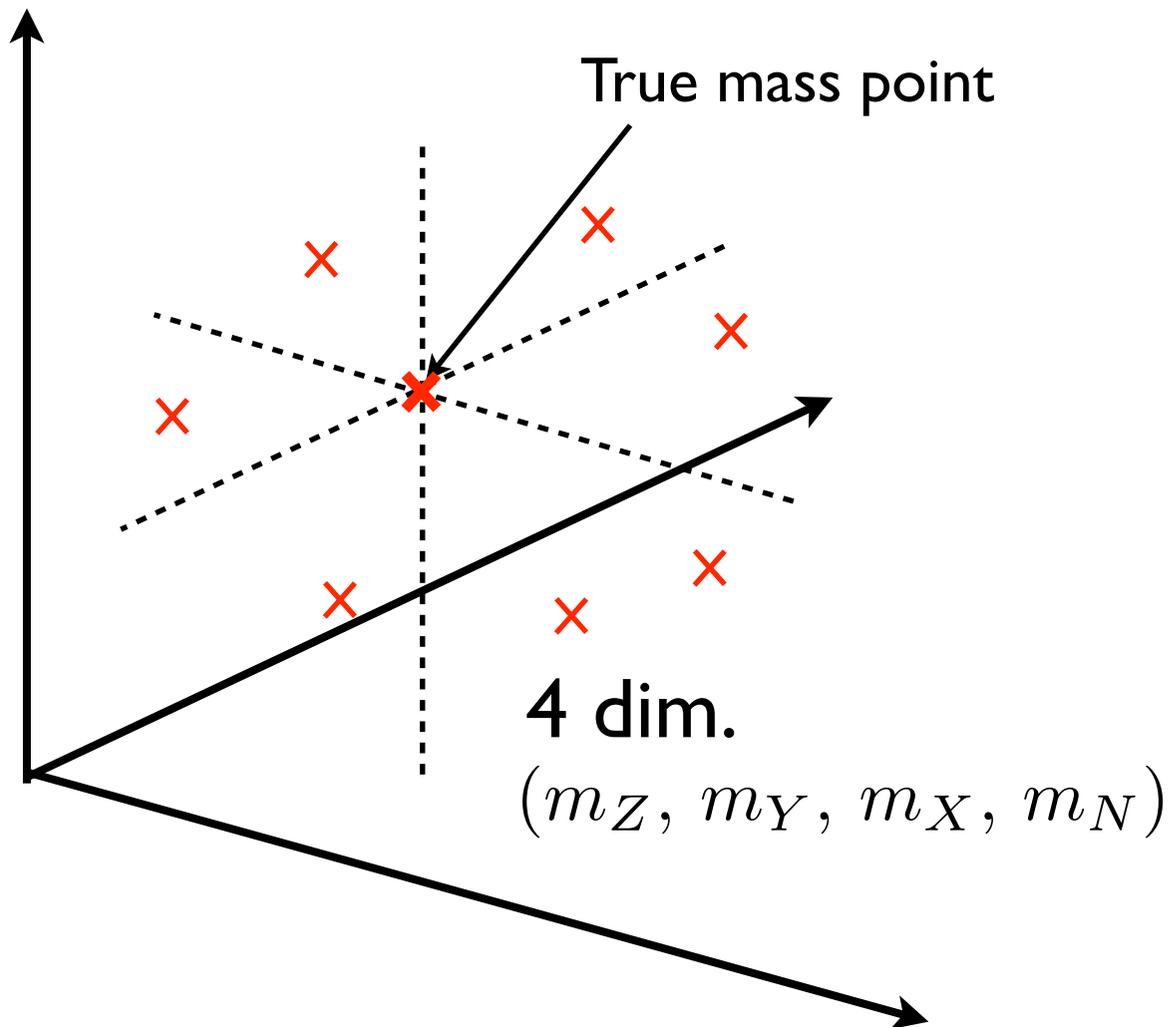


N events	1	2	3	...
diff	-2	0	2	...

**Full reconstruction needs at least two events.**

# Event reconstruction

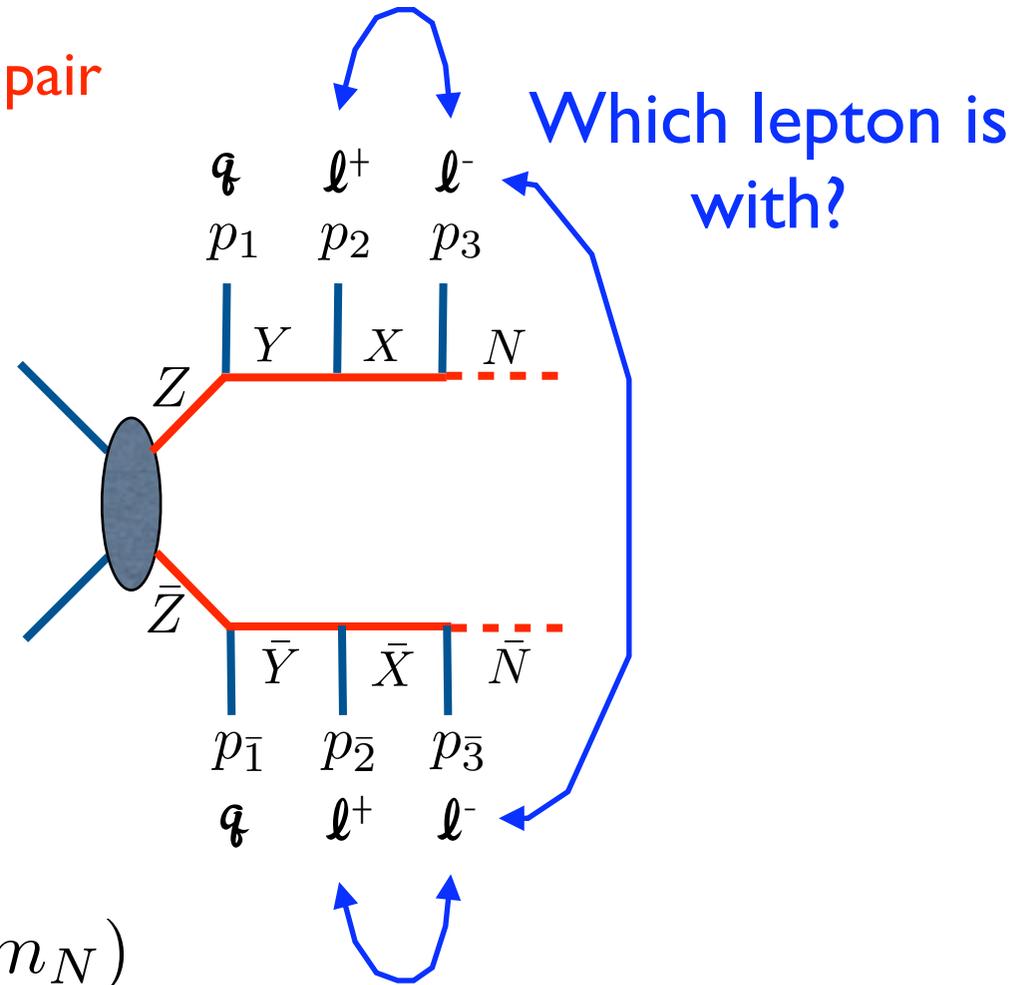
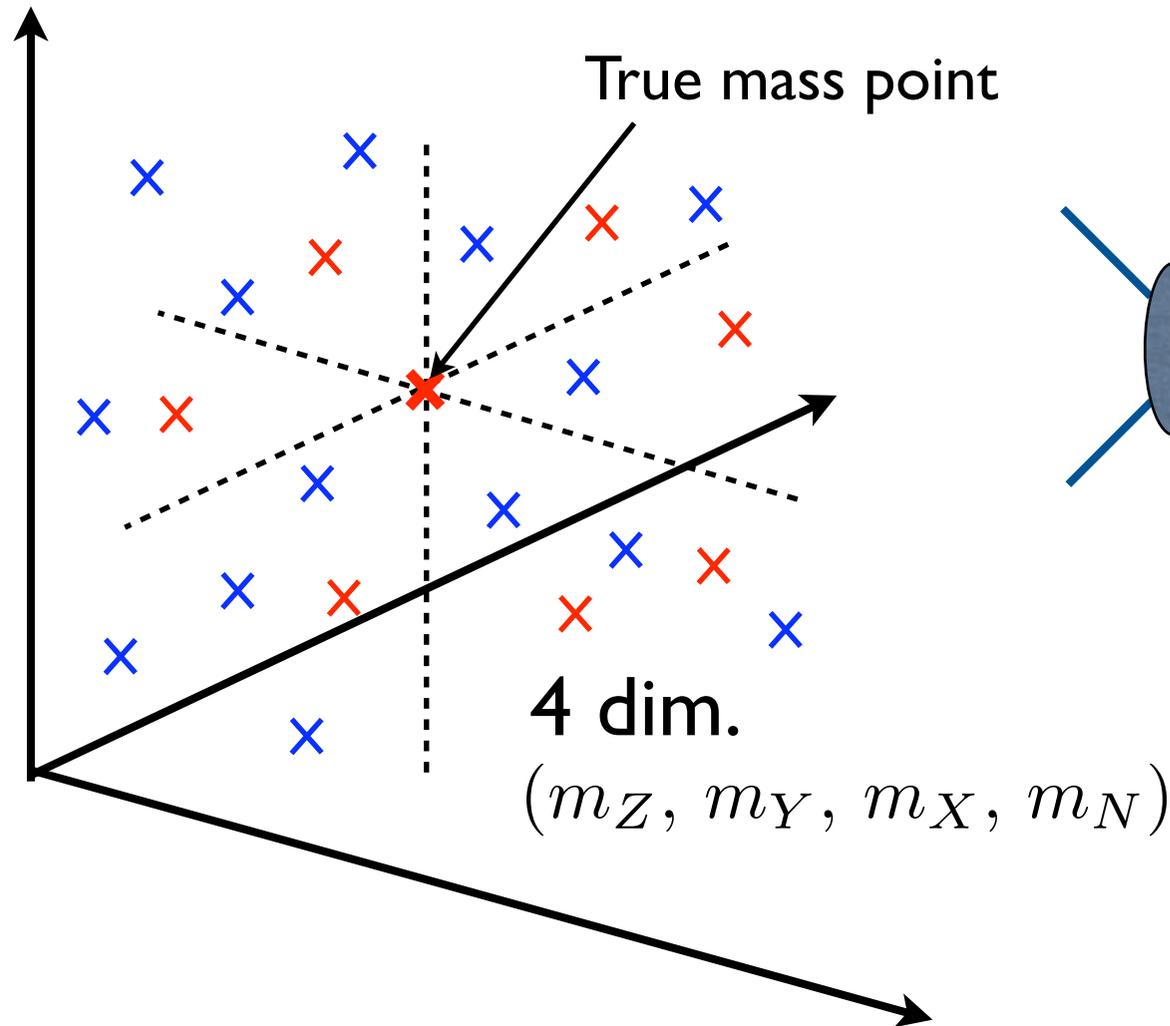
x solutions (up to eight) from an event pair



# Event reconstruction

× solutions with a wrong assignment

× solutions (up to eight) from an event pair

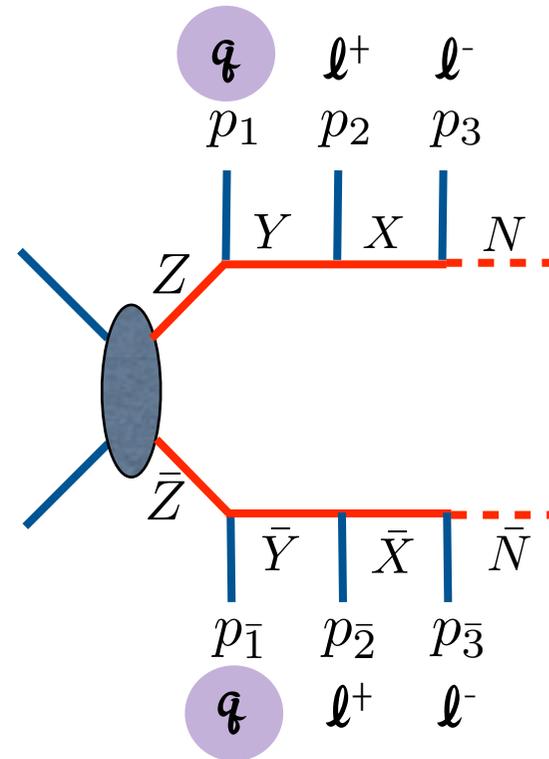
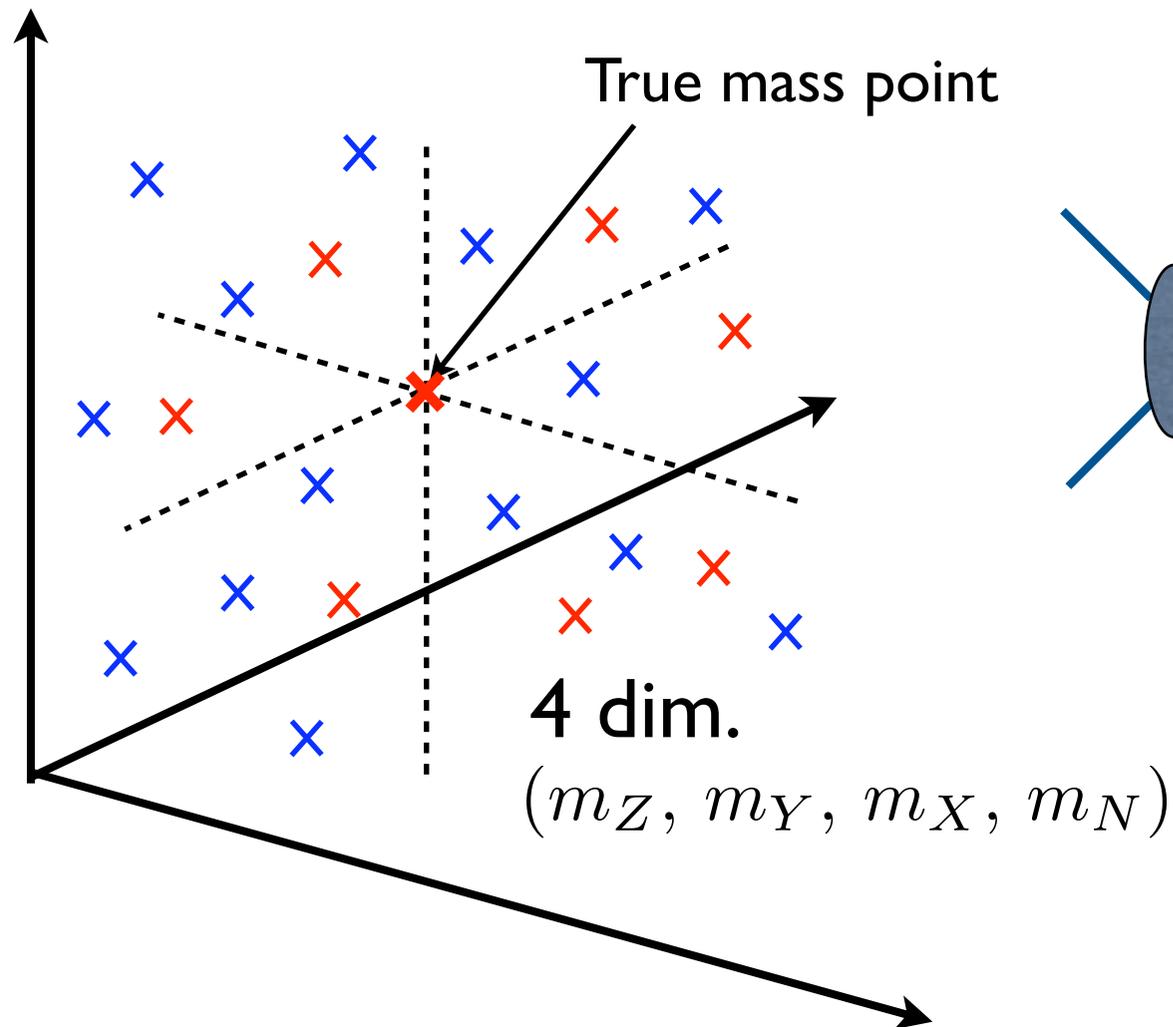


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**detector resolution  
distorts jet momentum**

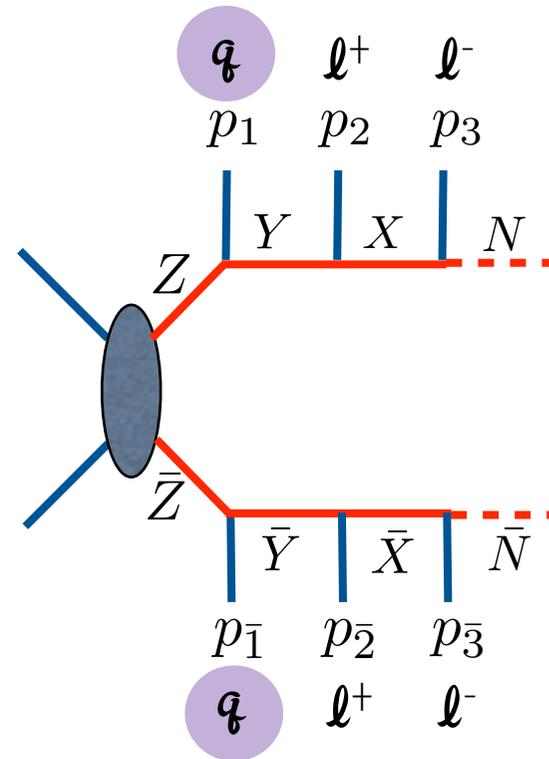
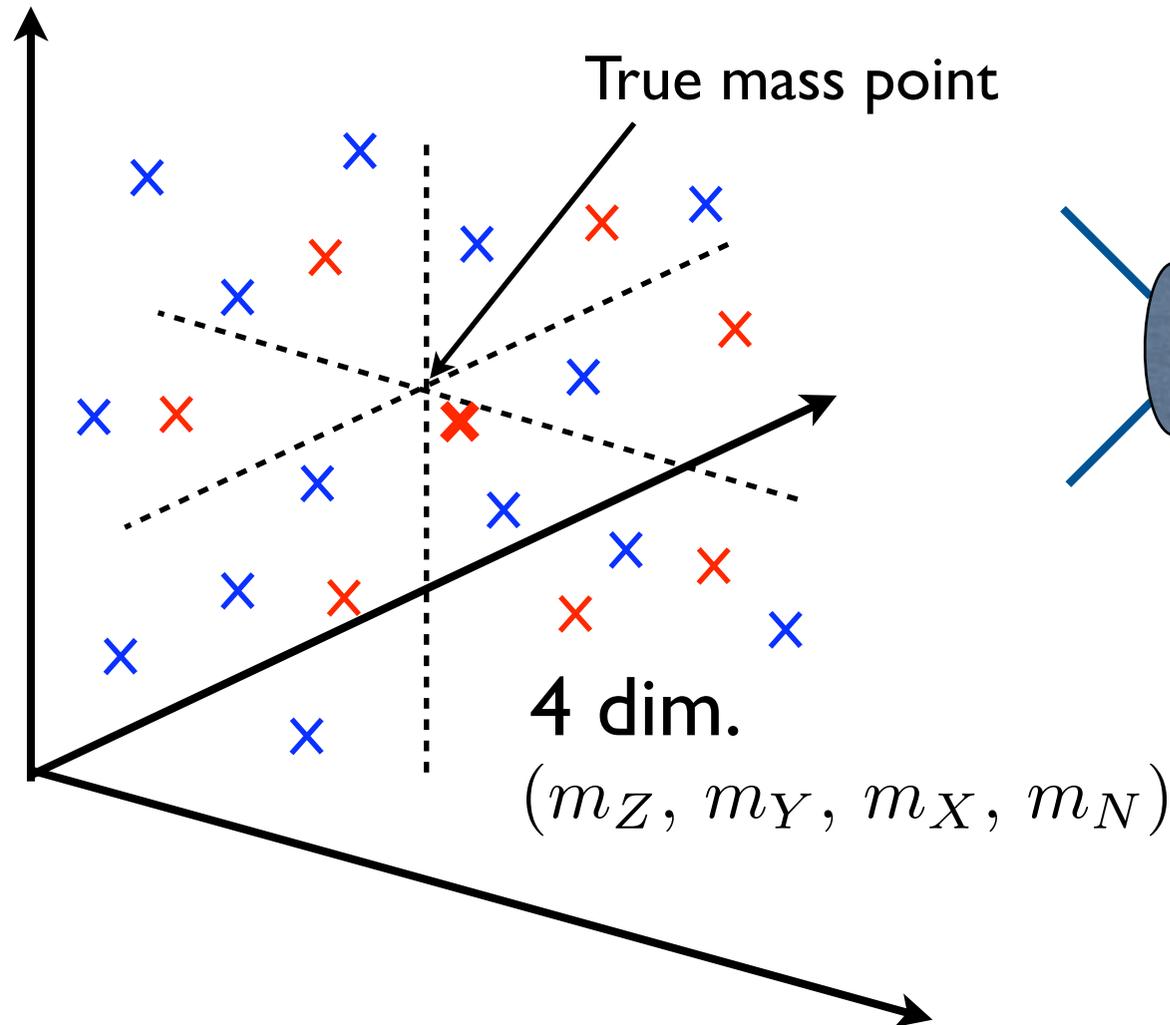


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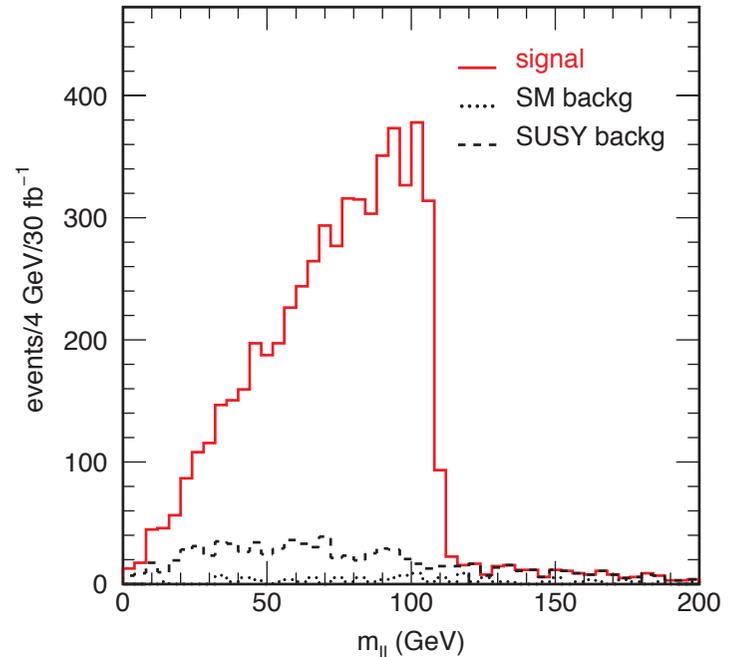
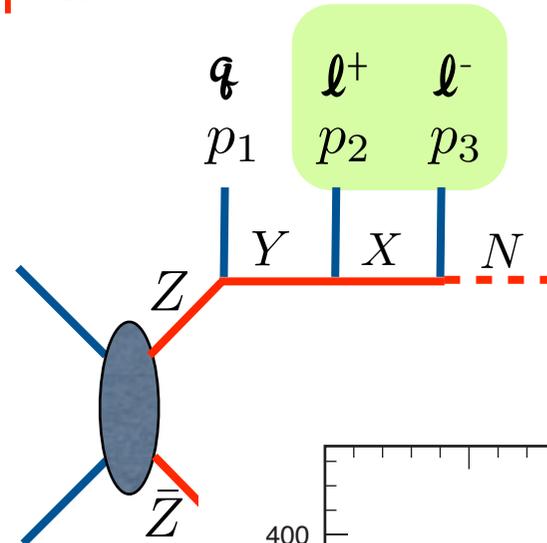
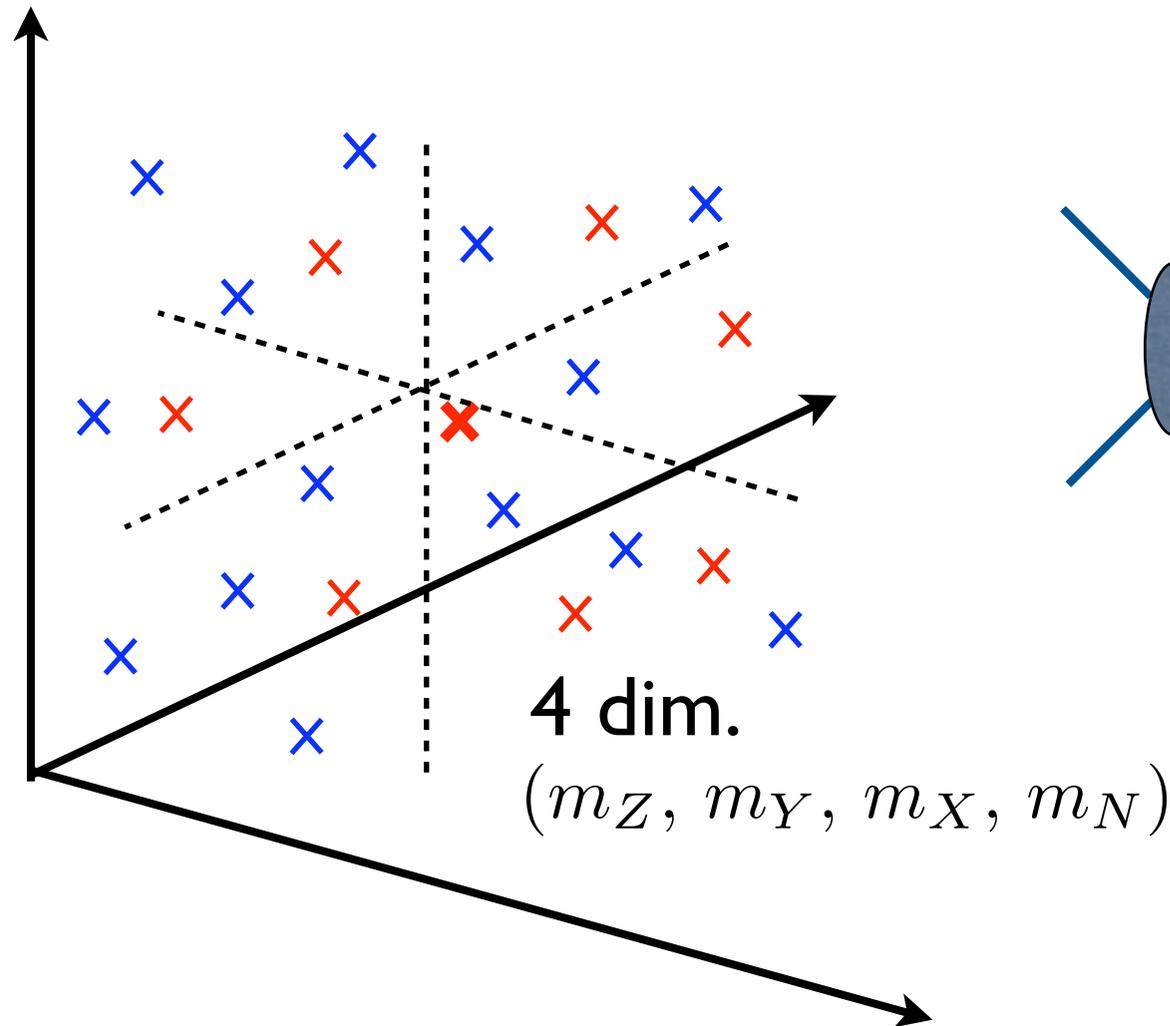


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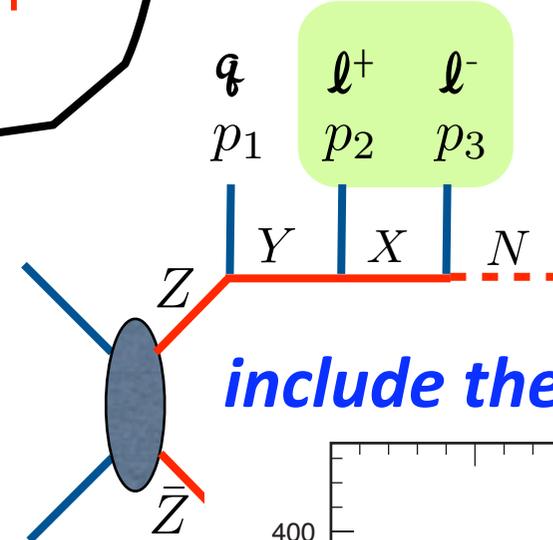
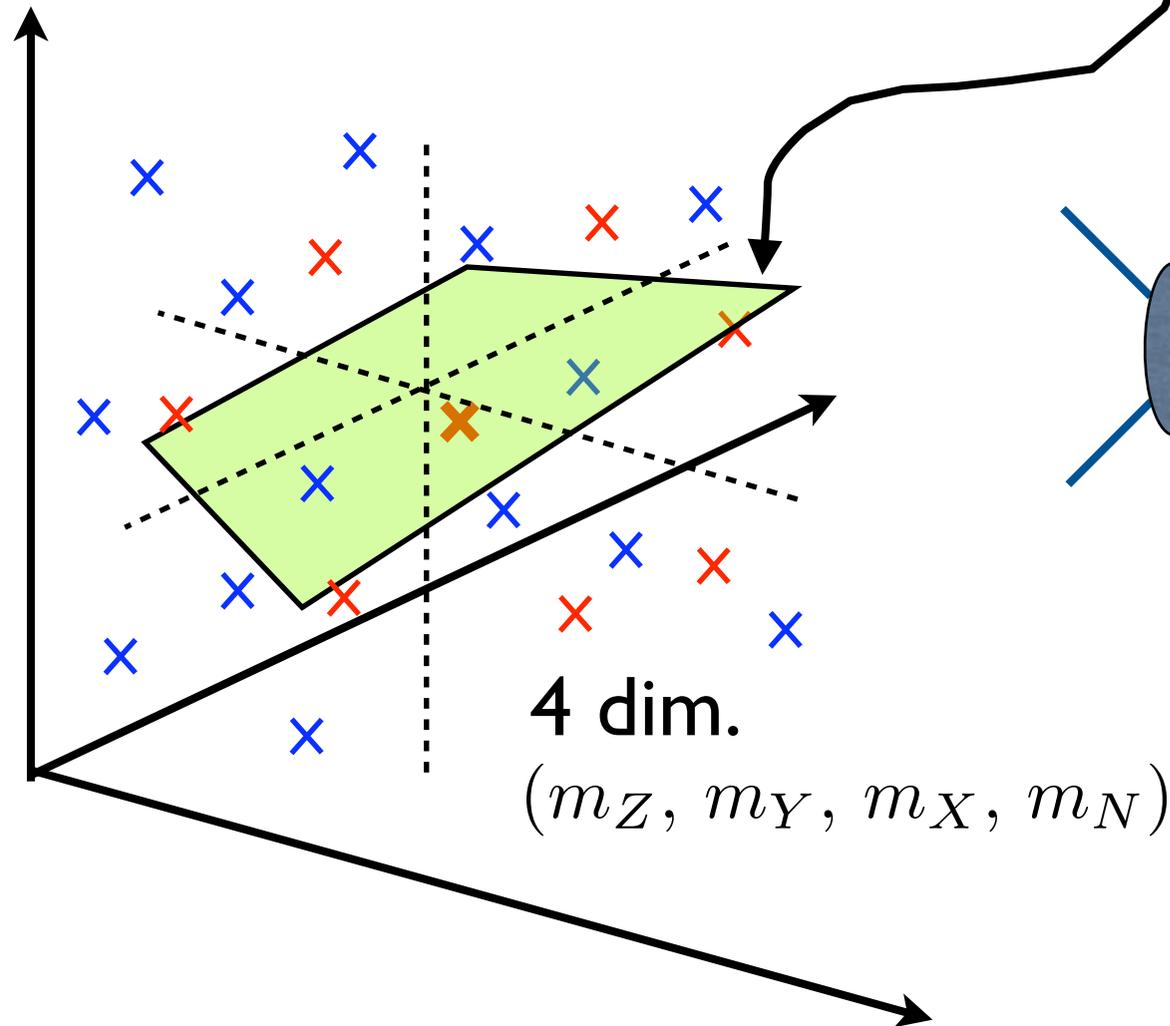


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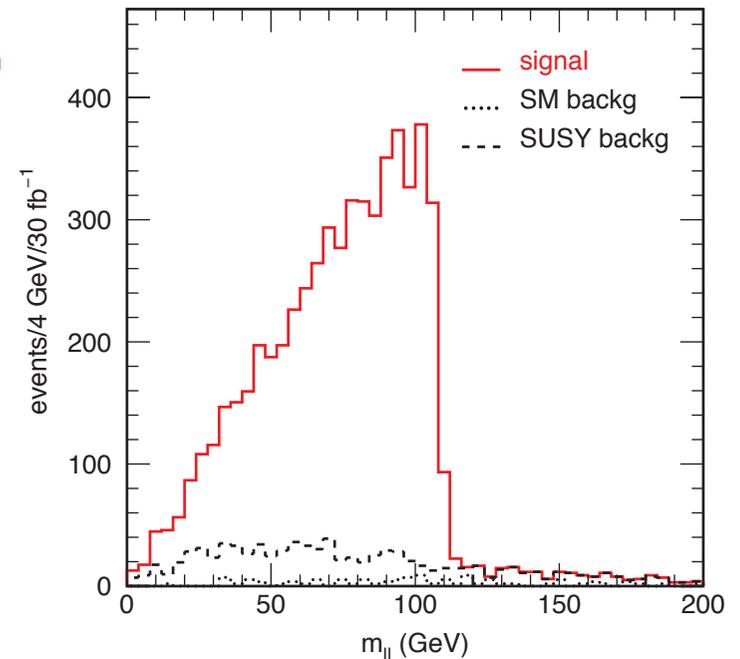
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$$(M_{ll}^2)^{\max} = \frac{1}{m_X^2} (m_Y^2 - m_X^2)(m_X^2 - m_N^2)$$



*include the additional info.*

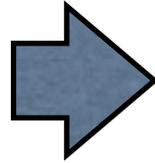


# How to incorporate constraints effectively?

- give priority to dilepton edge constraints

transform variables

$m_Z, m_Y, m_X, m_N$



$$M_1 = m_Z^2 - m_Y^2$$

$$M_2 = m_Y^2 - m_X^2$$

$$M_3 = m_X^2 - m_N^2$$

$$M_{ll} = \frac{(m_Y^2 - m_X^2)(m_X^2 - m_N^2)}{m_X^2}$$

fix by dilepton edge

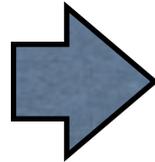


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3 unknowns  
(can be visualise)

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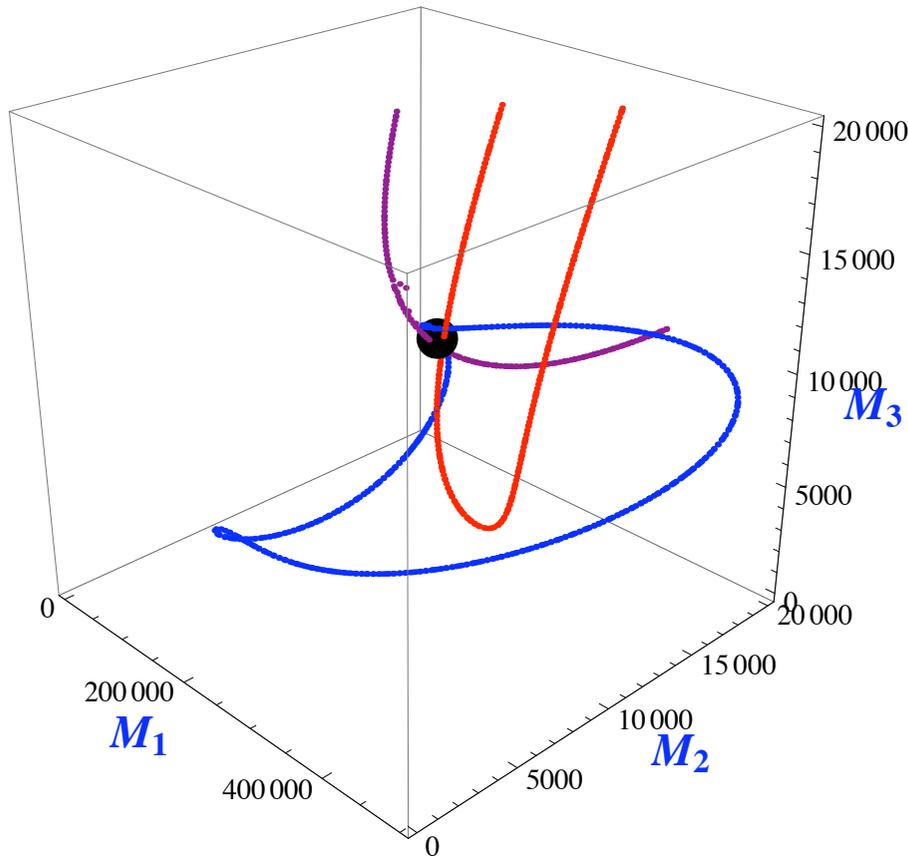
$$M_2 = m_Y^2 - m_X^2$$

$$M_3 = m_X^2 - m_N^2$$

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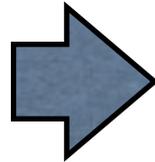
- each event-assignment gives a curve
- curves from different events intersect at the true mass point

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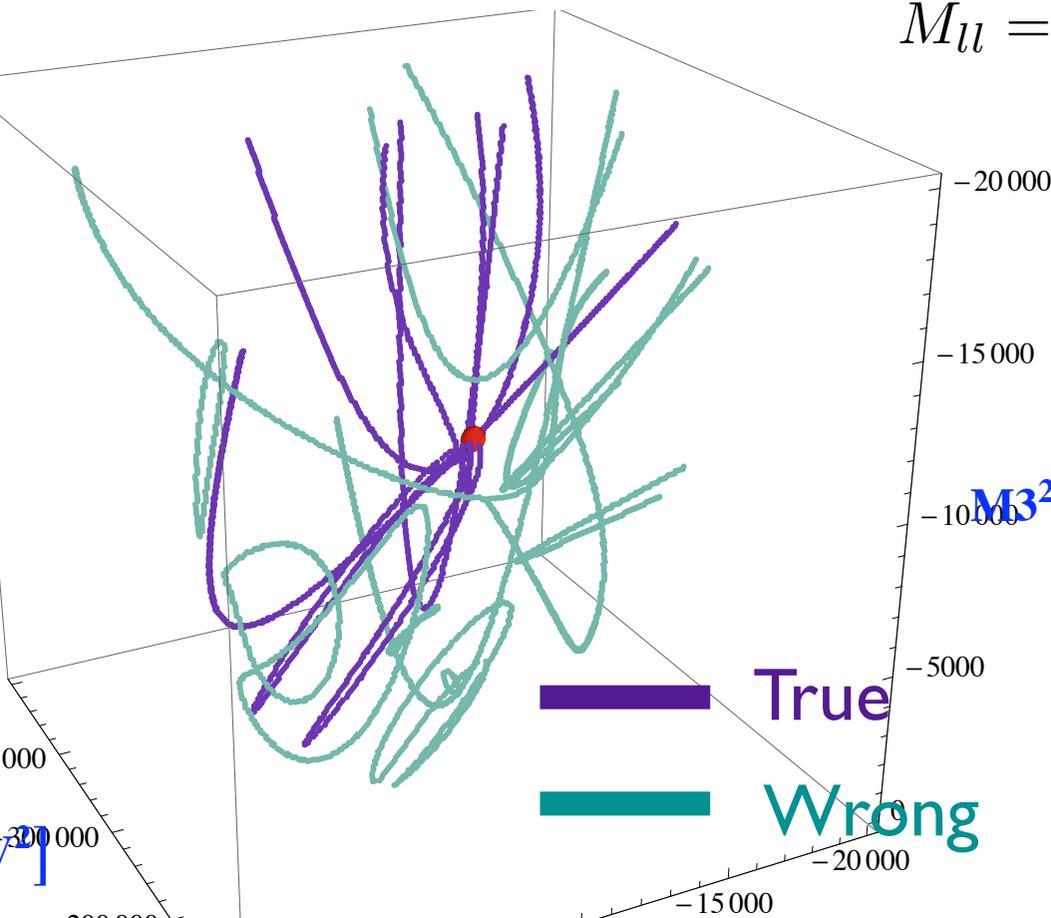
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- each event-assignment gives a curve

- curves from different events intersect at the true mass point

- kill wrong solutions and assignments efficiently

# How to incorporate constraints effectively?

- take account of detector resolution

For each observed event, we generate 1000 “fake” events whose momenta of jets and missing are deviate from observed ones.

The deviations are followed by Gaussian functions with the same error as the detector resolutions

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$\rho_i(\mathbf{M})$ : dens. of curves of event  $i$

$f_i(\mathbf{M})$ : probability dens. of event  $i$

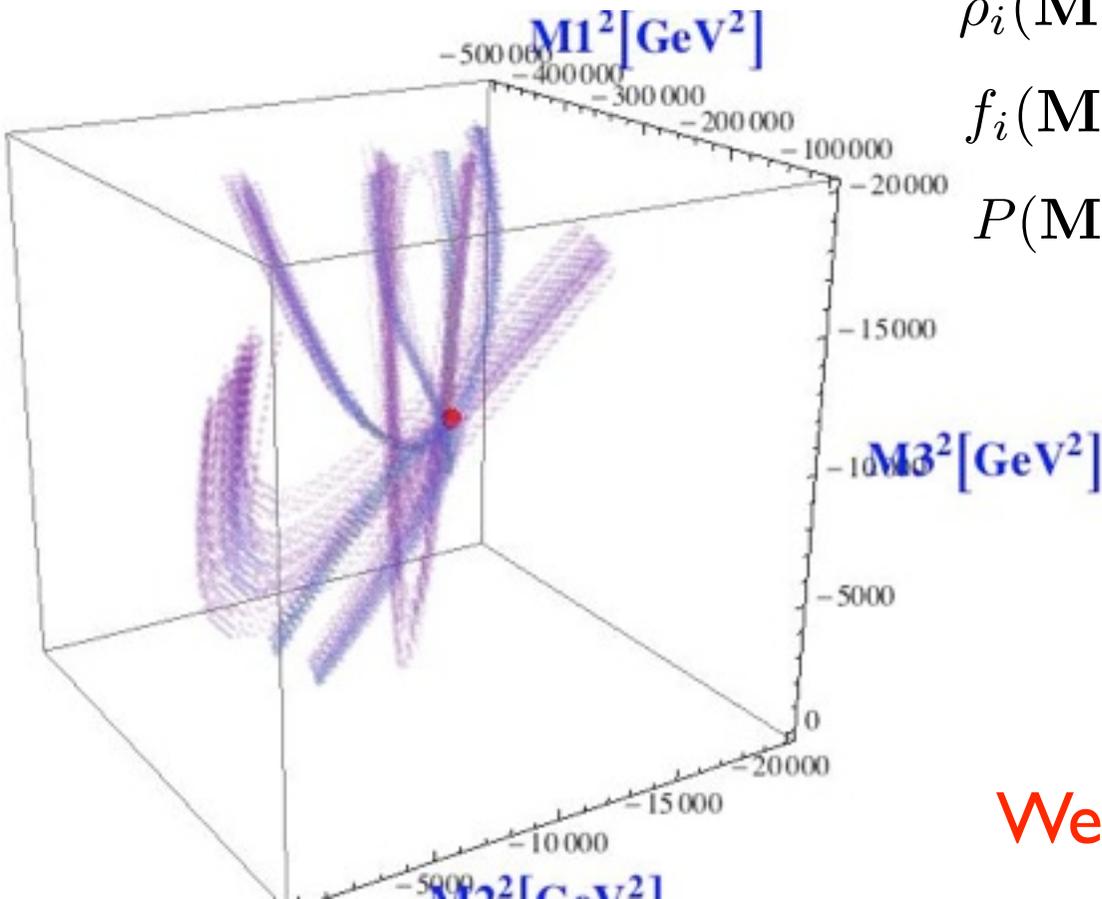
$P(\mathbf{M})$ : probability dens. of the total events

for  $N_{\text{fake}} \rightarrow \infty$

$$f_i(\mathbf{M}) \propto \rho_i(\mathbf{M})$$

$$P(\mathbf{M}) \propto \prod_i f_i(\mathbf{M}) \propto \prod_i \rho_i(\mathbf{M})$$

We can reasonably estimate errors.



# MC simulation

- 3 model points are examined

	$m_0$	$m_{1/2}$	$A_0$	$\tilde{\chi}_1^0$	$\tilde{e}_R$	$\tilde{\chi}_2^0$	$\tilde{u}_L$
Point A	110	220	0	86	142	161	504
Point B	100	250	-100	99	141	186	563
Point C	140	260	0	103	174	193	592

$$m_0^{\text{3rd gene.}} = 300 \text{ GeV}$$

to forbid  $\tilde{\chi}_2^0 \rightarrow \tilde{\tau}_1 \tau \rightarrow \tilde{\chi}_1^0 \tau^+ \tau^-$

- 500,000 inclusive SUSY events are generated by Herwig, corresponding to 10, 15 and 20 fb<sup>-1</sup> for Points A, B and C, respectively
- Effects of SUSY BG, hadronisation, parton shower, underlying events and detector resolution (AcerDET) are included
- The parameter space is divided into cells:

$$\Delta M_1 = 5000, \quad \Delta M_2 = 400, \quad \Delta M_3 = 600 \text{ in GeV}^2$$

# Cut

- The following cuts have been applied to reduce BG

(i)  $M_{\text{eff}} \equiv \sum_{i=1}^4 p_T^{\text{jet},i} + \sum_{i=1}^4 p_T^{\text{lep},i} + E_T^{\text{miss}} > 400 \text{ GeV} ;$

(ii)  $E_T^{\text{miss}} > \max(200 \text{ GeV}, 0.2M_{\text{eff}}) ;$

(iii) At least two jets with  $p_T^{\text{jet},1} > 100 \text{ GeV}$  and  $p_T^{\text{jet},2} > 50 \text{ GeV}$  within  $|\eta| < 2.5 ;$

(iv) Two pairs of opposite sign same flavour leptons with  $p_T > 20 \text{ GeV}$  and  $|\eta| < 3 ;$

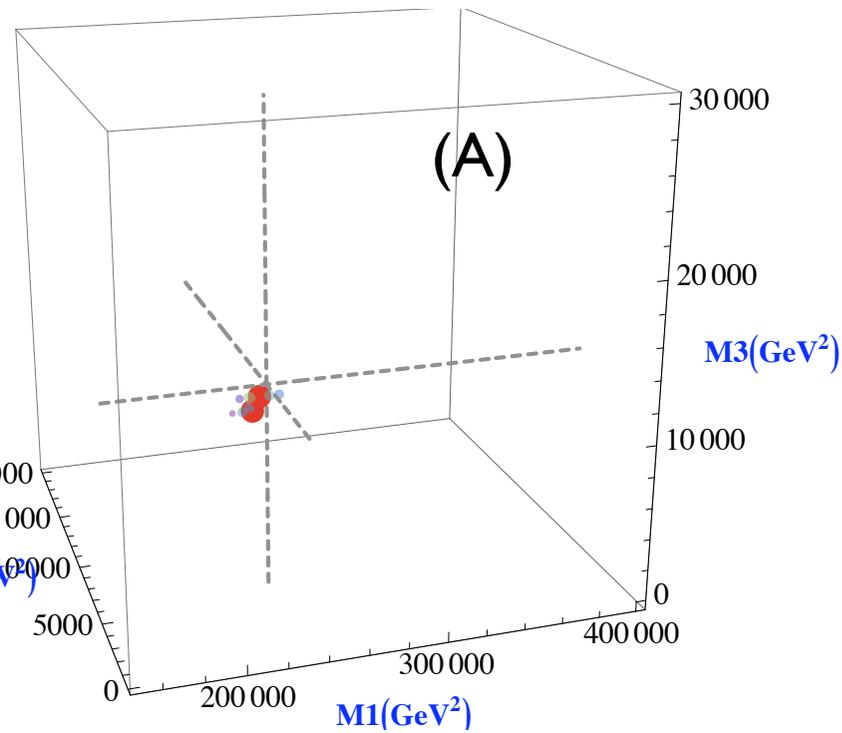
(v) No  $b$  jet with  $p_T > 30 \text{ GeV}$  and  $|\eta| < 3 .$

- The main SM-BG is  $t\bar{t} \rightarrow b\bar{b}W^+W^- \rightarrow 2l^+2l^-2j + E_T^{\text{miss}} .$

It is negligible after the cut. (about 10% of SUSY-BG)

# Results: (10-20 fb<sup>-1</sup>)

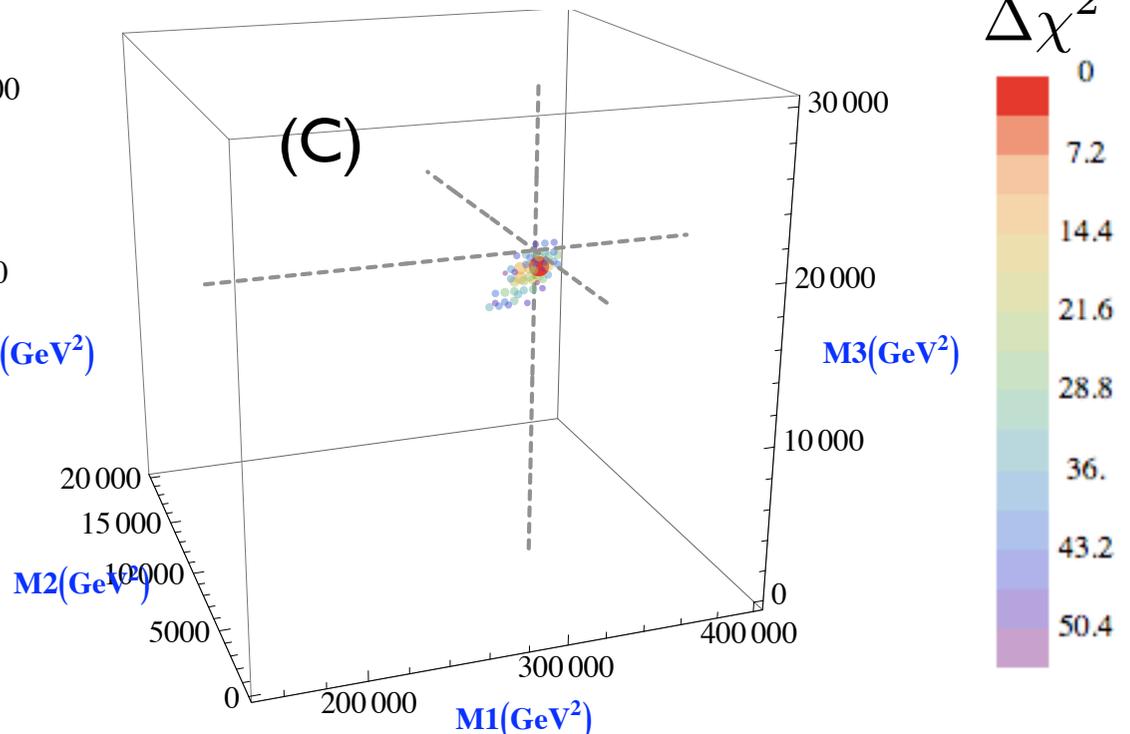
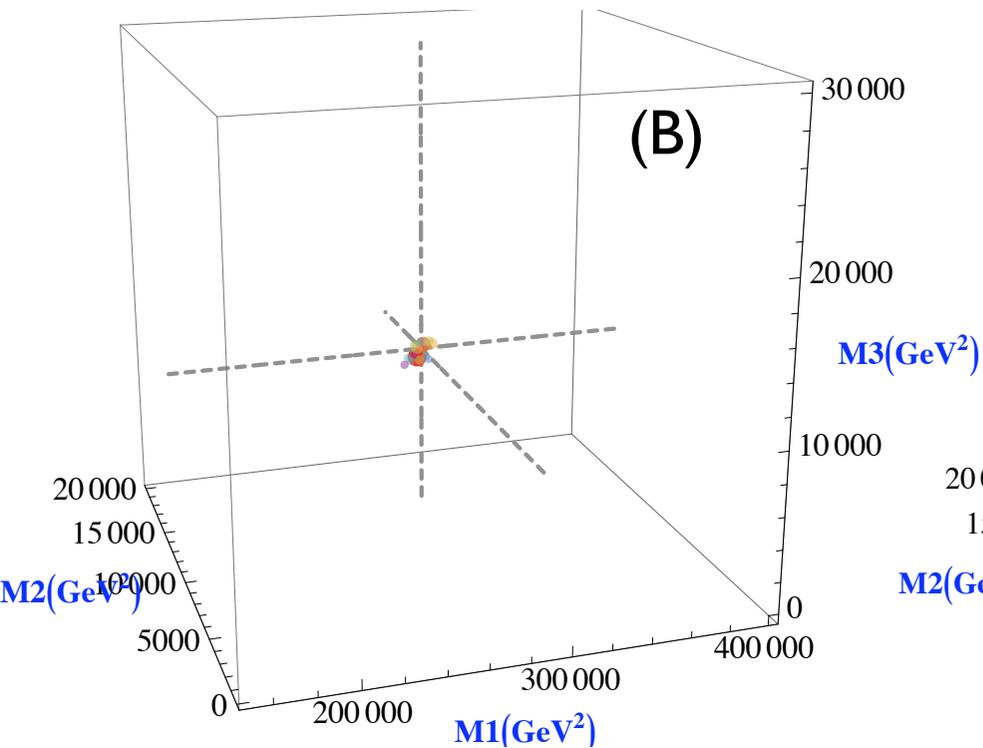
Nojiri, KS, Webber (1005.2532)



	$\tilde{\chi}_1^0$	$\tilde{e}_R$	$\tilde{\chi}_2^0$	$\tilde{u}_L$
Point A	$68.2^{+16.2}_{-5.8}$	$127.9^{+12.6}_{-4.2}$	$146.1^{+13.0}_{-4.4}$	$493.8^{+11.5}_{-3.8}$
Point B	$94.5^{+8.5}_{-2.8}$	$137.2^{+9.1}_{-3.1}$	$181.7^{+8.5}_{-2.8}$	$561.7^{+9.4}_{-3.1}$
Point C	$95.6^{+5.1}_{-5.3}$	$167.4^{+3.9}_{-3.9}$	$186.1^{+4.0}_{-4.0}$	$593.4^{+3.4}_{-3.4}$

	$\tilde{\chi}_1^0$	$\tilde{e}_R$	$\tilde{\chi}_2^0$	$\tilde{u}_L$
Point A	86	142	161	504
Point B	99	141	186	563
Point C	103	174	193	592

Input:



# Summary

- We proposed a new method of the kinematic reconstruction in a specific type of a pair of decay chains.
- In the method, a constraint from the dilepton mass edge is incorporated.
- Wrong solutions and assignments are effectively removed.
- The use of fake events enable us to estimate errors.

	Point A	Point B	Point C
Events (S/B)	326 (4.2)	499 (4.5)	292 (2.8)
Sharing (S/B)	219 (8.1)	341 (9.7)	172 (4.9)
$M_1$ (True ; Best)	231890 ; 222500	286157 ; 282500	316274 ; 317500
$M_2$ (True ; Best)	5624 ; 5000	14520 ; 14200	6815 ; 6600
$M_3$ (True ; Best)	12872 ; 11700	10293 ; 9900	19812 ; 18900

Signal / background ratios are enhanced at the best fit cell.

# Statistical approach

- $\Delta\chi^2$  is obtained from the log likelihood function as follows:

$$\ln L(\mathbf{M}) = \sum_{i_{ev}}^N \ln f_{i_{ev}}(\mathbf{M}) \quad \Delta\chi^2(\mathbf{M}) = 2(\ln L(\mathbf{M})_{\max} - \ln L(\mathbf{M})),$$

The relationship between  $\Delta\chi^2$  and CL, when  $\Delta\chi^2$  has 3 arguments



CL (%)	$\Delta\chi^2$
68.27	3.53
90.	6.25
95.	7.82
95.45	8.03
99.	11.34
99.73	14.16

