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Measuring masses of semi-invisibly decaying particles pair produced at hadron colliders

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Abstract

We introduce a variable useful for measuring masses of particles pair produced at hadron colliders, where each particle decays to one particle that is directly observable and another particle whose existence can only be inferred from missing transverse momenta. This variable is closely related to the transverse mass variable commonly used for measuring the W mass at hadron colliders, and like the transverse mass our variable extracts masses in a reasonably model independent way. Without considering either backgrounds or measurement errors we consider how our variable would perform measuring the mass of selectrons in a mSUGRA SUSY model at the LHC.

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Before 1983, e^+e^- colliders dominated the search for new particles, their clean final states making identification of new particles far easier than at hadron machines. However in 1983 this changed with the observation of the W and Z bosons by the UA1[1, 3] and UA2[2, 4] collaborations at the CERN $p\bar{p}$ collider. Despite the more difficult conditions where the initial state protons and anti-protons gave rise to an underlying event which complicates measurement of the final state, the higher energies available to a $p\bar{p}$ machine meant they discovered the W and Z bosons before other machines had enough energy to produce them. This trend continued in 1994/5 when CDF and DØ discovered the t quark at the Tevatron $p\bar{p}$ collider [5]. To date the only direct experimental evidence that we have for the t quark is from the Tevatron.

Clearly, the greater energies available to $p\bar{p}$ machines gives them an advantage in discovering fundamental particles, and we can hope that this trend will be continued at the LHC. Upon discovery of a new particle the first question that is asked is what is the mass of this particle. There are two fundamentally different ways of measuring the mass of a particle. Usually one tries to extract the mass of the particle directly from the observed events. For example UA1 and UA2 detected the Z boson in its decay to lepton pairs e^+e^- or $\mu^+\mu^-$. For each event that contains a pair of leptons one can construct the mass of the particle that produced the leptons as,

$$m^2 = (p_{l^+} + p_{l^-})^2 \tag{1}$$

and on an event by event basis obtain a direct estimate the mass of the particle. Alternatively one could make an indirect model dependent measure of the Z mass by measuring the "Z event" cross-section, and using a model that predicts the Z cross-section in terms of the Z mass, perform a fit to the measured cross-section. For measuring the Z mass the direct measurement is superior, if only for its lack of model dependence, however when measuring the mass of the W boson the case is less clear. At the CERN $p\bar{p}$ collider the W boson was detected in its decay to a charged lepton and a neutrino, however the neutrino goes unobserved and so only gives rise to missing momentum. This means that one can't directly observe the W mass from the lepton and neutrino momenta. UA1 [1] formed the transverse mass variable,

$$m_T^2 = 2(E_T^e \not{E}_T - \mathbf{p}_T^e \cdot \not{\mathbf{p}}_T), \tag{2}$$

where $\not{E}_T^2 \equiv \not{\mathbf{p}}_T^2$. This variable has the property that,

$$m_T^2 \leq m_W^2 \tag{3}$$

with equality possible for events where the lepton and neutrino are produced with the same rapidity. Thus although one can't obtain the W mass from a single event, one can obtain a lower limit on the W mass. In addition if one obtains the lower mass bound from many events, this can approach the W mass, and so one can extract the W mass in a model independent way. UA1 [1] along with UA2 [2] also performed a model depenendent fit to the p_T spectrum for the lepton, extracting the W mass from that model.

In practice, though, measuring the W mass with m_T does have some small model dependence, as the precise fraction of events which occur with m_T close to m_W is still dependent

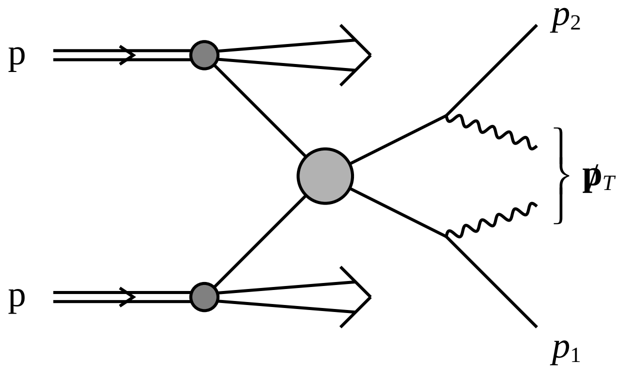

Figure 1: Diagram of the generic process that we consider. A hadronic collision that leads to a pair of particles being produced, which each decay into one particle that is observed with momenta p_1 and p_2 respectively; and one particle (shown as a wavy lines) that is not directly detected, and whose presence can only be inferred from the missing transverse momentum, $\not{\mathbf{p}}_T$.

on the physics processes which produce the W boson, and how the W boson decays. In addition, the missing transverse momentum is poorly measured experimentally compared with $\mathbf{p}_T(l)$, so the theoretical model dependence of the measurement of m_W from the $\mathbf{p}_T(l)$ spectrum is balanced by the experimental error on extracting m_W from the edge of the m_T spectrum.

In this paper we wish to introduce a variable which measures particle masses, which like transverse mass has little dependence on exactly how such massive particles are produced. The variable is used for the generic process shown in figure 1, where a hadronic collision pair produces a massive particle whose dominant decay is into one observed and one unobserved particle. This unobserved particle can only be detected from the missing momentum that it carries away, and that the massive particle is pair produced means that we can only measure the missing momentum of the pair of invisible particles. Although this may sound like an unusual process to look for new particles, it naturally occurs in any theory where there is an (approximately) conserved charge, and the lightest particle with that charge is only weakly interacting. Two examples of where such a situation can occur are SUSY models and a 4th lepton generation. In R-parity conserving SUSY models, sparticles are pair produced, and cascade decay to the lightest sparticle, which must be stable and is expected to not be directly detectable. Slepton production and decay can often follow this route:

$$pp \to X + \tilde{l}_R^+ \tilde{l}_R^- \to X + l^+ l^- \tilde{\chi}_1^0 \tilde{\chi}_1^0. \tag{4}$$

In such an event the pair of lightest SUSY particles, $\tilde{\chi}$, go unobserved and only leave their signature as missing transverse momentum.

For a 4th generation lepton the charged lepton would be pair produced in a Drell-Yan type process, decaying to a neutrino and a W boson,

$$pp \to X + l_4^+ l_4^- \to X + \bar{\nu}_{l_4} W^+ \nu_{l_4} W^- \tag{5}$$

the (probably massive) ν_{l_4} going unobserved, while the W bosons could be detected in their decays to either $l\nu$ or to jets.

We now look specifically at the process given in equation 4, although the variable which we define would work identically in any process where a particle is pair produced and decays to one visible and one invisible particle.

The variable that we wish to introduce is closely related to m_T, however the standard definition of m_T, given in equation 2, assumes that the unobserved particle is massless, so we return to the derivation of this variable. For the decay,

$$\tilde{l} \to l\tilde{\chi} \tag{6}$$

for arbitrary momenta we can write,

$$m_{\tilde{l}}^2 = m_l^2 + m_{\tilde{\chi}}^2 + 2(E_{Tl}E_{T\tilde{\chi}}\cosh(\Delta\eta) - \mathbf{p}_{Tl}\cdot\mathbf{p}_{T\tilde{\chi}}) \tag{7}$$

where $E_T = \sqrt{\mathbf{p}_T^2 + m^2}$ and $\Delta\eta$ is the difference in rapidity, $\eta = \frac{1}{2}\ln[(E+p_z)/(E-p_z)]$, between between the l and $\tilde{\chi}$.

Now as $\cosh\eta \geq 1$ we have,

$$m_{\tilde{l}}^2 \geq m_T^2(\mathbf{p}_{Tl}, \mathbf{p}_{T\tilde{\chi}}) \equiv m_l^2 + m_{\tilde{\chi}}^2 + 2(E_{Tl}E_{T\tilde{\chi}} - \mathbf{p}_{Tl}\cdot\mathbf{p}_{T\tilde{\chi}}). \tag{8}$$

This gives a version of transverse mass valid for arbitrary masses, with equality when the l and $\tilde{\chi}$ are produced with the same rapidity. Notice that E_{Tl} and $E_{T\tilde{\chi}}$ depend on m_l^2 and $m_{\tilde{\chi}}^2$ respectively.

The transverse mass can't be formed directly from the process in equation (4), as both the neutralinos give rise to missing momentum, however we can experimentally measure the sum of their transverse momenta as the missing transverse momenta in the event,

$$\not{\mathbf{p}}_T = \mathbf{p}_{T\tilde{\chi}_a} + \mathbf{p}_{T\tilde{\chi}_b}. \tag{9}$$

If $\mathbf{p}_{T\tilde{\chi}_a}$ and $\mathbf{p}_{T\tilde{\chi}_b}$ were obtainable, then one could form two transverse masses, and using the relationship (8) obtain,

$$m_{\tilde{l}}^2 \geq \max\{m_T^2(\mathbf{p}_{Tl^-}, \mathbf{p}_{T\tilde{\chi}_a}), m_T^2(\mathbf{p}_{Tl^+}, \mathbf{p}_{T\tilde{\chi}_b})\} \tag{10}$$

However, not knowing the form of the splitting (9), the best we can say is that:

$$m_{\tilde{l}}^2 \geq M_{T2}^2 \equiv \min_{\not{\mathbf{p}}_1 + \not{\mathbf{p}}_2 = \not{\mathbf{p}}_T} \left[\max\{m_T^2(\mathbf{p}_{Tl^-}, \not{\mathbf{p}}_1), m_T^2(\mathbf{p}_{Tl^+}, \not{\mathbf{p}}_2)\}\right] \tag{11}$$

With the minimization over all possible 2-momenta, $\not{\mathbf{p}}_{1,2}$, such that their sum gives the observed missing transverse momentum, $\not{\mathbf{p}}_T$. This is the variable, called M_{T2}, that we wish to introduce. This bound we can obtain directly from experimentally measured parameters. Although not totally transparent, for particular momenta, M_{T2} can be equal to $m_{\tilde{l}}$; the requirement being that for both slepton decays the lepton and neutralino are produced at the same rapidity (although the sleptons themselves can be at differing rapidities), and in addition,

$$\left(\frac{\mathbf{p}_{Tl^-}}{E_{Tl^-}} - \frac{\mathbf{p}_{T\tilde{\chi}_a}}{E_{T\tilde{\chi}_a}}\right) \propto \left(\frac{\mathbf{p}_{Tl^+}}{E_{Tl^+}} - \frac{\mathbf{p}_{T\tilde{\chi}_b}}{E_{T\tilde{\chi}_b}}\right). \tag{12}$$

We have not managed to derive a general analytic expression for the minimization over splittings of $\not{\mathbf{p}}_T$; largely because an experimental measurement of $\not{\mathbf{p}}_T$ only measures the missing transverse momentum, and neither the missing energy nor the missing longitudinal momenta. This means that $\not{\mathbf{p}}_T$ is not a 4 vector, which means that M_{T2} can not be calculated in a manifestly Lorentz invariant manner. The complication that this introduces to the minimization is enough to making an analytic solution non trivial. However if we take one of the parameters that we minimize over to be $m^2 = \not{p}_T^2$ (*i.e.* that $\not{E}_T = \sqrt{m^2 + \not{\mathbf{p}}_T^2}$), and in addition note that longitudinal momenta play no part in the definition of M_{T2}, then the remaining minimization becomes Lorentz invariant, and this means that it can be solved analytically. This leaves one minimization over m^2 that we performed numerically. The form of this minimization is not particularly illuminating, and so we do not give it here; however a computer code for evaluating M_{T2}^2 is available from the authors.

Of course, the variable M_{T2} is only good at extracting particle masses from processes having many events close to the maximum allowed value, and this depends on the physics of the process being measured. Hence, a priori, we can not say that M_{T2} is useful for measuring particle masses in all processes. However we expect typical physics processes to have reasonable numbers of events close to the maximum value allowed for M_{T2} and hence expect it to be a useful variable. To illustrate the variable in use, we consider a model which allows the process shown in equation (4), at the expected LHC centre of mass energy $\sqrt{s} = 14$TeV. Our SUSY model is the fifth minimal supergravity model (mSUGRA) [6] point selected by the LHC Committee in 1996 for detailed study by the ATLAS and CMS collaborations [7]. This model is characterized by,

$$\{\tan\beta = 2.1,\ m_{1/2} = 300 \text{ GeV},\ m_0 = 100 \text{ GeV},\ A_0 = 300 \text{ GeV},\ \mu > 0\}. \tag{13}$$

We generate events using the SUSY version of the Herwig generator [8]. For our purposes the only important features of the model that we use to produce our events are,

$$m_{\tilde{l}_R} = 157.1 \text{ GeV} \qquad m_{\tilde{\chi}_1^0} = 121.5 \text{ GeV}. \tag{14}$$

The angular distributions for the sleptons are given by the Drell-Yan process that produces spin-0 particles, while the decay of the sleptons is isotropic due again to their being spinless. Herwig serves to build up a reasonably realistic underlying event, *i.e.* that $\not{\mathbf{p}}_T \neq \mathbf{p}_{Tl^+} + \mathbf{p}_{Tl^-}$. Defining the missing momenta as,

$$\not{\mathbf{p}}_T = \mathbf{p}_{T\tilde{\chi}_a} + \mathbf{p}_{T\tilde{\chi}_b}, \tag{15}$$

we generate 1105 events, which corresponds to an integrated luminosity of about 30 fb^{-1}, which the LHC should collect in approximately one year of low luminosity running.

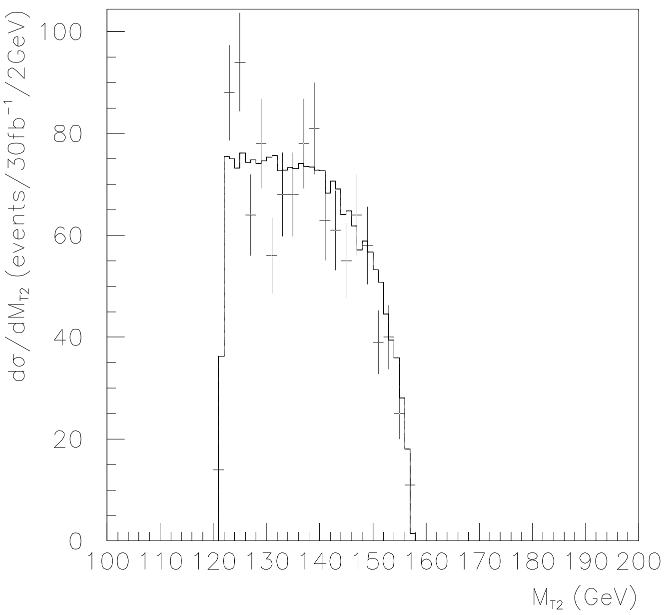

Figure 2: M_{T2} distribution for the process $pp \to X + \tilde{l}_R^+ \tilde{l}_R^- \to X + l^+ l^- \tilde{\chi}_1^0 \tilde{\chi}_1^0$ at the LHC. With $m_{\tilde{l}} = 157.1$ GeV and $m_{\tilde{\chi}} = 121.5$ GeV, assuming the actual value for $m_{\tilde{\chi}}$ when calculating M_{T2}. The data with error bars are 1105 events, that the LHC would collect in approximately 1 year of running at low luminosity, *i.e.* $\mathcal{L} \simeq 30$ fb^{-1}. The histogram represents $\mathcal{L} \simeq 500$ fb^{-1} to show the shape of the distribution that would be obtained with huge statistics, with the normalization modified to be the same as $\mathcal{L} \simeq 30$ fb^{-1}.

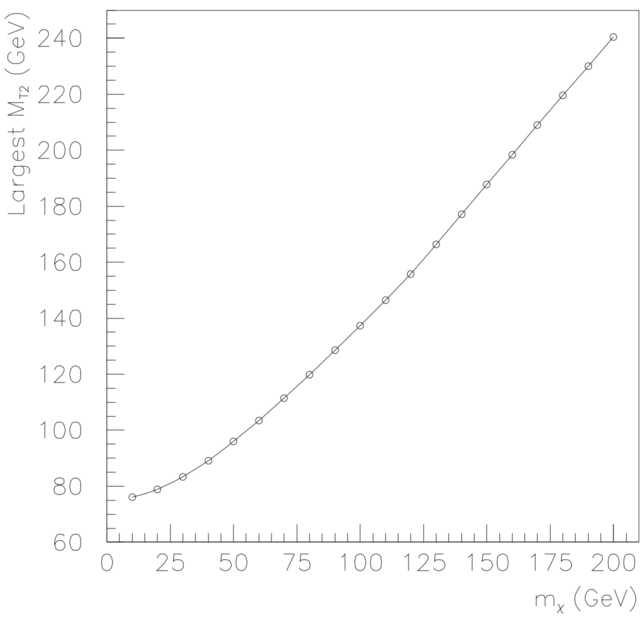

Figure 3: Values of $m_{\tilde{l}}$ that would be obtained from that largest M_{T2} value observed, where differing values of $m_{\tilde{\chi}}$ are used in the calculation of M_{T2}, for the 1105 events shown in figure 2. All events have been generated with $m_{\tilde{l}} = 157.1$ GeV and $m_{\tilde{\chi}} = 121.5$ GeV.

In figure 2 we show the M_{T2} distribution for this model. Quite clearly the maximum M_{T2} value allowed corresponds well to the mass of the selectron, with the M_{T2} distribution tending smoothly to zero at that point. While such an edge is not as easy to measure as a vertical drop, we still expect that such an edge would be detectable experimentally, and hence a viable means to measure the selectron mass. The lower bound on M_{T2} is $m_{\tilde{\chi}}$, however this does not give a means to measure the LSP mass, the lower bound on M_{T2} follows directly from equation (8), and this is a parameter that goes into the calculation of M_{T2}. Indeed if the value of $m_{\tilde{\chi}}$ is unknown in addition to $m_{\tilde{l}}$ one can only obtain a relationship between the two. In figure 3 we show the value of $m_{\tilde{l}}$ that would have been obtained from the same events shown in figure 2, using differing input values for $m_{\tilde{\chi}}$ in the calculation of M_{T2}. One can see that slepton mass that one would extract from the events behaves approximatly as,

$$m_{\tilde{l}} \simeq m_{\tilde{\chi}} + \text{constant}, \tag{16}$$

and so any uncertainty in the LSP mass will be directly reflected as an error in the extracted selectron mass. It should also be noted that the edge of the M_{T2} distribution becomes hard to fit as the input $\tilde{\chi}$ mass deviates from the physical $\tilde{\chi}$ mass as the M_{T2} distributions develops a tail at the largest M_{T2} values.

To conclude, in this paper we have introduced a new variable for measuring masses of particles produced at hadron colliders, where the longitudinal momentum of the hard scattering is typically unmeasured. It may be used when particles are pair produced, with each decaying to one particle that is directly observed and one particle that is not directly observed. This variable is analogous to the transverse mass variable, m_T, commonly used for measuring the W mass in its decay to $l\nu$ at $p\bar{p}$ colliders, except that it works where the particle being measured is pair produced and where the unseen particle is massive. We expect that the masses extracted using our variable, like those from the transverse mass, will be largely independent of the physics processes which produce the particles, and hence give a viable means of extracting masses in a model independent way. As an illustration of this variable in action we consider measuring the mass of selectrons in SUSY at the LHC. The results look promising however as we have not considered the effects of any background processes or experimental mis-measurement errors further study is required[9].

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