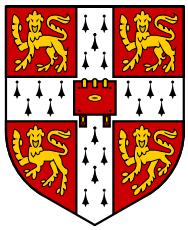
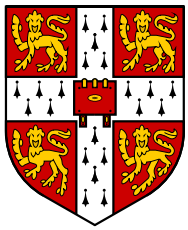


Simulation of SUSY

Peter Richardson
(Cavendish Laboratory and DAMTP, Cambridge)

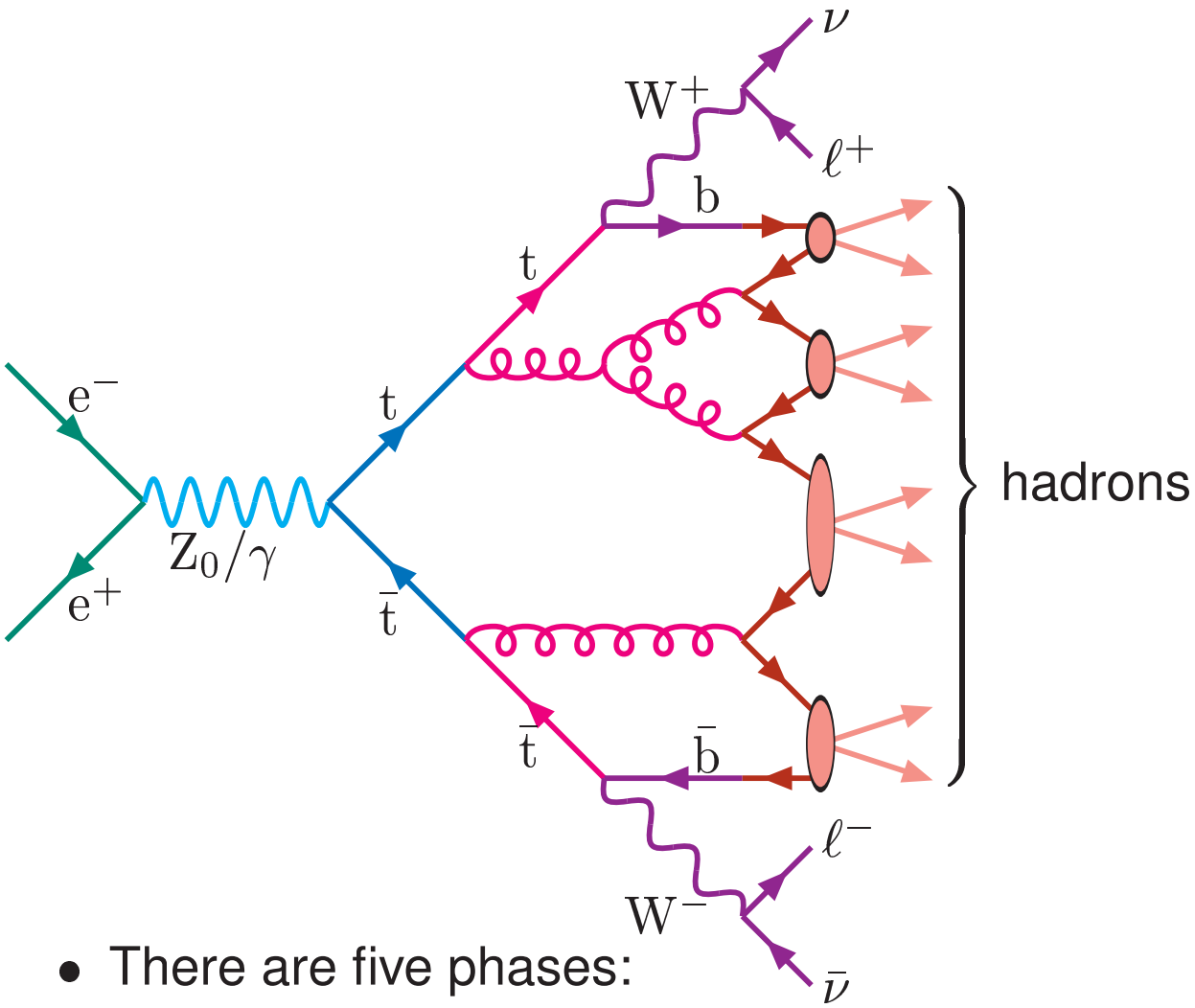
- Introduction to Monte Carlo Simulations
- SUSY Models
- SUSY spectrum generation and decay rate calculation
- Spin Correlations in particle decays
- Examples
- Conclusions



Introduction to Monte Carlo Simulations

- Monte Carlo simulations are used by all experimental collaborations both to compare their data and theoretical predictions, and in data analysis.
- They are often treated as black boxes.
- It's important to understand the assumptions and approximations involved in these simulations.
- Should the simulation really describe the physics I'm looking at?
- Is what I'm seeing in the simulation really physics?
  - Is it a result of the approximations used?
  - Is it a bug?
- Is the result of an experiment really physics or the result of the simulation used?

Monte Carlo Event Generation



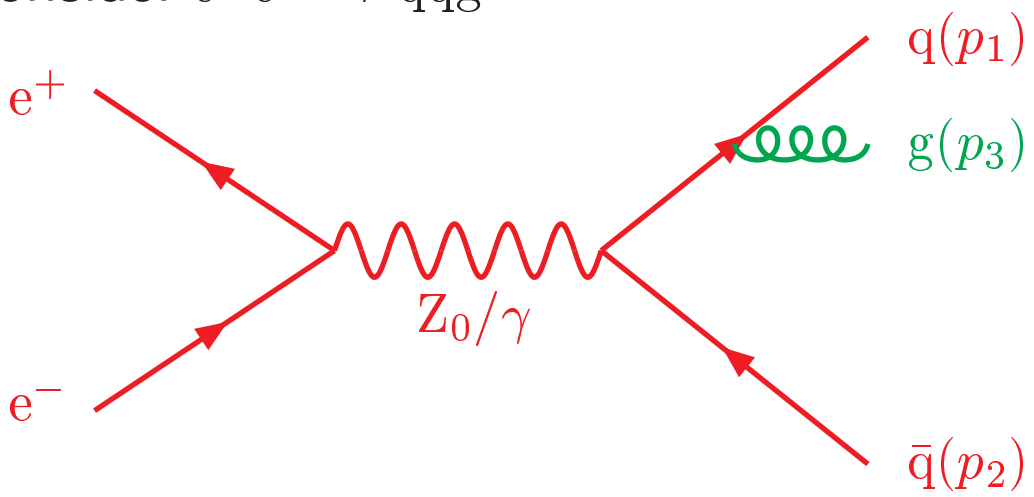
- There are five phases:
  - Hard Process;
  - Parton Shower;
  - Heavy particle decays;
  - Hadronization;
  - Hadron decays.
- I will mainly discuss the details of the HERWIG event generator.

QCD Radiation

- It is impossible to calculate and integrate the matrix elements for processes involving large numbers of partons.

- Instead we treat the regions where the emission of QCD radiation is enhanced.

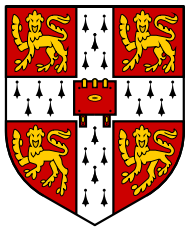
- Consider $\mathrm{e}^+\mathrm{e}^- \to \mathrm{q}\bar{\mathrm{q}}\mathrm{g}$



- The cross section is given by

$$\frac{d\sigma_{q\bar{q}g}}{dx_1 dx_2} = N_c \sigma_0 Q_q^2 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

- This diverges as $x_{1,2} \to 1$ where $x_i = 2E_i/\sqrt{s}$.



QCD Radiation

- Physically this happens when

1. The gluon and one of the quarks are collinear

$$(1 - x_1) = \frac{E_2 E_3}{Q^2}(1 - \cos\theta_{2g})$$

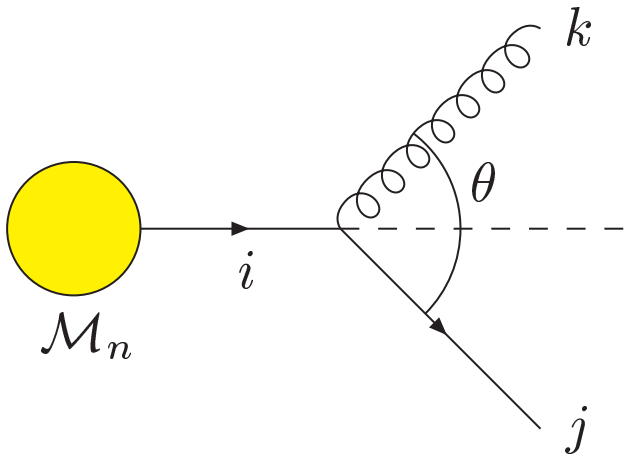
Collinear singularity

2. The energy of the gluon goes to zero

$$x_1 + x_2 + x_3 = 2, x_1, x_2 \to 1 \Rightarrow x_3 \to 0$$

Soft Singularity

Collinear Singularities



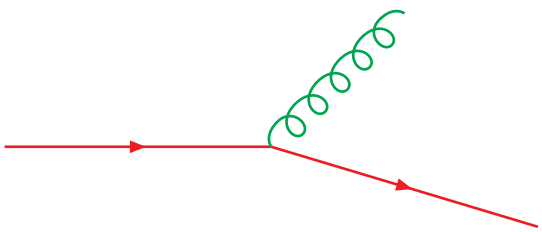
- In the collinear limit the cross section for a process factorizes

$$d\sigma_{n+1} = d\sigma_n \frac{d\theta^2}{\theta^2}\, dz \frac{\alpha_s}{2\pi} P_{ji}(z),$$

- P_{ji} is the DGLAP splitting function.
- This is singular as $\theta \to 0$.
- What is a parton? (difference between collinear pair and a parton)

Collinear Singularities

- Introduce resolution criterion *e.g.* $k_T > Q_0$.
- Combine virtual corrections & unresolvable emission.



Resolvable Emmission

Finite

Unresolvable Emission

Finite

- Unitary: Unresolved + Resolved = 1.

Monte Carlo Procedure

- Probability of Emission between $q^2 + dq^2$ and q^2

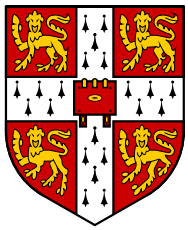
$$d\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz P(z) \equiv \frac{dq^2}{q^2}\bar{P}(q^2)$$

- Define probability of no emission between Q^2 and q^2 $\Delta(Q^2, q^2)$

$$\frac{d\Delta(Q^2, q^2)}{dq^2} = \Delta(Q^2, q^2)\frac{d\mathcal{P}}{dq^2}$$

$$\Rightarrow \Delta(Q^2, q^2) = \exp\left[-\int_{q^2}^{Q^2}\frac{dk^2}{k^2}\bar{P}(k^2)\right]$$

- $\Delta(Q^2, Q_0^2) \equiv \Delta(Q^2)$ The Sudakov Form Factor probability of emitting no resolvable radiation.



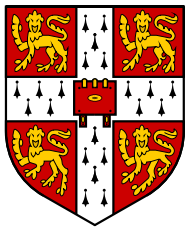
Monte Carlo Procedure

- We can implement this in the following way:
  - Generate a random number $0 < \rho < 1$;
  - If $\rho < \Delta(Q^2)$ no resolvable emission;
  - Otherwise solve

$$\rho = \Delta(Q^2, q^2)$$

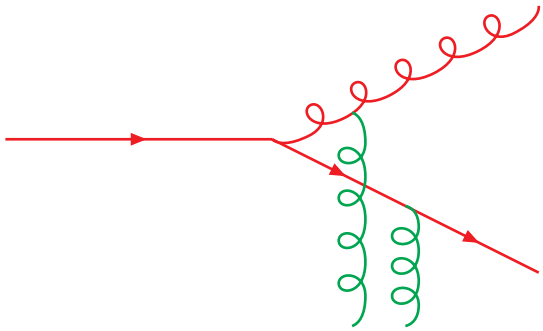
    to give the emission scale q^2.

- A number of formally free choices
  - Evolution scale: $q^2/k_\perp/\theta$
  - z: Energy, light-cone momentum.



Soft Emission

- There are two regions where the emission of QCD radiation is enhanced.
- The soft limit also factorizes but at the amplitude level.

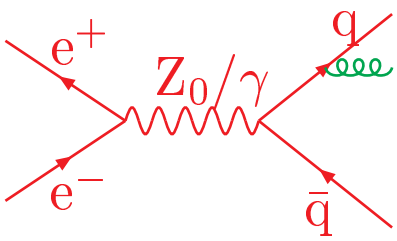


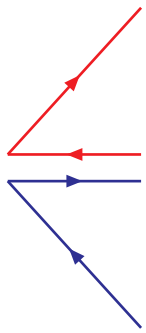
- Soft gluons come from everywhere in the event.
- There is quantum interference between these gluons.
- Does this spoil the parton shower picture?

Angular Ordering

- HERWIG uses the angular-ordered parton shower to resum both the collinear and soft logarithms.

- *e.g.* $e^+e^- \longrightarrow q\bar{q}g$ with radiation of an extra gluon

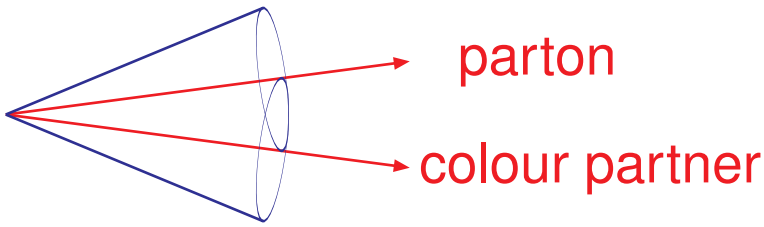




Colour Flow

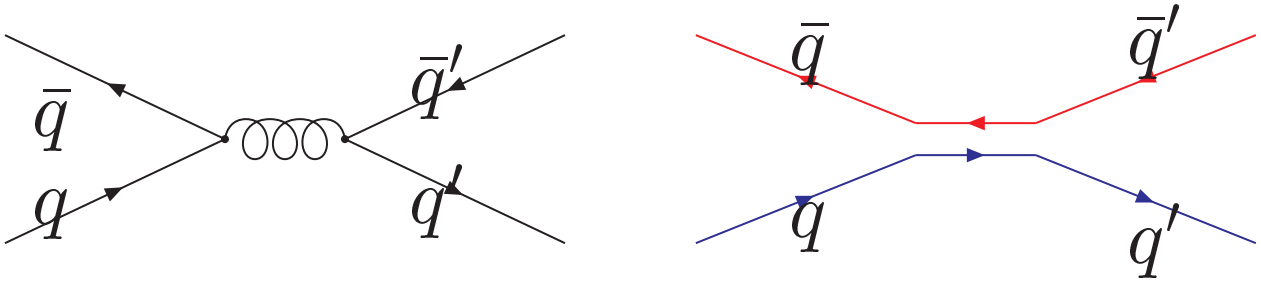
- We then get that
  - q and the g are colour connected.
  - $\bar{q}$ and the g are anticolour connected.

- QCD radiation only occurs in a cone up to the direction of the colour partner.
(Marchesini & Webber)



Colour Coherence Effects

- There have been a number of studies of colour coherence effects
  - CDF Phys. Rev. D50 5562;
  - D0 Phys. Lett. B414 419.



- If we define the jets in the order of their E_t
  - η_3 is the pseudo-rapidity of the third jet,
  - $\Delta\eta = \eta_3 - \eta_2$ pseudo-rapidity difference,
  - $\Delta\phi = \phi_3 - \phi_2$ azimuthal difference,
  - $R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$ distance in (η, ϕ),
  - $\Delta H = sgn(\eta_2)\Delta\eta$,
  - $\alpha = \tan^{-1}(\Delta H/|\Delta\phi|)$ azimuthal angle in $(|\Delta\phi|, \Delta H)$.

- Consider three jet events where the leading jet has $E_T > 110$ GeV.

Colour Coherence Effects

- η_3 Distributions

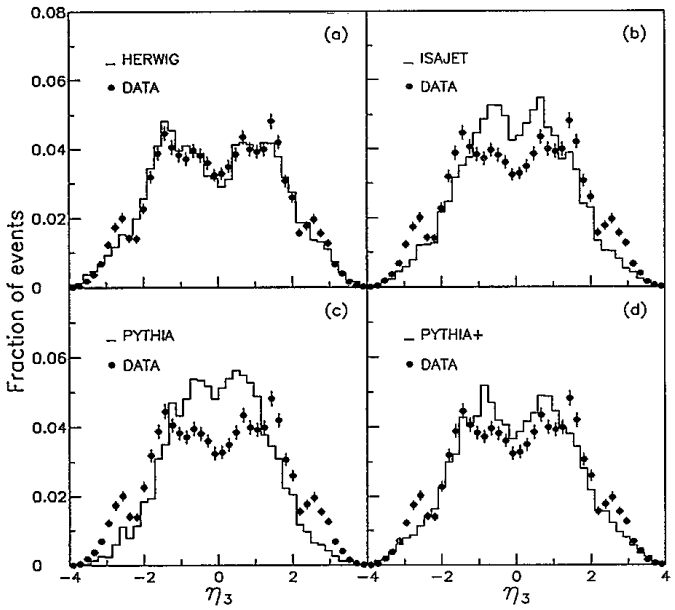


FIG. 13. Observed η_3 distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.

- α Distributions

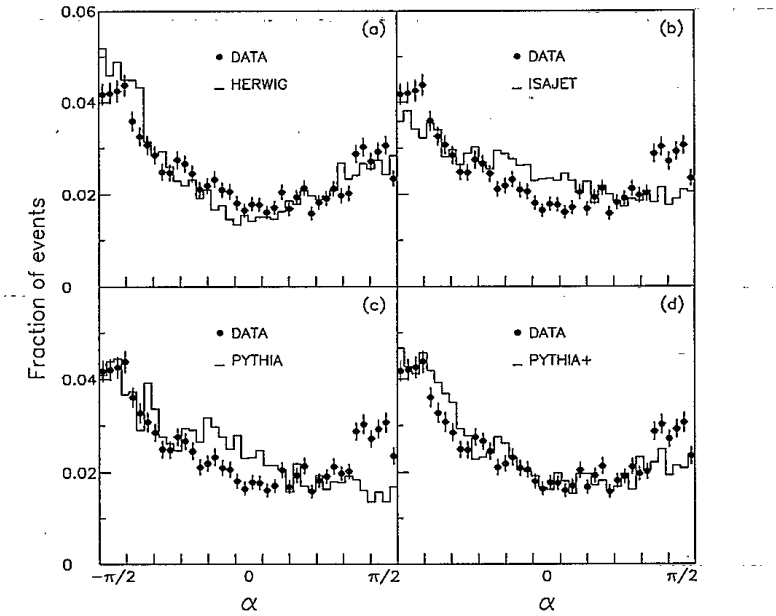
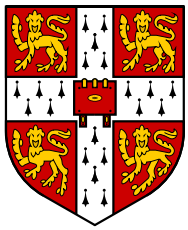


FIG. 15. Observed α distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.



Heavy Particle Decays

- After the parton shower phase those particles which decay before hadronization, *e.g.* the top quark and SUSY particles, are decayed.

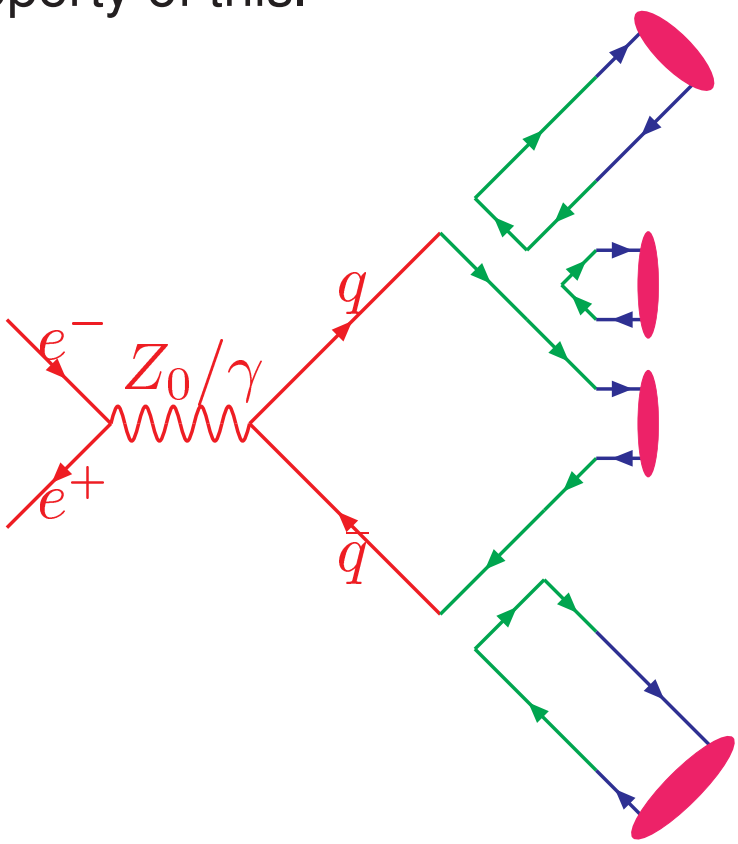
- These decays are usually performed using matrix elements to give the distributions of the decay products.

- However the production and decay of the particles are performed independently.

- This neglects spin correlation effects which may be important.

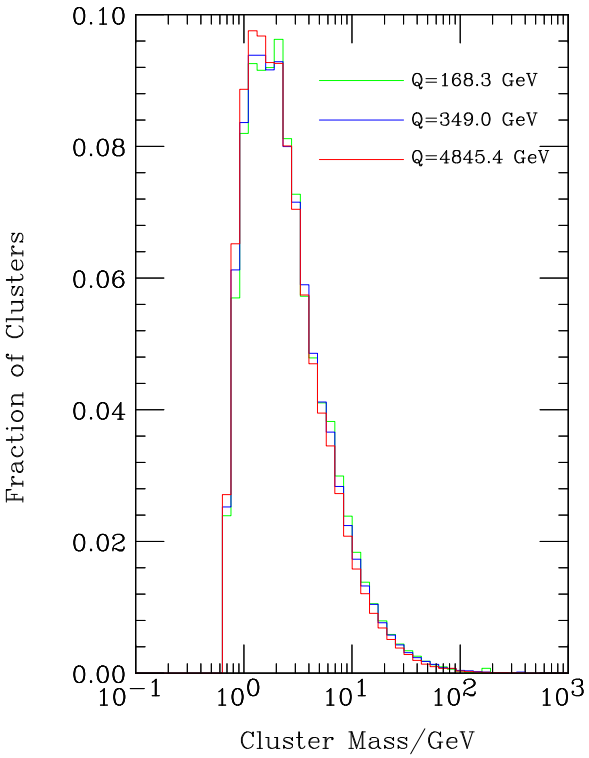
Cluster Hadronization Model

- HERWIG includes angular-ordered parton showers.
- The hadronization model is based on one property of this.

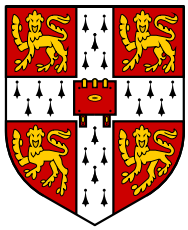


- First split the gluons into $\mathrm{q\bar{q}}$ pairs.
- Each quark is then uniquely paired with an antiquark in a colour singlet.
- Form these into colour singlet clusters.

Cluster Hadronization Model



- The mass spectrum of pairs of colour singlet clusters after gluon splitting.
- Called colour 'pre-confinement'.
- Assume these clusters are superpositions of resonances.
- Decay according to phase space to hadrons.
- The hadrons decays are then performed using phase space and the measured branching ratios.

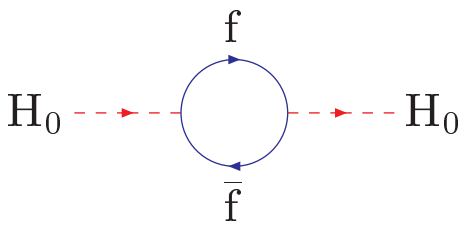


Need for BSM physics

- Why do we need physics beyond the Standard Model?
- The Standard Model is in remarkable agreement with all the experimental data.
- However it doesn't explain:
  - The masses of the particles and other parameters of the model.
  - What about gravity?
- These are hard questions and theorists haven't found solutions.
- Instead there's been a great deal of work answering
  - Why is the electroweak scale so much less than the GUT/Planck scales?
- This is called the hierarchy problem.

Hierarchy Problem

- If we look at the Higgs mass there are quantum corrections.

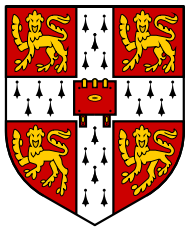


- This gives a correction to the Higgs mass,

$$\delta M_{H\,f}^2 = i\frac{|g_f|^2}{4}\int \frac{d^4k}{(2\pi)^4}\frac{\mathrm{tr}\left[(\not{k}+\not{p}+m_f)(\not{k}+m_f)\right]}{\left[(k+p)^2-m_f^2\right]\left[k^2-m_f^2\right]}.$$

- Need to introduce an ultra-violet cut-off to regularize the integral

$$\delta M_{H\,f}^2 = \frac{|g_f|^2}{16\pi^2}\left[-2\Lambda^2+6m_f^2\ln\left(\Lambda/m_f\right)\right].$$



Hierarchy Problem

- So either the Higgs mass is the GUT/Planck scale or there is a cancellation

$$M_H^2 = M_{H\,\mathrm{bare}}^2 + \delta M_H^2,$$

  of over 30 orders of magnitude to have a light Higgs.

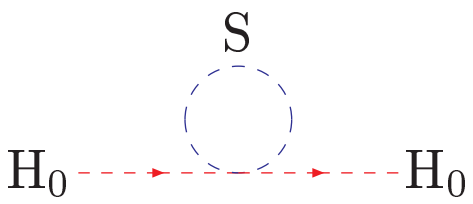
- This worries a lot of theorists.

- A number of solutions have been proposed.

- I will only consider Supersymmetry.

Supersymmetry

- If there is a scalar loop in the Higgs propagator



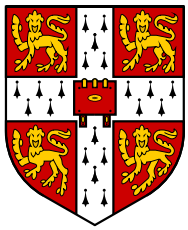
- We get a new contribution to the Higgs mass,

$$\delta M_{HS}^2 = \frac{\lambda_s}{16\pi^2}\left(\Lambda^2 - 2M_S^2 \ln\left(\Lambda/M_S\right)\right).$$

- So if there are two scalars for every fermion, with the same mass and $\lambda_s = |g_f|^2$ the quadratic dependence cancels.

- Theorists like to have symmetries to explain cancellations like this: Supersymmetry.

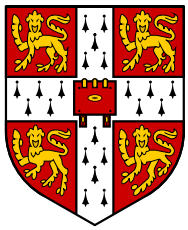
- For every fermion degree of freedom there is a bosonic degree of freedom.



Supersymmetric Particles

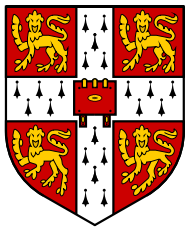
SM particle	Spin	SUSY particle	Spin	
Electron	1/2	Selectron	0	
Neutrino	1/2	Sneutrino	0	
Up	1/2	Sup	0	
Down	1/2	Sdown	0	
Gluon	1	Gluino	1/2	
Photon	1	Photino	1/2	
Z	1	Zino	1/2	Neutralinos
Higgs	0	Higgsino	1/2	
W^+	1	Wino	1/2	Charginos
H^+	0	Higgsino	1/2	

- One problem, the masses of the SUSY particles are the same as the SM ones.
- But they haven't been observed.
- Need to break SUSY in such a way that the Higgs mass doesn't depend quadratically on the cut-off.
- Called soft SUSY breaking.
- Introduces over 120 parameters into the model.



Supersymmetry breaking

- Need a model of where these parameters come from.
- Can't use the MSSM in simulations: too many parameters.
- Instead use models which predict these parameters from physics at higher energy scales.
- In all these models SUSY is broken in a hidden sector.
- The models differ in how this SUSY breaking is transmitted to the visible sector.

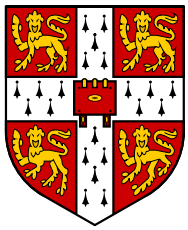


SUGRA

- SUSY breaking is transmitted via gravity.
  - All scalar masses (M_0) unified at the GUT scale.
  - All gaugino masses ($M_{1/2}$) unified at the GUT scale.
  - Universal A and B terms.
  - Use the known value of M_Z to constrain μ and B, leaves $\tan\beta = v_1/v_2$ as a parameter.

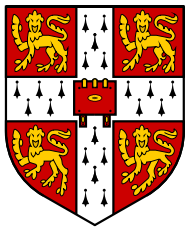
- Five parameters give the mass spectrum: M_0, $M_{1/2}$, $\tan\beta$, $\mathrm{sgn}\,\mu$, A.

- Gluino mass correlated with $M_{1/2}$ and slepton mass with M_0.



GMSB

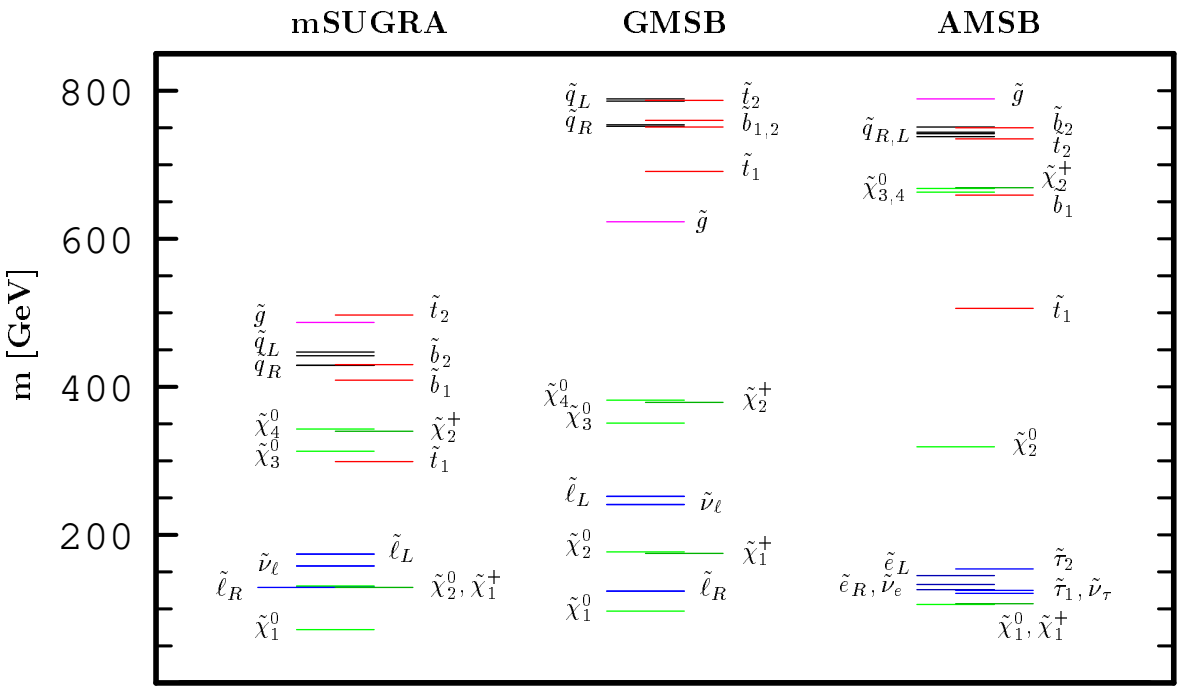
- Solves the flavour-changing neutral current problem by using gauge fields instead to gravity to transmit the SUSY breaking.
- Messenger particles, X, transmit the SUSY breaking.
- The simplest choice is a complete $SU(5)$ $\mathbf{5}$ or $\mathbf{10}$ to preserve the GUT symmetry.
- Fundamental SUSY breaking scale $\lesssim 10^{10}$ GeV.
- Gaugino masses occur at 1-loop, $M_{\tilde{g}} \sim \alpha_s N_X \Lambda$.
- Scalar masses at 2-loop, $M_{\tilde{q}} \sim \alpha_s^2 \sqrt{N_X} \Lambda$.
- True LSP almost massless gravitino.
- Lightest superpartner is unstable and decays to gravitino.
- Can be neutral, *e.g.* $\tilde{\chi}_1^0$, or charged, *e.g.* $\tilde{\tau}_1$.



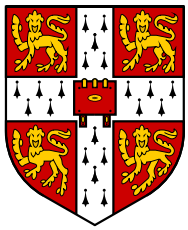
AMSB

- Superconformal anomaly is always present.
- Predicts sparticle masses in terms of $M_{3/2}$.
- Simplest version predicts tachyonic particles.
- Need another SUSY breaking mechanism to get a realistic spectrum.
- *e.g.* Add universal scalar masses (M_0) .
- The model has 4 parameters M_0, $M_{3/2}$, $\tan\beta$ and $\mathrm{sgn}\,\mu$.
- In this model the lightest chargino is almost degenerate with the lightest neutralino.

Mass Spectrum

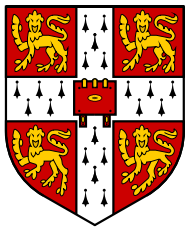


- The mass spectrum in the models is different.
- Main differences:
  - Mass splitting between gluino and electroweak gauginos;
  - Mass splitting of the squarks and sleptons;
  - Nature of the LSP.



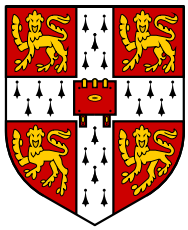
SUSY spectrum generation

- The first step of any SUSY simulation is to generate the full spectrum of the model.
- In most cases, particularly in hadron collisions, it is not possible to only consider one particle at a time.
- All the processes which are kinematically allowed must be simulated.
- In virtually all LHC simulations the main background to SUSY is SUSY.



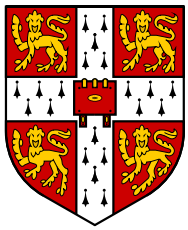
SUSY spectrum generation

- How do we calculate the spectrum
- Start with a model of SUSY breaking at the GUT scale.
- Evolve the parameters to the electroweak scale and imposes the condition of electroweak symmetry breaking and the measured value of M_Z. This changes the parameters.
- Evolve back to the GUT scale and impose the conditions of the model.
- Iterate until there is convergence.



SUSY Simulations

- Many of the problems in SUSY simulation are the same as simulating the SM.
- At the moment it is probably better to try and improve the simulation of the SM rather than simulate improve the simulation of SUSY.
- The problems of simulation SUSY are different for different experiments.
- At the Tevatron, where the signal is likely to be small (if there is one) understanding the SM backgrounds is critical.
- At the LHC, where (hopefully) the signals will have large cross sections and therefore hard cuts against the SM can be used, the main problem is simulating the QCD properties of the SUSY events themselves.
- At the NLC where SUSY will have hopefully already been discovered by the LHC and only the lower mass states will be accessible accurate simulation of the decays and correlations is vital.

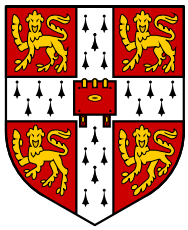


SUSY Simulations

- There are a number of SUSY simulations available.
  - ISAJET Many models and decays implemented, good treatment of SUSY but known to be poor in describing the SM.
  - PYTHIA Fewer models implemented and less sopisticated treatment of SUSY but much better at describing the SM.
  - HERWIG No implicit models, uses ISAJET to calculate spectrum and decay rates, now has good treatment for the decays in the event generation. Generally gives good (but not as good as PYTHIA) agreement with the SM.
  - SUSYGEN Mainly for lepton collisions. Sophisticated treatment of production and decay in e^+e^- collisions, uses PYTHIA for parton showers and hadronization.

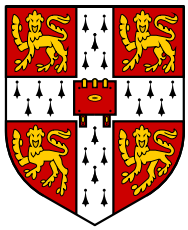
- Of all these generators only SUSYGEN had correlations between the production and decay of SUSY particles.

- However this treatment is limited as it relies on $2 \to n$ cross sections.



Spin Correlations

- In the production of heavy particles there are correlations between the production and decay of the particles due to their spin.
- These correlations are known to be important in tau production.
- Run II of the Tevatron should have sufficient statistics to measure the correlations in $\mathrm{t\bar{t}}$ production.
- These correlations are also important for $\mathrm{t\bar{t}}$ production at a future linear collider.
- One potentially important application is in gaugino production at a future linear collider.
  - Here both spin and polarization effects are important.
  - This affects the distribution of the decay products.
  - These distributions can be used to measure the fundamental parameters of the model.



Spin Correlations

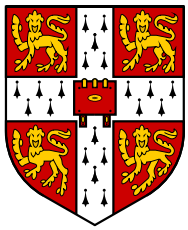
- There have been many studies of spin correlations
  - Gaugino Production in e^+e^- collisions
    G. Moortgat-Pick, *et.al*, Phys. Rev. D59 (1999) 015016, Eur. Phys. J. C7 (1999) 113, Eur. Phys. J. C9 (1999) 521, Eur. Phys. J. C18 (2000) 379.
  - Top production in hadron collisions
    A. Brandenburg Phys. Lett. B338 (1996) 626.

- Most of these studies used the same formalism.

- In the limit that the width of a particle is small relative to its mass we can neglect the width and keep the particles on mass shell.

- We can write the matrix element for a $2 \to 2$ process, including decays, as

$$\mathcal{M}_{\text{total}} = \frac{1}{m_a\Gamma_a}\frac{1}{m_b\Gamma_b}\sum_{\lambda_c\lambda_d}\mathcal{M}^{\lambda_c\lambda_d}_{ab\to cd}\mathcal{M}^{\lambda_c}_{c\ \text{decay}}\mathcal{M}^{\lambda_d}_{d\ \text{decay}}$$



Spin Correlations

- This means the amplitude squared can be written as

$$|\mathcal{M}_{\rm total}| = \frac{1}{m_a^2\Gamma_a^2}\frac{1}{m_b^2\Gamma_b^2}\rho^{\lambda_c\lambda_c'\lambda_d\lambda_d'}D_c^{\lambda_c\lambda_c'}D_d^{\lambda_d\lambda_d'}$$

  where

$$\rho_{\rm prod}^{\lambda_c\lambda_c'\lambda_d\lambda_d'} = \mathcal{M}_{ab\to cd}^{\lambda_c\lambda_d}\mathcal{M}_{ab\to cd}^{*\lambda_c'\lambda_d'},$$

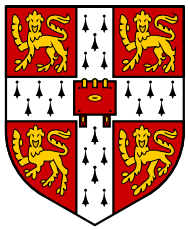
  is the unnormalized spin density matrix, and

$$D_c^{\lambda_c\lambda_c'} = \mathcal{M}_{c\ \rm decay}^{\lambda_c}\mathcal{M}_{c\ \rm decay}^{*\lambda_c'},$$

  the unnormalized decay matrix.

- Most authors then chose to evaluate these matrices analytically.

- However it will be easier for our purposes to evaluate the matrix elements using helicity amplitude techniques and then calculate the matrices numerically.

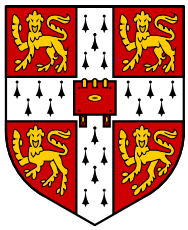


Spin Correlations

- These approaches require the production and decay of the particle to be generated at the same time.

- This is difficult if:
  - We want to generate QCD radiation from the particles before they decay;
  - The particle has many decay modes;
  - There are many sequential decays;
  - There are many final state particles.

- For a Monte Carlo event generator we need an algorithm which allows
  - The production and decay to be generated separately,
  - Has a complexity which only grows linearly with the number of final-state particles.



Helicity Amplitudes

- We will calculate all the cross sections and decays we will need as helicity amplitudes.

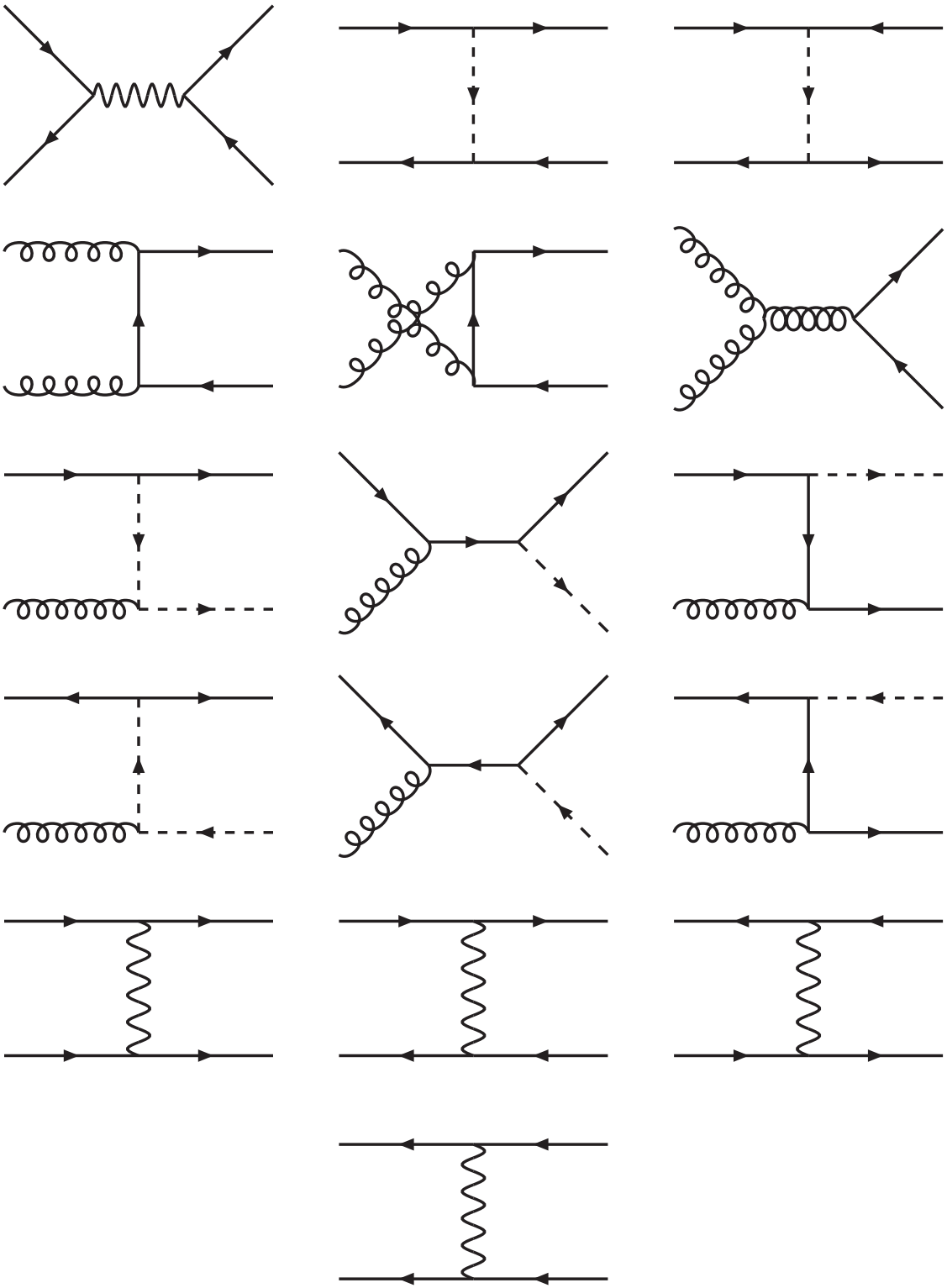
- The massive fermions will be defined as follows

$$u(p,\lambda) = \frac{1}{\sqrt{2p\cdot\ell}}(\not{p}+m)\,u_{-\lambda}(l),$$
$$v(p,\lambda) = -\frac{1}{\sqrt{2p\cdot\ell}}(\not{p}-m)\,u_{-\lambda}(l).$$

- Massive gauge bosons will not be included but replaced by their decay products.

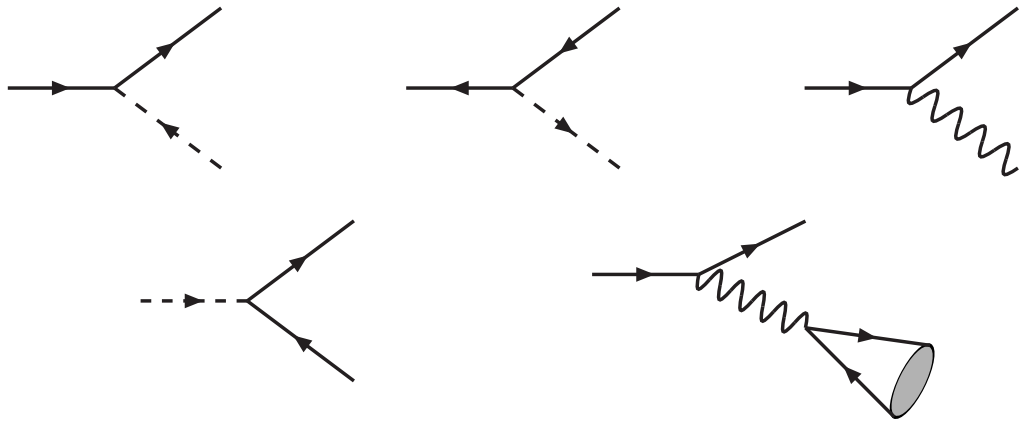
- This enables us to include the correlations by calculating a small number of helicity amplitudes for the production and decay of the particles.

Production Feynman Diagrams

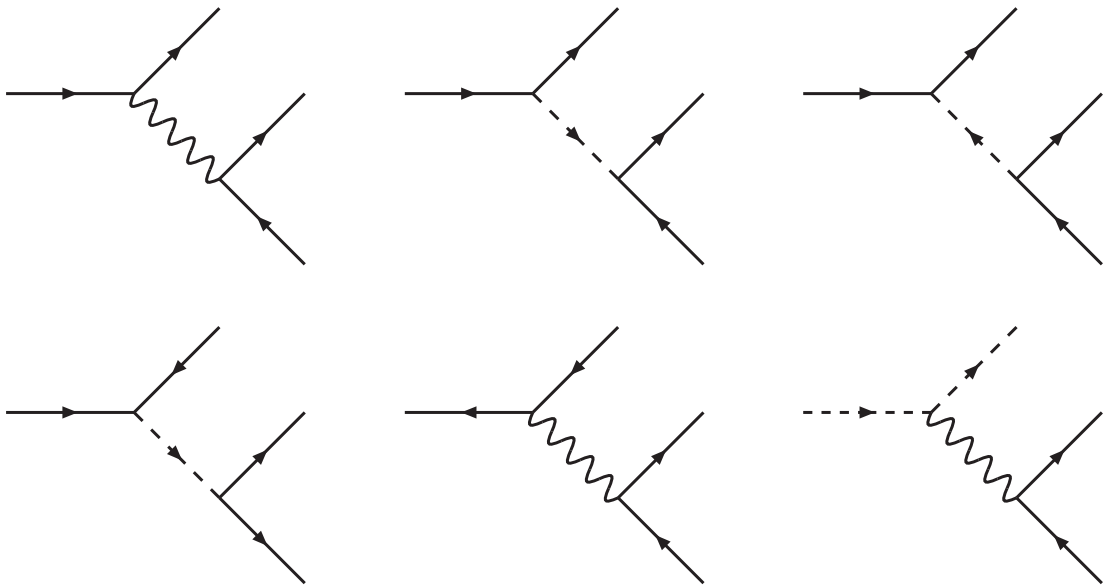


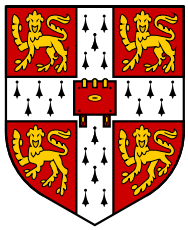
Decay Feynman Diagrams

- Two body decays



- Three body decays





Spin Correlations in the Decay of heavy particles

- HERWIG already includes correlations in and between jets using the algorithm of Collins and Knowles

  J. C. Collins, Nucl. Phys. B304, (1988) 794.

  I. G. Knowles, Nucl. Phys. B304, (1988) 767.

  I. G. Knowles, Nucl. Phys. B310, (1988) 571.

  I. G. Knowles, Comput. Phys. Commum. 58 (1990) 271.

- The algorithm for heavy particle decays is virtually the same.

Spin Correlations in the Decay of heavy particles

- The algorithm is as follows
- First the momenta of the particles in the hard collision are generated using

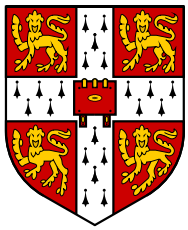
$$\rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\kappa_1'\kappa_2';\lambda_1'\ldots\lambda_n'}\prod_{i=1,n}D^i_{\lambda_i\lambda_i'}$$

  where $D^i_{\lambda_i\lambda_i'}=\delta_{\lambda_i\lambda_i'}$.

- One of the outgoing partons is selected at random and a spin density matrix calculated.

$$\begin{aligned}\rho_{\lambda_j\lambda_j'} &= \frac{1}{N_\rho}\rho^1_{\kappa_1\kappa_1'}\rho^2_{\kappa_2\kappa_2'}\prod_{i\neq j}D^i_{\lambda_i\lambda_i'} \\ &\quad \mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_j\ldots\lambda_n}\mathcal{M}^*_{\kappa_1'\kappa_2';\lambda_1'\ldots\lambda_j'\ldots\lambda_n'}\end{aligned}$$

  In the initial step we take $D^i_{\lambda_i\lambda_i'}=\delta_{\lambda_i\lambda_i'}$.
  The normalization is chosen so that $\mathrm{Tr}\rho=1$.



Spin Correlations in the Decay of heavy particles

- The decay mode of the particle is selected using the branching ratios.

- The momenta of the particles produced in the n-body decay are generated using

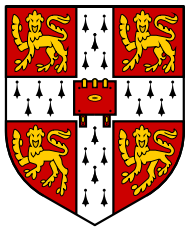
$$\rho_{\lambda_0\lambda_0'}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda_0';\lambda_1'\ldots\lambda_n'}\prod_{i=1,n}D^i_{\lambda_i\lambda_i'}$$

  where $D^i_{\lambda_i\lambda_i'}=\delta_{\lambda_i\lambda_i'}$.

- One of the particles is selected and a spin density matrix calculated

$$\rho_{\lambda_j\lambda_j'} = \frac{1}{N_{D\rho}}\rho_{\lambda_0\lambda_0'}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_j\ldots\lambda_n}\mathcal{M}^*_{\lambda_0';\lambda_1'\ldots\lambda_j'\ldots\lambda_n'}\prod_{i\neq j}D^i_{\lambda_i\lambda_i'}$$

  In the initial step we take $D^i_{\lambda_i\lambda_i'}=\delta_{\lambda_i\lambda_i'}$.
  The normalization is chosen so that $\mathrm{Tr}\rho=1$.



Spin Correlations in the Decay of heavy particles

- These steps are repeated until a stable particle is selected. The decay matrix of this particle is set to $\delta_{\lambda_i\lambda_i'}$ and another particle from the same decay selected to be decayed next.

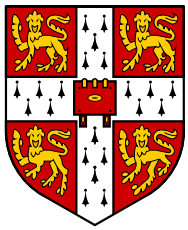
- When all the particles in a decay have been developed a decay matrix is calculated

$$D_{\lambda_0\lambda_0'} = \frac{1}{N_D}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda_0';\lambda_1'\ldots\lambda_n'}\prod_{i=1,n} D^i_{\lambda_i\lambda_i'}$$

  The normalization is chosen so that $\mathrm{Tr}D = 1$.

- We then select another particle from the decay which produced this particle and repeat the procedure and decay it.

- Eventually the hard process is reached and another particle selected. The whole procedure is repeated for this particle using the calculated decay matrices for those particles which have been decayed rather than the identity.



Examples

- Neutralino production in squark decay.
- Neutralino production in $\mathrm{e}^+\mathrm{e}^-$ collisions.
- Top quark pair production in $\mathrm{e}^+\mathrm{e}^-$ collisions.
- Chargino production in $\mathrm{e}^+\mathrm{e}^-$ collisions.

Neutralino Production in squark decay

- In most SUSY models the production of electroweak gauginos and sleptons at the LHC is dominated by production in squark decays rather than direct production.

- In many models the decay chain

$$\tilde{\mathrm{q}}_{\boldsymbol{L}} \to \mathrm{q}\tilde{\boldsymbol{\chi}}_{\boldsymbol{2}}^{\boldsymbol{0}} \to \mathrm{q}\tilde{\boldsymbol{\ell}}_{\boldsymbol{R}}^{\boldsymbol{\pm}}\boldsymbol{\ell}^{\mp} \to \mathrm{q}\boldsymbol{\ell}^{+}\boldsymbol{\ell}^{-}\tilde{\boldsymbol{\chi}}_{\boldsymbol{1}}^{\boldsymbol{0}}$$

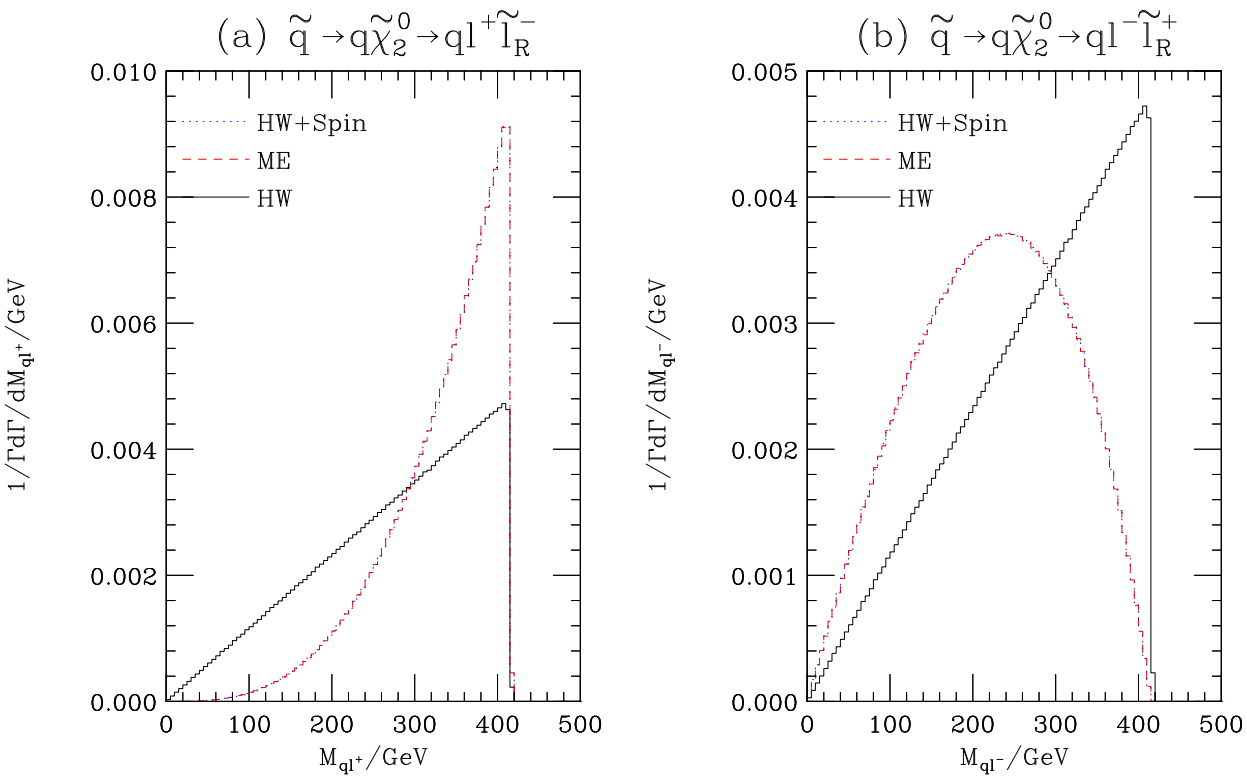
  is important as by measuring edges in the $\ell^+\ell^-$, $\mathrm{q}\ell$ and $\mathrm{q}\ell^+\ell^-$ distributions the masses of $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\ell}_R$ and $\tilde{\mathrm{q}}$ can be measured.

- Here the spin correlation algorithm must calculate the spin density matrix for $\tilde{\mathrm{q}}_L \to \mathrm{q}\tilde{\chi}_2^0$ and then use this to perform the decay $\tilde{\chi}_2^0 \to \tilde{\ell}_R^{\pm}\ell^{\mp}$.

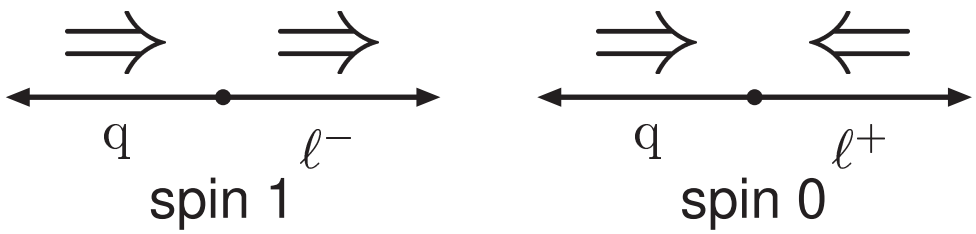
- The effects are most visible in the $\mathrm{q}\ell$ mass distribution.

- Used LHC SUGRA point 5 $M_0 = 200\ \mathrm{GeV}$, $M_{1/2} = 300\ \mathrm{GeV}$, $A_0 = 300\ \mathrm{GeV}$, $\tan\beta = 2.1$ and $\mathrm{sgn}\,\mu+$.

Neutralino Production in squark decay

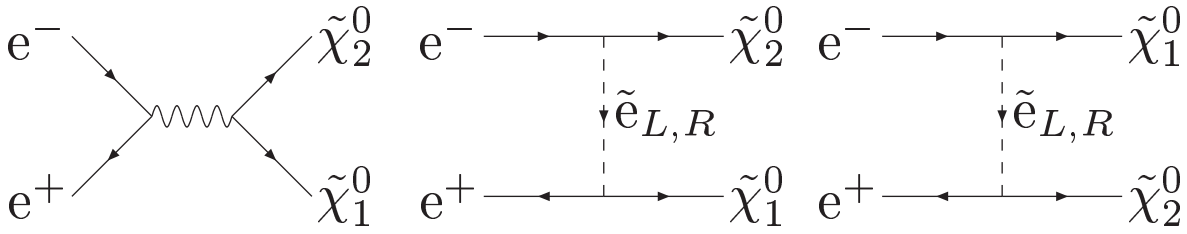


- The quark will always be left-handed and the lepton right-handed

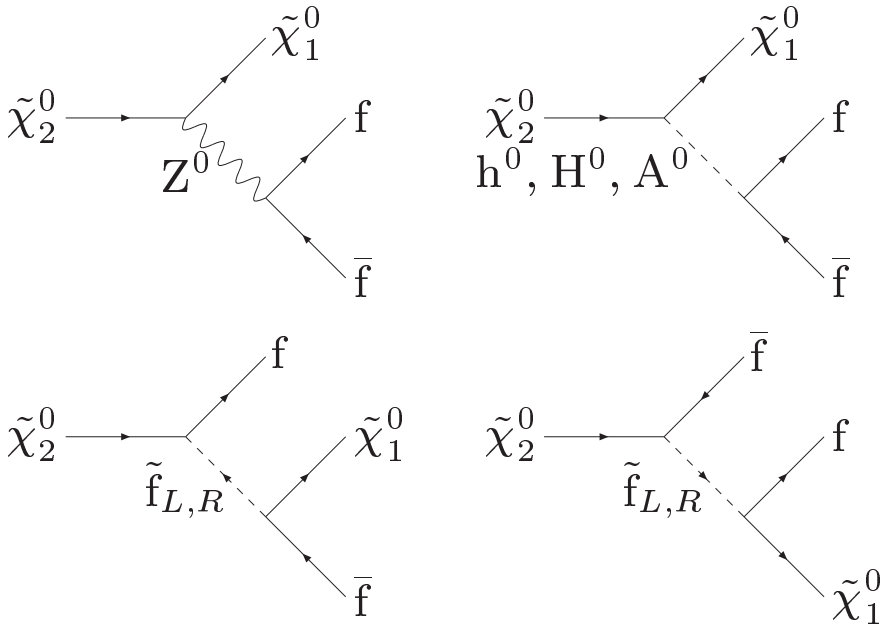


- As we can't experimentally distinguish quarks and antiquarks the average is observed.

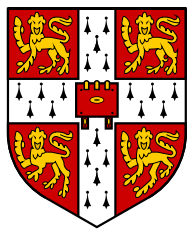
Neutralino Production in e^+e^- collisions



- This is a more complicated example.
- As the lightest neutralino is stable in R-parity conserving SUSY models the SUSY final state which requires the lowest centre-of-mass energy to produce it may be $\tilde{\chi}_2^0\tilde{\chi}_1^0$.
- The $\tilde{\chi}_2^0$ then decays $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 f\bar{f}$.



- The leptonic decay modes are the most important.

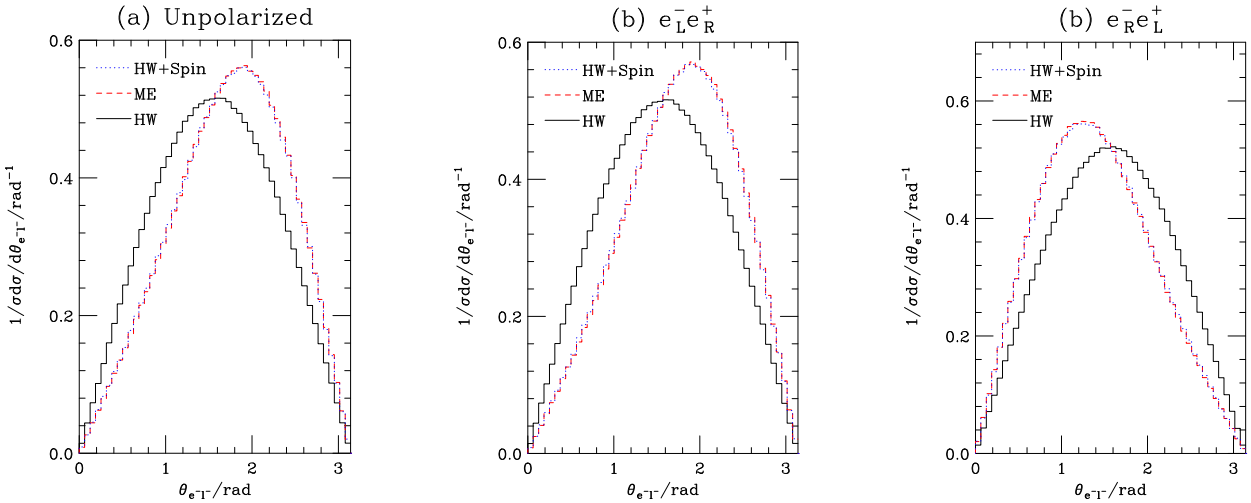


Neutralino Production in e^+e^- collisions

- Here the neutralino is produced in a $2 \to 2$ process, with the possibility of polarized beams so the calculation of the spin density matrix is more complicated.

- The decay of the neutralino, via a 2 or 3 body decay, is then performed using this decay matrix.

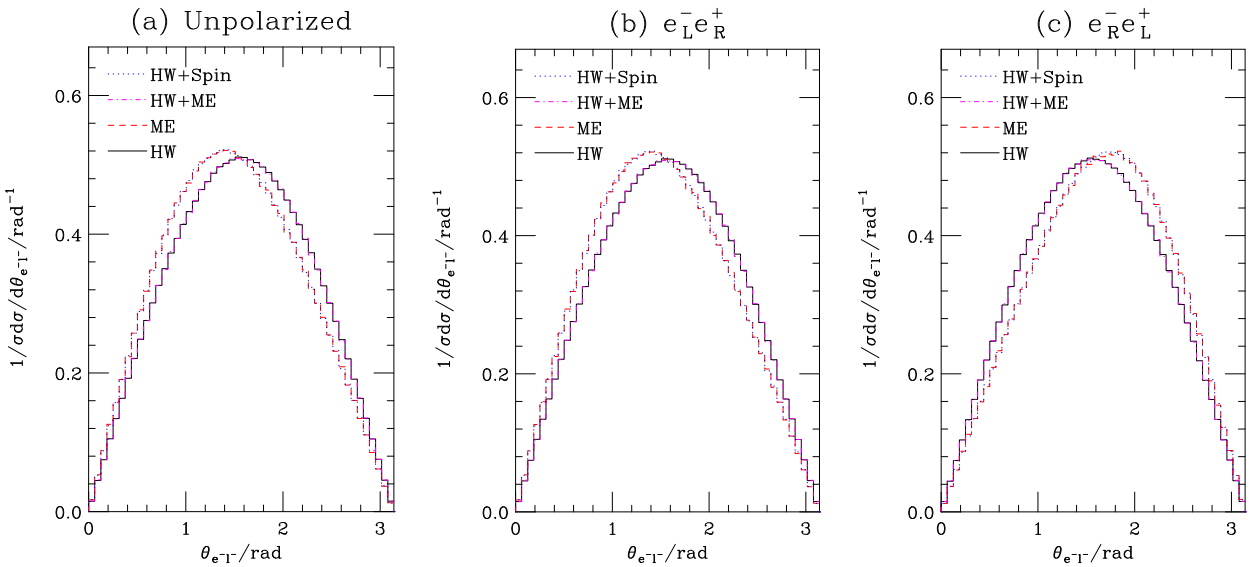
- First consider the two body decay $\tilde{\chi}_2^0 \to \tilde{\ell}_R^+ \ell^-$ and study the angle between the lepton and the beam, again using SUGRA point 5.



Neutralino Production in e^+e^- collisions

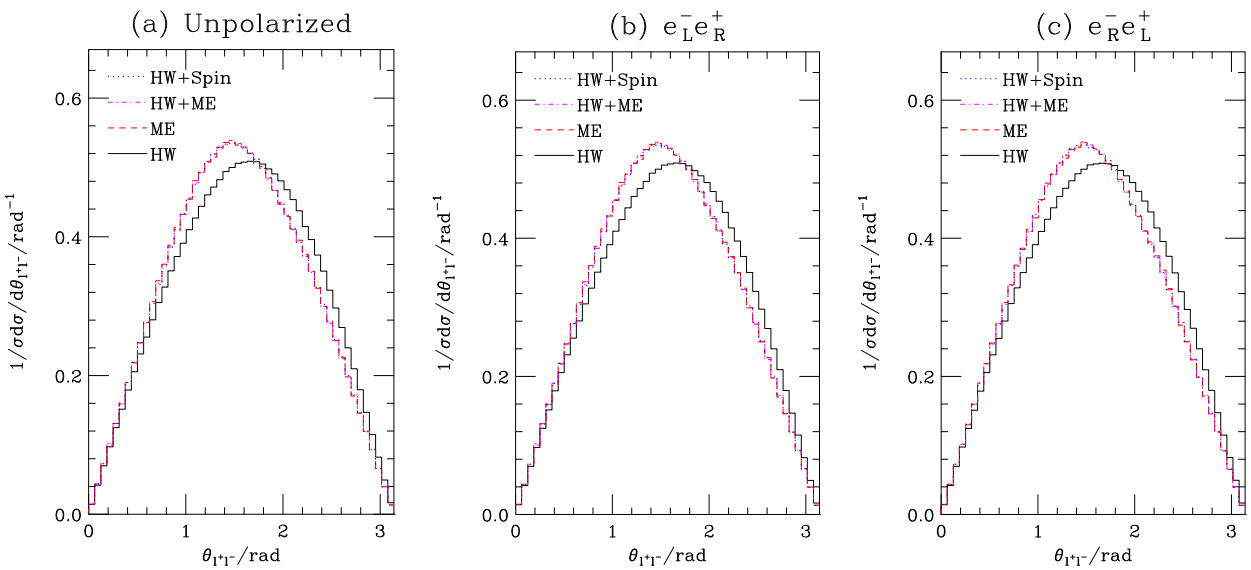
- We can also consider the three body decay $\tilde{\chi}_2^0 \to \ell^+\ell^-\tilde{\chi}_1^0$ using the SUSY parameters $M_0 = 210$ GeV, $A_0 = 0$ GeV, $\tan\beta = 10$, $M_1 = 450$ GeV, $M_2 = 350$ GeV, $M_3 = 350$ GeV.

- Study the angle between the lepton and the beam,



Neutralino Production in e^+e^- collisions

- and between the lepton and antilepton.

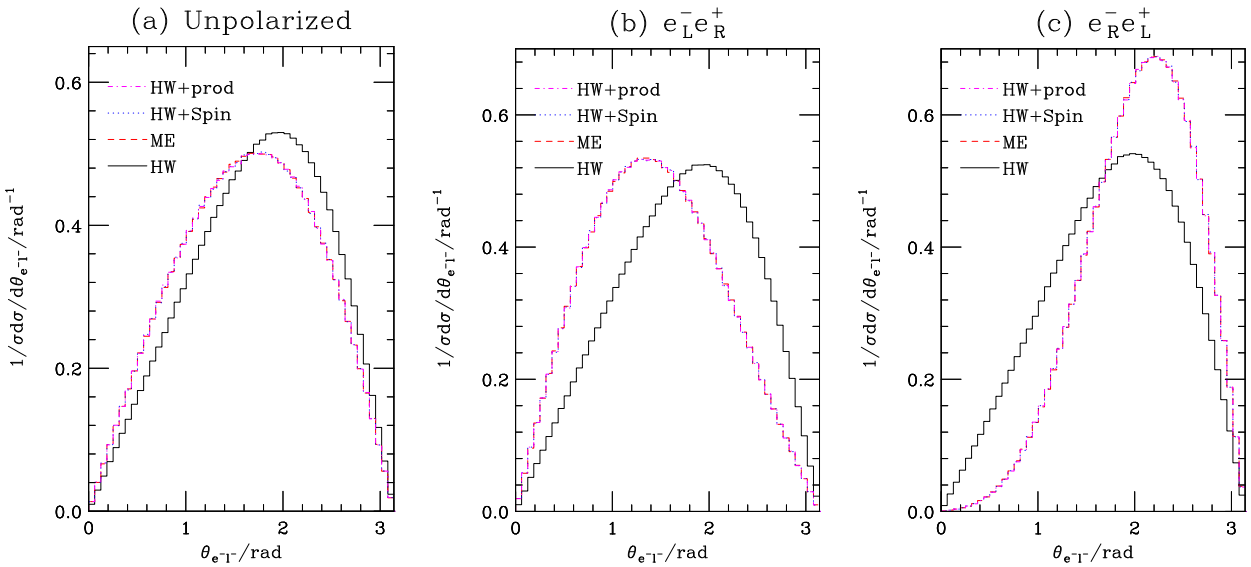


Top pair production in e^+e^- collisions

- In this example in addition to the calculation of the spin density matrix for the production we also need to calculate the decay matrix for the first decay and use this in performing the second decay.

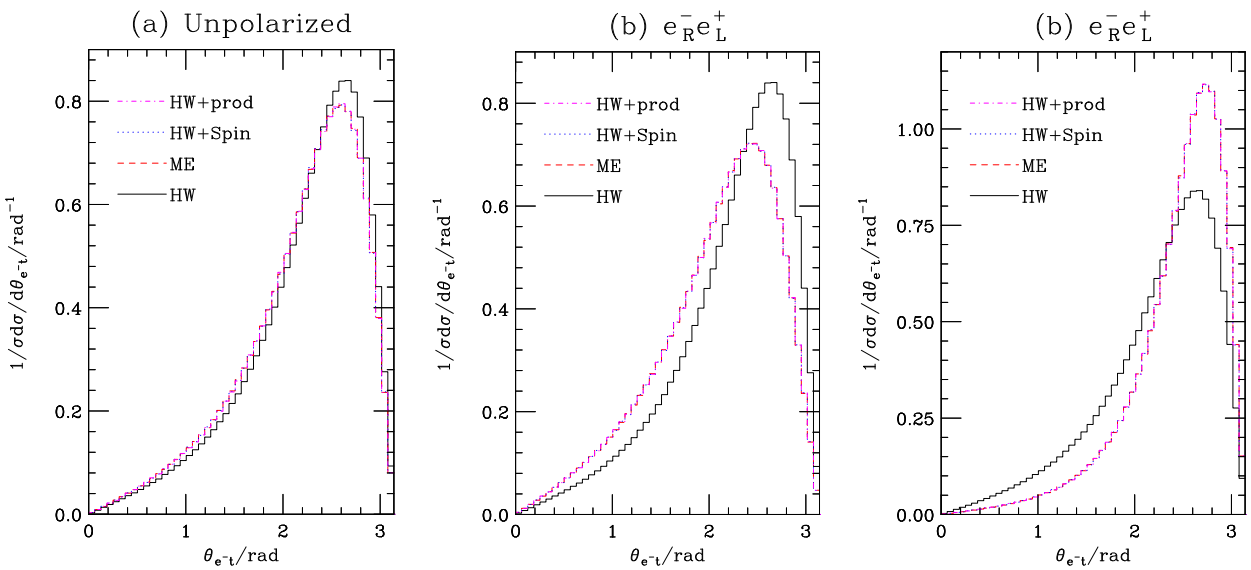
- Study the semi-leptonic decays of both top quarks and look at the correlations.

- Between the lepton produced in the antitop decay and the beam.

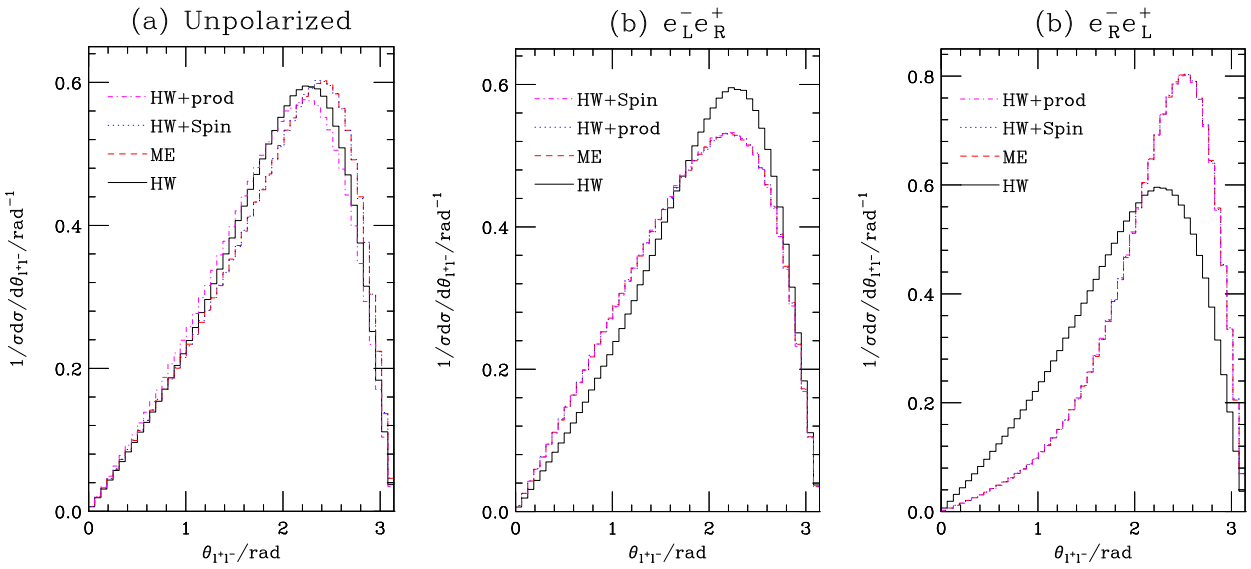


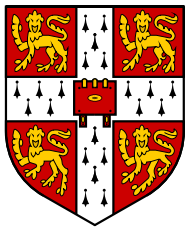
Top pair production in e^+e^- collisions

- Between the lepton and the top quark.



- Between the lepton and antilepton.





Top pair production in e^+e^- collisions

- The correlation between the lepton and the beam or top quark is only affected by the production mechanism.

- The correlation between the lepton and antilepton is also effected by the decay of the first quark.

Chargino production in e^+e^- collisions

- In a Monte Carlo simulation we often want to include the widths of the particles.
- Can't factorized the matrix element as before.
- We have two options
  - Use off-shell momenta and on-shell mass

$$\sum_\lambda u_\lambda(p)\bar{u}_\lambda(p) = \not{p}+m+\frac{\not{l}}{2p\cdot l}(m^2-p^2)$$

  - Use off-shell momenta and off-shell mass

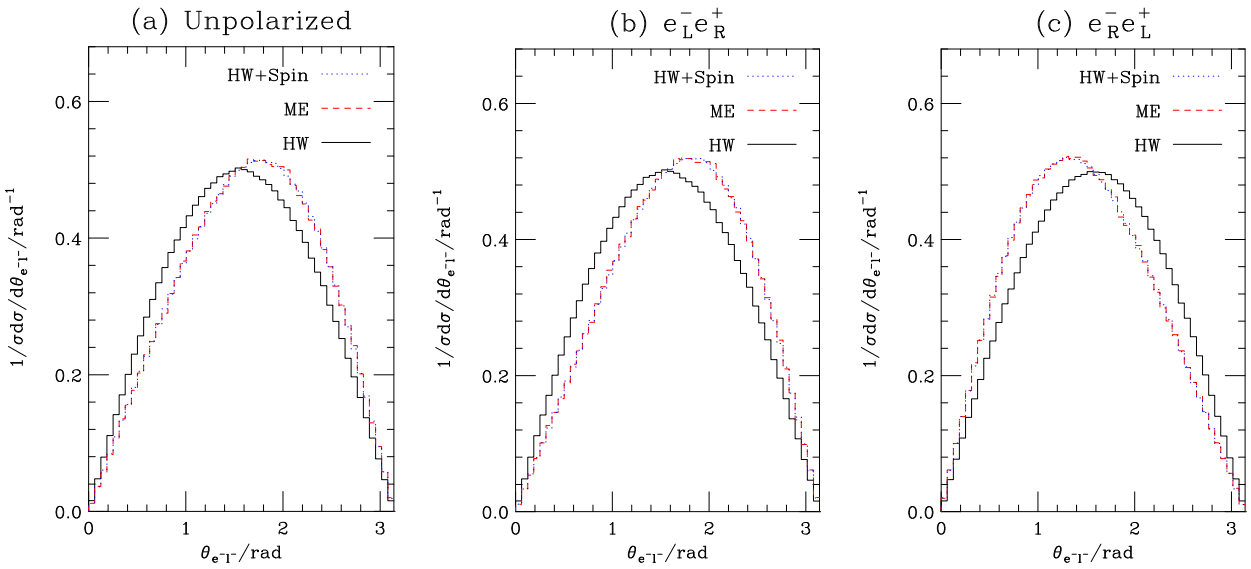
$$\sum_\lambda u_\lambda(p)\bar{u}_\lambda(p) = \not{p}+m_{\rm off},$$

- easiest to study effects in $\tilde{\chi}_1^\pm\tilde{\chi}_2^\mp$ production.
- At point we studied the $\tilde{\chi}_1^\pm$ width is 73 MeV and the $\tilde{\chi}_2^\pm$ width is 2.9 GeV. We also considered ten times the $\tilde{\chi}_2^\pm$ width. The $\tilde{\chi}_1^\pm$ mass is 127.8 GeV and the $\tilde{\chi}_2^\pm$ mass is 357.6 GeV.

Chargino production in e^+e^- collisions

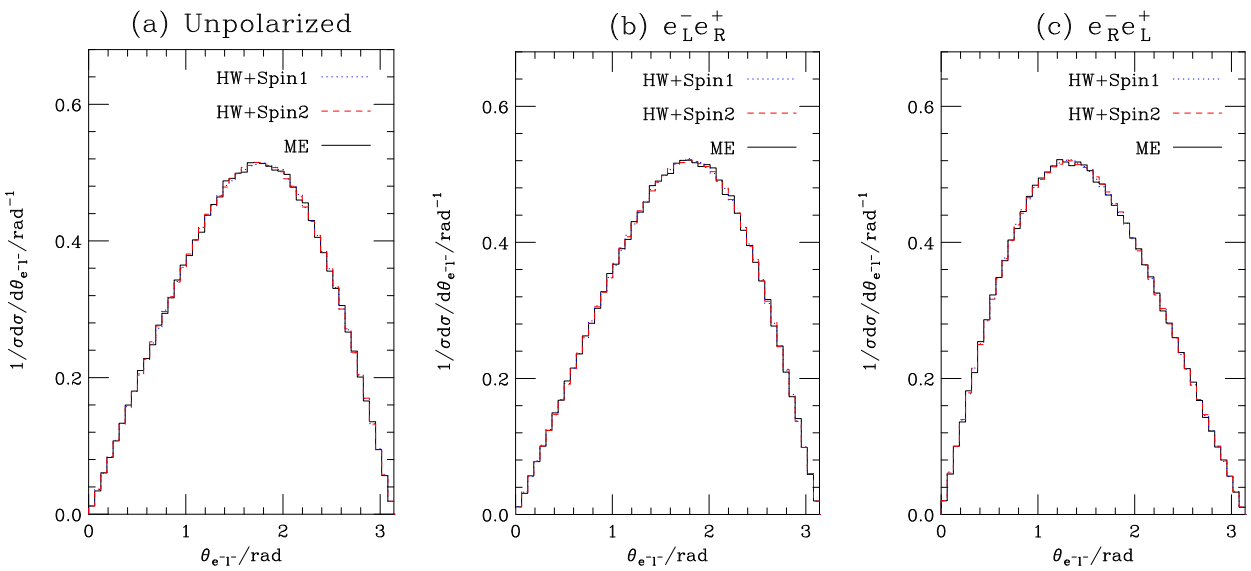
- Look at the angle between the beam and the lepton produced in decay of the $\tilde{\chi}_2^+$ via W exchange.

- No width

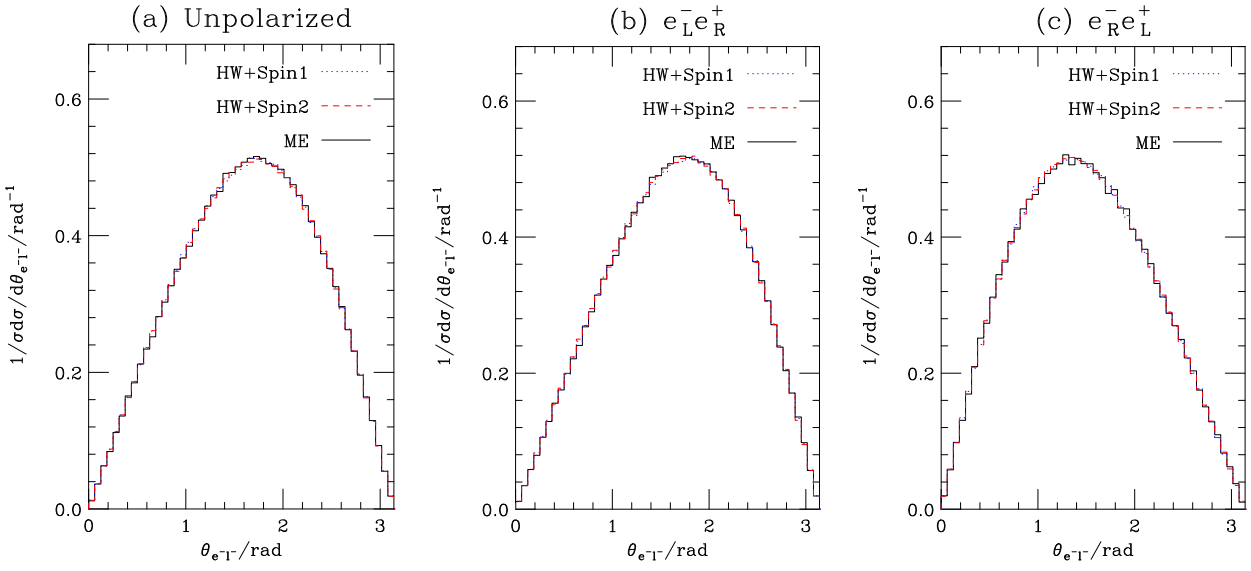


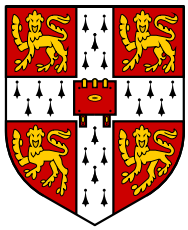
Chargino production in e^+e^- collisions

- Physical width



- Ten times physical width





Conclusions

- I have tried to explain the ideas behind event generation. It is important that people using these simulations have some idea of what the simulation is doing.

- The main principles of generating SUSY events are no different to generating SM events.

- I have shown an algorithm that allows us to include correlations between the production and decay of SUSY particles, and SM particles.

- The algorithm:
  - allows production and decay to be generated as separate steps;
  - has a complexity which only grows linearly with the number of particles.

- The results are in good agreement with matrix element calculations.

- This is now available in the new version of HERWIG *i.e.* 6.4.