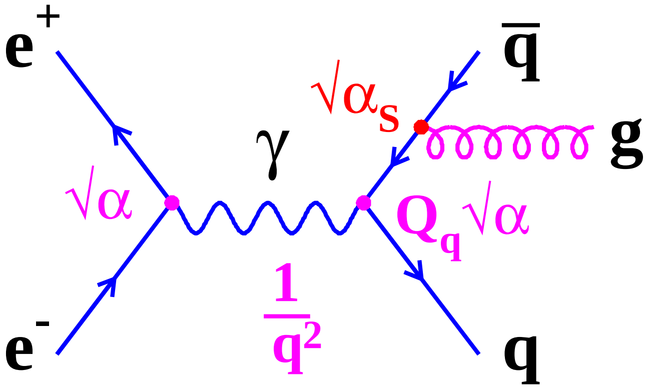
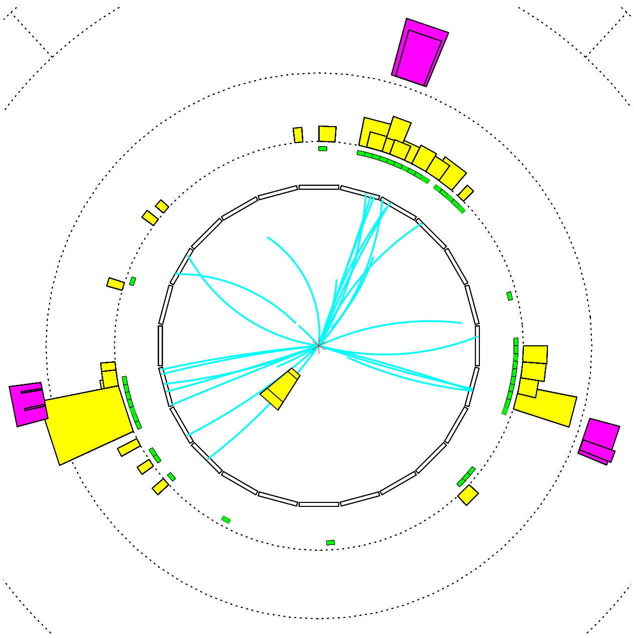
Particle Physics

Dr M.A. Thomson





Part II, Lent Term 2004

HANDOUT III

QCD

QUANTUM ELECTRODYNAMICS: is the quantum theory of the electromagnetic interaction.

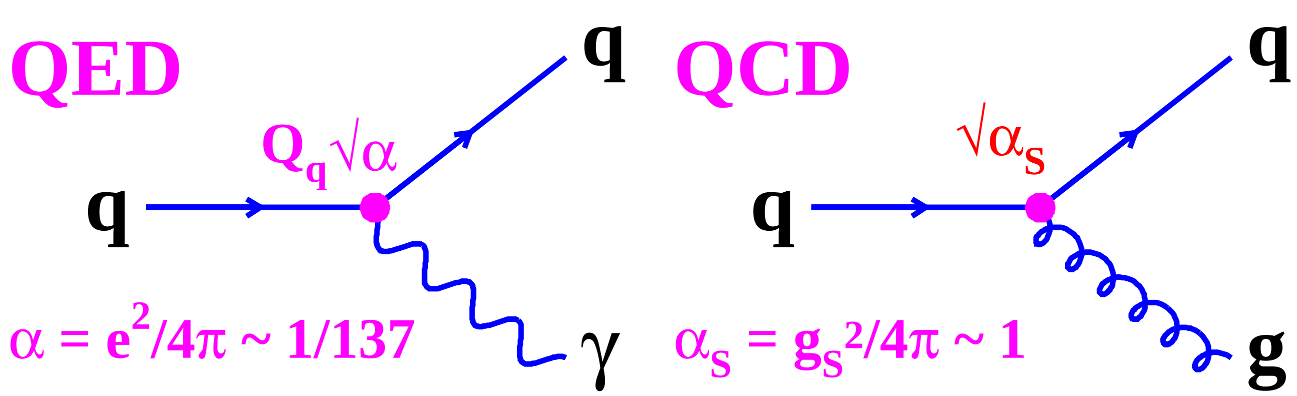
- mediated by massless photons
- photon couples to electric charge, e
- Strength of interaction : $\langle\psi_f|\hat{\mathrm{H}}|\psi_i\rangle \propto \sqrt{\alpha}$.

$$\alpha = \frac{e^2}{4\pi}$$

QUANTUM CHROMO-DYNAMICS: is the quantum theory of the strong interaction.

- mediated by massless gluons, *i.e.* $1/q^2$ propagator
- gluon couples to "strong" charge
- Only quarks have non-zero "strong" charge, therefore only quarks feel strong interaction

Basic QCD interaction looks like a stronger version of QED, $\alpha_S > \alpha_{EM}$



(subscript em is sometimes used to distinguish the α_{em} of electromagnetism from α_S).

COLOUR

In QED:

★ Charge of QED is electric charge.

★ Electric charge - conserved quantum number.

In QCD:

★ Charge of QCD is called "COLOUR"

★ COLOUR is a conserved quantum number with 3 VALUES labelled "red", "green" and "blue"

Quarks carry "COLOUR"	$r \; g \; b$
Anti-quarks carry "ANTI-COLOUR"	$\overline{r} \; \overline{g} \; \overline{b}$

Leptons, γ, $W^{\pm}$, Z^0 DO NOT carry colour, i.e. "have colour charge zero" $\longrightarrow$ DO NOT participate in STRONG interaction.

Note: Colour is just a label for states in a non-examinable SU(3) representation

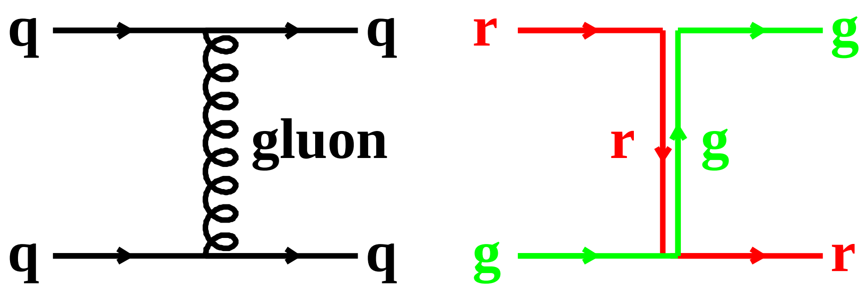
$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

GLUONS

In QCD:

★ Gluons are MASSLESS spin-1 bosons

Consider a red quark scattering off a green quark. Colour is exchanged but always conserved.



UNLIKE QED:

★ Gluons carry the charge of the interaction.

★ Gluons come in different colours.

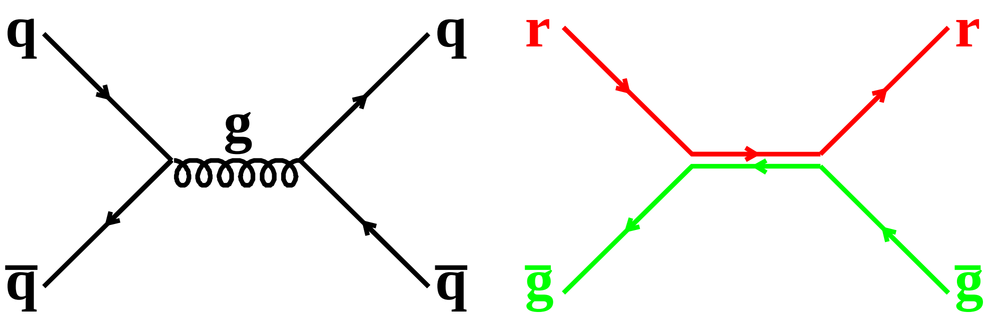
Expect 9 gluons (3 colours $\times$ 3 anti-colours)

$$r\overline{b}, r\overline{g}, g\overline{r}, g\overline{b}, b\overline{g}, b\overline{r}$$
$$r\overline{r}, g\overline{g}, b\overline{b}$$

However: Real gluons are orthogonal linear combinations of the above states. The combination $\frac{1}{\sqrt{3}}(r\overline{r} + g\overline{g} + b\overline{b})$ is colourless and does not take part in the strong interaction.

Colour at Work

EXAMPLE: $q\bar{q}$ Annihilation



Normally do not show colour on Feynman diagrams - colour is conserved.

QED POTENTIAL:

$$\boxed{V_{QED} = -\frac{\alpha}{r}}$$

QCD POTENTIAL:

At short distances QCD potential looks similar

$$\boxed{V_{QCD} = -\frac{4}{3}\frac{\alpha_S}{r}}$$

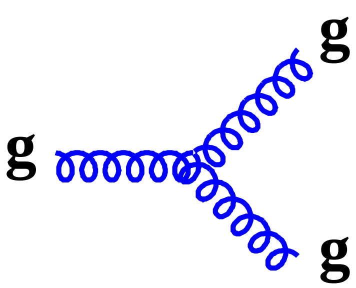
apart from $\frac{4}{3}$ colour factor.

Note: the colour factor (4/3) arises because more than one gluon can participate in the process $q \to qg$. Obtain colour factor from averaging over initial colour states and summing over final/intermediate colour states

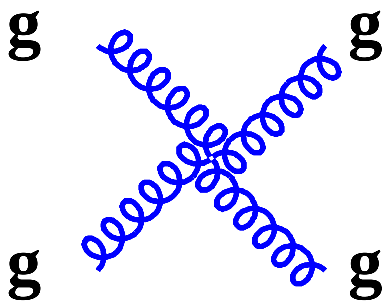
SELF-INTERACTIONS

At this point, QCD looks like a stronger version of QED. This is true up to a point. However, in practice QCD behaves very differently to QED. The similarities arise from the fact that both involve the exchange of **MASSLESS** spin-1 bosons. The big difference is that **GLUONS** carry **colour** "charge".

GLUONS CAN INTERACT WITH OTHER GLUONS:

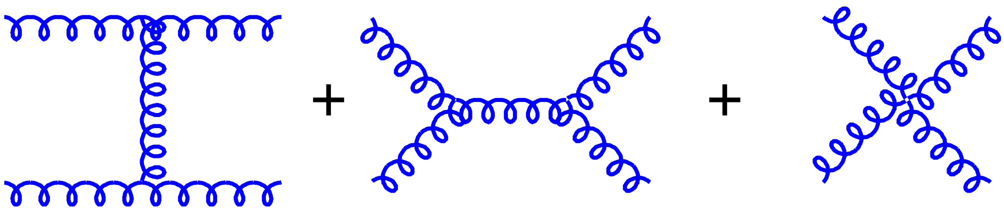


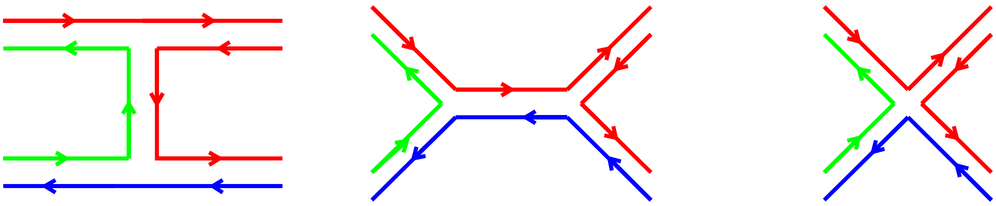
3 GLUON VERTEX



4 GLUON VERTEX

EXAMPLE: Gluon-Gluon Scattering $gg \rightarrow gg$





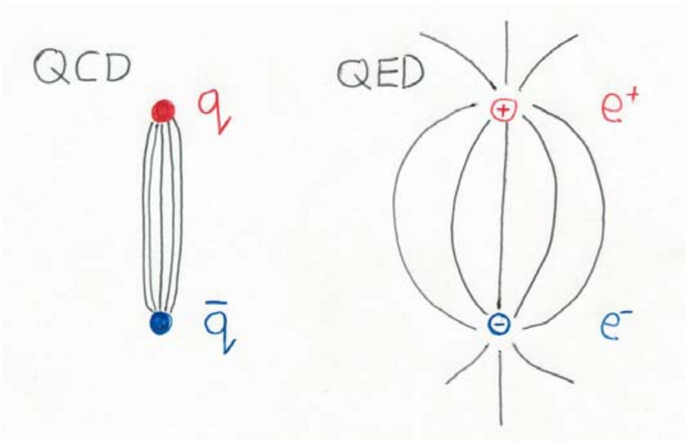
e.g. $r\bar{g} + g\bar{b} \rightarrow r\bar{r} + r\bar{b}$

CONFINEMENT

<u>NEVER OBSERVE:</u> **single FREE quarks/gluons**

- ★ **quarks are always confined within hadrons**
- ★ **This is a consequence of the strong self-interactions of gluons.**

Qualitatively, picture the colour field between two quarks. The gluons mediating the force act as additional sources of the colour field - they attract each other. The gluon-gluon interaction pulls the lines of colour force into a narrow tube or STRING. In this model the string has a 'tension' and as the quarks separate the string stores potential energy.



Energy stored per unit length $\sim$ constant.

$$V(r) \propto r$$

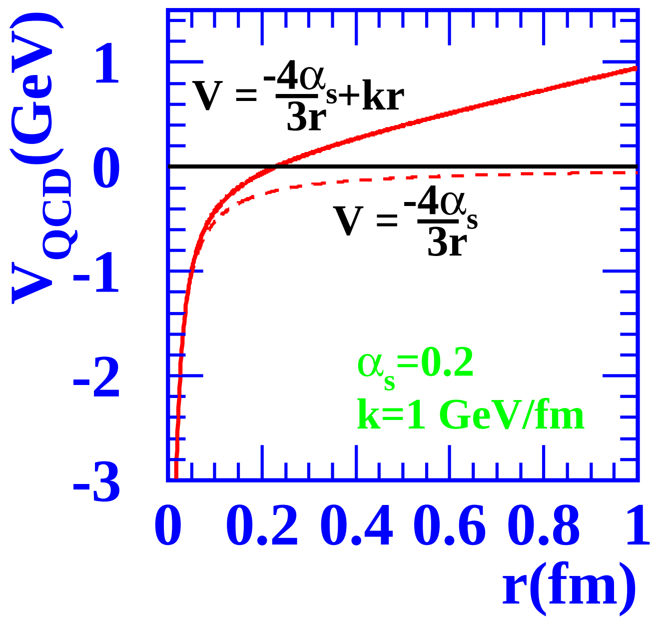
★ **Requires infinite energy to separate two quarks. Quarks always come in combinations with zero net colour charge: CONFINEMENT.**

How Strong is Strong ?

QCD Potential between quarks has two components:

★ "COULOMB"-LIKE TERM : $-\frac{4}{3}\frac{\alpha_S}{r}$

★ LINEAR TERM : $+kr$



Force between two quarks at separated by 10 m:

$$V_{QCD} = -\frac{\alpha_S}{r} + kr$$

with $k \approx 1\,\mathrm{GeV/fm}$

$$F = -\frac{dV}{dr} = \frac{\alpha_S}{r^2} + k$$

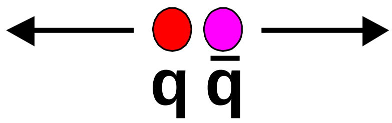
at large r $\quad F = k = \frac{1.6\times10^{-10}}{10^{-15}}\,N$

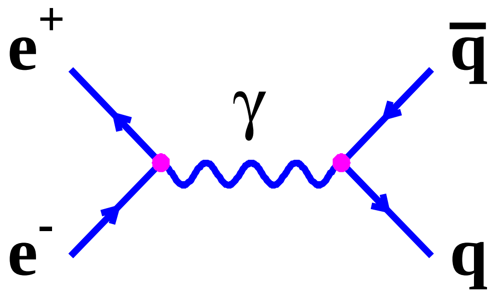
$$= 160000\,N$$

Equivalent to the weight of approximately 65 Widdecombes.

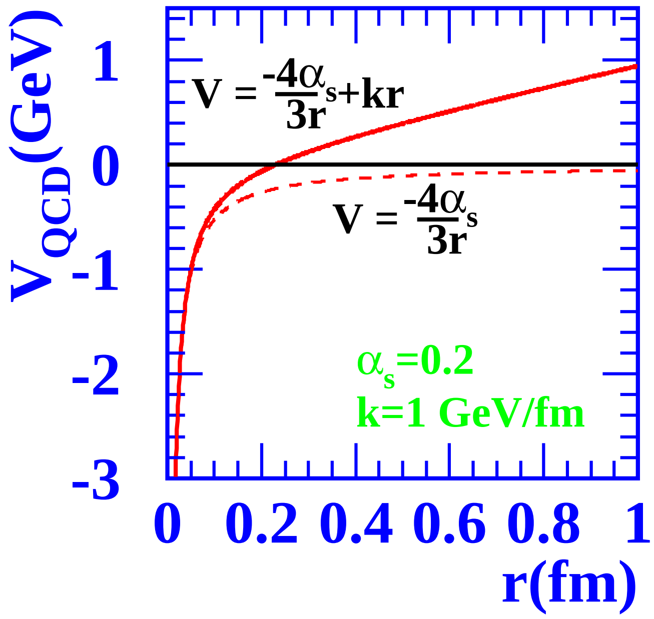
JETS

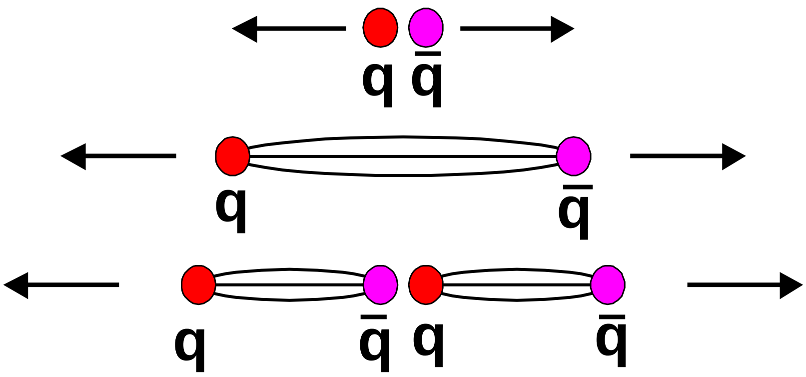
Consider the $q\bar{q}$ pair produced in $e^+e^- \to q\bar{q}$:



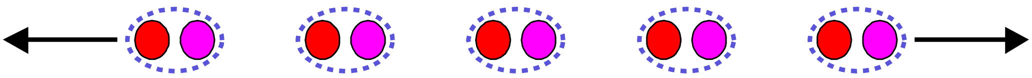


As the quarks separate, the energy stored in the colour field ('string') starts to increase linearly with separation. When $E_{stored} > 2m_q$ new $q\bar{q}$ pairs can be created.

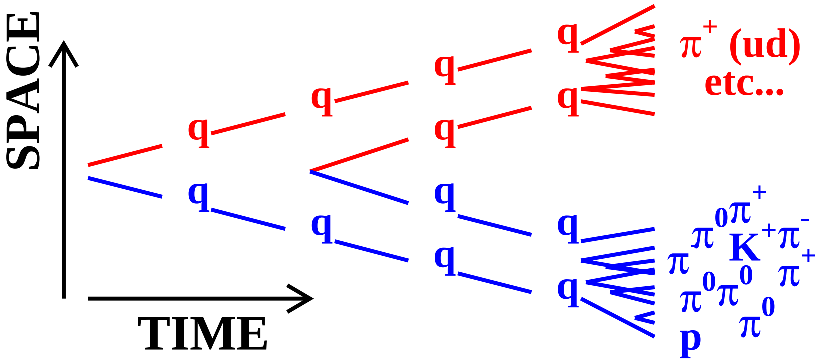


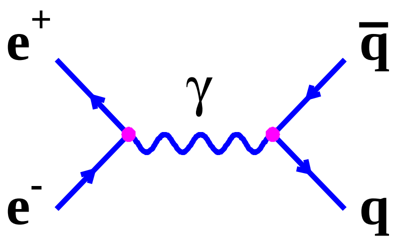


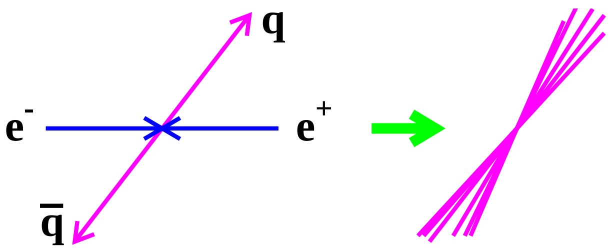
as energy decreases... hadrons freeze out



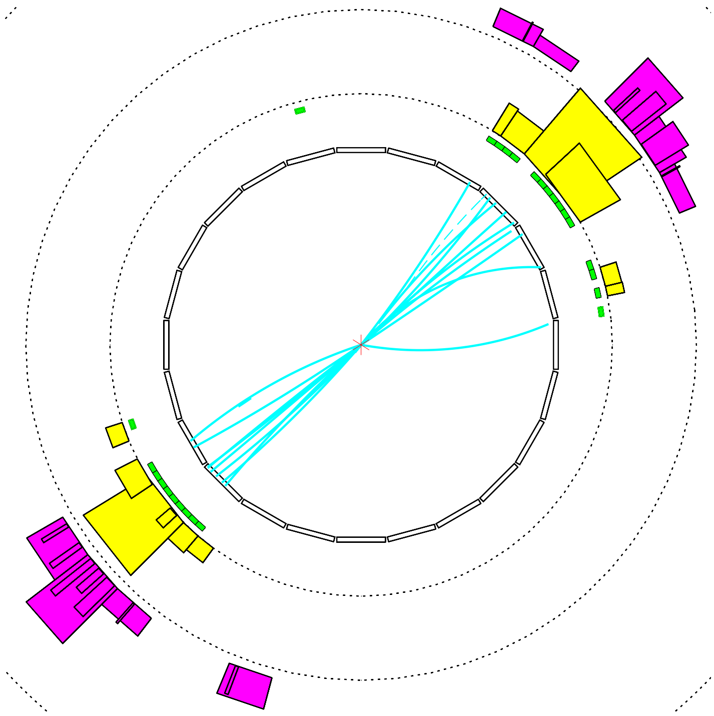
As quarks separate, more $q\overline{q}$ pairs are produced from the potential energy of the colour field. This process is called HADRONIZATION. Start out with quarks and end up with narrowly collimated JETS of HADRONS





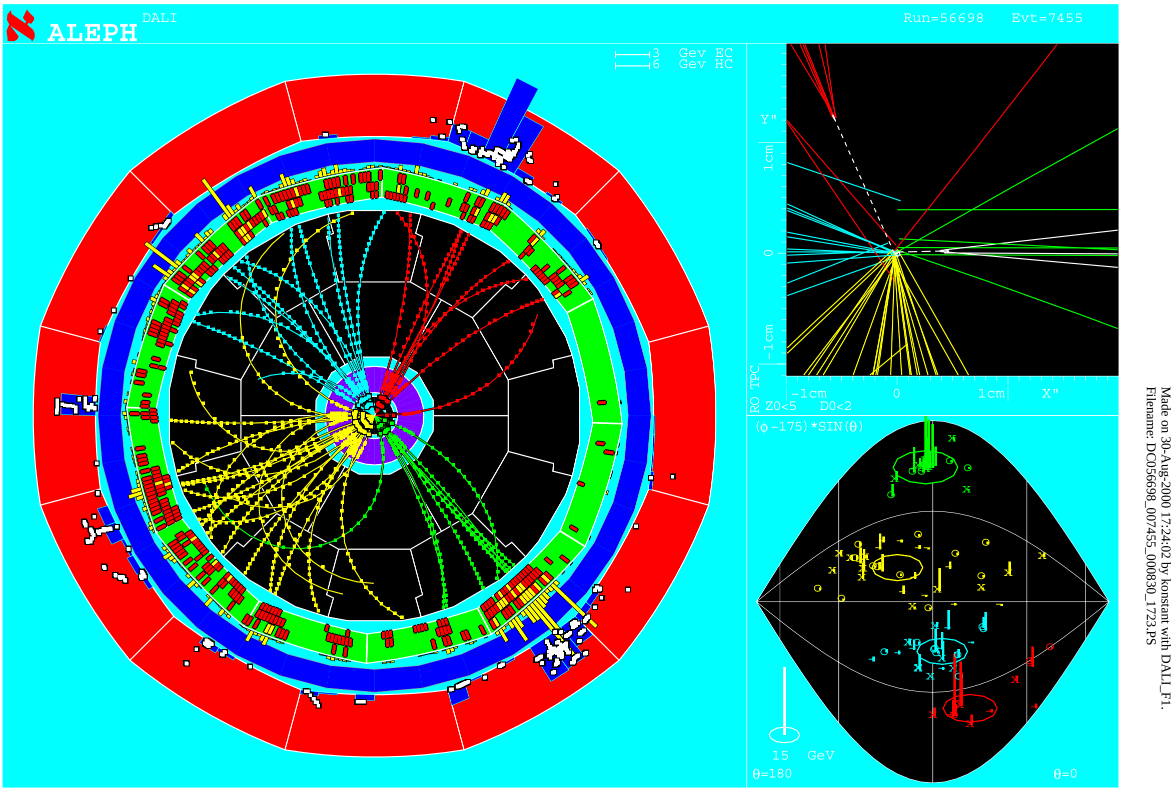


Typical $e^+e^- \rightarrow q\overline{q}$ Event



The hadrons in a quark(anti-quark) jet follow the direction of the original quark(anti-quark). Consequently $e^+e^- \rightarrow q\overline{q}$ is observed as a pair of back-to-back jets of hadrons

aside.....



★ **You will now recognize the "Higgs" event from the cover of Handout I as**

$$e^+e^- \rightarrow \textbf{something} \rightarrow q\bar{q}q\bar{q}$$

Running of α_S

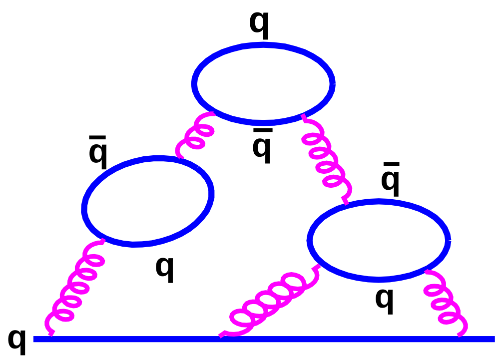
★ α_S specifies the strength of the strong interaction

★ BUT just as in QED, α_S isn't a constant, it "runs"

★ In QED the bare electron charge is screened by a cloud of virtual electron-positron pairs.

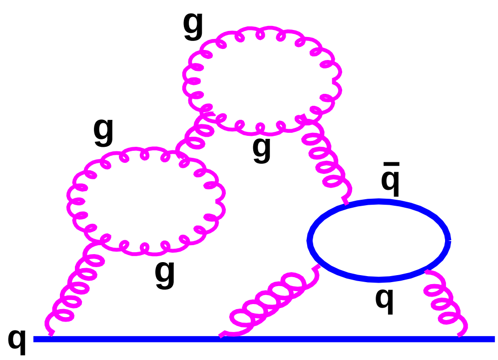
★ In QCD a similar effect occurs.

In QCD quantum fluctuations lead to a 'cloud' of virtual $q\overline{q}$ pairs



one of many (an infinite set) such diagrams analogous to those for QED.

In QCD the gluon self-interactions ALSO lead to a 'cloud' of virtual gluons



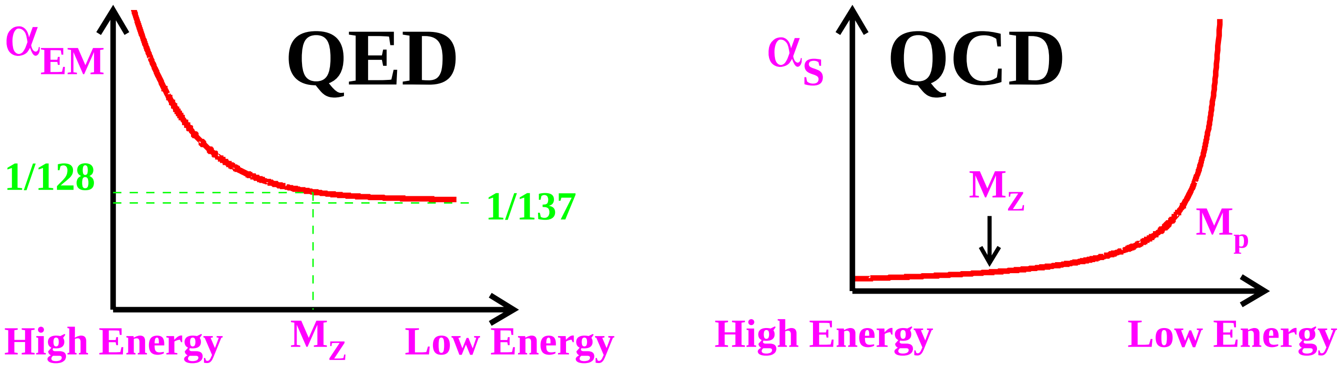
one of many (an infinite set) such diagrams. Here there is no analogy in QED, photons don't have self-interactions since they don't carry the charge of the interaction.

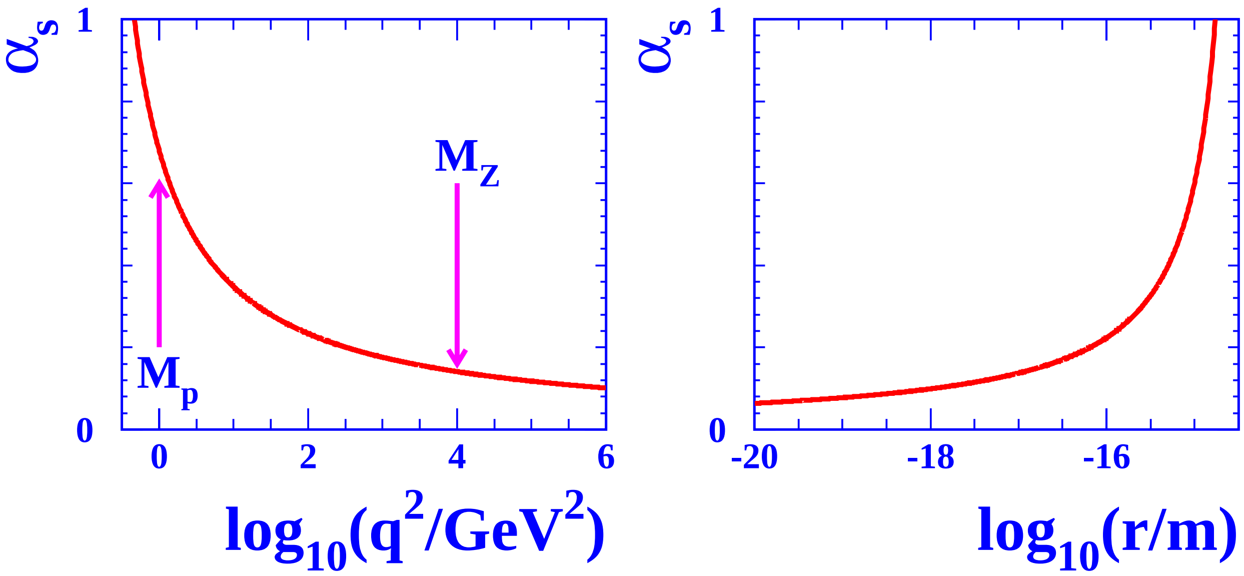
Colour Anti-Screening

★ Due to the gluon self-interactions bare colour charge is screened by both virtual quarks and virtual gluons

★ The cloud of virtual gluons carries colour charge and the effective colour charge INCREASES with distance !

★ At low energies (large distances) α_S becomes large $\longrightarrow$ can't use perturbation theory (not a weak perturbation)





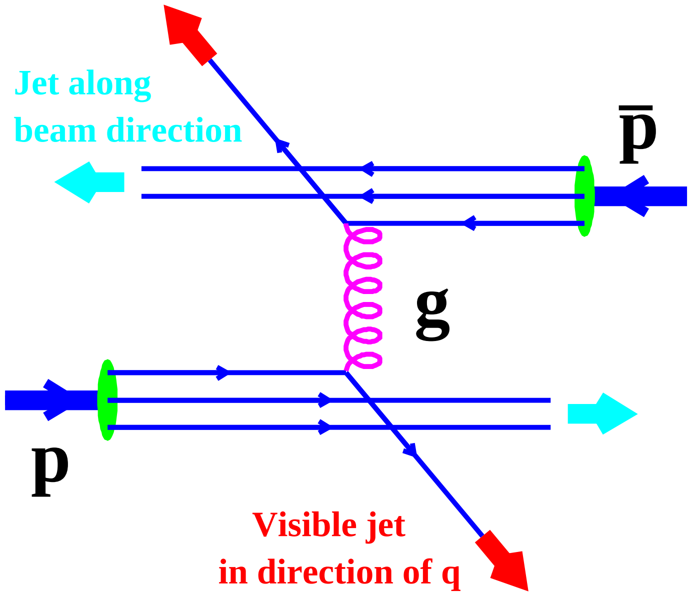
★ At High energies (short distances) α_S is small. In this regime treat quarks as free particles and can use perturbation theory

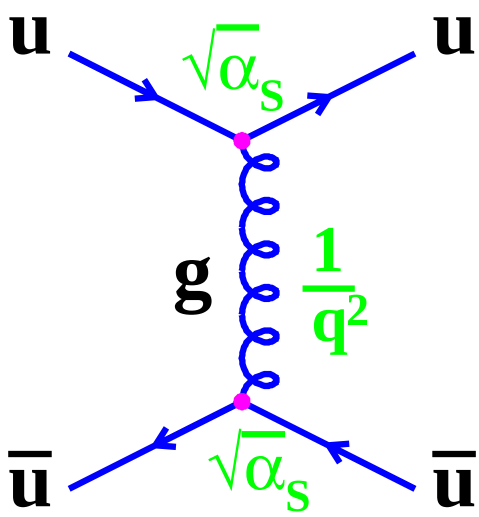
ASYMPTOTIC FREEDOM

★ At $\sqrt{s} = 100\ \mathrm{GeV}, \alpha_S = 0.12.$

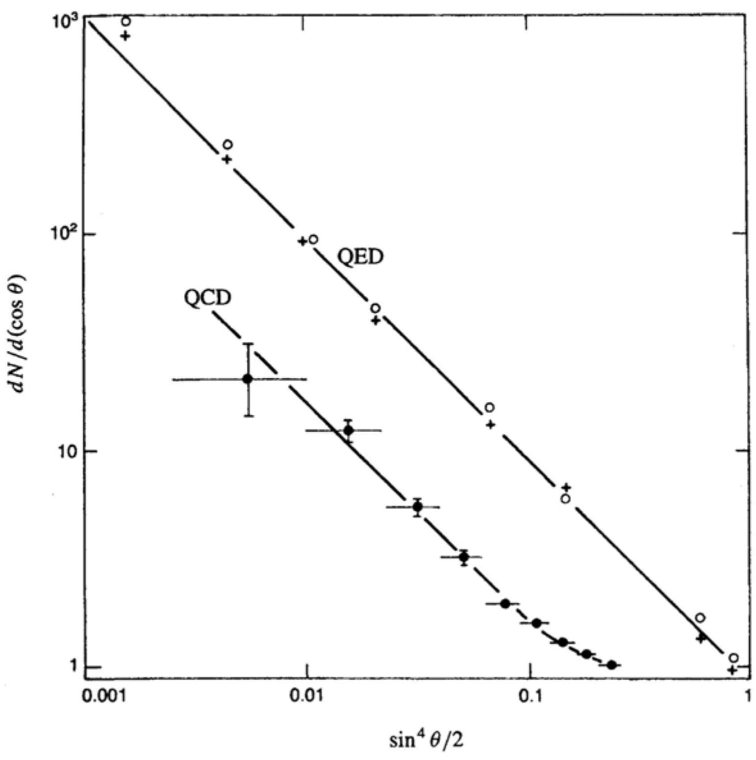
Scattering in QCD

EXAMPLE: High energy proton-antiproton scattering



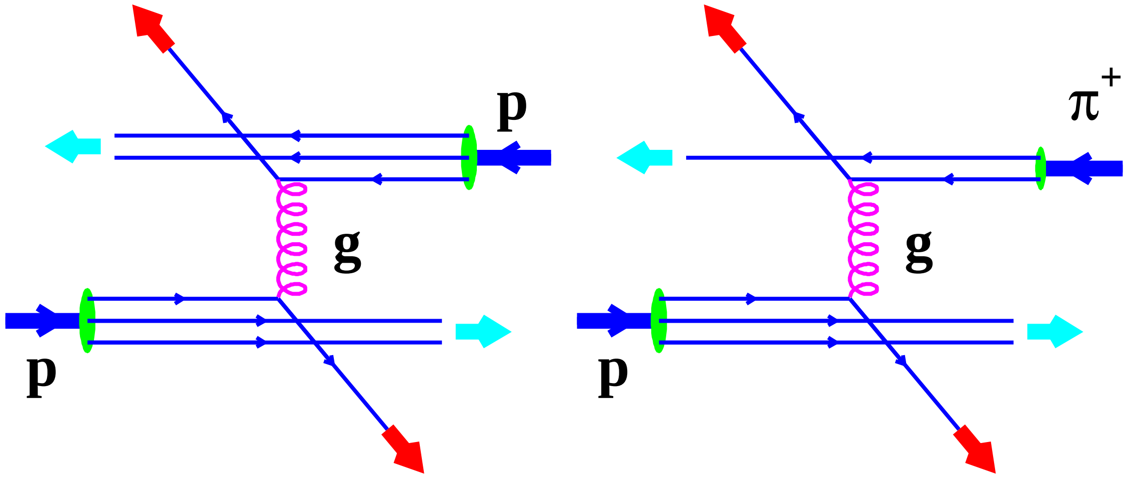


$$M \sim \frac{1}{q^2}\sqrt{\alpha_S}\sqrt{\alpha_S} \Rightarrow \frac{d\sigma}{d\Omega} \sim \frac{(\alpha_S)^2}{\sin^4\theta/2}$$



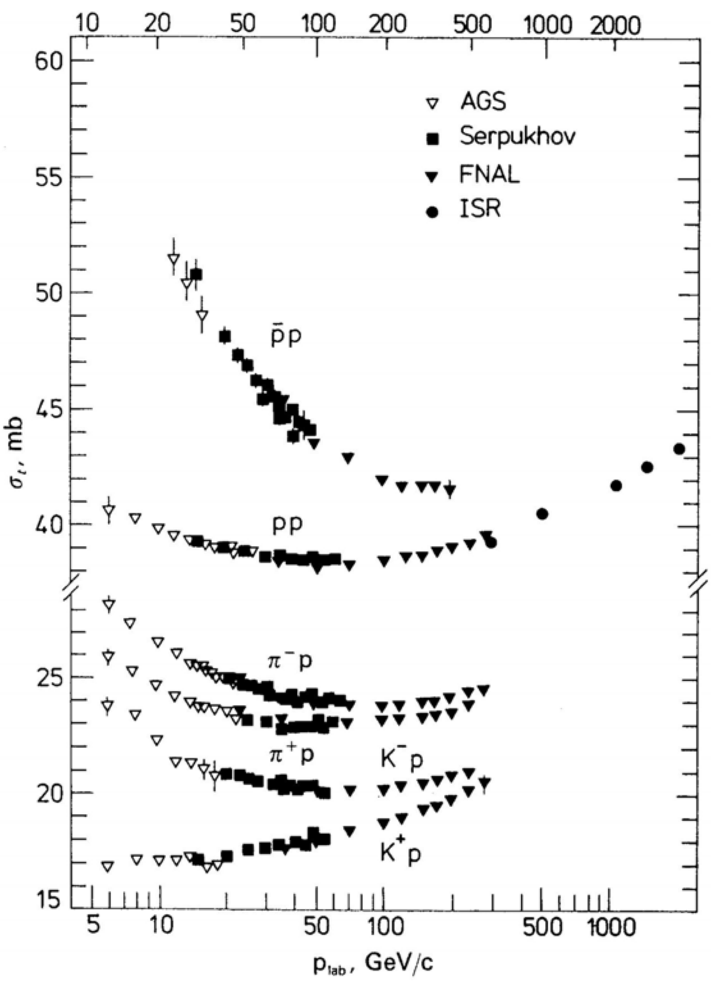
The upper points are the Geiger and Marsden data (1911) for the elastic scattering of α particles as they traverse thin gold and silver foils. The lower points show the angular distribution of the quark jets observed in proton-antiproton scattering at $q^2 = 2000\mathrm{GeV}^2$ Both follow the Rutherford formula for elastic scattering: $\sin^4\frac{\theta}{2}$.

EXAMPLE: pp vs $\pi^+ p$ scattering



Calculate ratio of $\sigma(pp)_{\text{total}}$ to $\sigma(\pi^+ p)_{\text{total}}$

★ **QCD does not distinguish between quark flavours, only COLOUR charge of quarks matters.**



At high energy (E $\gg$ binding energy of quarks within hadrons) ratio of pp and $p\pi$ total cross sections depends on number of possible quark-quark combinations.

Predict

$$\frac{\sigma(\pi p)}{\sigma(pp)} = \frac{2\times3}{3\times3} = \frac{2}{3}$$

Experiment

$$\frac{\sigma(\pi p)}{\sigma(pp)} \approx \frac{24\ mb}{38\ mb}$$

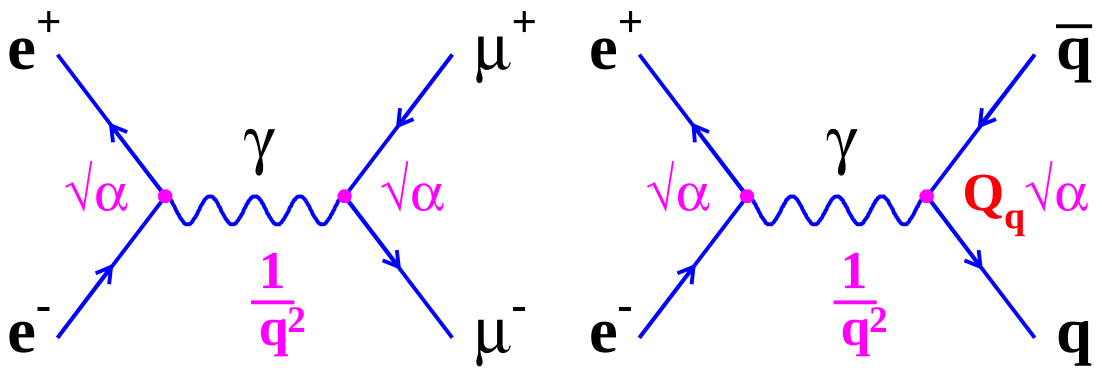
$$\approx 0.63$$

QCD in e^+e^- Annihilation

Direct evidence for the existence of colour comes from e^+e^- Annihilation.

★ **Compare $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to q\bar{q}$:**

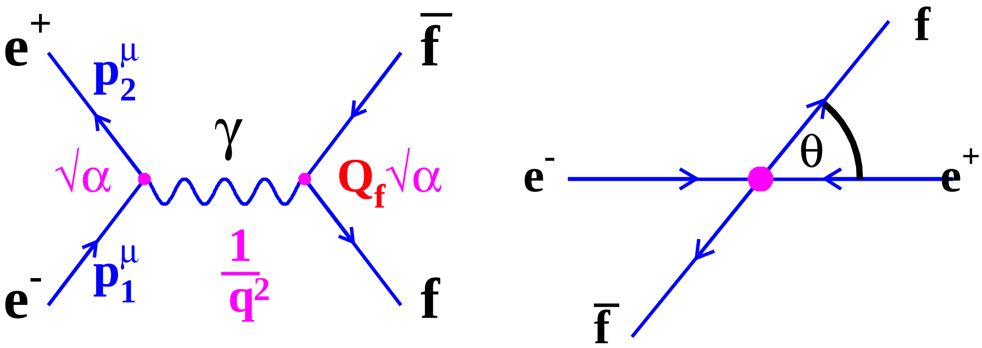
$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



If we neglect the masses of the final state quarks/muons then the ONLY difference is the charge of the final state particles ($Q_\mu = -1$, Q_q = $+\frac{2}{3}$ or $-\frac{1}{3}$)

Start by calculating the cross section for the process $\sigma(e^+e^- \to f\bar{f})$. ($f\bar{f}$ represent a fermion-antifermion pair *e.g.* $\mu^+\mu^-$ or $q\bar{q}$).

see Handout II for the case where $f\bar{f} = \mu^+\mu^-$



Electron/Positron beams along z-axis

$$
\begin{aligned}
p_1^\mu &= (E, p_x, p_y, p_z) \\
p_1^\mu &= (E, 0, 0, E) \quad \text{neglecting } m_e \\
p_2^\mu &= (E, 0, 0, -E) \\
q^\mu &= p_1^\mu + p_2^\mu \\
&= (2E, 0, 0, 0) \\
q^2 &= 4E^2 = s
\end{aligned}
$$

where s is (centre-of-mass energy)^2.

Fermi's Golden rule and Born Approximation

$$
\frac{d\sigma}{d\Omega} = 2\pi |M|^2 \frac{d\rho(E_f)}{d\Omega}
$$

Matrix element M:

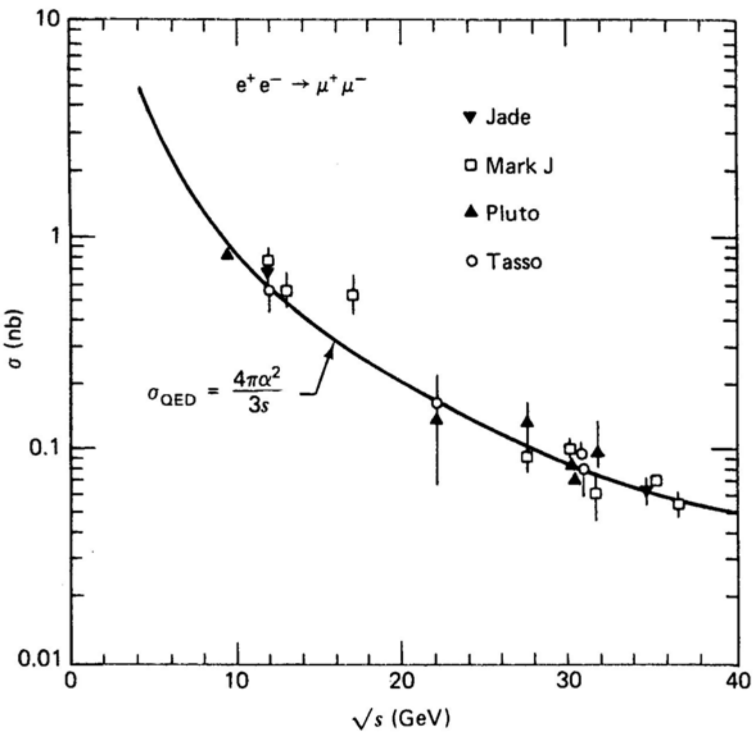
$$
\begin{aligned}
M &= \langle v_{e^+} | Q_e e | u_{e^-} \rangle \frac{1}{q^2} \langle v_{\bar{f}} | Q_f e | u_f \rangle \\
&= \frac{-4\pi\alpha Q_e Q_f}{q^2} \quad \text{with} \quad \alpha = \frac{e^2}{4\pi} \\
\frac{d\sigma}{d\Omega} &= 2\pi \frac{(-4\pi\alpha Q_e Q_f)^2}{q^4} \frac{E^2}{(2\pi)^2} \frac{1}{4} (1 + \cos^2\theta) \\
&= \frac{\alpha^2 Q_f^2}{4s} (1 + \cos^2\theta)
\end{aligned}
$$

★ $(1+\cos^2\theta)$ comes from spin-1 photon "decaying" to two spin-half fermions. see lecture on Dirac equation

Total cross section for $e^+e^- \to f\bar{f}$

$$\begin{aligned}
\sigma &= \int \frac{d\sigma}{d\Omega} d\Omega \\
&= \int_0^{2\pi}\int_0^{\pi} \frac{\alpha^2 Q_f^2}{4s}(1+\cos^2\theta)\sin\theta d\theta d\phi \\
&= \frac{\pi\alpha^2 Q_f^2}{2s}\int_{-1}^{+1}(1+y^2)dy \qquad (y=\cos\theta) \\
&= \frac{4\pi\alpha^2 Q_f^2}{3s}
\end{aligned}$$

$$\boxed{\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}}$$



$\sigma(e^+e^- \to \mu^+\mu^-)$ for e^+e^- collider data at centre-of-mass energies 8-36 GeV

Back to

$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

For a single quark flavour of a given colour

$$R = Q_q^2$$

However, we measure $e^+e^- \to \text{jets}$ not $e^+e^- \to u\bar{u}$. A jet from a u-quark looks just like a jet from a d-quark... Need to sum over flavours (u,d,c,s,t,b) and colours (r, g, b).

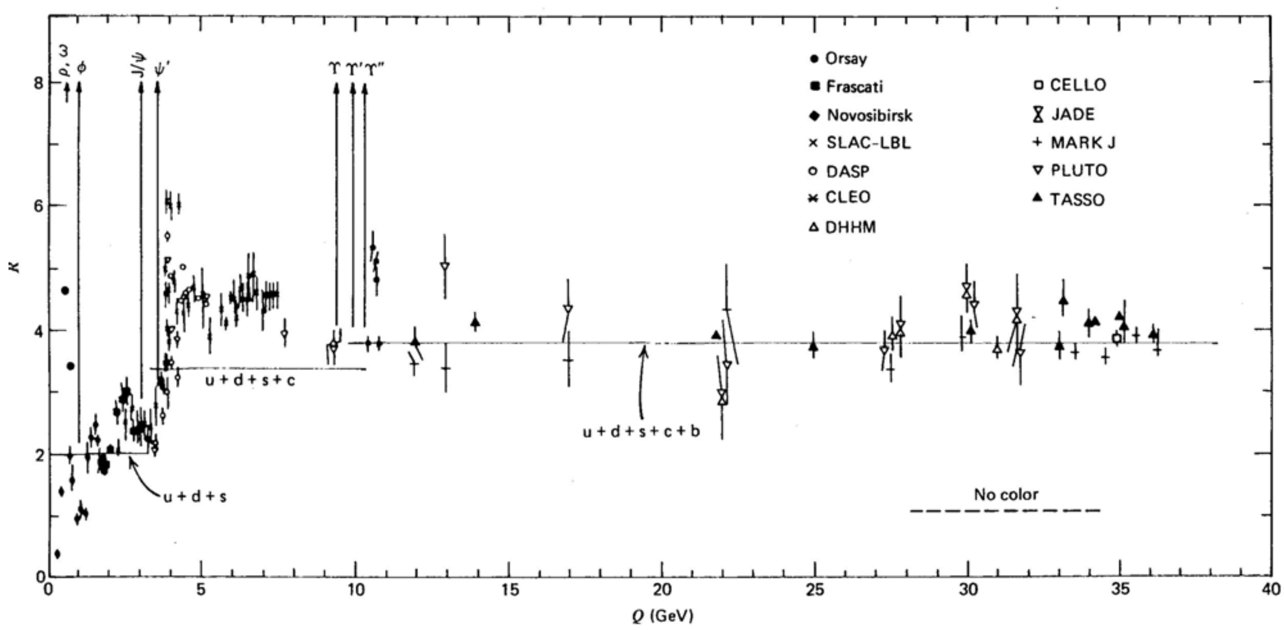
$$R = 3\sum_i Q_i^2 \qquad (3\text{colours})$$

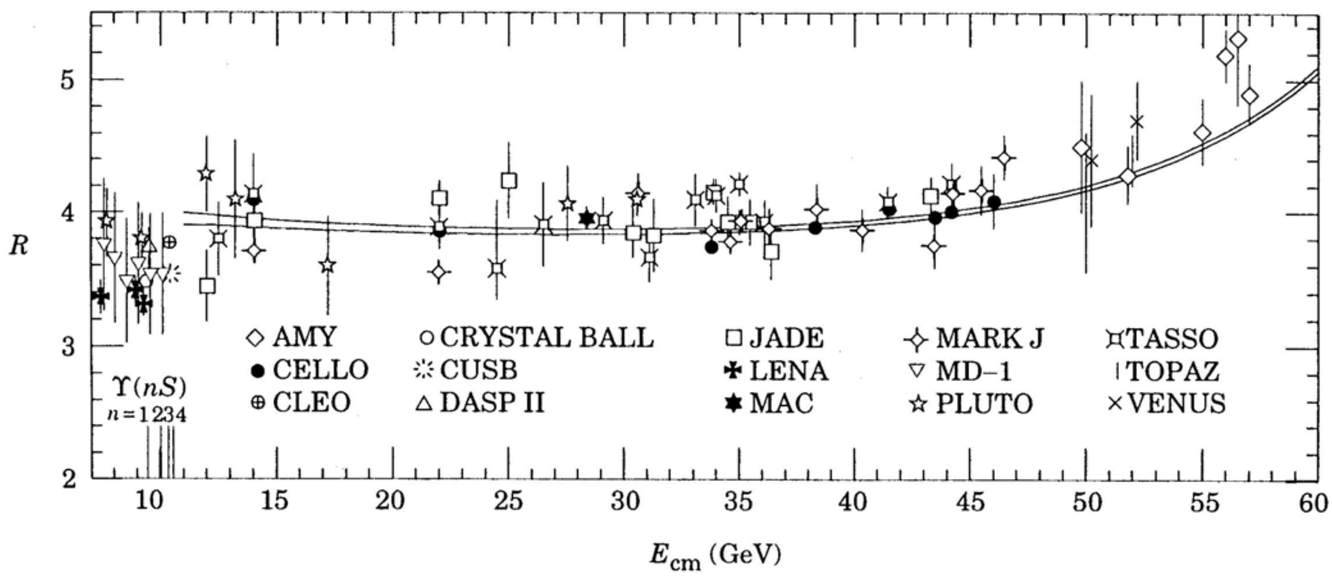
where the sum is over all quark flavours kinematically accessible at centre-of-mass energy, $\sqrt{s}$, of collider.

Energy	Ratio R	
$\sqrt{s} > 2m_s$ ∼1 GeV	$3(\frac{4}{9}+\frac{1}{9}+\frac{1}{9})$ u,d,s	= 2
$\sqrt{s} > 2m_c$ ∼4 GeV	$3(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9})$ u,d,s,c	= $3\frac{1}{3}$
$\sqrt{s} > 2m_b$ ∼10 GeV	$3(..+\frac{1}{9})$ u,d,s,c,b	= $3\frac{2}{3}$
$\sqrt{s} > 2m_t$ ∼350 GeV	$3(..+\frac{4}{9})$ u,d,s,c,b,t	= 5

$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Data: $\sqrt{s}$ from $0 - 40$ GeV





★ R_μ **increases in steps with $\sqrt{s}$**

★ $\sqrt{s} < 11\ GeV$ **region complicated by resonances: charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$).**

★ R_μ **Data exclude 'no colour' hypothesis.**

STRONG EVIDENCE for COLOUR

Experimental Evidence for Colour

★ R_μ

★ The existence of the $\Omega^-(sss)$

The $\Omega^-(sss)$ is a (L=0) spin-$\frac{3}{2}$ baryon consisting of 3 strange-quarks. The wave-function

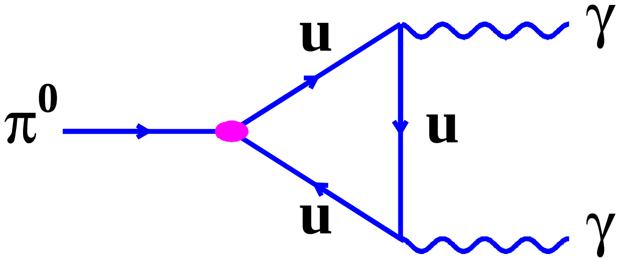
$$\psi = s\uparrow s\uparrow s\uparrow$$

is SYMMETRIC under particle interchange. However quarks are FERMIONS, therefore require an ANTI-SYMMETRIC wave-function, *i.e.* need another degree of freedom, namely COLOUR.

$$\begin{aligned} \psi &= (s\uparrow s\uparrow s\uparrow)\psi_{colour} \\ \psi_{colour} &= \frac{1}{\sqrt{6}}(rgb+gbr+brg-grb-rbg-bgr) \end{aligned}$$

★ $\pi^0 \to \gamma\gamma$ decay rate

Need colour to explain $\pi^0 \to \gamma\gamma$ decay rate.



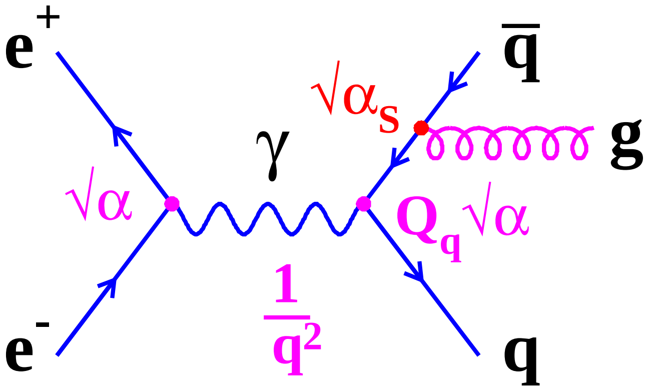
$$\Gamma(\pi^0 \to \gamma\gamma) \quad \propto \quad N^2_{colour}$$

$$\textbf{EXPT :} \quad N_{colour} = 2.99 \pm 0.12$$

Evidence for Gluons

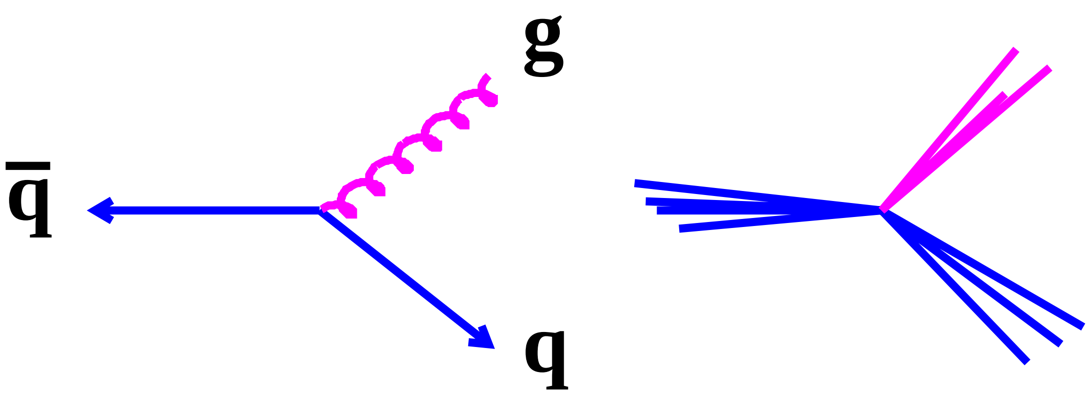
In QED, electrons can radiate photons. In QCD quarks can radiate gluons.

$e^+e^- \rightarrow q\bar{q}g$



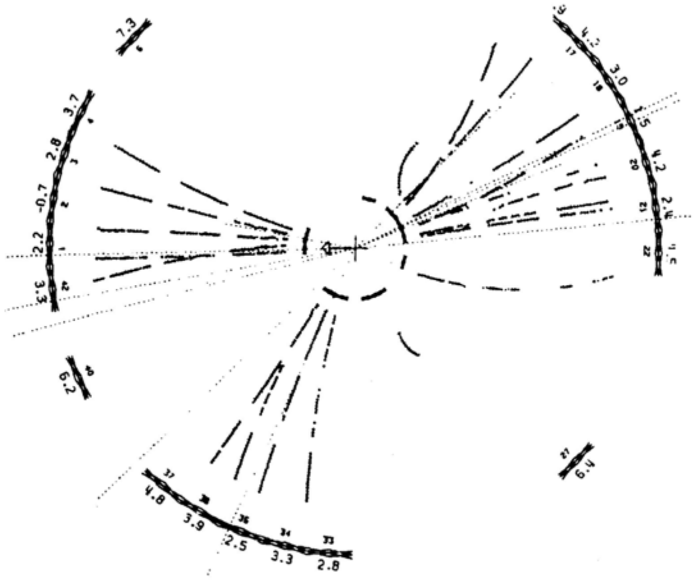
giving an extra factor of $\sqrt{\alpha_S}$ in the matrix element, i.e. an extra factor of α_S in cross section.

In QED we can detect the photons. In QCD we never see free gluons due to confinement.
Experimentally detect gluons as an additional jet: 3-Jet Events.

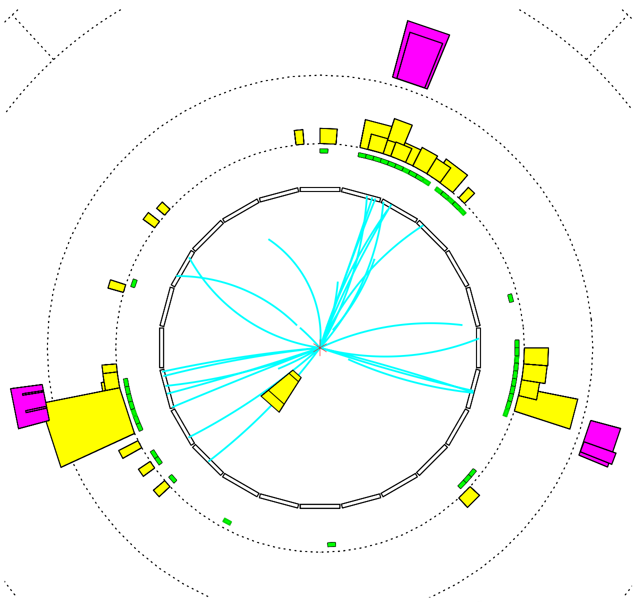


★ Angular distribution of gluon jet depends on gluon spin

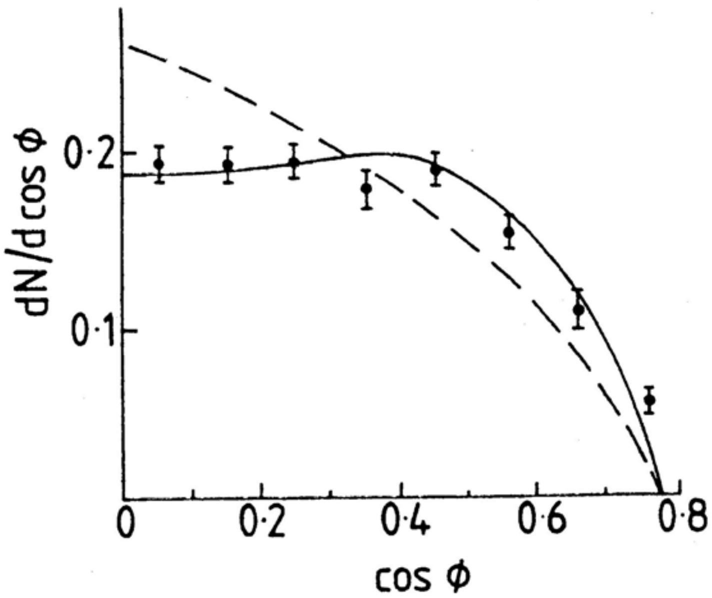
3-Jet Events and Gluon Spin



JADE $\sqrt{s} = 31$ GeV
Direct Evidence for Gluons (1978)



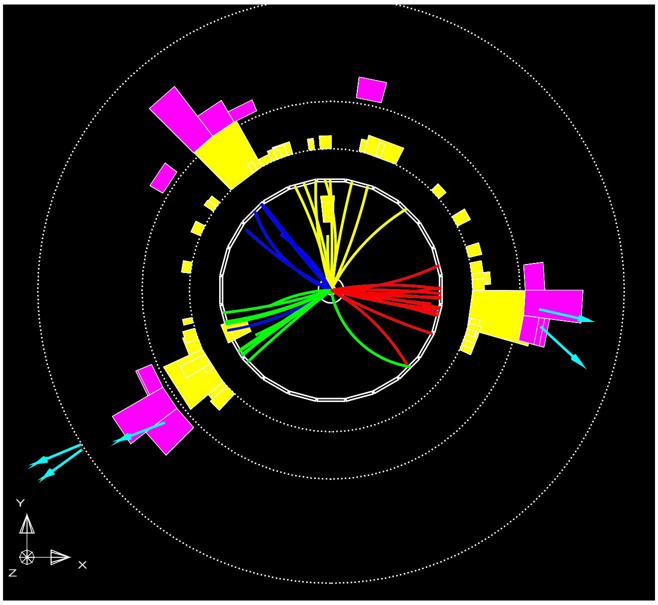
OPAL $\sqrt{s} = 91$ GeV (1990)



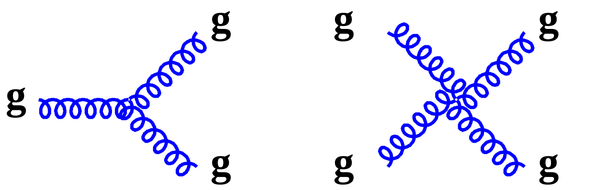
Distribution of the angle, ϕ between the highest energy jet (assumed to be one of the quarks) relative to the flight direction of the other two (in their cms frame). ϕ depends on the spin of the gluon.

$\Rightarrow$ **GLUON is SPIN-1**

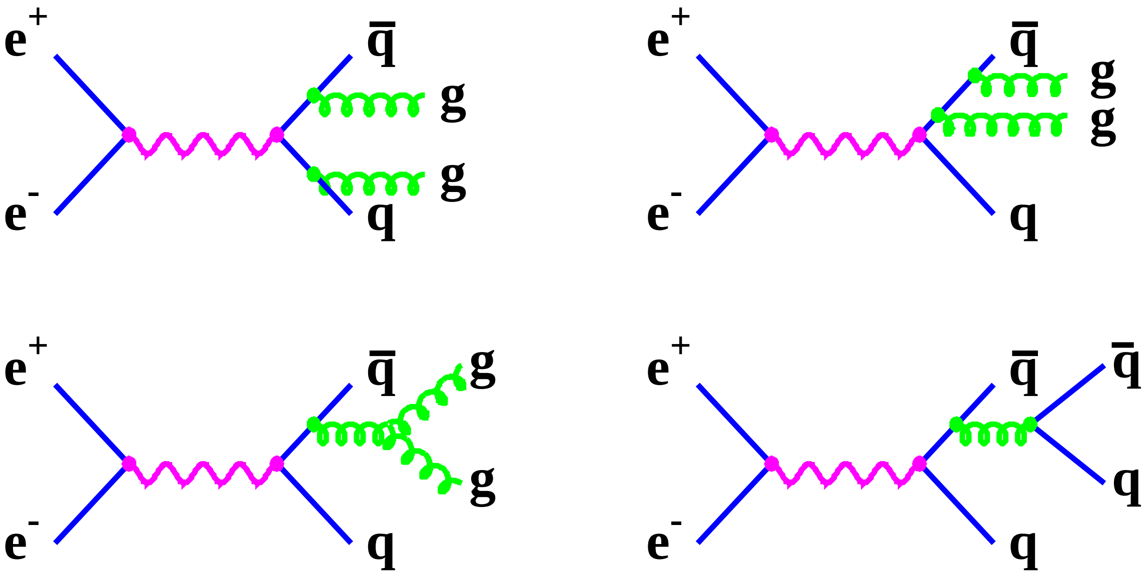
Gluon Self-Interactions



Direct Evidence for the existence of the gluon self-interactions from 4-JET events.



3 GLUON VERTEX   4 GLUON VERTEX



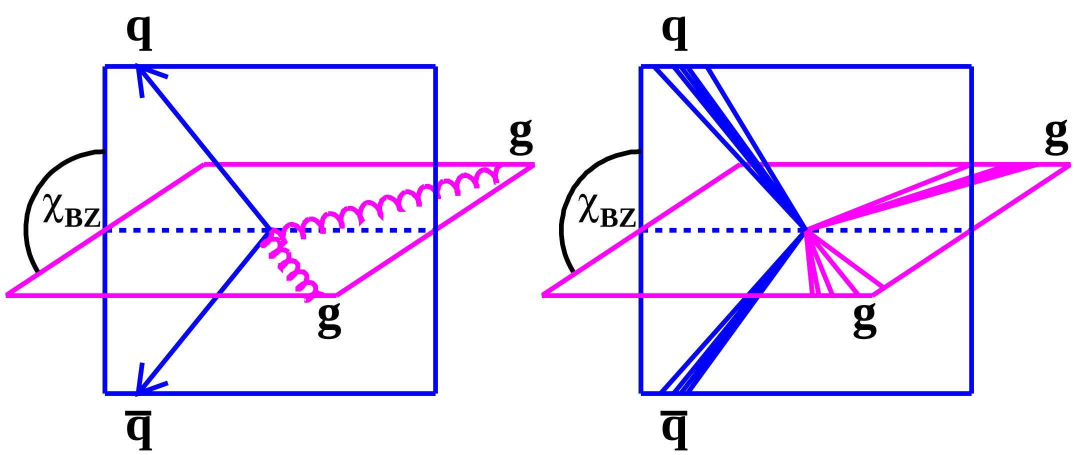
★ **Angular distribution of jets is sensitive to existence triple gluon vertex:**

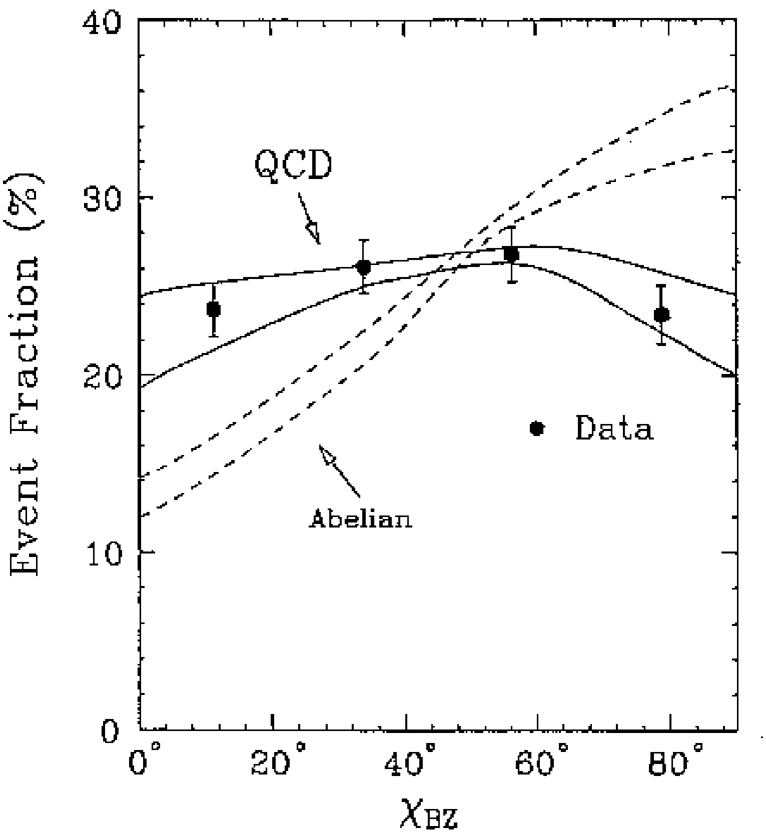
- ★ $q\bar{q}g$ **vertex consists of 2 spin-$\frac{1}{2}$ quarks and a spin-1 gluon.**
- ★ ggg **vertex consists of 3 spin-1 gluons, $\therefore$ different angular distribution.**

Experimentally:

★ **Define the two lowest energy jets as the gluons. (gluon jets are more likely to be low energy than quark jets)**

★ **Measure angle between the plane containing the 'quark' jets and the plane containing the 'gluon' jets, χ_{BZ}**

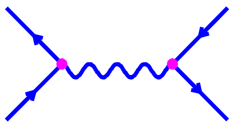




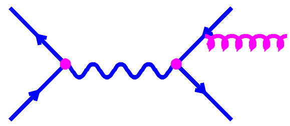
Gluon self-interactions are required to describe the experimental data. Theory without self-interactions (ABELIAN) is inconsistent with observations

Measuring α_S

α_S can be measured in many ways. The cleanest is from R_μ: In practice measure



+



+....

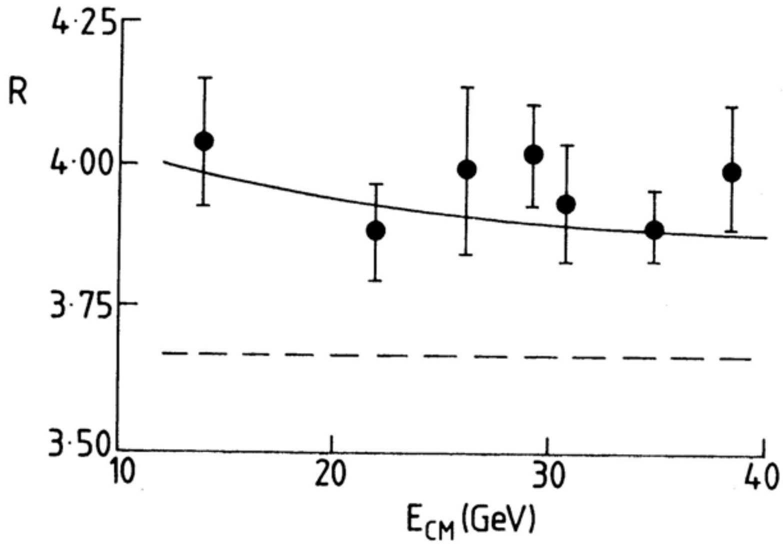
i.e. don't distinguish 2/3 jets. So measure

$$R_\mu = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\text{not} \quad R_\mu = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

When gluon radiation is included :

$$R_\mu = 3\sum Q_q^2\left(1 + \frac{\alpha_S}{\pi}\right)$$



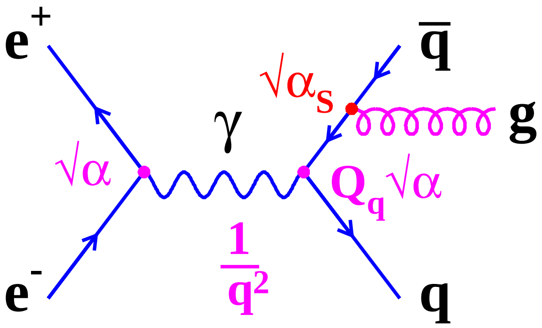
$$\sum_q Q_q^2 = 3\tfrac{2}{3}$$

Therefore $(1 + \frac{\alpha_S}{\pi}) \approx \frac{3.9}{3.66}$

giving $\quad \alpha_S(q^2 = 25^2) \approx 0.20$

Many other ways to measure α_S ....

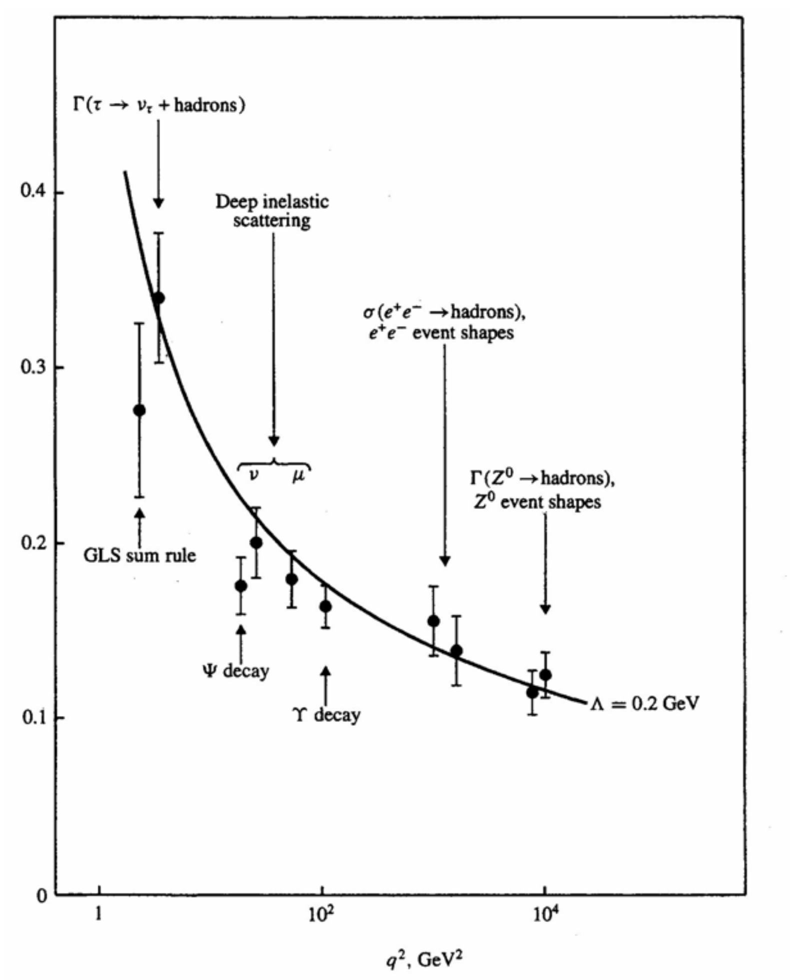
e.g. **3 jet rate :** $\mathrm{e^+e^-} \to \mathrm{q\overline{q}}g$



$$\frac{\sigma(\text{3 jet})}{\sigma(\text{2 jet})} = \frac{\sigma(\mathrm{q\overline{q}}g)}{\sigma(\mathrm{q\overline{q}})}$$

$$\propto \quad \alpha_S$$

α_S Summary



Summary of α_S measurements

α_S **RUNS !**

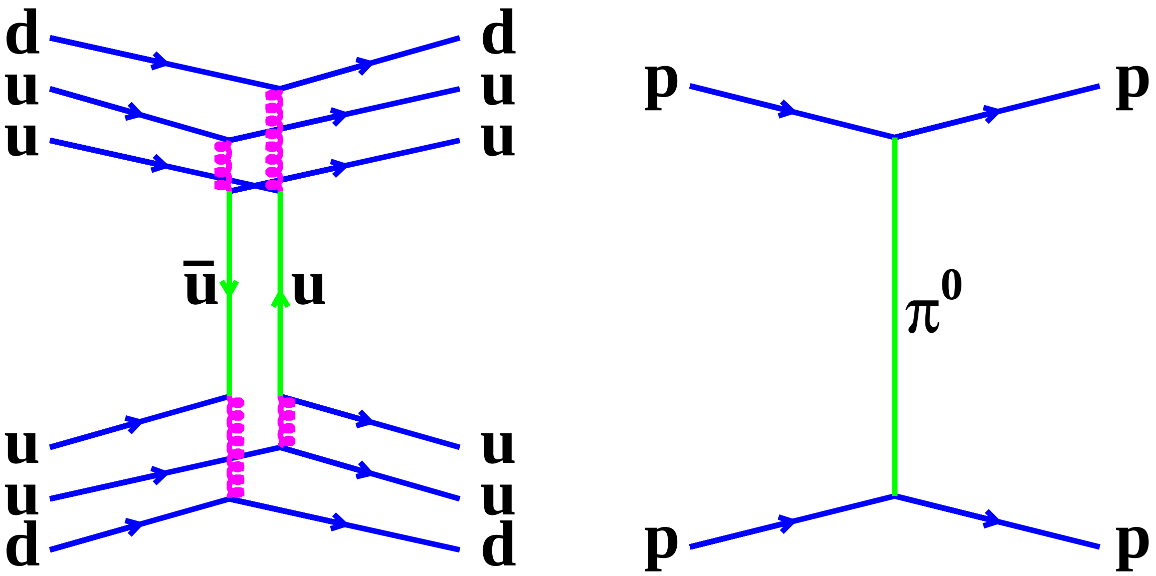
Nucleon-Nucleon Interactions

★ Bound qqq states (e.g. Protons and Neutrons) are COLOURLESS (COLOUR SINGLETS).

★ They can only interact via COLOURLESS intermediate states - i.e. not by single gluons. (conservation of colour charge)

★ Interact by exchange of PIONS

★ One possible diagram shown below :



★ Nuclear potential is YUKAWA potential with

$$V(\tilde{r}) = -\frac{g^2}{4\pi r} e^{-m_\pi r}$$

★ Short range force : range $\sim (m_\pi)^{-1}$

$$\begin{aligned} \text{Range} \quad R &= (0.140\,\mathrm{GeV})^{-1} \\ &= 7\,\mathrm{GeV}^{-1} \\ &= 7\hbar c/(1.6 \times 10^{-10})\,m \\ &= 1.4\,fm \end{aligned}$$