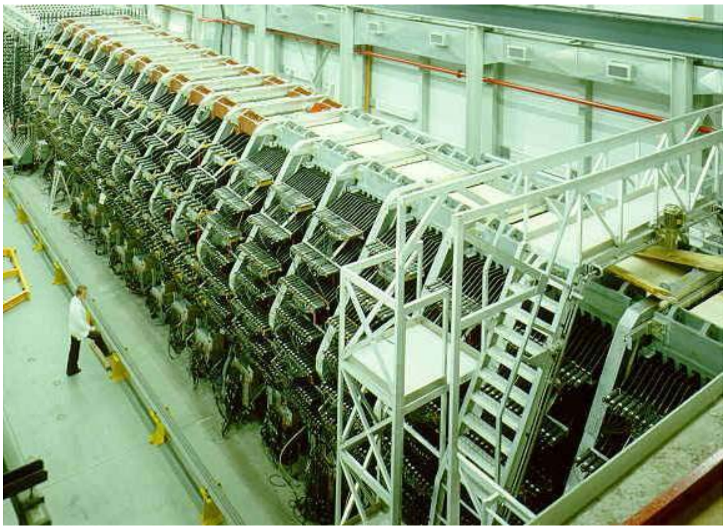
Particle Physics

Michaelmas Term 2009
Prof Mark Thomson



Handout 10 : Leptonic Weak Interactions and Neutrino Deep Inelastic Scattering

Aside : Neutrino Flavours

★ Recent experiments (see Handout 11) ➡ neutrinos have mass (albeit very small)

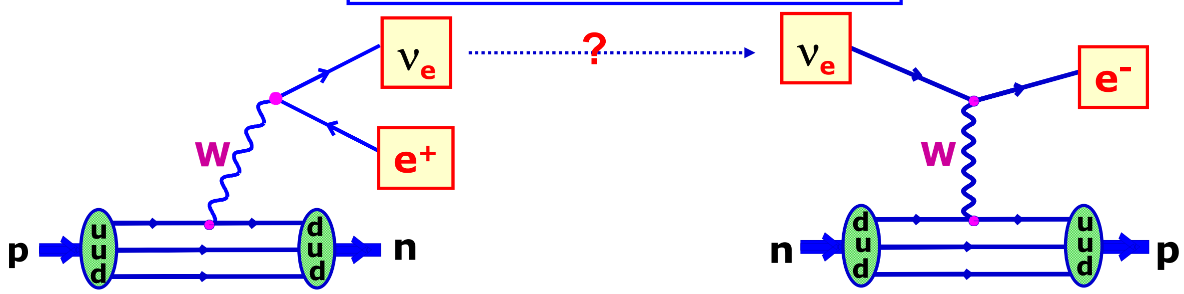
★ The textbook neutrino states, ν_e, ν_μ, ν_τ, are not the fundamental particles; these are ν_1, ν_2, ν_3

★ Concepts like "electron number" conservation are now known **not** to hold.

★ So what are ν_e, ν_μ, ν_τ ?

★ Never **directly** observe neutrinos – can only detect them by their weak interactions. Hence by **definition** ν_e is the neutrino state produced along with an electron. Similarly, charged current weak interactions of the state ν_e produce an electron

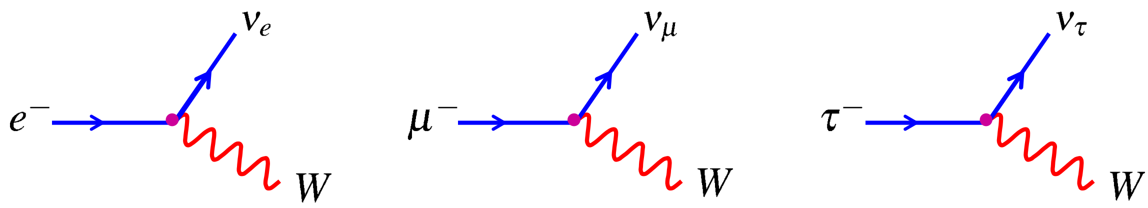
➡ $\boxed{\nu_e, \nu_\mu, \nu_\tau \text{ = weak eigenstates}}$



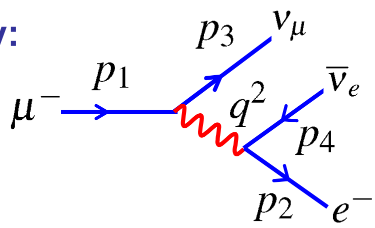
★ Unless dealing with <u>very large</u> distances: the neutrino produced with a positron will interact to produce an electron. For the discussion of the **weak interaction** continue to use ν_e, ν_μ, ν_τ as if they were the fundamental particle states.

Muon Decay and Lepton Universality

★The leptonic **charged current** ($W^{\pm}$) interaction vertices are:



★Consider muon decay:



•It is straight-forward to write down the matrix element

$$M_{fi} = \frac{g_W^{(e)} g_W^{(\mu)}}{8m_W^2}[\overline{u}(p_3)\gamma^\mu(1-\gamma^5)u(p_1)]g_{\mu\nu}[\overline{u}(p_2)\gamma^\nu(1-\gamma^5)v(p_4)]$$

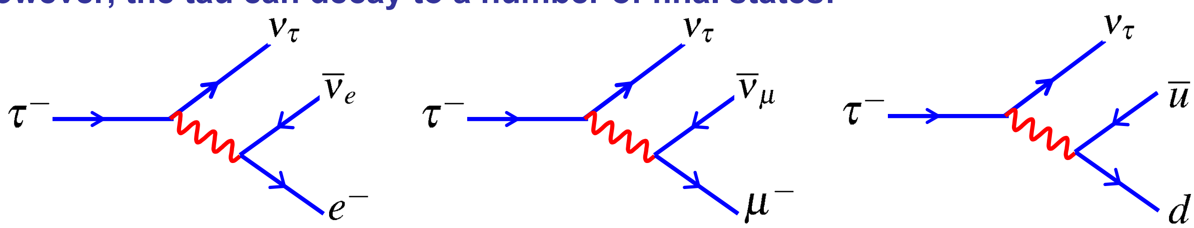
Note: for lepton decay $q^2 \ll m_W^2$ so propagator is a constant $1/m_W^2$
i.e. in limit of Fermi theory

•Its evaluation and subsequent treatment of a three-body decay is rather tricky (and not particularly interesting). Here will simply quote the result

•The muon to electron rate $\Gamma(\mu \to e\nu\nu) = \frac{G_F^e G_F^\mu m_\mu^5}{192\pi^3} = \frac{1}{\tau_\mu}$ with $G_F = \frac{g_W^2}{2\sqrt{2}m_W^2}$

•Similarly for tau to electron $\Gamma(\tau \to e\nu\nu) = \frac{G_F^e G_F^\tau m_\tau^5}{192\pi^3}$

•However, the tau can decay to a number of final states:



$Br(\tau \to e\nu\nu) = 0.1784(5)$ $Br(\tau \to \mu\nu\nu) = 0.1736(5)$

•Recall total width (total transition rate) is the sum of the partial widths

$$\Gamma = \sum_i \Gamma_i = \frac{1}{\tau}$$

•Can relate partial decay width to total decay width and therefore lifetime:

$$\Gamma(\tau \to e\nu\nu) = \Gamma_\tau Br(\tau \to e\nu\nu) = Br(\tau \to e\nu\nu)/\tau_\tau$$

•Therefore predict

$$\tau_\mu = \frac{192\pi^3}{G_F^e G_F^\mu m_\mu^5} \qquad \tau_\tau = \frac{192\pi^3}{G_F^e G_F^\tau m_\tau^5} Br(\tau \to e\nu\nu)$$

•All these quantities are precisely measured:

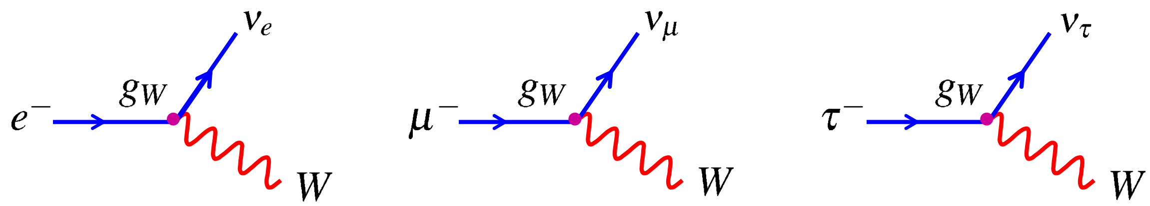
$$m_\mu = 0.1056583692(94)\,\text{GeV} \qquad \tau_\mu = 2.19703(4)\times 10^{-6}\,\text{s}$$
$$m_\tau = 1.77699(28)\,\text{GeV} \qquad \tau_\tau = 0.2906(10)\times 10^{-12}\,\text{s}$$
$$Br(\tau\to e\nu\nu) = 0.1784(5)$$

$$\Rightarrow \boxed{\frac{G_F^\tau}{G_F^\mu} = \frac{m_\mu^5 \tau_\mu}{m_\tau^5\tau_\tau} Br(\tau\to e\nu\nu) = 1.0024\pm 0.0033}$$

•Similarly by comparing $Br(\tau\to e\nu\nu)$ and $Br(\tau\to\mu\nu\nu)$

$$\boxed{\frac{G_F^e}{G_F^\mu} = 1.000\pm 0.004}$$

★Demonstrates the weak charged current is the same for all leptonic vertices



$\Rightarrow$ **Charged Current Lepton Universality**

Neutrino Scattering

•In handout 6 considered electron-proton **Deep Inelastic Scattering** where a virtual photon is used to probe nucleon structure

•Can also consider the weak interaction equivalent: **Neutrino Deep Inelastic Scattering** where a virtual W-boson probes the structure of nucleons

$\Rightarrow$ additional information about parton structure functions

+ provides a good example of calculations of weak interaction cross sections.

★ **Neutrino Beams:**

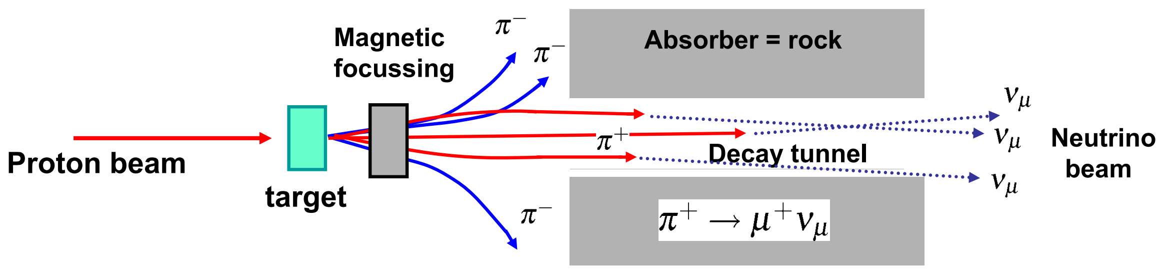
•Smash high energy protons into a fixed target $\Rightarrow$ hadrons

•Focus positive pions/kaons

•Allow them to decay $\pi^+\to\mu^+\nu_\mu$ + $K^+\to\mu^+\nu_\mu$ $(BR\approx 64\%)$

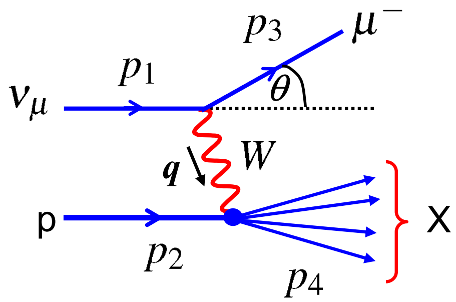
•Gives a beam of "collimated" ν_μ

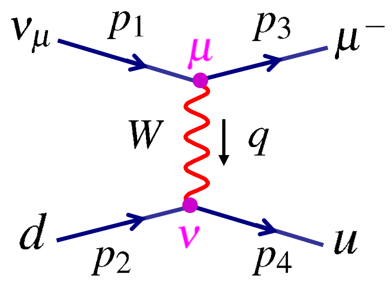
•Focus negative pions/kaons to give beam of $\overline{\nu}_\mu$



Neutrino-Quark Scattering

★For ν_μ-proton Deep Inelastic Scattering the underlying process is $\nu_\mu d \to \mu^- u$





★In the limit $q^2 \ll m_W^2$ the W-boson propagator is $\approx i g_{\mu\nu}/m_W^2$

•The Feynman rules give:

$$-iM_{fi} = \left[-i\frac{g_W}{\sqrt{2}}\overline{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1)\right]\frac{ig_{\mu\nu}}{m_W^2}\left[-i\frac{g_W}{\sqrt{2}}\overline{u}(p_4)\tfrac{1}{2}\gamma^\nu(1-\gamma^5)u(p_2)\right]$$

$$\boxed{M_{fi} = \frac{g_W^2}{2m_W^2}g_{\mu\nu}\left[\overline{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1)\right]\left[\overline{u}(p_4)\tfrac{1}{2}\gamma^\nu(1-\gamma^5)u(p_2)\right]}$$

•Evaluate the matrix element in the extreme relativistic limit where the muon and quark masses can be neglected

•In this limit the helicity states are equivalent to the chiral states and

$$\tfrac{1}{2}(1-\gamma^5)u_\uparrow(p_1) = 0 \qquad \tfrac{1}{2}(1-\gamma^5)u_\downarrow(p_1) = u_\downarrow(p_1)$$

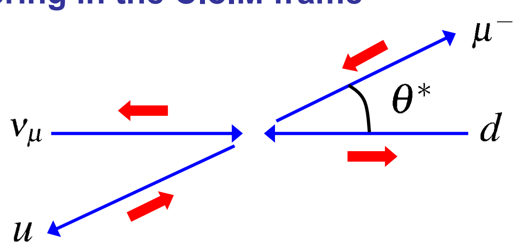
➡ $M_{fi} = 0$ for $u_\uparrow(p_1)$ and $u_\uparrow(p_2)$

•Since the weak interaction "conserves the helicity", the only helicity combination where the matrix element is non-zero is

$$M_{fi} = \frac{g_W^2}{2m_W^2}g_{\mu\nu}\left[\overline{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1)\right]\left[\overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)\right]$$

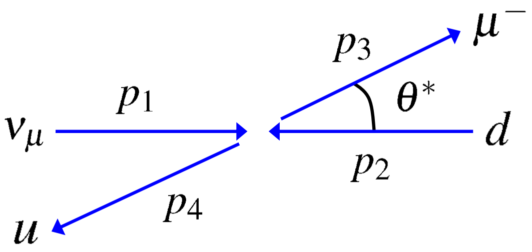
NOTE: we could have written this down straight away as in the ultra-relativistic limit only LH helicity particle states participate in the weak interaction.

★Consider the scattering in the C.o.M frame



Evaluation of Neutrino-Quark Scattering ME

- Go through the calculation in gory detail (fortunately only one helicity combination)
- In CMS frame, neglecting particle masses:



$$p_1 = (E,0,0,E),$$
$$p_2 = (E,0,0,-E)$$
$$p_3 = (E, E\sin\theta^*, 0, E\cos\theta^*)$$
$$p_4 = (E, -E\sin\theta^*, 0, -E\cos\theta^*)$$

- Dealing with LH helicity particle spinors. From handout 3 (p.80), for a massless particle travelling in direction (θ,ϕ):

$$u_\downarrow = \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix} \qquad c = \cos\tfrac{\theta}{2}; \quad s = \sin\tfrac{\theta}{2}$$

- Here $(\theta_1,\phi_1) = (0,0);\ (\theta_2,\phi_2) = (\pi,0);\ (\theta_3,\phi_3) = (\theta^*,0);\ (\theta_4,\phi_4) = (\pi-\theta^*,\pi)$

giving:

$$u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix};\ u_\downarrow(p_2) = \sqrt{E}\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix};\ u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix};\ u_\downarrow(p_4) = \sqrt{E}\begin{pmatrix} -c \\ -s \\ c \\ s \end{pmatrix}$$

- To calculate

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu}\left[\overline{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1)\right]\left[\overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)\right]$$

need to evaluate two terms of form

$$\begin{aligned}
\overline{\psi}\gamma^0\phi &= \psi^\dagger\gamma^0\gamma^0\phi = \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4 \\
\overline{\psi}\gamma^1\phi &= \psi^\dagger\gamma^0\gamma^1\phi = \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1 \\
\overline{\psi}\gamma^2\phi &= \psi^\dagger\gamma^0\gamma^2\phi = -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1) \\
\overline{\psi}\gamma^3\phi &= \psi^\dagger\gamma^0\gamma^3\phi = \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2
\end{aligned}$$

- Using

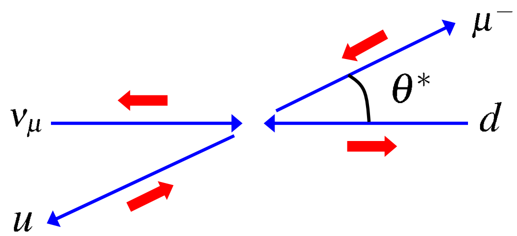
$$u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix};\ u_\downarrow(p_2) = \sqrt{E}\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix};\ u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix};\ u_\downarrow(p_4) = \sqrt{E}\begin{pmatrix} -c \\ -s \\ c \\ s \end{pmatrix}$$

$$\Rightarrow \boxed{\begin{aligned} \overline{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1) &= 2E(c, s, -is, c) \\ \overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2) &= 2E(c, -s, -is, -c) \end{aligned}}$$

$$\Rightarrow M_{fi} = \frac{g_W^2}{2m_W^2} 4E^2(c^2+s^2+s^2+c^2) = \frac{g_W^2\hat{s}}{m_W^2} \qquad \hat{s} = (2E)^2$$

★ Note the Matrix Element is isotropic

$$\boxed{M_{fi} = \frac{g_W^2}{m_W^2}\hat{s}}$$



we could have anticipated this since the helicity combination (spins anti-parallel) has $S_z = 0$ ➡ no preferred polar angle

★As before need to sum over all possible spin states and average over all possible initial state spin states. Here only one possible spin combination (LL→LL) and only 2 possible initial state combinations (the neutrino is always produced in a LH helicity state)

$$\langle |M_{fi}|^2 \rangle = \frac{1}{2} \cdot \left| \frac{g_W^2}{m_W^2}\hat{s} \right|^2$$

The factor of a half arises because half of the time the quark will be in a RH states and won't participate in the charged current Weak interaction

★From handout 1, in the extreme relativistic limit, the cross section for any 2→2 body scattering process is

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2\hat{s}}\langle |M_{fi}|^2 \rangle$$

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2\hat{s}}\langle |M_{fi}|^2 \rangle = \frac{1}{64\pi\hat{s}}\frac{1}{2}\left(\frac{g_W^2\hat{s}}{m_W^2}\right)^2 = \left(\frac{g_W^2}{8\sqrt{2}\pi m_W^2}\right)^2\hat{s}$$

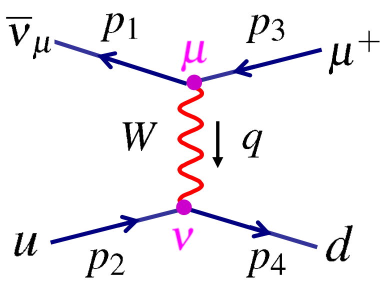
using $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$ ➡ $\boxed{\frac{d\sigma}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}}$

★Integrating this isotropic distribution over $d\Omega^*$

➡ $\boxed{\sigma_{\nu q} = \frac{G_F^2\hat{s}}{\pi}}$ (1)

•cross section is a Lorentz invariant quantity so this is valid in any frame

Antineutrino-Quark Scattering



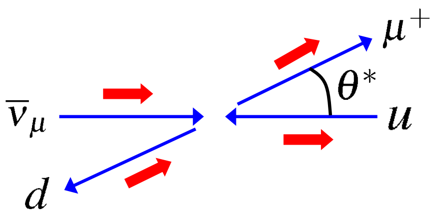
•In the ultra-relativistic limit, the charged-current interaction matrix element is:

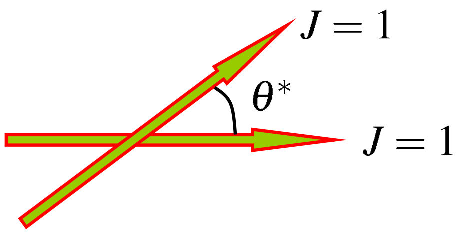
$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\overline{v}(p_1)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)v(p_3)\right] \left[\overline{u}(p_4)\gamma^\nu \tfrac{1}{2}(1-\gamma^5)u(p_2)\right]$$

★ In the extreme relativistic limit only **LH Helicity particles** and **RH Helicity anti-particles** participate in the charged current weak interaction:

$$\Rightarrow \quad M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\overline{v}_\uparrow(p_1)\gamma^\mu v_\uparrow(p_3)\right] \left[\overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)\right]$$

★ In terms of the particle spins it can be seen that the interaction occurs in a total angular momentum 1 state

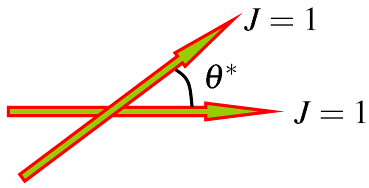




★In a similar manner to the neutrino-quark scattering calculation obtain:

$$\frac{d\sigma_{\overline{\nu}q}}{d\Omega^*} = \frac{d\sigma_{\nu q}}{d\Omega^*}\frac{1}{4}(1+\cos\theta^*)^2$$

•The factor $\frac{1}{4}(1+\cos\theta^*)^2$ can be understood in terms of the overlap of the initial and final angular momentum wave-functions



★In a similar manner to the neutrino-quark scattering calculation obtain:

$$\frac{d\sigma_{\overline{\nu}q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$$

★Integrating over solid angle:

$$d\Omega = d\phi\sin\theta d\theta \rightarrow d\phi d(\cos\theta)$$

$$\int(1+\cos\theta^*)^2 d\Omega^* = \int(1+\cos\theta^*)^2 d(\cos\theta^*)d\phi = 2\pi\int_{-1}^{+1}(1+\cos\theta^*)^2 d(\cos\theta^*) = \frac{16\pi}{3}$$

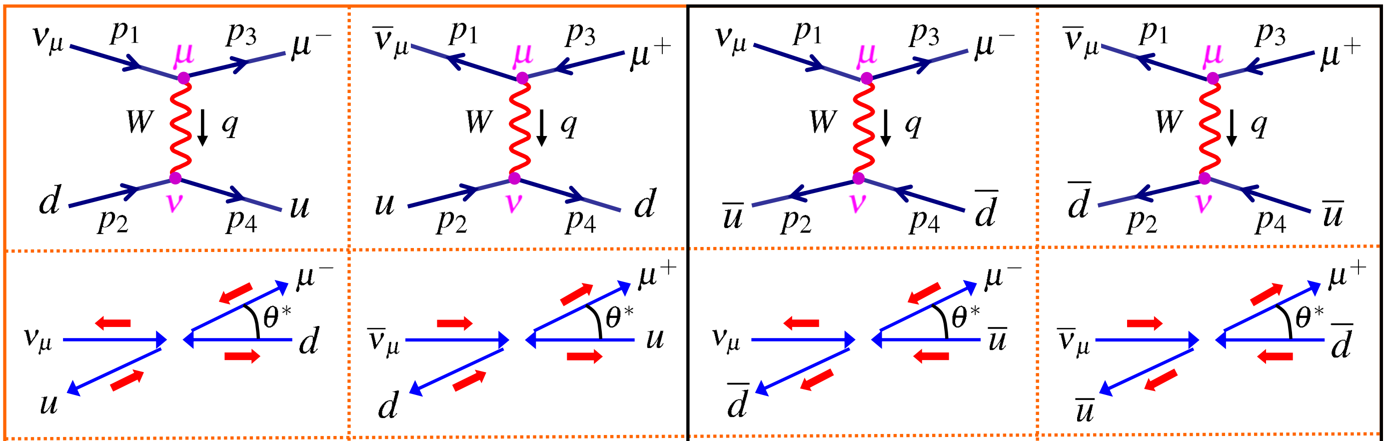
$$\Rightarrow \quad \boxed{\sigma_{\overline{\nu}q} = \frac{G_F^2\hat{s}}{3\pi}}$$

★This is a factor three smaller than the neutrino quark cross-section

$$\boxed{\frac{\sigma_{\overline{\nu}q}}{\sigma_{\nu q}} = \frac{1}{3}}$$

(Anti)neutrino-(Anti)quark Scattering

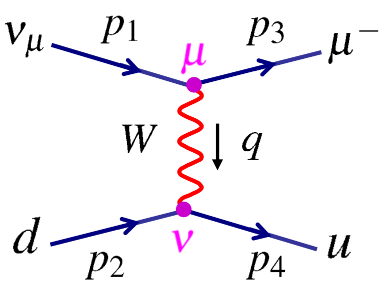
- Non-zero anti-quark component to the nucleon $\Rightarrow$ also consider scattering from $\overline{q}$
- Cross-sections can be obtained immediately by comparing with quark scattering and remembering to only include **LH particles** and **RH anti-particles**



$S_z = 0$	$S_z = +1$	$S_z = -1$	$S_z = 0$
$\frac{\mathrm{d}\sigma_{\nu q}}{\mathrm{d}\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$	$\frac{\mathrm{d}\sigma_{\overline{\nu} q}}{\mathrm{d}\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{\mathrm{d}\sigma_{\nu \overline{q}}}{\mathrm{d}\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$	$\frac{\mathrm{d}\sigma_{\overline{\nu}\overline{q}}}{\mathrm{d}\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$
$\sigma_{\nu q} = \frac{G_F^2\hat{s}}{\pi}$	$\sigma_{\overline{\nu} q} = \frac{G_F^2\hat{s}}{3\pi}$	$\sigma_{\nu \overline{q}} = \frac{G_F^2\hat{s}}{3\pi}$	$\sigma_{\overline{\nu}\overline{q}} = \frac{G_F^2\hat{s}}{\pi}$

Differential Cross Section $\mathrm{d}\sigma/\mathrm{d}y$

★ Derived differential neutrino scattering cross sections in C.o.M frame, can convert to Lorentz invariant form



- As for DIS use Lorentz invariant $y \equiv \frac{p_2.q}{p_2.p_1}$
- In relativistic limit y can be expressed in terms of the C.o.M. scattering angle

$$y = \tfrac{1}{2}(1-\cos\theta^*)$$

- In lab. frame $y = 1 - \frac{E_3}{E_1}$

★ Convert from $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^*} \longrightarrow \frac{\mathrm{d}\sigma}{\mathrm{d}y}$ using

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y} = \left|\frac{\mathrm{d}\cos\theta^*}{\mathrm{d}y}\right|\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta^*} = \left|\frac{\mathrm{d}\cos\theta^*}{\mathrm{d}y}\right|2\pi\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^*} = 4\pi\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^*}$$

- Already calculated (1) $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$

- Hence:

$$\boxed{\frac{\mathrm{d}\sigma_{\nu q}}{\mathrm{d}y} = \frac{\mathrm{d}\sigma_{\overline{\nu}\overline{q}}}{\mathrm{d}y} = \frac{G_F^2}{\pi}\hat{s}}$$

and $$\frac{d\sigma_{\overline{\nu}q}}{d\Omega^*} = \frac{d\sigma_{\nu\overline{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$$

becomes $$\frac{d\sigma_{\overline{\nu}q}}{dy} = \frac{d\sigma_{\nu\overline{q}}}{dy} = \frac{G_F^2}{4\pi}(1+\cos\theta^*)^2\hat{s}$$

from $y = \frac{1}{2}(1-\cos\theta^*) \rightarrow 1-y = \frac{1}{2}(1+\cos\theta^*)$

and hence $$\boxed{\frac{d\sigma_{\overline{\nu}q}}{dy} = \frac{d\sigma_{\nu\overline{q}}}{dy} = \frac{G_F^2}{\pi}(1-y)^2\hat{s}}$$

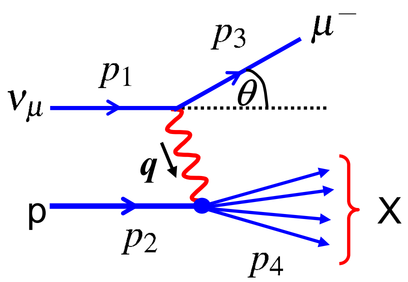
★For comparison, the electro-magnetic $e^{\pm}q \rightarrow e^{\pm}q$ cross section is:

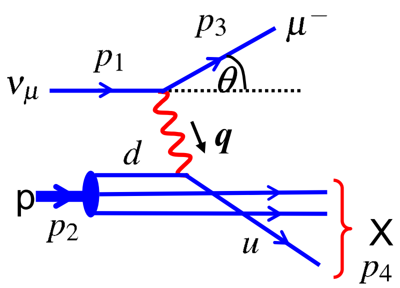
QED $$\frac{d\sigma_{e^{\pm}q}}{dy} = \underbrace{\frac{2\pi\alpha^2}{q^4}}_{\text{Interaction +propagator}} e_q^2 \underbrace{\left[1+(1-y)^2\right]}_{\text{Helicity Structure}}\hat{s}$$

DIFFERENCES:

WEAK $$\frac{d\sigma_{\overline{\nu}q}}{dy} = \frac{d\sigma_{\nu\overline{q}}}{dy} = \overbrace{\frac{G_F^2}{\pi}}^{\text{Interaction +propagator}}\overbrace{(1-y)^2}^{\text{Helicity Structure}}\hat{s}$$

Parton Model For Neutrino Deep Inelastic Scattering





Scattering from a proton with structure functions → Scattering from a point-like quark within the proton

★Neutrino-proton scattering can occur via scattering from a **down-quark** or from an **anti-up quark**

•In the parton model, number of down quarks within the proton in the momentum fraction range $x \rightarrow x+dx$ is $d^p(x)dx$. Their contribution to the neutrino scattering cross-section is obtained by multiplying by the $\nu_\mu d \rightarrow \mu^- u$ cross-section derived previously

$$\frac{d\sigma^{\nu p}}{dy} = \frac{G_F^2}{\pi}\hat{s}d^p(x)dx$$

where $\hat{s}$ is the centre-of-mass energy of the $\nu_\mu d$

•Similarly for the $\overline{u}$ contribution

$$\frac{\mathrm{d}\sigma^{\nu p}}{\mathrm{d}y} = \frac{G_F^2}{\pi}\hat{s}(1-y)^2\overline{u}^p(x)\mathrm{d}x$$

★Summing the two contributions and using $\hat{s} = xs$

➡ $$\frac{\mathrm{d}^2\sigma^{\nu p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2}{\pi}sx\left[d^p(x)+(1-y)^2\overline{u}^p(x)\right]$$

★ The anti-neutrino proton differential cross section can be obtained in the same manner:

$$\frac{\mathrm{d}^2\sigma^{\overline{\nu} p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2}{\pi}sx\left[(1-y)^2u^p(x)+\overline{d}^p(x)\right]$$

★ For (anti)neutrino – neutron scattering:

$$\frac{\mathrm{d}^2\sigma^{\nu n}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2}{\pi}sx\left[d^n(x)+(1-y)^2\overline{u}^n(x)\right]$$

$$\frac{\mathrm{d}^2\sigma^{\overline{\nu} n}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2}{\pi}sx\left[(1-y)^2u^n(x)+\overline{d}^n(x)\right]$$

•As before, define neutron distributions functions in terms of those of the proton

$$u(x) \equiv u^{\mathrm{p}}(x) = d^{\mathrm{n}}(x); \qquad d(x) \equiv d^{\mathrm{p}}(x) = u^{\mathrm{n}}(x)$$
$$\overline{u}(x) \equiv \overline{u}^{\mathrm{p}}(x) = \overline{d}^{\mathrm{n}}(x); \qquad \overline{d}(x) \equiv \overline{d}^{\mathrm{p}}(x) = \overline{u}^{\mathrm{n}}(x)$$

$$\frac{\mathrm{d}^2\sigma^{\nu p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2}{\pi}sx\left[d(x)+(1-y)^2\overline{u}(x)\right] \quad (2)$$

$$\frac{\mathrm{d}^2\sigma^{\overline{\nu} p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2}{\pi}sx\left[(1-y)^2u(x)+\overline{d}(x)\right] \quad (3)$$

$$\frac{\mathrm{d}^2\sigma^{\nu n}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2}{\pi}sx\left[u(x)+(1-y)^2\overline{d}(x)\right] \quad (4)$$

$$\frac{\mathrm{d}^2\sigma^{\overline{\nu} p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2}{\pi}sx\left[(1-y)^2d(x)+\overline{u}(x)\right] \quad (5)$$

★Because neutrino cross sections are very small, need massive detectors. These are usually made of Iron, hence, experimentally measure a **combination** of proton/neutron scattering cross sections

★ For an isoscalar target (i.e. equal numbers of protons and neutrons), the mean cross section per nucleon:

$$\frac{\mathrm{d}^2\sigma^{\nu N}}{\mathrm{d}x\mathrm{d}y} = \frac{1}{2}\left(\frac{\mathrm{d}^2\sigma^{\nu p}}{\mathrm{d}x\mathrm{d}y} + \frac{\mathrm{d}^2\sigma^{\nu n}}{\mathrm{d}x\mathrm{d}y}\right)$$

$$\Rightarrow \boxed{\frac{\mathrm{d}^2\sigma^{\nu N}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2}{2\pi}sx\left[u(x)+d(x)+(1-y)^2(\overline{u}(x)+\overline{d}(x))\right]}$$

•Integrate over momentum fraction x

$$\boxed{\frac{\mathrm{d}\sigma^{\nu N}}{\mathrm{d}y} = \frac{G_F^2}{2\pi}s\left[f_q+(1-y)^2 f_{\overline{q}}\right]} \qquad (6)$$

where f_q and $f_{\overline{q}}$ are the total momentum fractions carried by the quarks and by the anti-quarks within a nucleon

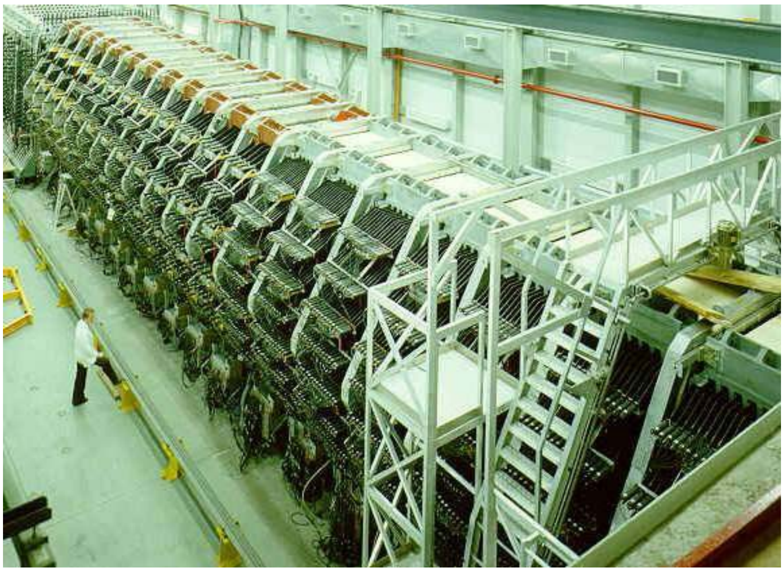
$$f_q \equiv f_d + f_u = \int_0^1 x\left[u(x)+d(x)\right]\mathrm{d}x; \quad f_{\overline{q}} \equiv f_{\overline{d}} + f_{\overline{u}} = \int_0^1 x\left[\overline{u}(x)+\overline{d}(x)\right]\mathrm{d}x$$

•Similarly

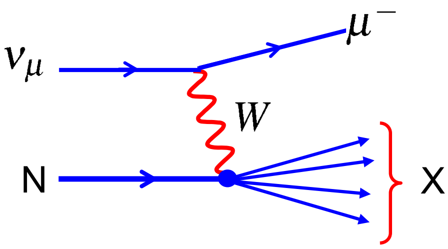
$$\boxed{\frac{\mathrm{d}\sigma^{\overline{\nu} N}}{\mathrm{d}y} = \frac{G_F^2}{2\pi}s\left[(1-y)^2 f_q+f_{\overline{q}}\right]} \qquad (7)$$

e.g. CDHS Experiment (CERN 1976-1984)

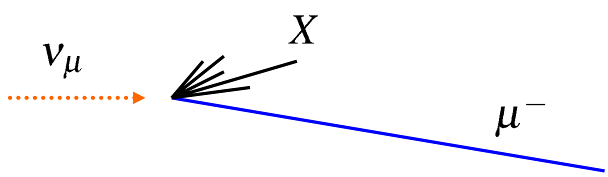
•1250 tons
•Magnetized iron modules
•Separated by drift chambers



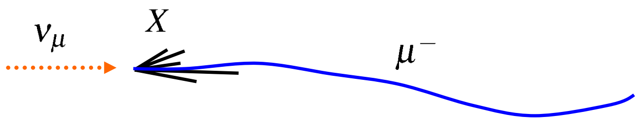
Study Neutrino Deep Inelastic Scattering

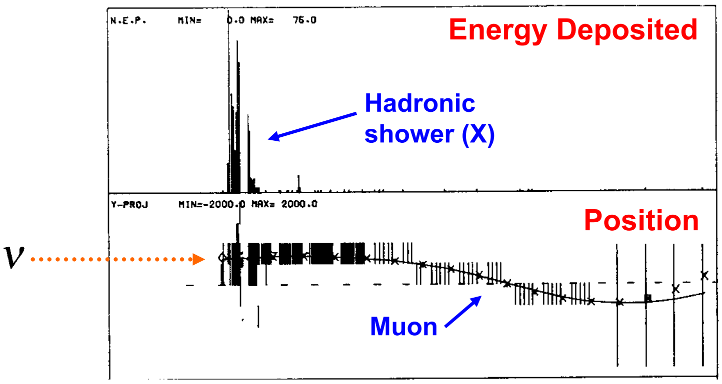


Experimental Signature:



Example Event:





- Measure energy of X

$$E_X$$

- Measure muon momentum from curvature in B-field

$$E_\mu$$

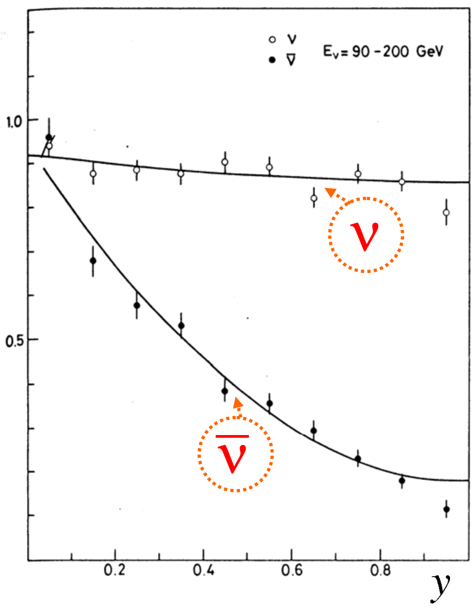
★ **For each event can determine neutrino energy and y !**

$$E_\nu = E_X + E_\mu$$

$$E_\mu = (1-y)E_\nu \;\rightarrow\; y = \left(1 - \frac{E_\mu}{E_\nu}\right)$$

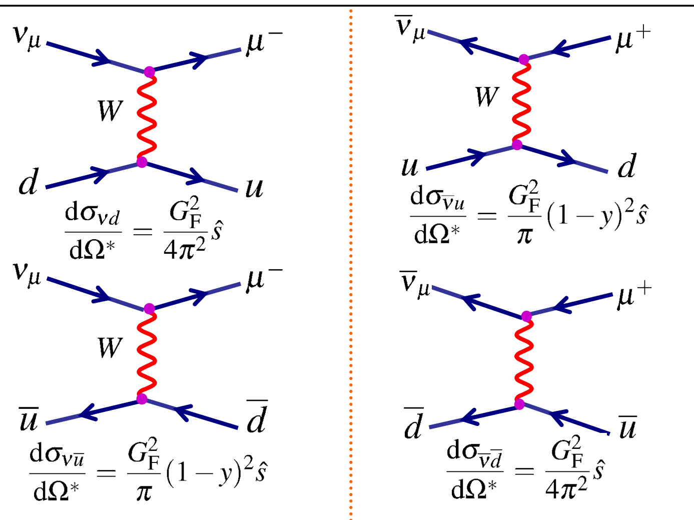
Measured y Distribution

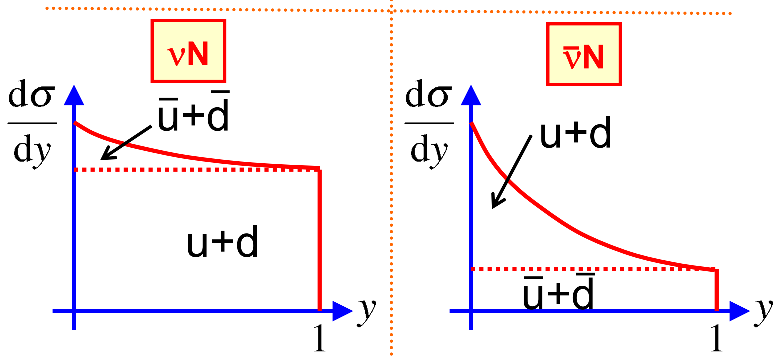
- **CDHS measured y distribution**



J. de Groot et al., Z.Phys. C1 (1979) 143

- **Shapes can be understood in terms of (anti)neutrino – (anti)quark scattering**





Measured Total Cross Sections

★ Integrating the expressions for $\frac{d\sigma}{dy}$ (equations (6) and (7))

$$\boxed{\sigma^{\nu N} = \frac{G_F^2 s}{2\pi}\left[f_q + \frac{1}{3}f_{\overline{q}}\right]} \qquad \boxed{\sigma^{\overline{\nu} N} = \frac{G_F^2 s}{2\pi}\left[\frac{1}{3}f_q + f_{\overline{q}}\right]}$$

$$\underset{(E_\nu,0,0,+E_\nu)}{\nu} \longrightarrow \underset{(m_p,0,0,0)}{p} \qquad s = (E_\nu + m_p)^2 - E_\nu^2 = 2E_\nu m_p + m_p^2 \approx 2E_\nu m_p$$

➡ **DIS cross section $\propto$ lab. frame neutrino energy**

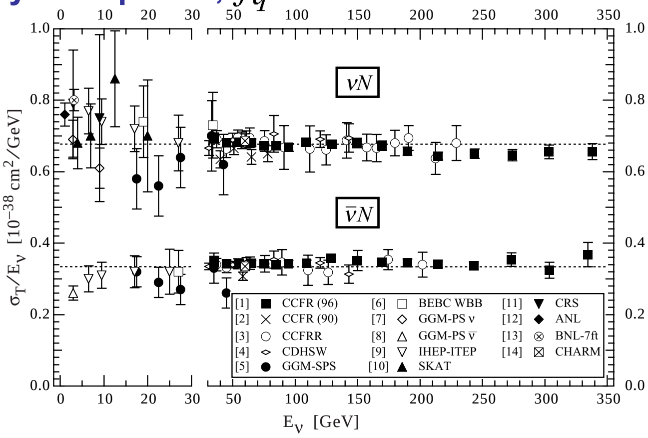
★ Measure cross sections can be used to determine fraction of protons momentum carried by quarks, f_q , and fraction carried by anti-quarks, $f_{\overline{q}}$

- Find: $f_q \approx 0.41; \quad f_{\overline{q}} \approx 0.08$
- ~50% of momentum carried by gluons (which don't interact with virtual W boson)
- If no anti-quarks in nucleons expect

$$\frac{\sigma^{\nu N}}{\sigma^{\overline{\nu} N}} = 3$$

- Including anti-quarks

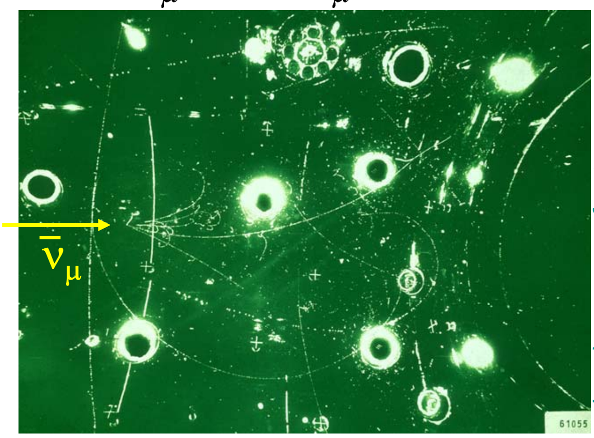
$$\boxed{\frac{\sigma^{\nu N}}{\sigma^{\overline{\nu} N}} \approx 2}$$



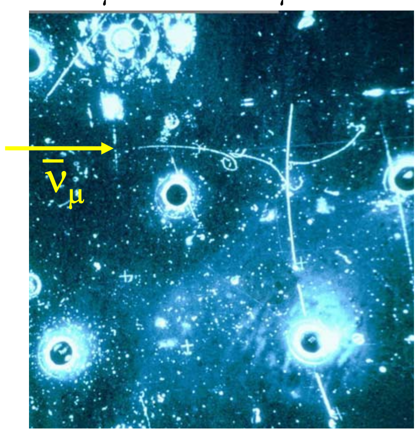
Weak Neutral Current

★ Neutrinos also interact via the Neutral Current. First observed in the Gargamelle bubble chamber in 1973. Interaction of muon neutrinos produce a final state muon

$$\overline{\nu}_\mu + N \rightarrow \overline{\nu}_\mu + \text{hadrons} \qquad \overline{\nu}_\mu + e^- \rightarrow \overline{\nu}_\mu + e^-$$

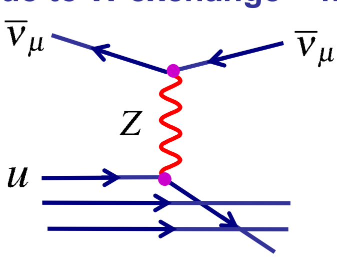


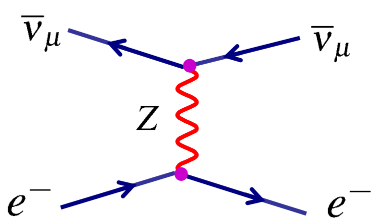
F.J. Hasert et al., Phys. Lett. 46B (1973) 138



F.J. Hasert et al., Phys. Lett. 46B (1973) 121

★ Cannot be due to W exchange - first evidence for Z boson





Summary

★ Derived neutrino/anti-neutrino – quark/anti-quark weak charged current (CC) interaction cross sections

★ Neutrino – nucleon scattering yields extra information about parton distributions functions:

- ν couples to d and $\overline{u}$; $\overline{\nu}$ couples to u and $\overline{d}$

  ➡ investigate flavour content of nucleon

- can measure anti-quark content of nucleon

  $\nu\overline{q}$ suppressed by factor $(1-y)^2$ compared with νq

  $\overline{\nu}q$ suppressed by factor $(1-y)^2$ compared with $\overline{\nu}\overline{q}$

★ Further aspects of neutrino deep-inelastic scattering (expressed in general structure functions) are covered in **Appendix II**

★ Finally observe that neutrinos interact via weak neutral currents (NC)

Appendix I

•For the adjoint spinors $\overline{u} = u^\dagger\gamma^0$ consider

$$\overline{\tfrac{1}{2}(1-\gamma^5)u} = [\tfrac{1}{2}(1-\gamma^5)u]^\dagger\gamma^0 = u^\dagger\tfrac{1}{2}(1-\gamma^5)\gamma^0 = u^\dagger\gamma^0\tfrac{1}{2}(1+\gamma^5) = \overline{u}\tfrac{1}{2}(1+\gamma^5)$$

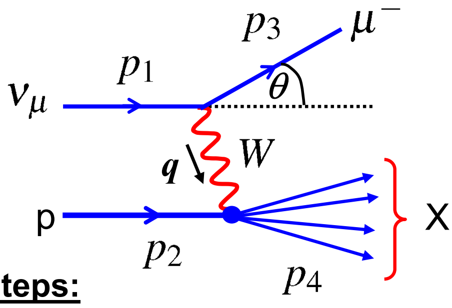
$$\tfrac{1}{2}(1-\gamma^5)u_\uparrow = 0 \quad \Rightarrow \quad \overline{u}\tfrac{1}{2}(1+\gamma^5) = 0$$

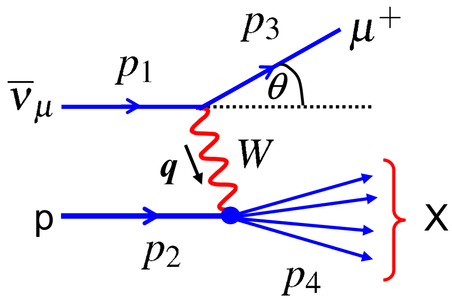
Using the fact that γ^5 and γ^μ anti-commute can rewrite ME:

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\nu}\left[\overline{u}(p_3)\tfrac{1}{2}(1+\gamma^5)\gamma^\mu u(p_1)\right]\left[\overline{u}(p_4)\tfrac{1}{2}(1+\gamma^5)\gamma^\nu u(p_2)\right]$$

➡ $M_{fi} = 0$ for $\overline{u}_\uparrow(p_3)$ and $\overline{u}_\uparrow(p_4)$

Appendix II: Deep-Inelastic Neutrino Scattering





Two steps:

- First write down most general cross section in terms of structure functions
- Then evaluate expressions in the quark-parton model

QED Revisited

★In the limit $s \gg M^2$ the most general electro-magnetic deep-inelastic cross section (from single photon exchange) can be written (Eq. 2 of handout 6)

$$\frac{\mathrm{d}^2\sigma_{e^\pm p}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]$$

- For neutrino scattering typically measure the energy of the produced muon $E_\mu = E_\nu(1-y)$ and differential cross-sections expressed in terms of $\mathrm{d}x\mathrm{d}y$
- Using $Q^2 = (s-M^2)xy \approx sxy$ $\Rightarrow$ $\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}y} = \left|\frac{\mathrm{d}Q^2}{\mathrm{d}y}\right|\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = sx\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2}$

- In the limit $s \gg M^2$ the general Electro-magnetic DIS cross section can be written

$$\boxed{\frac{\mathrm{d}^2\sigma^{e^\pm p}}{\mathrm{d}x\mathrm{d}y} = \frac{4\pi\alpha^2 s}{Q^4}\left[(1-y)F_2(x,Q^2) + y^2xF_1(x,Q^2)\right]}$$

- **NOTE:** This is the most general **Lorentz Invariant** parity conserving expression

★For neutrino DIS parity is violated and the general expression includes an additional term to allow for parity violation. New structure function $F_3(x,Q^2)$

$\boxed{\nu_\mu p \rightarrow \mu^- X}$
$$\boxed{\frac{\mathrm{d}^2\sigma^{\nu p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\nu p}(x,Q^2) + y^2xF_1^{\nu p}(x,Q^2) + y\left(1-\frac{y}{2}\right)xF_3^{\nu p}(x,Q^2)\right]}$$

- For anti-neutrino scattering new structure function enters with opposite sign

$\boxed{\overline{\nu}_\mu p \rightarrow \mu^+ X}$
$$\boxed{\frac{\mathrm{d}^2\sigma^{\overline{\nu} p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\overline{\nu} p}(x,Q^2) + y^2xF_1^{\overline{\nu} p}(x,Q^2) - y\left(1-\frac{y}{2}\right)xF_3^{\overline{\nu} p}(x,Q^2)\right]}$$

- Similarly for neutrino-neutron scattering

$\boxed{\nu_\mu n \rightarrow \mu^- X}$
$$\boxed{\frac{\mathrm{d}^2\sigma^{\nu n}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\nu n}(x,Q^2) + y^2xF_1^{\nu n}(x,Q^2) + y\left(1-\frac{y}{2}\right)xF_3^{\nu n}(x,Q^2)\right]}$$

$\boxed{\overline{\nu}_\mu n \rightarrow \mu^+ X}$
$$\boxed{\frac{\mathrm{d}^2\sigma^{\overline{\nu} n}}{\mathrm{d}x\mathrm{d}y} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\overline{\nu} n}(x,Q^2) + y^2xF_1^{\overline{\nu} n}(x,Q^2) - y\left(1-\frac{y}{2}\right)xF_3^{\overline{\nu} n}(x,Q^2)\right]}$$

Neutrino Interaction Structure Functions

★ **In terms of the parton distribution functions we found (2) :**

$$\frac{\mathrm{d}^2\sigma^{\nu p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_\mathrm{F}^2}{\pi} sx\left[d(x) + (1-y)^2\overline{u}(x)\right]$$

• **Compare coefficients of y with the general Lorentz Invariant form (p.321) and assume Bjorken scaling, i.e.** $F(x,Q^2) \to F(x)$

$$\frac{\mathrm{d}^2\sigma^{\nu p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_\mathrm{F}^2 s}{2\pi}\left[(1-y)F_2^{\nu p}(x) + y^2 x F_1^{\nu p}(x) + y\left(1-\frac{y}{2}\right)xF_3^{\nu p}(x)\right]$$

• **Re-writing (2)**

$$\frac{\mathrm{d}^2\sigma^{\nu p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_\mathrm{F}^2}{2\pi} s\left[2xd(x) + 2x\overline{u}(x) - 4xy\overline{u}(x) + 2xy^2\overline{u}(x)\right]$$

and equating powers of y

$$\left.\begin{aligned} 2xd + 2x\overline{u} &= F_2 \\ -4x\overline{u} &= -F_2 + xF_3 \\ 2\overline{u} &= F_1 - xF_3/2 \end{aligned}\right\}$$

gives:

$$\boxed{\begin{aligned} F_2^{\nu p} = 2xF_1^{\nu p} &= 2x[d(x) + \overline{u}(x)] \\ xF_3^{\nu p} &= 2x[d(x) - \overline{u}(x)] \end{aligned}}$$

<u>NOTE:</u> again we get the Callan-Gross relation $F_2 = 2xF_1$

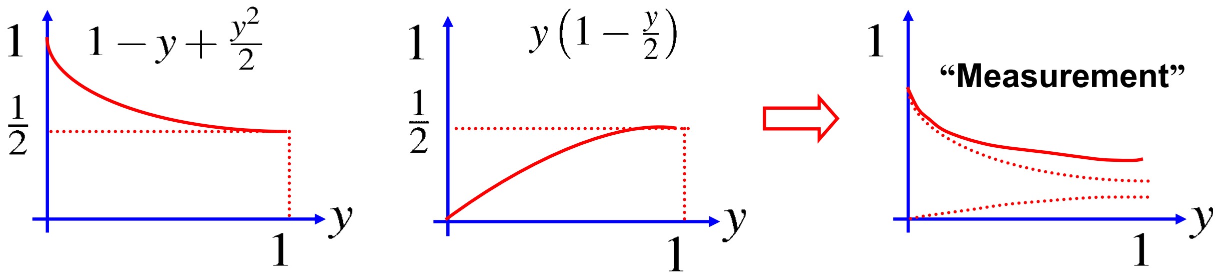
No surprise, underlying process is scattering from point-like spin-1/2 quarks

★ **Substituting back in to expression for differential cross section:**

$$\frac{\mathrm{d}^2\sigma^{\nu p}}{\mathrm{d}x\mathrm{d}y} = \frac{G_\mathrm{F}^2 s}{2\pi}\left[\left(1-y+\frac{y^2}{2}\right)F_2^{\nu p}(x) + y\left(1-\frac{y}{2}\right)xF_3^{\nu p}(x)\right]$$

★ **Experimentally measure** F_2 **and** F_3 **from** y **distributions at fixed** x

- **Different y dependencies (from different rest frame angular distributions) allow contributions from the two structure functions to be measured**



★ **Then use** $F_2^{\nu p} = 2x[d(x) + \overline{u}(x)]$ **and** $F_3^{\nu p} = 2[d(x) - \overline{u}(x)]$

➡ **Determine** $d(x)$ **and** $\overline{u}(x)$ **separately**

★ Neutrino experiments require large detectors (often iron) i.e. isoscalar target

$$\boxed{\begin{aligned} F_2^{\nu N} = 2xF_1^{\nu N} = \tfrac{1}{2}\left(F_2^{\nu p} + F_2^{\nu n}\right) &= x[u(x)+d(x)+\overline{u}(x)+\overline{d}(x)] \\ xF_3^{\nu N} = \tfrac{1}{2}\left(xF_3^{\nu p} + xF_3^{\nu n}\right) &= x[u(x)+d(x)-\overline{u}(x)-\overline{d}(x)] \end{aligned}}$$

★ For electron – nucleon scattering:

$$F_2^{ep} = 2xF_1^{ep} = x[\tfrac{4}{9}u(x) + \tfrac{1}{9}d(x) + \tfrac{4}{9}\overline{u}(x) + \tfrac{1}{9}\overline{d}(x)]$$

$$F_2^{en} = 2xF_1^{en} = x[\tfrac{4}{9}d(x) + \tfrac{1}{9}u(x) + \tfrac{4}{9}\overline{d}(x) + \tfrac{1}{9}\overline{u}(x)]$$

• For an isoscalar target

$$\boxed{F_2^{eN} = \tfrac{1}{2}\left(F_2^{ep} + F_2^{en}\right) = \tfrac{5}{18}x[u(x)+d(x)+\overline{u}(x)+\overline{d}(x)]}$$

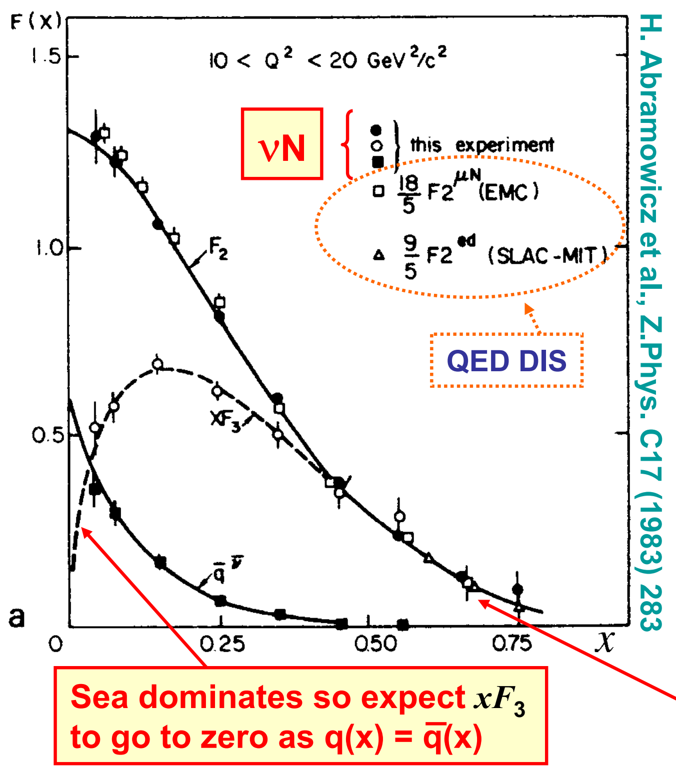
$$\Rightarrow \boxed{F_2^{\nu N} = \tfrac{18}{5}F_2^{eN}}$$

• Note that the factor $\frac{5}{18} = \frac{1}{2}\left(q_u^2 + q_d^2\right)$ and by comparing neutrino to electron scattering structure functions measure the sum of quark charges

Experiment: 0.29 ± 0.02

Measurements of $F_2(x)$ and $F_3(x)$

• CDHS Experiment $\nu_\mu + \mathrm{Fe} \rightarrow \mu^- + X$



$$\boxed{\begin{aligned} F_2^{\nu N} &= x[u(x)+d(x)+\overline{u}(x)+\overline{d}(x)] \\ xF_3^{\nu N} &= x[u(x)+d(x)-\overline{u}(x)-\overline{d}(x)] \end{aligned}}$$

$$\Rightarrow F_2^{\nu N} - xF_3^{\nu N} = 2x[\overline{u} + \overline{d}]$$

✶ Difference in neutrino structure functions measures anti-quark (sea) parton distribution functions

Valence Contribution

★ Separate parton density functions into sea and valence components

$$u(x) = u_V(x) + u_S(x) = u_V(x) + S(x)$$
$$d(x) = d_V(x) + d_S(x) = d_V(x) + S(x)$$
$$\overline{u}(x) = \overline{u}_S(x) = S(x)$$
$$\overline{d}(x) = \overline{d}_S(x) = S(x)$$

➡ $$\boxed{F_3^{\nu N} = [u(x) + d(x) - \overline{u}(x) - \overline{d}(x)] = u_V(x) + d_V(x)}$$

➡ $$\int_0^1 F_3^{\nu N}(x)\mathrm{d}x = \int_0^1 (u_V(x) + d_V(x))\,\mathrm{d}x = N_u^V + N_d^V$$

★ Area under measured function $F_3^{\nu N}(x)$ gives a measurement of the total number of valence quarks in a nucleon !

expect $$\int_0^1 F_3^{\nu N}(x)\mathrm{d}x = 3$$ "Gross – Llewellyn-Smith sum rule"

Experiment: 3.0±0.2

•Note: $F_2^{\overline{\nu}p} = F_2^{\nu n};\ F_2^{\overline{\nu}n} = F_2^{\nu p};\ F_3^{\overline{\nu}p} = F_3^{\nu n};\ F_3^{\overline{\nu}n} = F_3^{\nu p}$ and anti-neutrino structure functions contain same pdf information