# Particle Physics

**Michaelmas Term 2011**
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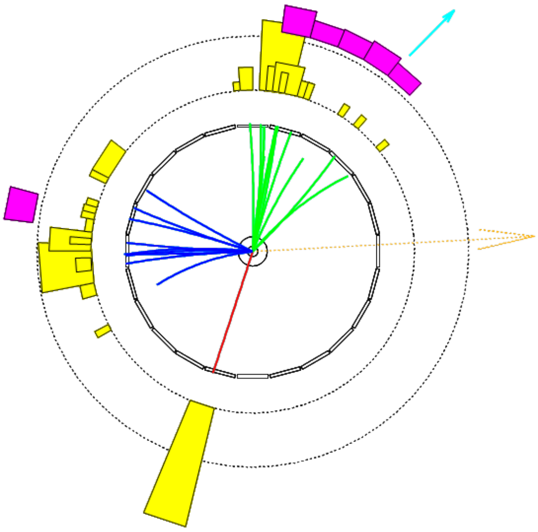


## Handout 13 : Electroweak Unification and the W and Z Bosons



# Boson Polarization States

★ In this handout we are going to consider the decays of W and Z bosons, for this we will need to consider the polarization. Here simply quote results although the justification is given in Appendices I and II

★ A real (i.e. not virtual) massless spin-1 boson can exist in two transverse polarization states, a massive spin-1 boson also can be longitudinally polarized

★ Boson wave-functions are written in terms of the polarization four-vector $\varepsilon^{\mu}$

$$B^{\mu} = \varepsilon^{\mu} e^{-ip.x} = \varepsilon^{\mu} e^{i(\vec{p}.\vec{x}-Et)}$$

★ For a spin-1 boson travelling along the z-axis, the polarization four vectors are:

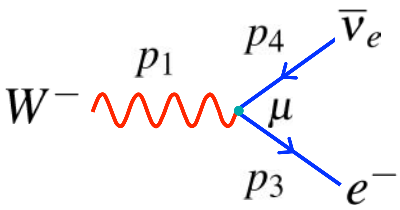
| $\varepsilon^{\mu}_{-} = \frac{1}{\sqrt{2}}(0,1,-i,0);$ | $\varepsilon_{L} = \frac{1}{m}(p_z,0,0,E)$ | $\varepsilon^{\mu}_{+} = -\frac{1}{\sqrt{2}}(0,1,i,0)$ |
|---|---|---|
| $S_z = -1$ | $S_z = 0$ | $S_z = +1$ |
| transverse | longitudinal | transverse |

Longitudinal polarization isn't present for on-shell massless particles, the photon can exist in two helicity states $h = \pm 1$ (LH and RH circularly polarized light)

# W-Boson Decay

★ To calculate the W-Boson decay rate first consider $W^- \to e^- \overline{\nu}_e$

★ Want matrix element for :




Incoming W-boson : $\varepsilon_\mu(p_1)$

Out-going electron : $\overline{u}(p_3)$

Out-going $\overline{\nu}_e$ : $v(p_4)$

Vertex factor : $-i\frac{g_W}{\sqrt{2}}\frac{1}{2}\gamma^\mu(1-\gamma^5)$

$$-iM_{fi} = \varepsilon_\mu(p_1).\overline{u}(p_3).-i\frac{g_W}{\sqrt{2}}\gamma^\mu\tfrac{1}{2}(1-\gamma^5).v(p_4)$$

Note, no propagator

$$\Rightarrow \quad M_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_\mu(p_1)\overline{u}(p_3)\gamma^\mu\tfrac{1}{2}(1-\gamma^5)v(p_4)$$

★ This can be written in terms of the four-vector scalar product of the W-boson polarization $\varepsilon_\mu(p_1)$ and the weak charged current $j^\mu$

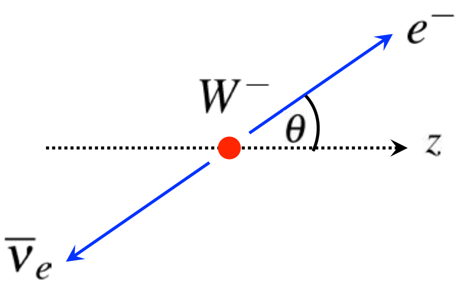
$$M_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_\mu(p_1).j^\mu \quad \text{with} \quad j^\mu = \overline{u}(p_3)\gamma^\mu\tfrac{1}{2}(1-\gamma^5)v(p_4)$$



# W-Decay : The Lepton Current

★ First consider the lepton current $j^\mu = \overline{u}(p_3)\gamma^\mu\tfrac{1}{2}(1-\gamma^5)v(p_4)$

★ Work in Centre-of-Mass frame




$$p_1 = (m_W, 0, 0, 0);$$
$$p_3 = (E, E\sin\theta, 0, E\cos\theta)$$
$$p_4 = (E, -E\sin\theta, 0, -E\cos\theta)$$

with $E = \frac{m_W}{2}$

★ In the ultra-relativistic limit only LH particles and RH anti-particles participate in the weak interaction so

$$j^\mu = \overline{u}(p_3)\gamma^\mu\tfrac{1}{2}(1-\gamma^5)v(p_4) = \overline{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4)$$

Note: $\tfrac{1}{2}(1-\gamma^5)v(p_4) = v_\uparrow(p_4)$ $\qquad \overline{u}(p_3)\gamma^\mu v_\uparrow(p_4) = \overline{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4)$

Chiral projection operator, e.g. see p.131 or p.294

"Helicity conservation", e.g. see p.133 or p.295

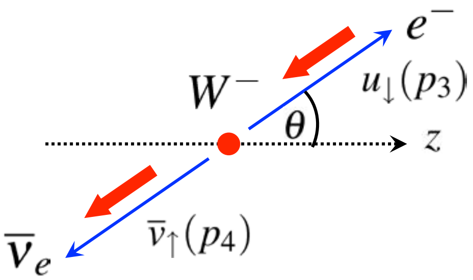
- We have already calculated the current

$$j^{\mu} = \bar{u}_{\downarrow}(p_3)\gamma^{\mu}v_{\uparrow}(p_4)$$

when considering $e^+e^- \to \mu^+\mu^-$

- From page 128 we have for $\mu_L^- \mu_R^+$

$$j^{\mu}_{\uparrow\downarrow} = 2E(0, -\cos\theta, -i, \sin\theta)$$

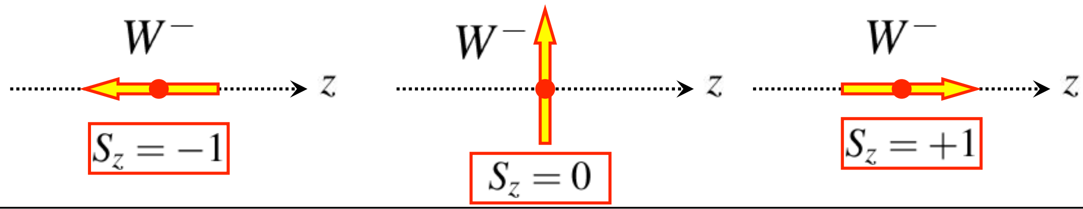



- For the charged current weak Interaction we only have to consider this single combination of helicities

$$j^{\mu} = \bar{u}(p_3)\gamma^{\mu}\tfrac{1}{2}(1-\gamma^5)v(p_4) = \bar{u}_{\downarrow}(p_3)\gamma^{\mu}v_{\uparrow}(p_4) = 2E(0, -\cos\theta, -i, \sin\theta)$$

and the three possible W-Boson polarization states:

$$\varepsilon^{\mu}_{-} = \frac{1}{\sqrt{2}}(0,1,-i,0); \quad \varepsilon_L = \frac{1}{m}(p_z,0,0,E) \quad \varepsilon^{\mu}_{+} = -\frac{1}{\sqrt{2}}(0,1,i,0)$$






★ For a W-boson at rest these become:

$$\varepsilon^{\mu}_{-} = \frac{1}{\sqrt{2}}(0,1,-i,0); \quad \varepsilon_L = (0,0,0,1) \quad \varepsilon^{\mu}_{+} = -\frac{1}{\sqrt{2}}(0,1,i,0)$$

★ Can now calculate the matrix element for the different polarization states

$$M_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_{\mu}(p_1)j^{\mu} \quad \text{with} \quad j^{\mu} = 2\frac{m_W}{2}(0, -\cos\theta, -i, \sin\theta)$$

Decay at rest : $E_e = E_\nu = m_W/2$

★ giving

$\varepsilon_-$ $$M_- = \frac{g_W}{\sqrt{2}}\frac{1}{\sqrt{2}}(0,1,-i,0).m_W(0,-\cos\theta,-i,\sin\theta) = \tfrac{1}{2}g_W m_W(1+\cos\theta)$$

$\varepsilon_L$ $$M_L = \frac{g_W}{\sqrt{2}}(0,0,0,1).m_W(0,-\cos\theta,-i,\sin\theta) = -\frac{1}{\sqrt{2}}g_W m_W \sin\theta$$

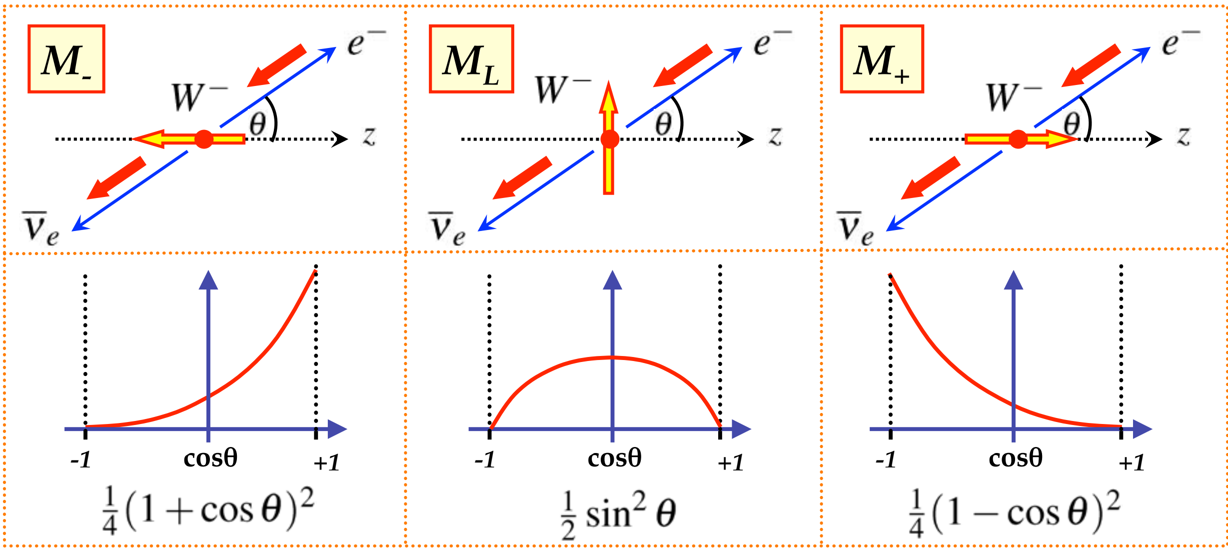
$\varepsilon_+$ $$M_+ = -\frac{g_W}{\sqrt{2}}\frac{1}{\sqrt{2}}(0,1,i,0).m_W(0,-\cos\theta,-i,\sin\theta) = \tfrac{1}{2}g_W m_W(1-\cos\theta)$$

$$\Rightarrow \quad |M_-|^2 = g_W^2 m_W^2 \tfrac{1}{4}(1+\cos\theta)^2$$

$$|M_L|^2 = g_W^2 m_W^2 \tfrac{1}{2}\sin^2\theta$$

$$|M_+|^2 = g_W^2 m_W^2 \tfrac{1}{4}(1-\cos\theta)^2$$

★ The angular distributions can be understood in terms of the spin of the particles




★ The differential decay rate (see page 26) can be found using:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{|p^*|}{32\pi^2 m_W^2}|M|^2$$

where p* is the C.o.M momentum of the final state particles, here $p^* = \frac{m_W}{2}$



★ Hence for the three different polarisations we obtain:

$$\frac{\mathrm{d}\Gamma_-}{\mathrm{d}\Omega} = \frac{g_W^2 m_w}{64\pi^2}\frac{1}{4}(1+\cos\theta)^2 \qquad \frac{\mathrm{d}\Gamma_L}{\mathrm{d}\Omega} = \frac{g_W^2 m_w}{64\pi^2}\frac{1}{2}\sin^2\theta \qquad \frac{\mathrm{d}\Gamma_+}{\mathrm{d}\Omega} = \frac{g_W^2 m_w}{64\pi^2}\frac{1}{4}(1-\cos\theta)^2$$

★ Integrating over all angles using

$$\int \tfrac{1}{4}(1 \pm \cos\theta)^2 \mathrm{d}\phi\,\mathrm{d}\cos\theta = \int \tfrac{1}{2}\sin^2\theta\,\mathrm{d}\phi\,\mathrm{d}\cos\theta = \frac{4\pi}{3}$$

★ Gives

$$\boxed{\Gamma_- = \Gamma_L = \Gamma_+ = \frac{g_W^2 m_W}{48\pi}}$$

★ The total W-decay rate is independent of polarization; this has to be the case as the decay rate cannot depend on the arbitrary definition of the z-axis

★ For a sample of unpolarized W boson each polarization state is equally likely, for the average matrix element sum over all possible matrix elements and average over the three initial polarization states

$$\begin{aligned}\langle |M_{fi}|^2 \rangle &= \tfrac{1}{3}(|M_-|^2 + |M_L|^2 + |M_+|^2) \\ &= \tfrac{1}{3} g_W^2 m_W^2 \left[\tfrac{1}{4}(1+\cos\theta)^2 + \tfrac{1}{2}\sin^2\theta + \tfrac{1}{4}(1-\cos\theta)^2\right] \\ &= \tfrac{1}{3} g_W^2 m_W^2\end{aligned}$$

★ For a sample of unpolarized W-bosons, the decay is isotropic (as expected)

★ For this isotropic decay

$$\frac{d\Gamma}{d\Omega} = \frac{|p^*|}{32\pi^2 m_W^2}\langle|M|^2\rangle \Rightarrow \Gamma = \frac{4\pi|p^*|}{32\pi^2 m_W^2}\langle|M|^2\rangle$$

$$\Rightarrow \boxed{\Gamma(W^- \to e^-\overline{\nu}) = \frac{g_W^2 m_W}{48\pi}}$$

★ The calculation for the other decay modes (neglecting final state particle masses) is same. For quarks need to account for colour and CKM matrix. No decays to top – the top mass (175 GeV) is greater than the W-boson mass (80 GeV)

| | | | | |
|---|---|---|---|---|
| $W^- \to e^-\overline{\nu}_e$ | $W^- \to d\overline{u}$ | $\times 3\lvert V_{ud}\rvert^2$ | $W^- \to d\overline{c}$ | $\times 3\lvert V_{cd}\rvert^2$ |
| $W^- \to \mu^-\overline{\nu}_\mu$ | $W^- \to s\overline{u}$ | $\times 3\lvert V_{us}\rvert^2$ | $W^- \to s\overline{c}$ | $\times 3\lvert V_{cs}\rvert^2$ |
| $W^- \to \tau^-\overline{\nu}_\tau$ | $W^- \to b\overline{u}$ | $\times 3\lvert V_{ub}\rvert^2$ | $W^- \to b\overline{c}$ | $\times 3\lvert V_{cb}\rvert^2$ |

★ Unitarity of CKM matrix gives, e.g. $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

★ Hence $BR(W \to qq') = 6BR(W \to e\nu)$

and thus the total decay rate :

$$\boxed{\Gamma_W = 9\Gamma_{W\to e\nu} = \frac{3g_W^2 m_W}{16\pi} = 2.07\,\text{GeV}}$$

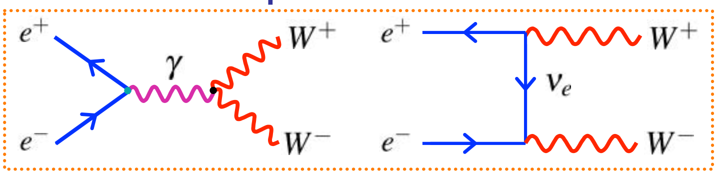
**Experiment: 2.14±0.04 GeV**
(our calculation neglected a 3% QCD correction to decays to quarks )

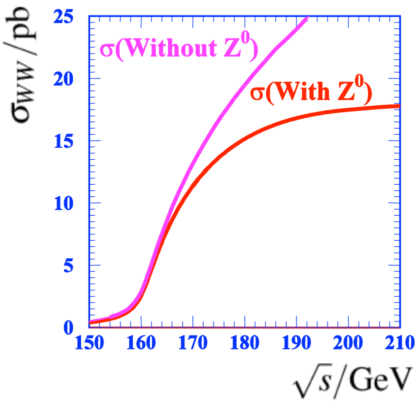


# From W to Z

★ The W± bosons carry the EM charge - suggestive Weak are EM forces are related.

★ W bosons can be produced in e⁺e⁻ annihilation

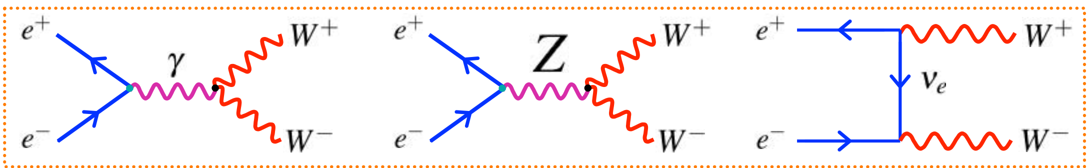



★ With just these two diagrams there is a problem: the cross section increases with C.o.M energy and at some point violates QM unitarity

**UNITARITY VIOLATION: when QM calculation gives larger flux of W bosons than incoming flux of electrons/positrons**

★ Problem can be "fixed" by introducing a new boson, the Z. The new diagram interferes negatively with the above two diagrams fixing the unitarity problem

⇒ 


$$|M_{\gamma WW} + M_{ZWW} + M_{\nu WW}|^2 < |M_{\gamma WW} + M_{\nu WW}|^2$$

★ Only works if Z, γ, W couplings are related: need ELECTROWEAK UNIFICATION

# SU(2)$_L$ : The Weak Interaction

★ The Weak Interaction arises from SU(2) local phase transformations

$$\psi \to \psi' = \psi e^{i\vec{\alpha}(x).\frac{\vec{\sigma}}{2}}$$

where the $\vec{\sigma}$ are the generators of the SU(2) symmetry, i.e the three Pauli spin matrices ⇒ **3 Gauge Bosons** $W_1^\mu,\ W_2^\mu,\ W_3^\mu$

★ The wave-functions have two components which, in analogy with isospin, are represented by "weak isospin"

★ The fermions are placed in isospin doublets and the local phase transformation corresponds to

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix} \to \begin{pmatrix}\nu_e\\ e^-\end{pmatrix}' = e^{i\vec{\alpha}(x).\frac{\vec{\sigma}}{2}}\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}$$

★ Weak Interaction only couples to LH particles/RH anti-particles, hence only place LH particles/RH anti-particles in weak isospin doublets: $I_W = \frac{1}{2}$
RH particles/LH anti-particles placed in weak isospin singlets: $I_W = 0$

**Weak Isospin**

$I_W = \frac{1}{2}$ $\quad \begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L, \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L, \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L, \begin{pmatrix}u\\ d'\end{pmatrix}_L, \begin{pmatrix}c\\ s'\end{pmatrix}_L, \begin{pmatrix}t\\ b'\end{pmatrix}_L$ ← $I_W^3 = +\frac{1}{2}$ (upper), $I_W^3 = -\frac{1}{2}$ (lower)

$I_W = 0$ $\quad (\nu_e)_R,\ (e^-)_R, \ldots (u)_R,\ (d)_R, \ldots$

Note: RH/LH refer to chiral states



★ For simplicity only consider $\chi_L = \begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L$

•The gauge symmetry specifies the form of the interaction: one term for each of the 3 generators of SU(2) – [note: here include interaction strength in current]

$$j_\mu^1 = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2}\sigma_1 \chi_L \qquad j_\mu^2 = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2}\sigma_2 \chi_L \qquad j_\mu^3 = g_W \overline{\chi}_L \gamma^\mu \tfrac{1}{2}\sigma_3 \chi_L$$

★The charged current $W^+/W^-$ interaction enters as a linear combinations of $W_1$, $W_2$

$$W^{\pm\mu} = \tfrac{1}{\sqrt{2}}(W_1^\mu \pm W_2^\mu)$$

★ The $W^\pm$ interaction terms

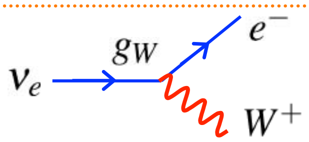
$$j_\pm^\mu = \tfrac{g_W}{\sqrt{2}}(j_1^\mu \pm i j_2^\mu) = \tfrac{g_W}{\sqrt{2}}\overline{\chi}_L \gamma^\mu \tfrac{1}{2}(\sigma_1 \pm i\sigma_2)\chi_L$$

★ Express in terms of the weak isospin ladder operators $\sigma_\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$

$$j_\pm^\mu = \tfrac{g_W}{\sqrt{2}}\overline{\chi}_L \gamma^\mu \sigma_\pm \chi_L$$

} Origin of $\frac{1}{\sqrt{2}}$ in Weak CC

**W⁺**




corresponds to $j_+^\mu = \frac{g_W}{\sqrt{2}}\overline{\chi}_L \gamma^\mu \sigma_+ \chi_L$

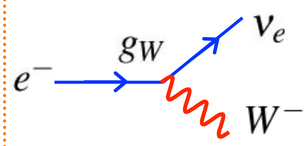
Bars indicates adjoint spinors

which can be understood in terms of the weak isospin doublet

$$j_+^\mu = \frac{g_W}{\sqrt{2}}\overline{\chi}_L \gamma^\mu \sigma_+ \chi_L = \frac{g_W}{\sqrt{2}}(\overline{\nu}_L, \overline{e}_L)\gamma^\mu \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}\nu\\ e\end{pmatrix}_L = \frac{g_W}{\sqrt{2}}\overline{\nu}_L \gamma^\mu e_L = \frac{g_W}{\sqrt{2}}\overline{\nu}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)e$$

★ Similarly

W⁻




corresponds to $j^\mu_- = \frac{g_W}{\sqrt{2}}\overline{\chi}_L\gamma^\mu\sigma_-\chi_L$

$$j^\mu_- = \frac{g_W}{\sqrt{2}}\overline{\chi}_L\gamma^\mu\sigma_-\chi_L = \frac{g_W}{\sqrt{2}}(\overline{\nu}_L,\overline{e}_L)\gamma^\mu\begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}\nu\\e\end{pmatrix}_L = \frac{g_W}{\sqrt{2}}\overline{e}_L\gamma^\mu\nu_L = \frac{g_W}{\sqrt{2}}\overline{e}\gamma^\mu\tfrac{1}{2}(1-\gamma^5)\nu$$

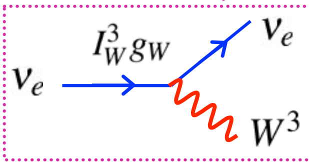
★ However have an **additional interaction** due to W³

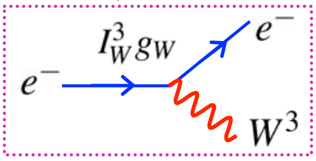
$$j^\mu_3 = g_W\overline{\chi}_L\gamma^\mu\tfrac{1}{2}\sigma_3\chi_L$$

expanding this:

$I^3_W = \pm\frac{1}{2}$

$$j^\mu_3 = g_W\frac{1}{2}(\overline{\nu}_L,\overline{e}_L)\gamma^\mu\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}\nu\\e\end{pmatrix}_L = g_W\tfrac{1}{2}\overline{\nu}_L\gamma^\mu\nu_L - g_W\tfrac{1}{2}\overline{e}_L\gamma^\mu e_L$$




NEUTRAL CURRENT INTERACTIONS !



# Electroweak Unification

★ Tempting to identify the $W^3$ as the $Z$

★ However this is not the case, have two physical neutral spin-1 gauge bosons, $\gamma, Z$ and the $W^3$ is a mixture of the two,

★ Equivalently write the photon and $Z$ in terms of the $W^3$ and a new neutral spin-1 boson the $B$

★ The physical bosons (the $Z$ and photon field, $A$ ) are:

$$A_\mu = B_\mu\cos\theta_W + W^3_\mu\sin\theta_W$$
$$Z_\mu = -B_\mu\sin\theta_W + W^3_\mu\cos\theta_W$$

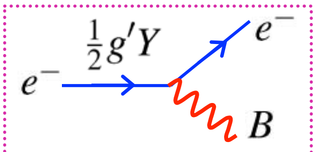
$\theta_W$ is the weak mixing angle

★ The new boson is associated with a new gauge symmetry similar to that of electromagnetism : $U(1)_Y$

★ The charge of this symmetry is called WEAK HYPERCHARGE $Y$

$$Y = 2Q - 2I^3_W$$

Q is the EM charge of a particle
$I^3_W$ is the third comp. of weak isospin




•By convention the coupling to the $B_\mu$ is $\frac{1}{2}g'Y$

$e_L: \; Y = 2(-1) - 2(-\frac{1}{2}) = -1$ $\qquad \nu_L: \; Y = +1$

$e_R: \; Y = 2(-1) - 2(0) = -2$ $\qquad \nu_R: \; Y = 0$

(this identification of hypercharge in terms of Q and $I_3$ makes all of the following work out)

★ For this to work the coupling constants of the **W³**, **B**, and **photon** must be related

**e.g. consider contributions involving the neutral interactions of electrons:**

γ $$j^{em}_\mu = e\overline{\psi}Q_e\gamma_\mu\psi = e\overline{e}_L Q_e\gamma_\mu e_L + e\overline{e}_R Q_e\gamma_\mu e_R$$

W³ $$j^{W^3}_\mu = -\tfrac{g_W}{2}\overline{e}_L\gamma_\mu e_L$$

B $$j^{Y}_\mu = \tfrac{g'}{2}\overline{\psi}Y_e\gamma_\mu\psi = \tfrac{g'}{2}\overline{e}_L Y_{e_L}\gamma_\mu e_L + \tfrac{g'}{2}\overline{e}_R Y_{e_R}\gamma_\mu e_R$$

★ The relation $A_\mu = B_\mu\cos\theta_W + W^3_\mu\sin\theta_W$ is equivalent to requiring

$$j^{em}_\mu = j^Y_\mu\cos\theta_W + j^{W^3}_\mu\sin\theta_W$$

•Writing this in full:

$$e\overline{e}_L Q_e\gamma_\mu e_L + e\overline{e}_R Q_e\gamma_\mu e_R = \tfrac{1}{2}g'\cos\theta_W[\overline{e}_L Y_{e_L}\gamma_\mu e_L + \overline{e}_R Y_{e_R}\gamma_\mu e_R] - \tfrac{1}{2}g_W\sin\theta_W[\overline{e}_L\gamma_\mu e_L]$$

$$-e\overline{e}_L\gamma_\mu e_L - e\overline{e}_R\gamma_\mu e_R = \tfrac{1}{2}g'\cos\theta_W[-\overline{e}_L\gamma_\mu e_L - 2\overline{e}_R\gamma_\mu e_R] - \tfrac{1}{2}g_W\sin\theta_W[\overline{e}_L\gamma_\mu e_L]$$

which works if: $$e = g_W\sin\theta_W = g'\cos\theta_W$$ (i.e. equate coefficients of L and R terms)

★ Couplings of electromagnetism, the weak interaction and the interaction of the $U(1)_Y$ symmetry are therefore related.



# The Z Boson

★In this model we can now derive the couplings of the Z Boson

$$Z_\mu = -B_\mu\sin\theta_W + W^3_\mu\cos\theta_W$$

$I^3_W$ for the electron $I^3_W = \frac{1}{2}$

$$j^Z_\mu = -\tfrac{1}{2}g'\sin\theta_W[\overline{e}_L Y_{e_L}\gamma_\mu e_L + \overline{e}_R Y_{e_R}\gamma_\mu e_R] - \tfrac{1}{2}g_W\cos\theta_W[\overline{e}_L\gamma_\mu e_L]$$

•Writing this in terms of weak isospin and charge:

$$j^Z_\mu = -\tfrac{1}{2}g'\sin\theta_W[\overline{e}_L(2Q - 2I^3_W)\gamma_\mu e_L + \overline{e}_R(2Q)\gamma_\mu e_R] + I^3_W g_W\cos\theta_W[\overline{e}_L\gamma_\mu e_L]$$

For RH chiral states $I_3=0$

•Gathering up the terms for LH and RH chiral states:

$$j^Z_\mu = \left[g' I^3_W\sin\theta_W - g'Q\sin\theta_W + g_W I^3_W\cos\theta_W\right]\overline{e}_L\gamma_\mu e_L - \left[g'Q\sin\theta_W\right]\overline{e}_R\gamma_\mu e_R$$

•Using: $e = g_W\sin\theta_W = g'\cos\theta_W$ gives

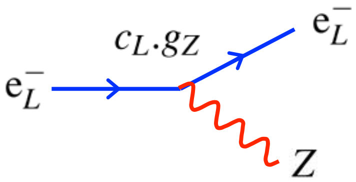
$$j^Z_\mu = \left[g'\frac{(I^3_W - Q\sin^2\theta_W)}{\sin\theta_W}\right]\overline{e}_L\gamma_\mu e_L - \left[g'\frac{Q\sin^2\theta_W}{\sin\theta_W}\right]\overline{e}_R\gamma_\mu e_R$$

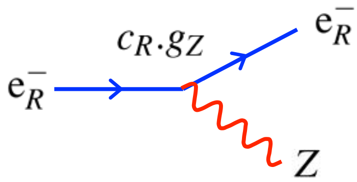
$$j^Z_\mu = g_Z(I^3_W - Q\sin^2\theta_W)[\overline{e}_L\gamma_\mu e_L] - g_Z Q\sin^2\theta_W[\overline{e}_R\gamma_\mu e_R]$$

with $e = g_Z\cos\theta_W\sin\theta_W$ i.e. $g_Z = \dfrac{g_W}{\cos\theta_W}$

★ Unlike for the Charged Current Weak interaction (W) the Z Boson couples to both LH and RH chiral components, **but not equally…**

$$j^Z_\mu = g_Z(I^3_W - Q\sin^2\theta_W)[\bar{e}_L\gamma_\mu e_L] - g_Z Q\sin^2\theta_W[\bar{e}_R\gamma_\mu e_R]$$
$$= g_Z c_L[\bar{e}_L\gamma_\mu e_L] + g_Z c_R[\bar{e}_R\gamma_\mu e_R]$$




$$c_L = I^3_W - Q\sin^2\theta_W \qquad c_R = -Q\sin^2\theta_W$$

$W^3$ part of Z couples only to LH components (like $W^\pm$)

$B_\mu$ part of Z couples equally to LH and RH components

★ Use projection operators to obtain vector and axial vector couplings

$$\bar{u}_L\gamma_\mu u_L = \bar{u}\gamma_\mu \tfrac{1}{2}(1-\gamma_5)u \qquad \bar{u}_R\gamma_\mu u_R = \bar{u}\gamma_\mu \tfrac{1}{2}(1+\gamma_5)u$$

$$j^Z_\mu = g_Z\bar{u}\gamma_\mu\left[c_L\tfrac{1}{2}(1-\gamma_5) + c_R\tfrac{1}{2}(1+\gamma_5)\right]u$$



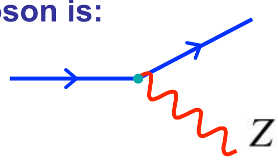
$$j^Z_\mu = \frac{g_Z}{2}\bar{u}\gamma_\mu\left[(c_L+c_R) + (c_R-c_L)\gamma_5\right]u$$

★ Which in terms of **V** and **A** components gives: $j^Z_\mu = \frac{g_Z}{2}\bar{u}\gamma_\mu\left[c_V - c_A\gamma_5\right]u$

with $c_V = c_L + c_R = I^3_W - 2Q\sin^2\theta_W$ $\qquad c_A = c_L - c_R = I^3_W$

★ Hence the vertex factor for the Z boson is:

$$-ig_Z\tfrac{1}{2}\gamma_\mu\left[c_V - c_A\gamma_5\right]$$



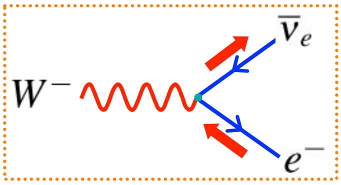

★ Using the experimentally determined value of the weak mixing angle:

$\sin^2\theta_W \approx 0.23$

| Fermion | $Q$ | $I^3_W$ | $c_L$ | $c_R$ | $c_V$ | $c_A$ |
|---|---|---|---|---|---|---|
| $\nu_e, \nu_\mu, \nu_\tau$ | 0 | $+\frac{1}{2}$ | $+\frac{1}{2}$ | 0 | $+\frac{1}{2}$ | $+\frac{1}{2}$ |
| $e^-, \mu^-, \tau^-$ | $-1$ | $-\frac{1}{2}$ | $-0.27$ | 0.23 | $-0.04$ | $-\frac{1}{2}$ |
| $u, c, t$ | $+\frac{2}{3}$ | $+\frac{1}{2}$ | 0.35 | $-0.15$ | $+0.19$ | $+\frac{1}{2}$ |
| $d, s, b$ | $-\frac{1}{3}$ | $-\frac{1}{2}$ | $-0.42$ | 0.08 | $-0.35$ | $-\frac{1}{2}$ |

# Z Boson Decay : $\Gamma_Z$

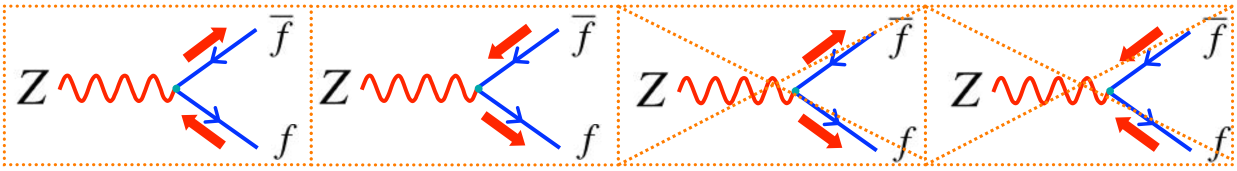
★ In W-boson decay only had to consider one helicity combination of (assuming we can neglect final state masses: helicity states = chiral states)




W-boson couples:
to LH particles
and RH anti-particles

★ But Z-boson couples to LH and RH particles (with different strengths)

★ Need to consider only two helicity (or more correctly chiral) combinations:

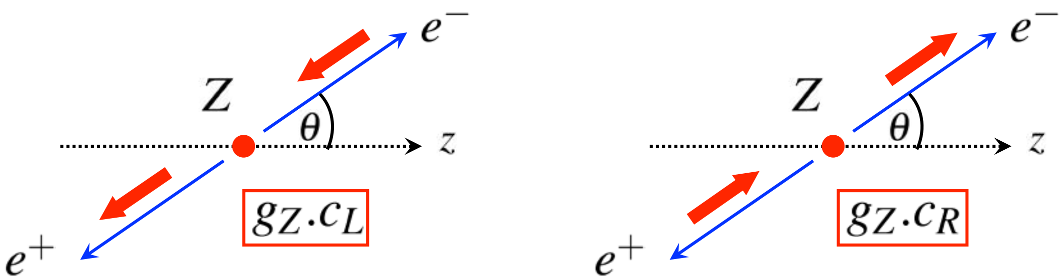



This can be seen by considering either of the combinations which give zero

e.g. $$\overline{u}_R\gamma^\mu(c_V+c_A\gamma_5)v_R = u^\dagger\tfrac{1}{2}(1+\gamma^5)\gamma^0\gamma^\mu(c_V+c_A\gamma^5)\tfrac{1}{2}(1-\gamma^5)v$$
$$=\tfrac{1}{4}u^\dagger\gamma^0(1-\gamma^5)\gamma^\mu(1-\gamma^5)(c_V+c_A\gamma^5)v$$
$$=\tfrac{1}{4}\overline{u}\gamma^\mu(1+\gamma^5)(1-\gamma^5)(c_V+c_A\gamma_5)v=0$$



★ In terms of left and right-handed combinations need to calculate:




★ For unpolarized Z bosons: (Question 26)

$$\langle|M_{fi}|^2\rangle = \tfrac{1}{3}[2c_L^2g_Z^2m_Z^2+2c_R^2g_Z^2m_Z^2] = \tfrac{2}{3}g_Z^2m_Z^2(c_L^2+c_R^2)$$

average over polarization

★ Using $c_V^2+c_A^2=2(c_L^2+c_R^2)$ and $\dfrac{d\Gamma}{d\Omega}=\dfrac{|p^*|}{32\pi^2m_W^2}|M|^2$

$$\Gamma(Z\to e^+e^-)=\frac{g_Z^2m_Z}{48\pi}(c_V^2+c_A^2)$$

## Z Branching Ratios (Question 27)

★ (Neglecting fermion masses) obtain the same expression for the other decays

$$\Gamma(Z \to f\overline{f}) = \frac{g_Z^2 m_Z}{48\pi}(c_V^2 + c_A^2)$$

•Using values for $c_V$ and $c_A$ on page 471 obtain:

$$Br(Z \to e^+e^-) = Br(Z \to \mu^+\mu^-) = Br(Z \to \tau^+\tau^-) \approx 3.5\%$$

$$Br(Z \to \nu_1\overline{\nu}_1) = Br(Z \to \nu_2\overline{\nu}_2) = Br(Z \to \nu_3\overline{\nu}_3) \approx 6.9\%$$

$$Br(Z \to d\overline{d}) = Br(Z \to s\overline{s}) = Br(Z \to b\overline{b}) \approx 15\%$$

$$Br(Z \to u\overline{u}) = Br(Z \to c\overline{c}) \approx 12\%$$

•The Z Boson therefore predominantly decays to hadrons

$$Br(Z \to \text{hadrons}) \approx 69\%$$

Mainly due to factor 3 from colour

•Also predict total decay rate (total width)

$$\Gamma_Z = \sum_i \Gamma_i = 2.5\,\text{GeV}$$

Experiment: $\Gamma_Z = 2.4952 \pm 0.0023\,\text{GeV}$



## Summary

★ The Standard Model interactions are mediated by spin-1 gauge bosons
★ The form of the interactions are completely specified by the assuming an underlying local phase transformation ➡ GAUGE INVARIANCE

| | | |
|---|---|---|
| $U(1)_{em}$ | ⇨ | QED |
| $SU(2)_L$ | ⇨ | Charged Current Weak Interaction + $W^3$ |
| $SU(3)_{col}$ | ⇨ | QCD |

★ In order to "unify" the electromagnetic and weak interactions, introduced a new symmetry gauge symmetry : U(1) hypercharge

$U(1)_Y$ ⇨ $B_\mu$

★ The physical Z boson and the photon are mixtures of the neutral W boson and B determined by the Weak Mixing angle

$$\sin\theta_W \approx 0.23$$

★ Have we really unified the EM and Weak interactions ? Well not really…
•Started with two independent theories with coupling constants $g_W, e$
•Ended up with coupling constants which are related but at the cost of introducing a new parameter in the Standard Model $\theta_W$
•Interactions not unified from any higher theoretical principle… but it works!

# Appendix I : Photon Polarization

(Non-examinable)

- For a free photon (i.e. $j^\mu = 0$) equation (A7) becomes

$$\Box^2 A^\mu = 0 \quad \text{(B1)}$$

(note have chosen a gauge where the Lorentz condition is satisfied)

★ Equation (A8) has solutions (i.e. the wave-function for a free photon)

$$A^\mu = \varepsilon^\mu(q)e^{-iq.x}$$

where $\varepsilon^\mu$ is the four-component polarization vector and $q$ is the photon four-momentum

$$0 = \Box^2 A^\mu = -q^2 \varepsilon^\mu e^{-iq.x}$$

$$\Rightarrow \quad q^2 = 0$$

★ Hence equation (B1) describes a massless particle.

★ But the solution has four components – might ask how it can describe a spin-1 particle which has three polarization states?

★ But for (A8) to hold we must satisfy the Lorentz condition:

$$0 = \partial_\mu A^\mu = \partial_\mu(\varepsilon^\mu e^{-iq.x}) = \varepsilon^\mu \partial_\nu(e^{-iq.x}) = -i\varepsilon^\mu q_\mu e^{-iq.x}$$

Hence the Lorentz condition gives $\boxed{q_\mu \varepsilon^\mu = 0}$ (B2)

i.e. only 3 independent components.



★ However, in addition to the Lorentz condition still have the addional gauge freedom of $A_\mu \to A'_\mu = A_\mu + \partial_\mu \Lambda$ with (A8) $\Box^2 \Lambda = 0$

- Choosing $\Lambda = iae^{-iq.x}$ which has $\Box^2 \Lambda = q^2 \Lambda = 0$

$$\begin{aligned} A_\mu \to A'_\mu = A_\mu + \partial_\mu \Lambda &= \varepsilon_\mu e^{-iq.x} + ia\partial_\mu e^{-iq.x} \\ &= \varepsilon_\mu e^{-iq.x} + ia(-iq_\mu)e^{-iq.x} \\ &= (\varepsilon_\mu + aq_\mu)e^{-iq.x} \end{aligned}$$

★ Hence the electromagnetic field is left unchanged by

$$\varepsilon_\mu \to \varepsilon'_\mu = \varepsilon_\mu + aq_\mu$$

★ Hence the two polarization vectors which differ by a mulitple of the photon four-momentum describe the same photon. Choose $a$ such that the time-like component of $\varepsilon_\mu$ is zero, i.e. $\varepsilon_0 \equiv 0$

★ With this choice of gauge, which is known as the **COULOMB GAUGE**, the Lorentz condition (B2) gives

$$\boxed{\vec{\varepsilon} \cdot \vec{q} = 0} \quad \text{(B3)}$$

i.e. only 2 independent components, both transverse to the photons momentum

★ A massless photon has two transverse polarisation states. For a photon travelling in the z direction these can be expressed as the transversly polarized states:

$$\epsilon_1^\mu = (0,1,0,0); \qquad \epsilon_2^\mu = (0,0,1,0)$$

★ Alternatively take linear combinations to get the circularly polarized states

$$\epsilon_-^\mu = \frac{1}{\sqrt{2}}(0,1,-i,0); \qquad \epsilon_+^\mu = -\frac{1}{\sqrt{2}}(0,1,i,0)$$

★ It can be shown that the $\epsilon_+$ state corresponds the state in which the photon spin is directed in the **+z** direction, i.e. $S_z = +1$



# Appendix II : Massive Spin-1 particles

**(Non-examinable)**

•For a massless photon we had (before imposing the Lorentz condition) we had from equation (A5)

$$\Box^2 A^\mu - \partial^\mu(\partial_\nu A^\nu) = j^\mu$$

★The Klein-Gordon equation for a spin-0 particle of mass m is

$$(\Box^2 + m^2)\phi = 0$$

suggestive that the appropriate equations for a massive spin-1 particle can be obtained by replacing $\Box^2 \rightarrow \Box^2 + m^2$

★ This is indeed the case, and from QFT it can be shown that for a massive spin 1 particle equation (A5) becomes

$$(\Box^2 + m^2)B^\mu - \partial^\mu(\partial_\nu B^\nu) = j^\mu$$

★ Therefore a free particle must satisfy

$$(\Box^2 + m^2)B^\mu - \partial^\mu(\partial_\nu B^\nu) = 0 \qquad \text{(B4)}$$

• Acting on equation (B4) with $\partial_\nu$ gives

$$\begin{aligned}(\Box^2+m^2)\partial_\mu B^\mu - \partial_\mu\partial^\mu(\partial_\nu B^\nu) &= 0\\ (\Box^2+m^2)\partial_\mu B^\mu - \Box^2(\partial_\nu B^\nu) &= 0\\ m^2\partial_\mu B^\mu &= 0 \qquad \text{(B5)}\end{aligned}$$

★ Hence, for a massive spin-1 particle, unavoidably have $\partial_\mu B^\mu = 0$ ; note this is not a relation that reflects to choice of gauge.

• Equation (B4) becomes

$$(\Box^2+m^2)B^\mu = 0 \qquad \text{(B6)}$$

★ For a free spin-1 particle with 4-momentum, $p^\mu$ , equation (B6) admits solutions

$$B_\mu = \varepsilon_\mu e^{-ip.x}$$

★ Substituting into equation (B5) gives

$$p_\mu \varepsilon^\mu = 0$$

★ The four degrees of freedom in $\varepsilon^\mu$ are reduced to three, but for a massive particle, equation (B6) does <u>not</u> allow a choice of gauge and we can not reduce the number of degrees of freedom any further.



★ Hence we need to find three orthogonal polarisation states satisfying

$$p_\mu \varepsilon^\mu = 0 \qquad \text{(B7)}$$

★ For a particle travelling in the z direction, can still admit the circularly polarized states.

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0,1,-i,0); \qquad \varepsilon_+^\mu = -\frac{1}{\sqrt{2}}(0,1,i,0)$$

★ Writing the third state as

$$\varepsilon_L^\mu = \frac{1}{\sqrt{\alpha^2+\beta^2}}(\alpha,0,0,\beta)$$

equation (B7) gives $\alpha E - \beta p_z = 0$

$$\Rightarrow \quad \varepsilon_L^\mu = \frac{1}{m}(p_z,0,0,E)$$

★ This <u>longitudinal</u> polarisation state is only present for massive spin-1 particles, i.e. there is no analogous state for a free photon (although off-mass shell virtual photons can be longitudinally polarized – a fact that was alluded to on page 114).