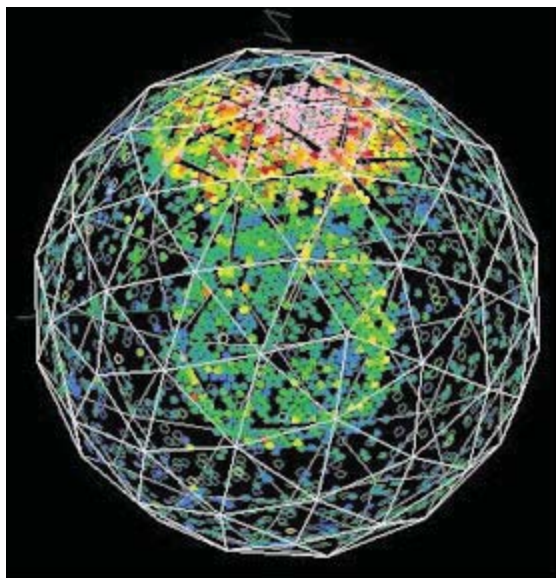


# Particle Physics

Michaelmas Term 2010

Prof Mark Thomson



## Handout 9 : The Weak Interaction and V-A

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Michaelmas 2010

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### Parity

- ★ The parity operator performs spatial inversion through the origin:

$$\psi'(\vec{x}, t) = \hat{P}\psi(\vec{x}, t) = \psi(-\vec{x}, t)$$

- applying  $\hat{P}$  twice:  $\hat{P}\hat{P}\psi(\vec{x}, t) = \hat{P}\psi(-\vec{x}, t) = \psi(\vec{x}, t)$

$$\text{so} \quad \hat{P}\hat{P} = I \quad \rightarrow \quad \hat{P}^{-1} = \hat{P}$$

- To preserve the normalisation of the wave-function

$$\langle \psi | \psi \rangle = \langle \psi' | \psi' \rangle = \langle \psi | \hat{P}^\dagger \hat{P} | \psi \rangle$$

$$\hat{P}^\dagger \hat{P} = I \quad \rightarrow \quad \hat{P} \quad \text{Unitary}$$

- But since  $\hat{P}\hat{P} = I$   $\hat{P} = \hat{P}^\dagger \quad \rightarrow \quad \hat{P} \quad \text{Hermitian}$

which implies Parity is an **observable** quantity. If the interaction Hamiltonian commutes with  $\hat{P}$ , parity is an **observable** conserved quantity

- If  $\psi(\vec{x}, t)$  is an eigenfunction of the parity operator with eigenvalue  $P$

$$\hat{P}\psi(\vec{x}, t) = P\psi(\vec{x}, t) \quad \rightarrow \quad \hat{P}\hat{P}\psi(\vec{x}, t) = P\hat{P}\psi(\vec{x}, t) = P^2\psi(\vec{x}, t)$$

$$\text{since } \hat{P}\hat{P} = I \quad P^2 = 1$$

$$\rightarrow \text{Parity has eigenvalues } P = \pm 1$$

- ★ QED and QCD are invariant under parity

- ★ Experimentally observe that **Weak Interactions** do not conserve parity

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## Intrinsic Parities of fundamental particles:

### Spin-1 Bosons

- From Gauge Field Theory can show that the gauge bosons have  $P = -1$

$$P_\gamma = P_g = P_{W^+} = P_{W^-} = P_Z = -1$$

### Spin-1/2 Fermions

- From the Dirac equation showed (handout 2):

**Spin 1/2 particles have opposite parity to spin 1/2 anti-particles**

- Conventional choice: spin 1/2 particles have  $P = +1$

$$P_{e^-} = P_{\mu^-} = P_{\tau^-} = P_\nu = P_q = +1$$

and anti-particles have opposite parity, i.e.

$$P_{e^+} = P_{\mu^+} = P_{\tau^+} = P_{\bar{\nu}} = P_{\bar{q}} = -1$$

- ★ For Dirac spinors it was shown (handout 2) that the parity operator is:

$$\hat{P} = \gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

## Parity Conservation in QED and QCD

- Consider the QED process  $e^- q \rightarrow e^- q$

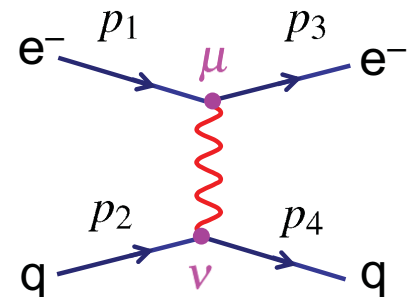
- The Feynman rules for QED give:

$$-iM = [\bar{u}_e(p_3)ie\gamma^\mu u_e(p_1)] \frac{-ig_{\mu\nu}}{q^2} [\bar{u}_q(p_4)ie\gamma^\nu u_q(p_2)]$$

- Which can be expressed in terms of the electron and quark 4-vector currents:

$$M = -\frac{e^2}{q^2} g_{\mu\nu} j_e^\mu j_q^\nu = -\frac{e^2}{q^2} j_e \cdot j_q$$

with  $j_e = \bar{u}_e(p_3)\gamma^\mu u_e(p_1)$  and  $j_q = \bar{u}_q(p_4)\gamma^\mu u_q(p_2)$



- ★ Consider the what happen to the matrix element under the parity transformation

- ♦ Spinors transform as

$$u \xrightarrow{\hat{P}} \hat{P}u = \gamma^0 u$$

- ♦ Adjoint spinors transform as

$$\bar{u} = u^\dagger \gamma^0 \xrightarrow{\hat{P}} (\hat{P}u)^\dagger \gamma^0 = u^\dagger \gamma^{0\dagger} \gamma^0 = u^\dagger \gamma^0 \gamma^0 = \bar{u} \gamma^0$$

$$\bar{u} \xrightarrow{\hat{P}} \bar{u} \gamma^0$$

- ♦ Hence  $j_e = \bar{u}_e(p_3)\gamma^\mu u_e(p_1) \xrightarrow{\hat{P}} \bar{u}_e(p_3)\gamma^0 \gamma^\mu \gamma^0 u_e(p_1)$

★ Consider the components of the four-vector current

0:  $j_e^0 \xrightarrow{\hat{P}} \bar{u} \gamma^0 \gamma^0 \gamma^0 u = \bar{u} \gamma^0 u = j_e^0$  since  $\gamma^0 \gamma^0 = 1$

$k=1,2,3$ :  $j_e^k \xrightarrow{\hat{P}} \bar{u} \gamma^0 \gamma^k \gamma^0 u = -\bar{u} \gamma^k \gamma^0 \gamma^0 u = -\bar{u} \gamma^k u = -j_e^k$  since  $\gamma^0 \gamma^k = -\gamma^k \gamma^0$

- The time-like component remains unchanged and the space-like components change sign

• Similarly  $j_q^0 \xrightarrow{\hat{P}} j_q^0$   $j_q^k \xrightarrow{\hat{P}} -j_q^k$   $k = 1, 2, 3$

★ Consequently the four-vector scalar product

$j_e \cdot j_q = j_e^0 j_q^0 - j_e^k j_q^k \xrightarrow{\hat{P}} j_e^0 j_q^0 - (-j_e^k)(-j_q^k) = j_e \cdot j_q$   $k = 1, 3$

or  $j^\mu \xrightarrow{\hat{P}} j_\mu$   
 $j^\mu \cdot j^\nu \xrightarrow{\hat{P}} j_\mu \cdot j_\nu$   
 $\xrightarrow{\hat{P}} j^\mu \cdot j^\nu$

**QED Matrix Elements are Parity Invariant**



**Parity Conserved in QED**

★ The QCD vertex has the same form and, thus,

**Parity Conserved in QCD**

## Parity Violation in $\beta$ -Decay

★ The parity operator  $\hat{P}$  corresponds to a discrete transformation  $x \rightarrow -x$ , etc.

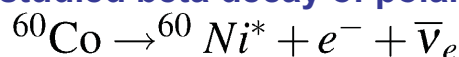
★ Under the parity transformation:

**Vectors change sign**  $\begin{cases} \vec{r} \xrightarrow{\hat{P}} -\vec{r} \\ \vec{p} \xrightarrow{\hat{P}} -\vec{p} \end{cases} \quad (p_x = \frac{\partial}{\partial x}, \text{ etc.})$

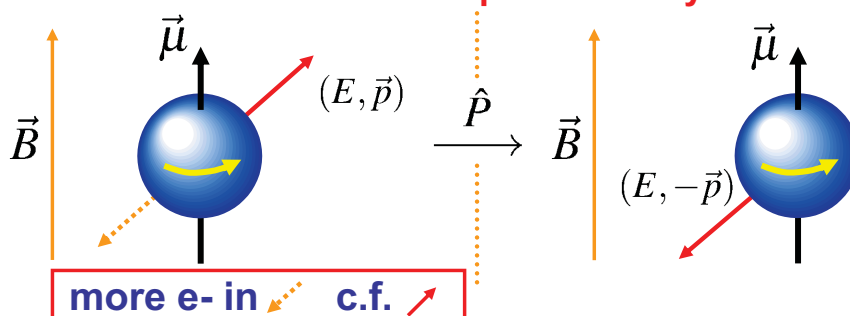
**Axial-Vectors unchanged**  $\begin{cases} \vec{L} \xrightarrow{\hat{P}} \vec{L} \\ \vec{\mu} \xrightarrow{\hat{P}} \vec{\mu} \end{cases} \quad (\vec{L} = \vec{r} \wedge \vec{p})$   
 $(\vec{\mu} \propto \vec{L})$

**Note B is an axial vector**  
 $d\vec{B} \propto \vec{J} \wedge \vec{r} d^3\vec{r}$

★ 1957: C.S.Wu et al. studied beta decay of polarized cobalt-60 nuclei:



★ Observed **electrons emitted preferentially** in direction opposite to applied field



If parity were conserved: expect equal rate for producing  $e^-$  in directions along and opposite to the nuclear spin.

★ Conclude **parity is violated** in WEAK INTERACTION

→ that the WEAK interaction vertex is **NOT** of the form  $\bar{u}_e \gamma^\mu u_\nu$

# Bilinear Covariants

- ★ The requirement of Lorentz invariance of the matrix element severely restricts the form of the interaction vertex. QED and QCD are “**VECTOR**” interactions:

$$j^\mu = \bar{\psi} \gamma^\mu \phi$$

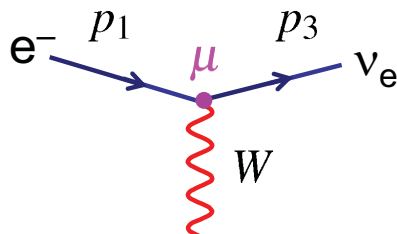
- ★ This combination transforms as a 4-vector (Handout 2 appendix V)
- ★ In general, there are only 5 possible combinations of two spinors and the gamma matrices that form Lorentz invariant currents, called “bilinear covariants”:

Type	Form	Components	“Boson Spin”
♦ <b>SCALAR</b>	$\bar{\psi} \phi$	<b>1</b>	<b>0</b>
♦ <b>PSEUDOSCALAR</b>	$\bar{\psi} \gamma^5 \phi$	<b>1</b>	<b>0</b>
♦ <b>VECTOR</b>	$\bar{\psi} \gamma^\mu \phi$	<b>4</b>	<b>1</b>
♦ <b>AXIAL VECTOR</b>	$\bar{\psi} \gamma^\mu \gamma^5 \phi$	<b>4</b>	<b>1</b>
♦ <b>TENSOR</b>	$\bar{\psi} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) \phi$	<b>6</b>	<b>2</b>

- ★ Note that in total the sixteen components correspond to the 16 elements of a general 4x4 matrix: “decomposition into Lorentz invariant combinations”
- ★ In QED the factor  $g_{\mu\nu}$  arose from the sum over polarization states of the virtual photon (2 transverse + 1 longitudinal, 1 scalar) = (2J+1) + 1
- ★ Associate **SCALAR** and **PSEUDOSCALAR** interactions with the exchange of a **SPIN-0** boson, etc. – no spin degrees of freedom

## V-A Structure of the Weak Interaction

- ★ The most general form for the interaction between a fermion and a boson is a linear combination of bilinear covariants
- ★ For an interaction corresponding to the exchange of a spin-1 particle the most general form is a linear combination of **VECTOR** and **AXIAL-VECTOR**
- ★ The form for **WEAK** interaction is determined from experiment to be **VECTOR - AXIAL-VECTOR (V - A)**



$$j^\mu \propto \bar{u}_{\nu_e} (\gamma^\mu - \gamma^\mu \gamma^5) u_e$$

**V - A**

- ★ Can this account for parity violation?
- ★ First consider parity transformation of a pure **AXIAL-VECTOR** current

$$j_A = \bar{\psi} \gamma^\mu \gamma^5 \phi \quad \text{with} \quad \gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3; \quad \gamma^5 \gamma^0 = -\gamma^0 \gamma^5$$

$$j_A = \bar{\psi} \gamma^\mu \gamma^5 \phi \xrightarrow{\hat{P}} \bar{\psi} \gamma^0 \gamma^\mu \gamma^5 \gamma^0 \phi = -\bar{\psi} \gamma^0 \gamma^\mu \gamma^0 \gamma^5 \phi$$

$$j_A^0 \xrightarrow{\hat{P}} -\bar{\psi} \gamma^0 \gamma^0 \gamma^0 \gamma^5 \phi = -\bar{\psi} \gamma^0 \gamma^5 \phi = -j_A^0$$

$$j_A^k \xrightarrow{\hat{P}} -\bar{\psi} \gamma^0 \gamma^k \gamma^0 \gamma^5 \phi = +\bar{\psi} \gamma^k \gamma^5 \phi = +j_A^k \quad k = 1, 2, 3$$

or  $j_A^\mu \xrightarrow{\hat{P}} -j_{A\mu}$

- The space-like components remain unchanged and the time-like components change sign (the opposite to the parity properties of a vector-current)

$$j_A^0 \xrightarrow{\hat{P}} -j_A^0; \quad j_A^k \xrightarrow{\hat{P}} +j_A^k; \quad j_V^0 \xrightarrow{\hat{P}} +j_V^0; \quad j_V^k \xrightarrow{\hat{P}} -j_V^k$$

- Now consider the matrix elements

$$M \propto g_{\mu\nu} j_1^\mu j_2^\nu = j_1^0 j_2^0 - \sum_{k=1,3} j_1^k j_2^k$$

- For the combination of a two axial-vector currents

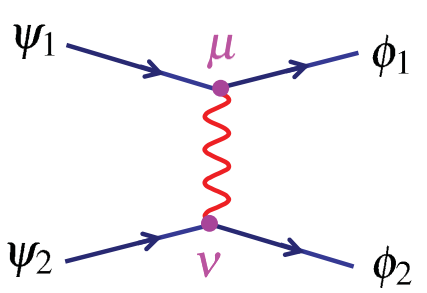
$$j_{A1} \cdot j_{A2} \xrightarrow{\hat{P}} (-j_1^0)(-j_2^0) - \sum_{k=1,3} (j_1^k)(j_2^k) = j_{A1} \cdot j_{A2}$$

- Consequently parity is conserved for both a pure vector and pure axial-vector interactions
- However the combination of a vector current and an axial vector current

$$j_{V1} \cdot j_{A2} \xrightarrow{\hat{P}} (j_1^0)(-j_2^0) - \sum_{k=1,3} (-j_1^k)(j_2^k) = -j_{V1} \cdot j_{A2}$$

changes sign under parity – can give parity violation !

- ★ Now consider a general linear combination of VECTOR and AXIAL-VECTOR (note this is relevant for the Z-boson vertex)



$$j_1 = \bar{\phi}_1 (g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) \psi_1 = g_V j_1^V + g_A j_1^A$$

$$j_2 = \bar{\phi}_2 (g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) \psi_2 = g_V j_2^V + g_A j_2^A$$

$$\frac{g_{\mu\nu}}{q^2 - m^2}$$

$$M_{fi} \propto j_1 \cdot j_2 = g_V^2 j_1^V \cdot j_2^V + g_A^2 j_1^A \cdot j_2^A + g_V g_A (j_1^V \cdot j_2^A + j_1^A \cdot j_2^V)$$

- Consider the parity transformation of this scalar product

$$j_1 \cdot j_2 \xrightarrow{\hat{P}} g_V^2 j_1^V \cdot j_2^V + g_A^2 j_1^A \cdot j_2^A - g_V g_A (j_1^V \cdot j_2^A + j_1^A \cdot j_2^V)$$

- If either  $g_A$  or  $g_V$  is zero, Parity is conserved, i.e. parity conserved in a pure VECTOR or pure AXIAL-VECTOR interaction

- Relative strength of parity violating part  $\propto \frac{g_V g_A}{g_V^2 + g_A^2}$

**Maximal Parity Violation for V-A (or V+A)**

# Chiral Structure of QED (Reminder)

- ★ Recall (Handout 4) introduced CHIRAL projections operators

$$P_R = \frac{1}{2}(1 + \gamma^5); \quad P_L = \frac{1}{2}(1 - \gamma^5)$$

project out **chiral** right- and left- handed states

- ★ In the ultra-relativistic limit, **chiral states** correspond to **helicity states**
- ★ Any spinor can be expressed as:

$$\psi = \frac{1}{2}(1 + \gamma^5)\psi + \frac{1}{2}(1 - \gamma^5)\psi = P_R\psi + P_L\psi = \psi_R + \psi_L$$

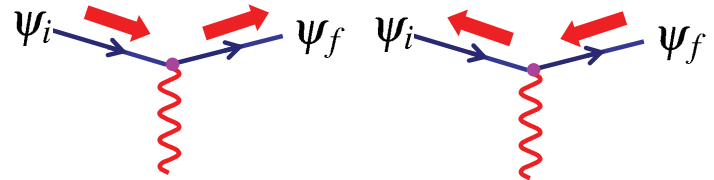
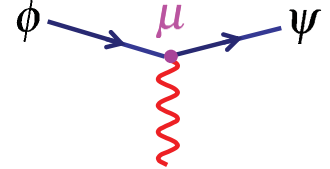
- **The QED vertex**  $\bar{\psi}\gamma^\mu\phi$  in terms of chiral states:

$$\bar{\psi}\gamma^\mu\phi = \bar{\psi}_R\gamma^\mu\phi_R + \bar{\psi}_R\gamma^\mu\phi_L + \bar{\psi}_L\gamma^\mu\phi_R + \bar{\psi}_L\gamma^\mu\phi_L$$

conserves chirality, e.g.

$$\begin{aligned} \bar{\psi}_R\gamma^\mu\phi_L &= \frac{1}{2}\psi^\dagger(1 + \gamma^5)\gamma^0\gamma^\mu\frac{1}{2}(1 - \gamma^5)\phi \\ &= \frac{1}{4}\psi^\dagger\gamma^0(1 - \gamma^5)\gamma^\mu(1 - \gamma^5)\phi \\ &= \frac{1}{4}\bar{\psi}\gamma^\mu(1 + \gamma^5)(1 - \gamma^5)\phi = 0 \end{aligned}$$

- ★ In the ultra-relativistic limit only two helicity combinations are non-zero



## Helicity Structure of the WEAK Interaction

- ★ The charged current ( $W^\pm$ ) weak vertex is:

$$\frac{-ig_w}{\sqrt{2}} \frac{1}{2} \gamma^\mu (1 - \gamma^5)$$

- ★ Since  $\frac{1}{2}(1 - \gamma^5)$  projects out left-handed **chiral** particle states:

$$\bar{\psi} \frac{1}{2} \gamma^\mu (1 - \gamma^5) \phi = \bar{\psi} \gamma^\mu \phi_L$$

(question 16)

- ★ Writing  $\bar{\psi} = \bar{\psi}_R + \bar{\psi}_L$  and from discussion of QED,  $\bar{\psi}_R\gamma^\mu\phi_L = 0$  gives

$$\bar{\psi} \frac{1}{2} \gamma^\mu (1 - \gamma^5) \phi = \bar{\psi}_L \gamma^\mu \phi_L$$



Only the **left-handed chiral** components of **particle** spinors and **right-handed chiral** components of **anti-particle** spinors participate in charged current weak interactions

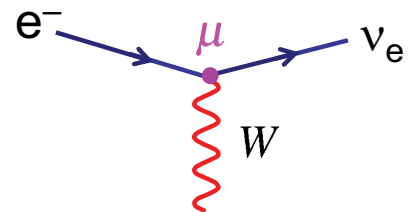
- ★ At very high energy ( $E \gg m$ ), the **left-handed chiral** components are helicity eigenstates :

$$\frac{1}{2}(1 - \gamma^5)u \Rightarrow \text{left-pointing arrow}$$

$$\frac{1}{2}(1 - \gamma^5)v \Rightarrow \text{right-pointing arrow}$$

LEFT-HANDED PARTICLES  
Helicity = -1

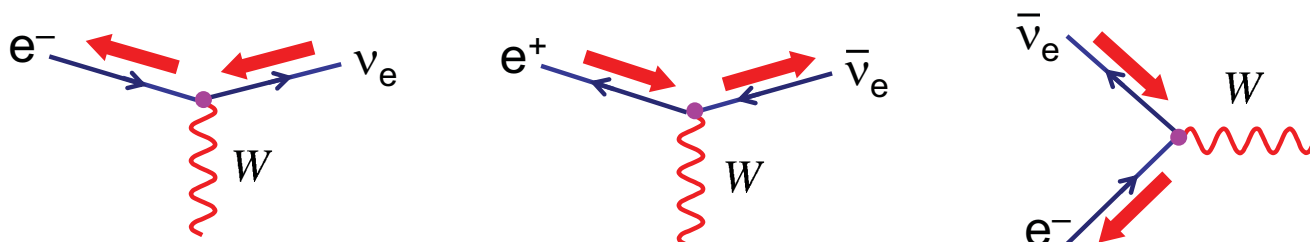
RIGHT-HANDED ANTI-PARTICLES  
Helicity = +1





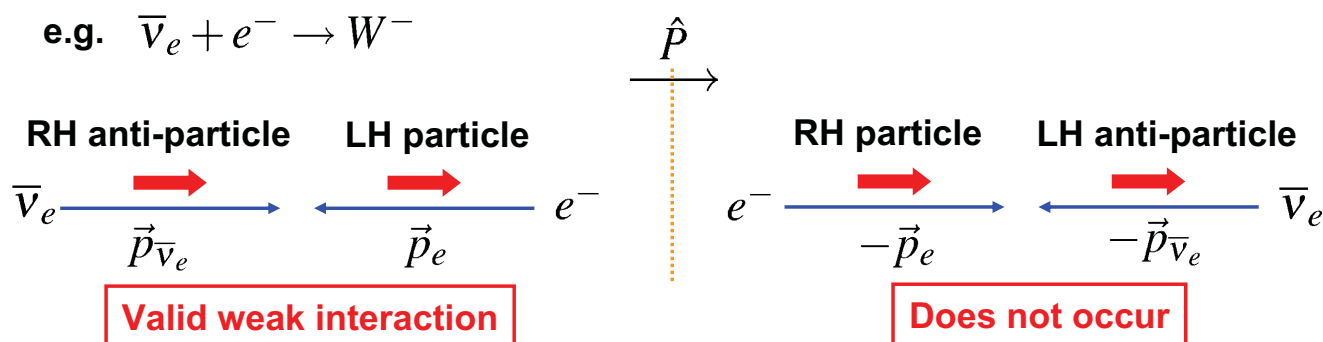
In the ultra-relativistic limit only **left-handed particles** and **right-handed antiparticles** participate in charged current weak interactions

e.g. In the relativistic limit, the only possible electron - neutrino interactions are:



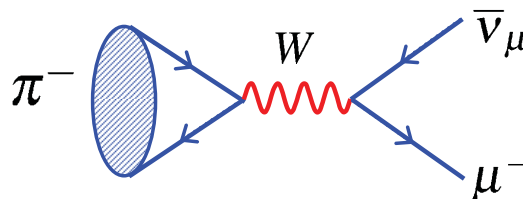
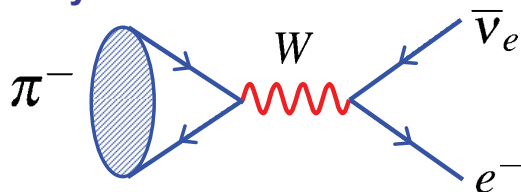
★ The helicity dependence of the weak interaction  $\longleftrightarrow$  parity violation

e.g.  $\bar{\nu}_e + e^- \rightarrow W^-$



## Helicity in Pion Decay

★ The decays of charged pions provide a good demonstration of the role of helicity in the weak interaction



**EXPERIMENTALLY:** 
$$\frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} = 1.23 \times 10^{-4}$$

• Might expect the decay to electrons to dominate - due to increased phase space.... The opposite happens, the electron decay is helicity suppressed

★ Consider decay in pion rest frame.

- Pion is spin zero: so the spins of the  $\bar{\nu}$  and  $\mu$  are opposite
- Weak interaction only couples to **RH chiral** anti-particle states. Since neutrinos are (almost) massless, must be in **RH Helicity** state
- Therefore, to conserve angular mom. muon is emitted in a **RH HELICITY** state



• But only **left-handed CHIRAL particle states** participate in weak interaction



★ The general **right-handed helicity** solution to the Dirac equation is

$$u_{\uparrow} = N \begin{pmatrix} c \\ e^{i\phi} s \\ \frac{|\vec{p}|}{E+m} c \\ \frac{|\vec{p}|}{E+m} e^{i\phi} s \end{pmatrix} \quad \text{with} \quad c = \cos \frac{\theta}{2} \quad \text{and} \quad s = \sin \frac{\theta}{2}$$

- project out the **left-handed chiral** part of the wave-function using  $P_L = \frac{1}{2}(1 - \gamma^5) = \frac{1}{2} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$

giving  $P_L u_{\uparrow} = \frac{1}{2} N \left(1 - \frac{|\vec{p}|}{E+m}\right) \begin{pmatrix} c \\ e^{i\phi} s \\ -c \\ -e^{i\phi} s \end{pmatrix} = \frac{1}{2} N \left(1 - \frac{|\vec{p}|}{E+m}\right) u_L$

In the limit  $m \ll E$  this tends to zero

- similarly  $P_R u_{\uparrow} = \frac{1}{2} N \left(1 + \frac{|\vec{p}|}{E+m}\right) \begin{pmatrix} c \\ e^{i\phi} s \\ c \\ e^{i\phi} s \end{pmatrix} = \frac{1}{2} N \left(1 + \frac{|\vec{p}|}{E+m}\right) u_R$

In the limit  $m \ll E$ ,  $P_R u_{\uparrow} \rightarrow u_R$

★ Hence  $u_{\uparrow} = P_R u_{\uparrow} + P_L u_{\uparrow} = \frac{1}{2} \left(1 + \frac{|\vec{p}|}{E+m}\right) u_R + \frac{1}{2} \left(1 - \frac{|\vec{p}|}{E+m}\right) u_L$

RH Helicity
RH Chiral
LH Chiral

- In the limit  $E \gg m$ , as expected, the RH chiral and helicity states are identical
- Although only LH chiral particles participate in the weak interaction the contribution from RH **Helicity** states is not necessarily zero !

$$\bar{\nu}_{\mu} \leftarrow \bullet \rightarrow \mu^{-}$$

$m_{\nu} \approx 0$ : RH Helicity  $\equiv$  RH Chiral

$m_{\mu} \neq 0$ : RH Helicity has LH Chiral Component

- Expect matrix element to be proportional to **LH chiral component of RH Helicity** electron/muon **spinor**

$$M_{fi} \propto \frac{1}{2} \left(1 - \frac{|\vec{p}|}{E+m}\right) = \frac{m_{\mu}}{m_{\pi} + m_{\mu}} \quad \leftarrow \text{from the kinematics of pion decay at rest}$$

- Hence because the electron mass is much smaller than the pion mass the decay  $\pi^{-} \rightarrow e^{-} \bar{\nu}_e$  is **heavily suppressed**.



# Evidence for V-A

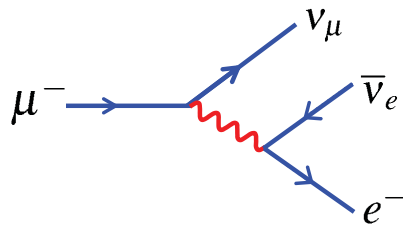
- ★ The V-A nature of the charged current weak interaction vertex fits with experiment

## EXAMPLE charged pion decay

(question 17)

- Experimentally measure:  $\frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} = (1.230 \pm 0.004) \times 10^{-4}$
- Theoretical predictions (depend on Lorentz Structure of the interaction)
  - V-A**  $(\bar{\psi}\gamma^\mu(1-\gamma^5)\phi)$  or **V+A**  $(\bar{\psi}\gamma^\mu(1+\gamma^5)\phi)$   $\rightarrow \frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} \approx 1.3 \times 10^{-4}$
  - Scalar**  $(\bar{\psi}\phi)$  or **Pseudo-Scalar**  $(\bar{\psi}\gamma^5\phi)$   $\rightarrow \frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} = 5.5$

## EXAMPLE muon decay

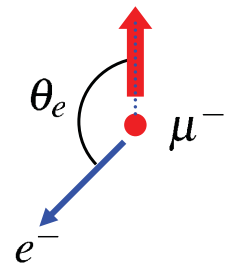


e.g. TWIST expt:  $6 \times 10^9$   $\mu$  decays  
Phys. Rev. Lett. 95 (2005) 101805

**V-A Prediction:**  $\rho = 0.75$

Measure **electron** energy and angular distributions relative to muon spin direction. Results expressed in terms of general **S+P+V+A+T** form in “Michel Parameters”

$$\rho = 0.75080 \pm 0.00105$$



# Weak Charged Current Propagator

- ★ The charged-current Weak interaction is different from QED and QCD in that it is mediated by massive W-bosons (**80.3 GeV**)
- ★ This results in a more complicated form for the propagator:
  - in handout 4 showed that for the exchange of a massive particle:

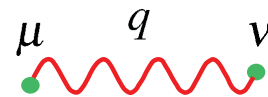
$$\frac{1}{q^2} \xrightarrow{\text{massless}} \frac{1}{q^2 - m^2} \quad \text{massive}$$

- In addition the sum over W boson polarization states modifies the numerator

## W-boson propagator

spin 1  $W^\pm$

$$\frac{-i [g_{\mu\nu} - q_\mu q_\nu / m_W^2]}{q^2 - m_W^2}$$



- ★ However in the limit where  $q^2$  is small compared with  $m_W = 80.3 \text{ GeV}$  the interaction takes a simpler form.

## W-boson propagator ( $q^2 \ll m_W^2$ )

$$\frac{ig_{\mu\nu}}{m_W^2}$$



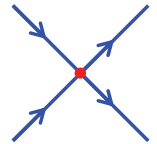
- The interaction appears point-like (i.e no  $q^2$  dependence)

# Connection to Fermi Theory

- ★ In 1934, before the discovery of parity violation, Fermi proposed, in analogy with QED, that the invariant matrix element for  $\beta$ -decay was of the form:

$$M_{fi} = G_F g_{\mu\nu} [\bar{\psi} \gamma^\mu \psi] [\bar{\psi} \gamma^\nu \psi]$$

where  $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$



- Note the absence of a propagator : i.e. this represents an interaction at a point
- ★ After the discovery of parity violation in 1957 this was modified to

$$M_{fi} = \frac{G_F}{\sqrt{2}} g_{\mu\nu} [\bar{\psi} \gamma^\mu (1 - \gamma^5) \psi] [\bar{\psi} \gamma^\nu (1 - \gamma^5) \psi]$$

(the factor of  $\sqrt{2}$  was included so the numerical value of  $G_F$  did not need to be changed)

- ★ Compare to the prediction for W-boson exchange

$$M_{fi} = \left[ \frac{g_W}{\sqrt{2}} \bar{\psi} \frac{1}{2} \gamma^\mu (1 - \gamma^5) \psi \right] \frac{g_{\mu\nu} - q_\mu q_\nu / m_W^2}{q^2 - m_W^2} \left[ \frac{g_W}{\sqrt{2}} \bar{\psi} \frac{1}{2} \gamma^\nu (1 - \gamma^5) \psi \right]$$

which for  $q^2 \ll m_W^2$  becomes:

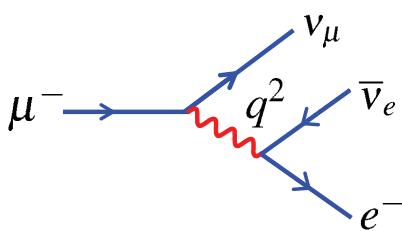
$$M_{fi} = \frac{g_W^2}{8m_W^2} g_{\mu\nu} [\bar{\psi} \gamma^\mu (1 - \gamma^5) \psi] [\bar{\psi} \gamma^\nu (1 - \gamma^5) \psi]$$

$$\Rightarrow \frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$$

Still usually use  $G_F$  to express strength of weak interaction as this is the quantity that is precisely determined in muon decay

## Strength of Weak Interaction

- ★ Strength of weak interaction most precisely measured in muon decay



- Here  $q^2 < m_\mu$  (0.106 GeV)
- To a very good approximation the W-boson propagator can be written

$$\frac{-i [g_{\mu\nu} - q_\mu q_\nu / m_W^2]}{q^2 - m_W^2} \approx \frac{i g_{\mu\nu}}{m_W^2}$$

- In muon decay measure  $g_W^2 / m_W^2$
- Muon decay  $\Rightarrow G_F = 1.16639(1) \times 10^{-5} \text{ GeV}^{-2}$

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$$

- ★ To obtain the intrinsic strength of weak interaction need to know mass of W-boson:  $m_W = 80.403 \pm 0.029 \text{ GeV}$  (see handout 14)

$$\Rightarrow \alpha_W = \frac{g_W^2}{4\pi} = \frac{8m_W^2 G_F}{4\sqrt{2}\pi} = \frac{1}{30}$$



The intrinsic strength of the weak interaction is similar to, but greater than, the EM interaction ! It is the massive W-boson in the propagator which makes it appear weak. For  $q^2 \gg m_W^2$  weak interactions are more likely than EM.

# Summary

- ★ Weak interaction is of form **Vector – Axial-vector (V-A)**

$$\frac{-ig_w}{\sqrt{2}} \frac{1}{2} \gamma^\mu (1 - \gamma^5)$$

- ★ Consequently only left-handed chiral particle states and right-handed chiral anti-particle states participate in the weak interaction



**MAXIMAL PARITY VIOLATION**

- ★ Weak interaction also violates Charge Conjugation symmetry
- ★ At low  $q^2$  weak interaction is only weak because of the large W-boson mass

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$$

- ★ Intrinsic strength of weak interaction is similar to that of QED