Prof Mark Thomson

Particle Physics Major Option

EXAMPLES SHEET 4

THE Z BOSON

26. Consider the decay of the $\mathrm{Z^0}$ to a fermion-antifermion pair, $\mathrm{Z^0} \rightarrow \mathrm{f\overline{f}}$, where the fermion couples to the $\mathrm{Z^0}$ with vector and axial vector coupling constants c_V and c_A:

a) Use the Feynman rules to show that the matrix element for the decay $\mathrm{Z^0} \rightarrow \mathrm{f\overline{f}}$ can be written in the form

$$\begin{aligned} M_\mathrm{fi} &= c_\mathrm{L} \cdot g_\mathrm{Z}\epsilon_\mu(p_1)\overline{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)v(p_4) + c_\mathrm{R} \cdot g_\mathrm{Z}\epsilon_\mu(p_1)\overline{u}(p_3)\gamma^\mu \tfrac{1}{2}(1+\gamma^5)v(p_4) \\ &\equiv c_\mathrm{L} \cdot M_\mathrm{L} + c_\mathrm{R} \cdot M_\mathrm{R} \end{aligned}$$

where p_1 is the $\mathrm{Z^0}$ 4-momentum, p_3 and p_4 are the 4-momenta of the fermion and antifermion, and $c_\mathrm{L} = \frac{1}{2}(c_\mathrm{V} + c_\mathrm{A})$, $c_\mathrm{R} = \frac{1}{2}(c_\mathrm{V} - c_\mathrm{A})$.

b) Assuming the fermion mass can be neglected, draw diagrams illustrating the spin configurations which result in non-zero values of M_L and M_R.

c) Use the results of the calculation of the $\mathrm{W^-} \rightarrow \mathrm{e^-}\overline{\nu}_\mathrm{e}$ decay rate in the lectures to show that, for unpolarised $\mathrm{Z^0}$'s,

$$\langle |M_\mathrm{fi}|^2 \rangle = \tfrac{2}{3} g_\mathrm{Z}^2 m_\mathrm{Z}^2 (c_\mathrm{L}^2 + c_\mathrm{R}^2)$$

and hence that the decay rate is

$$\Gamma(\mathrm{Z^0} \rightarrow \mathrm{f\overline{f}}) = \frac{g_\mathrm{Z}^2 m_\mathrm{Z}}{48\pi}(c_\mathrm{V}^2 + c_\mathrm{A}^2)\ .$$

27. a) Use the result of the previous question to compute the total width of the $\mathrm{Z^0}$, and compare to experiment. [Take $\sin^2\theta_\mathrm{W} = 0.23$, and remember that quarks have three colour states].

b) What will be the value of

$$R = \frac{\sigma(\mathrm{e^+e^-} \rightarrow \text{hadrons})}{\sigma(\mathrm{e^+e^-} \rightarrow \mu^+\mu^-)}$$

at the peak of the $\mathrm{Z^0}$ resonance ?

c) Calculate the cross section for $\mathrm{e^+e^-} \rightarrow \mathrm{Z^0}$ at the resonance peak, and show that the cross-section for $\mathrm{e^+e^-} \rightarrow \mu^+\mu^-$ is increased by a factor of ≈ 200 relative to the QED cross section.

d) The width $\Gamma(\mathrm{Z^0} \rightarrow \mathrm{b\overline{b}})$ has been measured at LEP to be 0.378 GeV. Show that the weak isospin of the b quark is compatible with a value of -0.5. Explain why this result effectively guaranteed the existence of the top quark, even before it was directly discovered.

[$G_\mathrm{F} = 1.166 \times 10^{-5}\,\mathrm{GeV^{-2}}$.]

28. a) It was shown in the lectures that the centre of mass frame differential cross section $\mathrm{d}\sigma_{\mathrm{LR}}/\mathrm{d}\cos\theta$ for the process $\mathrm{e^+e^-} \to \mathrm{f\bar{f}}$ on the peak of the Z^0 resonance, for the case that the incoming electron is left-handed and the outgoing fermion is right-handed, is given by

$$\frac{\mathrm{d}\sigma_{\mathrm{LR}}}{\mathrm{d}\cos\theta} \propto (c_{\mathrm{L}}^{\mathrm{e}})^2(c_{\mathrm{R}}^{\mathrm{f}})^2(1-\cos\theta)^2 \ .$$

Show that the corresponding forward and backward cross sections $\sigma_{\mathrm{LR}}^{\mathrm{F}}$ and $\sigma_{\mathrm{LR}}^{\mathrm{B}}$ are given by

$$\sigma_{\mathrm{LR}}^{\mathrm{F}} \propto (c_{\mathrm{L}}^{\mathrm{e}})^2(c_{\mathrm{R}}^{\mathrm{f}})^2, \qquad \sigma_{\mathrm{LR}}^{\mathrm{B}} \propto 7(c_{\mathrm{L}}^{\mathrm{e}})^2(c_{\mathrm{R}}^{\mathrm{f}})^2 \ ,$$

and write down similar expressions for the cross sections $\sigma_{\mathrm{RL}}^{\mathrm{F}}$, $\sigma_{\mathrm{RL}}^{\mathrm{B}}$, $\sigma_{\mathrm{LL}}^{\mathrm{F}}$, $\sigma_{\mathrm{LL}}^{\mathrm{B}}$, $\sigma_{\mathrm{RR}}^{\mathrm{F}}$, $\sigma_{\mathrm{RR}}^{\mathrm{B}}$.

b) The asymmetry $A_{\mathrm{LR}}^{\mathrm{FB}}$ is defined as

$$A_{\mathrm{LR}}^{\mathrm{FB}} \equiv \frac{(\sigma_{\mathrm{L}}^{\mathrm{F}}-\sigma_{\mathrm{L}}^{\mathrm{B}})-(\sigma_{\mathrm{R}}^{\mathrm{F}}-\sigma_{\mathrm{R}}^{\mathrm{B}})}{(\sigma_{\mathrm{L}}^{\mathrm{F}}+\sigma_{\mathrm{L}}^{\mathrm{B}})+(\sigma_{\mathrm{R}}^{\mathrm{F}}+\sigma_{\mathrm{R}}^{\mathrm{B}})}$$

where $\sigma_{\mathrm{L}} \equiv \sigma_{\mathrm{LL}}+\sigma_{\mathrm{LR}}$ and $\sigma_{\mathrm{R}} \equiv \sigma_{\mathrm{RL}}+\sigma_{\mathrm{RR}}$ are the total cross sections for left-handed and right-handed incoming electrons, respectively. Show that

$$A_{\mathrm{LR}}^{\mathrm{FB}} = \frac{3}{4}\frac{(c_{\mathrm{L}}^{\mathrm{f}})^2-(c_{\mathrm{R}}^{\mathrm{f}})^2}{(c_{\mathrm{L}}^{\mathrm{f}})^2+(c_{\mathrm{R}}^{\mathrm{f}})^2} \equiv \frac{3}{4}A_{\mathrm{f}} \ ,$$

and compare with the similar predictions for the asymmetries A_{LR} and A_{FB}.

c) Using a polarised electron beam, the SLD experiment has recently measured $A_{\mathrm{LR}}^{\mathrm{FB}}$ for the process $\mathrm{e^+e^-} \to \mathrm{c\bar{c}}$, and obtained the result $A_{\mathrm{c}} = 0.6712 \pm 0.0274$. Determine the corresponding value of $\sin^2\theta_{\mathrm{W}}$ and (optionally) its error.

THE TOP QUARK

29. a) The top quark decays into final states containing 1) two quarks and an antiquark, or 2) a quark, a lepton and an antilepton. List the possible final states of each type and draw the generic leading order Feynman diagram for these decays. Explain why the total top quark decay rate is dominated by the rate for the decay $\mathrm{t} \to \mathrm{W}^+\mathrm{b}$ into a real W^+ boson and b quark.

b) Use the Feynman rules to show that the matrix element for the decay $\mathrm{t} \to \mathrm{W}^+\mathrm{b}$ is given by

$$M_{\mathrm{fi}} = \frac{g_\mathrm{W}}{\sqrt{2}}\epsilon^*_\mu(p_4)\overline{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1)$$

where p_1 is the 4-momentum of the top quark and p_3 and p_4 are the 4-momenta of the b quark and W^+, respectively.

c) Consider the decay $\mathrm{t} \to \mathrm{W}^+\mathrm{b}$ in the top quark rest frame, with the b quark travelling in the $+z$ direction. Neglect the b quark mass. Draw diagrams illustrating the two spin configurations which are allowed in this case. Show that, when the top quark spin points in the $+z$ direction, the matrix element M_{fi} is given by

$$M_\uparrow = -g_\mathrm{W}\sqrt{2m_\mathrm{t}p^*}$$

where $p^* = (m_\mathrm{t}^2 - m_\mathrm{W}^2)/2m_\mathrm{t}$ is the magnitude of the three-momenta of the W^+ and the b quark. Show that when the top quark spin points in the $-z$ direction, the matrix element becomes

$$M_\downarrow = -g_\mathrm{W}\frac{m_\mathrm{t}}{m_\mathrm{W}}\sqrt{m_\mathrm{t}p^*}\ .$$

d) Explain why the decay of an unpolarised sample of top quarks must be isotropic, and show that the total decay rate in this case is

$$\Gamma = \frac{G_\mathrm{F}m_\mathrm{t}^3}{8\pi\sqrt{2}}\left(1-\frac{m_\mathrm{W}^2}{m_\mathrm{t}^2}\right)^2\left(1+\frac{2m_\mathrm{W}^2}{m_\mathrm{t}^2}\right)\ .$$

e) Calculate the top quark lifetime. Use the uncertainty principle to estimate a typical hadronisation timescale and comment on the result.

THE HIGGS BOSON

30. a) Use the Feynman rules to show that the matrix element for the decay $\mathrm{H} \to \mathrm{W^+W^-}$ is

$$M_{\mathrm{fi}} = -g_{\mathrm{W}} m_{\mathrm{W}} g_{\mu\nu} \epsilon^{\mu}(p_2)^{*} \epsilon^{\nu}(p_3)^{*}$$

where p_2 and p_3 are the 4-momenta of the W^- and W^+, respectively.

b) Show that $M_{\mathrm{fi}} = -g_{\mathrm{W}} m_{\mathrm{W}}$ when both W bosons are left-handed or both are right-handed, that $M_{\mathrm{fi}} = (g_{\mathrm{W}}/m_{\mathrm{W}})(\frac{1}{2}m_{\mathrm{H}}^2 - m_{\mathrm{W}}^2)$ when both W bosons are longitudinally polarised, and that $M_{\mathrm{fi}} = 0$ for the six remaining combinations of W boson spin states.

c) Show that the $\mathrm{H} \to \mathrm{W^+W^-}$ decay rate is

$$\Gamma(\mathrm{H} \to \mathrm{W^+W^-}) = \frac{G_{\mathrm{F}} m_{\mathrm{H}}^3}{8\pi\sqrt{2}} \sqrt{1 - 4\lambda^2} \left(1 - 4\lambda^2 + 12\lambda^4\right)$$

where $\lambda = m_{\mathrm{W}}/m_{\mathrm{H}}$.

d) For $\mathrm{H} \to \mathrm{Z^0Z^0}$ decays, an extra factor of $\frac{1}{2}$ is required to account for the fact that the final state contains two identical particles. Show that

$$\Gamma(\mathrm{H} \to \mathrm{Z^0Z^0}) = \tfrac{1}{2}\Gamma(\mathrm{H} \to \mathrm{W^+W^-})|_{(m_{\mathrm{W}} \to m_{\mathrm{Z}})} \, .$$

e) For $\mathrm{H} \to \mathrm{f\bar{f}}$ decays into a fermion-antifermion pair, the decay rate is

$$\Gamma(\mathrm{H} \to \mathrm{f\bar{f}}) = N_c \frac{G_{\mathrm{F}}}{\sqrt{2}} \frac{m_{\mathrm{f}}^2 m_{\mathrm{H}}}{4\pi} \left(1 - \frac{4m_{\mathrm{f}}^2}{m_{\mathrm{H}}^2}\right)^{3/2}$$

where N_c is the number of colour degrees of freedom of the fermion f of mass m_{f} [See Tripos paper, Jan 2002, for a derivation of this result]. Compute the $\mathrm{H} \to \mathrm{W^+W^-}$, $\mathrm{H} \to \mathrm{Z^0Z^0}$ and $\mathrm{H} \to \mathrm{t\bar{t}}$ branching ratios and the total Higgs width Γ for a Higgs mass of 500 GeV. [Note that the decay rates into $\mathrm{f\bar{f}}$ final states other than $\mathrm{H} \to \mathrm{t\bar{t}}$ are negligibly small since $m_{\mathrm{f}} \ll m_{\mathrm{t}}$.]

NUMERICAL ANSWERS

27. a) $\Gamma_Z = 2.3\,\mathrm{GeV}$; b) $R = 20.1$; c) 61 nb

28. c) $\sin^2\theta_\mathrm{W} \approx 0.230 \pm 0.008$

29. e) $\tau \approx 4.0 \times 10^{-25}\,\mathrm{s}$, $\tau_\mathrm{had} \sim \times 10^{-23}\,\mathrm{s}$

30. e) $\mathrm{BR(H \to W^+W^-)} = 55.8$%, $\mathrm{BR(H \to Z^0Z^0)} = 26.7$%, $\mathrm{BR(H \to t\bar{t})} = 17.5$%;
$\Gamma = 62.9\,\mathrm{GeV}$