NATURAL SCIENCES TRIPOS Part III

Tuesday 12 January 2010 2.00pm to 4.00pm

EXPERIMENTAL AND THEORETICAL PHYSICS (4)
Particle Physics

Answer **two** *questions only. The approximate number of marks allotted to each part of a question is indicated in the right hand margin where appropriate. The paper contains FIVE sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
20 page answer book
Rough workpad
Metric graph paper

SPECIAL REQUIREMENTS
Mathematical formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 The Lorentz-invariant matrix element for the decay $\tau^- \to \pi^- \nu_\tau$ is given by

$$M_{\mathrm{fi}} = \kappa g_{\mu\nu} p_4^\nu \overline{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1),$$

where κ is a constant, and p_1, p_3 and p_4 are the respective four-momenta of the τ^-, ν_τ and π^-.

(a) Draw the leading-order Feynman diagram for the decay $\tau^- \to \pi^- \nu_\tau$. [2]

(b) Consider the τ^- decay in its rest frame. The τ^- spin is in the positive z direction and the π^- is produced at a polar angle θ^* and has energy and momentum, E^* and p^*, i.e. $p_4^\mu = (E^*, p^* \sin\theta^*, 0, p^*\cos\theta^*)$. Show that

$$\overline{u}_\uparrow(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u_1(p_1) = 0 \quad \text{and} \quad \overline{u}_\downarrow(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u_1(p_1) = \overline{u}_\downarrow(p_3)\gamma^\mu u_1(p_1),$$

where $\overline{u}_\uparrow$ and $\overline{u}_\downarrow$ are the adjoint spinors for the ν_τ right- and left-handed helicity states and u_1 is the spinor for the τ-lepton at rest with $S_z = +1/2$. [5]

(c) Show that

$$\overline{u}_\downarrow(p_3)\gamma^\mu u_1(p_1) = \sqrt{2m_\tau p^*}\left(-\cos\frac{\theta^*}{2}, \sin\frac{\theta^*}{2}, i\sin\frac{\theta^*}{2}, \cos\frac{\theta^*}{2}\right),$$

and hence

$$\frac{\mathrm{d}\Gamma_{\tau_\uparrow}}{\mathrm{d}(\cos\theta^*)} \propto \frac{(p^*)^2}{m_\tau}(1+\cos\theta^*),$$

where $\Gamma_{\tau_\uparrow}$ is the partial decay width when the τ^- spin is in the $+z$-direction. [*The polar angles of the tau neutrino are $\theta_3 = \pi - \theta^*$ and $\phi_3 = \pi$.*] [12]

(d) Sketch the form of $\mathrm{d}\Gamma_{\tau_\uparrow}/\mathrm{d}(\cos\theta^*)$ and explain it in terms of the spins of the particles involved. Without explicit calculation, sketch the expected form of $\mathrm{d}\Gamma_{\tau_\downarrow}/\mathrm{d}(\cos\theta^*)$, i.e. the polar angle distribution of the π^- in the rest frame of the τ^- for the case where the τ^- spin is in the $-z$-direction. [3]

(e) At LEP, τ leptons are produced in the process $e^+e^- \to Z \to \tau^+\tau^-$ at $\sqrt{s} = m_Z$. For τ^- decays where the τ^- has helicity $+1$, show that the energy distribution of the produced pion is approximately

$$\frac{\mathrm{d}\Gamma_{\tau_\uparrow}}{\mathrm{d}E_\pi} \propto \frac{E_\pi}{E_\tau},$$

where E_π and E_τ are the energies of the π^- and τ^- in the laboratory frame. [*Apply a Lorentz transformation to p_4 to obtain an expression for E_π in terms of E^*, p^* and $\cos\theta^*$, take $p^* \approx E^*$, and take the τ^- velocity to be $\beta_\tau \approx 1$.*] [5]

(f) In the process $e^+e^- \to Z \to \tau^+\tau^-$ at $\sqrt{s} = m_Z$, the energy distribution of the π^- from τ^- decays is found to be consistent with

$$\frac{\mathrm{d}N}{\mathrm{d}x} = 1.14 - 0.28x \equiv 0.86x + 1.14(1-x),$$

where $x = E_\pi/E_\tau$. Explain this observation. [3]

You may require the following information:

The helicity eigenstate spinors $u_\uparrow(p)$ and $u_\downarrow(p)$ for a particle of mass m and four-momentum $p^\mu = (E, |\boldsymbol{p}|\sin\theta\cos\phi, |\boldsymbol{p}|\sin\theta\sin\phi, |\boldsymbol{p}|\cos\theta)$ are

$$u_\uparrow(p) = \sqrt{E+m}\begin{pmatrix} \cos\theta/2 \\ e^{i\phi}\sin\theta/2 \\ \frac{|\boldsymbol{p}|}{E+m}\cos\theta/2 \\ \frac{|\boldsymbol{p}|}{E+m}e^{i\phi}\sin\theta/2 \end{pmatrix} \quad \text{and} \quad u_\downarrow(p) = \sqrt{E+m}\begin{pmatrix} -\sin\theta/2 \\ e^{i\phi}\cos\theta/2 \\ \frac{|\boldsymbol{p}|}{E+m}\sin\theta/2 \\ -\frac{|\boldsymbol{p}|}{E+m}e^{i\phi}\cos\theta/2 \end{pmatrix}.$$

For a particle with momentum along the z-direction, the spinor for the $S_z = +1/2$ eigenstate is

$$u_1(p) = \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ \frac{p}{E+m} \\ 0 \end{pmatrix}.$$

The Dirac matrices are given by

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix},$$

$$\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \text{and} \quad \gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

The differential decay rate in a particle's rest frame is given by

$$\frac{d\Gamma}{d\Omega} = \frac{p^*}{32\pi^2 m^2}|M_{fi}|^2.$$

For a Lorentz transformation along the z-direction

$$E = \gamma(E' + \beta p'_z),\ p_x = p'_x,\ p_y = p'_y, \text{and } p_z = \gamma(p'_z + \beta E').$$

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2 In electron-proton scattering, the Lorentz invariant quantities, s, Q^2, x and y are defined as

$$s = (p_1 + p_2)^2, \quad Q^2 = -q^2 = -(p_1 - p_3)^2, \quad x = \frac{Q^2}{2p_2.q} \quad \text{and} \quad y = \frac{p_2.q}{p_1.p_2},$$

where p_1 and p_2 are the four-momenta of the initial-state electron and proton, and p_3 is the four-momentum of the scattered electron. Neglecting the Q^2 dependence of the structure functions, F_1^{ep} and F_2^{ep}, the differential cross section for electron-proton deep inelastic scattering can be written as

$$\frac{\mathrm{d}^2\sigma^{ep}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2^{ep}(x)}{x} + y^2 F_1^{ep}(x)\right].$$

(a) For the case where the proton is at rest, express s, Q^2, x and y in terms of the proton mass, m_p, the electron scattering angle, θ, and the energies of the incoming and scattered electron, E_1 and E_3. [4]

(b) In the parton model, show that x can be interpreted as the fraction of the proton's momentum carried by the struck quark in a frame where the proton has infinite momentum. Explain any assumptions made. [4]

(c) The differential cross section for electron-quark scattering can be written as

$$\frac{\mathrm{d}^2\sigma^{eq}}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2 e_q^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right],$$

where e_q is the charge of the quark. Using the parton model, including contributions from the light quarks (u, d, s) only, show that

$$\frac{F_2^{ep}(x)}{x} = \frac{4}{9}[u(x) + \overline{u}(x)] + \frac{1}{9}[d(x) + \overline{d}(x) + s(x) + \overline{s}(x)],$$

where $u(x)$, $d(x)$ and $s(x)$ are the up-, down- and strange-quark parton distribution functions for the proton. Obtain a similar expression for the electron-neutron structure function, $F_2^{en}(x)$. [6]

(d) Stating clearly any assumptions made, show that

$$\int_0^1 \frac{[F_2^{ep}(x) - F_2^{en}(x)]}{x}\mathrm{d}x = \frac{1}{3} + \frac{2}{3}\int_0^1 [\overline{u}(x) - \overline{d}(x)]\,\mathrm{d}x,$$

and interpret the observed value of 0.24 ± 0.03. [6]

(e) In the parton model for neutrino-nucleon scattering the structure functions are

$$F_2^{\nu p}(x) = 2x[d(x) + s(x) + \overline{u}(x)] \quad \text{and} \quad F_2^{\nu n}(x) = 2x[u(x) + \overline{s}(x) + \overline{d}(x)].$$

Assuming $s(x) = \overline{s}(x)$, obtain an expression for $xs(x)$ in terms of the structure functions for neutrino- and electron-nucleon scattering,
$F_2^{\nu N}(x) = \frac{1}{2}(F_2^{\nu p}(x) + F_2^{\nu n}(x))$ and $F_2^{eN}(x) = \frac{1}{2}(F_2^{ep}(x) + F_2^{en}(x))$. [7]

(f) Provide possible physical explanations for why $\overline{d}(x) > \overline{u}(x) > \overline{s}(x)$. [3]

3 Answer **both** sections (i) **and** (ii).

(i) In the form of bullet points, write brief notes on **two** of the following topics:

(a) CP violation in the neutral kaon system. [12]

(b) Atmospheric and solar neutrino oscillations. [12]

(c) Electroweak unification. [12]

(d) QCD. [12]

(ii) Answer **two** of the following questions:

(a) Show that the decays $K^0 \to \pi^0\pi^0$ and $K^0 \to \pi^+\pi^-$ occur in CP even eigenstates. [*All the particles involved have* $J^P = 0^-$.] [3]

(b) In the laboratory frame, determine the neutrino threshold energies for the charged-current weak interaction processes $\nu_\mu + e^- \to \mu^- + \nu_e$ and $\nu_\tau + n \to \tau^- + p$. [*Take* $m_e = 0.5\,MeV$, $m_\mu = 106\,MeV$, $m_\tau = 1.78\,GeV$ *and* $m_n \approx m_p = 940\,MeV$. *Neglect the neutrino masses.*] [3]

(c) Draw the lowest order Feynman diagrams for $e^+e^- \to W^+W^-$. Assuming the universality of the weak charged current, what fraction of W^+W^- decays are fully-hadronic, i.e. $W^+W^- \to q_1\overline{q}_2 q_3\overline{q}_4$? [3]

(d) For the interaction between quarks, the QCD colour factor is defined as

$$C(ik \to jl) = \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{ji}\lambda^a_{lk},$$

where $i, j, k, l = r, g, b$ and the λ^a are the Gell-Mann matrices. Obtain the colour factors for $rr \to rr$, $rb \to rb$ and $rb \to br$. [3]

[*The standard representation of the SU(3) λ-matrices is*

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

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