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Tuesday 17 January 2006

2.00 to 3.30

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EXPERIMENTAL AND THEORETICAL PHYSICS (4)

Particle Physics

*Answer **two** questions only. The approximate number of marks allotted to each part of a question is indicated in the right margin where appropriate. The paper contains **four** sides (including this one) and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS

20 Page Answer Book

Metric Graph Paper

Rough Work Pad

SPECIAL REQUIREMENTS

Mathematical Formulae Handbook

Approved Calculators Allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

- 1 Write brief accounts of **two** of the following:
- (a) electron-nucleon elastic scattering; [15]
  - (b) isospin and SU(3) flavour symmetry; [15]
  - (c) experimental tests of electroweak unification; [15]
  - (d) CP violation in the neutral kaon system. [15]

- 2 Draw the leading-order, quark-level Feynman diagram for the decay  $\tau^- \rightarrow \pi^- \nu_\tau$  of the  $\tau$  lepton. [2]

The Lorentz-invariant matrix element  $M_{fi}$  for  $\tau^- \rightarrow \pi^- \nu_\tau$  decay is given by

$$M_{fi} = \sqrt{2}G_F f_\pi V_{ud} j \cdot p_4$$

where  $G_F$  is the Fermi constant,  $f_\pi$  is a constant,  $V_{ud}$  is a CKM matrix element,  $j \cdot p_4$  is the scalar product of the lepton current  $j^\mu = \bar{u}(p_3)\gamma^\mu \frac{1}{2}(1 - \gamma^5)u(p_1)$  with the  $\pi^-$  four-momentum  $p_4$ , and  $p_1$  and  $p_3$  are the four-momenta of the  $\tau^-$  and  $\nu_\tau$  respectively.

Working in the rest frame of the  $\tau^-$ , with the  $\tau^-$  spin fully-polarised in the  $+z$  direction, show that (up to an arbitrary overall phase factor) the lepton current is given by

$$j^\mu = \sqrt{2m_\tau p^*} (-\sin \theta/2, -\cos \theta/2, -i \cos \theta/2, \sin \theta/2)$$

where  $m_\tau$  is the  $\tau^-$  mass, and the  $\nu_\tau$  has four-momentum  $p_3^\mu = (p^*, p^* \sin \theta, 0, p^* \cos \theta)$ . Hence show that the magnitude of  $M_{fi}$  is

$$|M_{fi}| = 2G_F f_\pi |V_{ud}| m_\tau \sqrt{m_\tau p^*} \sin \theta/2 . \quad [8]$$

Explain, with the aid of a diagram, why  $M_{fi}$  vanishes when  $\theta = 0$ . Write down an equivalent expression for  $|M_{fi}|$  when the  $\tau^-$  spin is fully-polarised in the  $-z$  direction. [3]

Given that the rest-frame differential decay rate is

$$\frac{d\Gamma}{d\Omega} = \frac{p^*}{32\pi^2 m_\tau^2} |M_{fi}|^2 ,$$

show that the partial width for the decay of an unpolarised sample of  $\tau^-$  leptons is

$$\Gamma(\tau^- \rightarrow \pi^- \nu_\tau) = \frac{G_F^2 f_\pi^2}{16\pi} |V_{ud}|^2 m_\tau^3 \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 ,$$

where  $m_\pi$  is the  $\pi^-$  mass. [5]

The mean lifetime of the  $\tau^-$  is  $2.91 \times 10^{-13}$  s. Taking  $f_\pi = m_\pi$ , and making a suitable approximation for the value of  $V_{ud}$ , estimate the branching ratio of the  $\tau^- \rightarrow \pi^- \nu_\tau$  decay. [4]

Explain qualitatively why experimental studies of atmospheric neutrino interactions provide evidence for  $\nu_\mu \rightarrow \nu_\tau$  oscillations, and indicate how the values of the oscillation parameters  $\Delta m^2$  and  $\sin^2 2\theta$  can be inferred. Summarise briefly what can be deduced about the values of neutrino masses and the structure of the PMNS matrix from neutrino oscillation experiments. (Detailed descriptions of experiments, and precise numerical values are not expected).

[8]

*You may require the following information:*

The  $\pi^-$  has spin zero and a mass of 139.6 MeV. The  $\tau^-$  has a mass of 1.777 GeV.

$$G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}.$$

$$\hbar = 6.582 \times 10^{-25} \text{ GeV s}.$$

The helicity eigenstate spinors  $u_\uparrow(p)$  and  $u_\downarrow(p)$  for a particle of mass  $m$  and four-momentum  $p^\mu = (E, p \sin \theta \cos \phi, p \sin \theta \sin \phi, p \cos \theta)$  are

$$u_\uparrow(p) = \sqrt{E+m} \begin{pmatrix} \cos \theta/2 \\ e^{i\phi} \sin \theta/2 \\ \frac{p}{E+m} \cos \theta/2 \\ \frac{p}{E+m} e^{i\phi} \sin \theta/2 \end{pmatrix}, \quad u_\downarrow(p) = \sqrt{E+m} \begin{pmatrix} -\sin \theta/2 \\ e^{i\phi} \cos \theta/2 \\ \frac{p}{E+m} \sin \theta/2 \\ -\frac{p}{E+m} e^{i\phi} \cos \theta/2 \end{pmatrix}$$

The Dirac matrices are given by

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

(TURN OVER)

3 Top quark pairs are produced in hadron-hadron collisions predominantly via the processes  $q\bar{q} \rightarrow t\bar{t}$  and  $gg \rightarrow t\bar{t}$ . Draw (in total) three leading order QCD Feynman diagrams for these processes. [4]

Outline in general terms how the total cross section for  $t\bar{t}$  production can be calculated in terms of integrals over appropriate parton distribution functions, and explain how these distribution functions can be determined experimentally. [6]

The QCD vertex factor associated with the interaction of a quark and a gluon is proportional to  $\frac{1}{2}\lambda_{ij}^a$ , where  $i, j = 1, 2, 3$  (or r,g,b) are colour indices, and  $\lambda^a$  is an SU(3)  $\lambda$ -matrix. List the allowed colour configurations for the process  $q\bar{q} \rightarrow t\bar{t}$ , and determine the values of the corresponding colour factors, such as  $C(r\bar{r} \rightarrow r\bar{r})$ , contributing to the matrix element. [7]

Hence determine the colour factor associated with the  $q\bar{q} \rightarrow t\bar{t}$  component of the overall cross section for  $t\bar{t}$  production in high energy hadron-hadron collisions. [3]

A  $t\bar{t}$  pair is produced in a proton-antiproton interaction at the Tevatron collider, where proton and antiproton beams of energy 0.98 TeV collide head-on. The  $t$  quark is emitted at right-angles to the direction of each beam and has a momentum of 40 GeV. The  $\bar{t}$  antiquark is emitted at an angle of  $50^\circ$  to the incoming proton direction. The total momentum of the  $t\bar{t}$  system is directed along the incoming proton direction. Calculate the momentum fractions  $x_1$  and  $x_2$  of the partons which interacted to produce the  $t\bar{t}$  pair. What would be the values of  $x_1$  and  $x_2$  if this  $t\bar{t}$  pair were to be produced instead in a proton-proton collision at the LHC, where proton beams of energy 7 TeV will interact head-on? [8]

[The top quark has a mass of 175 GeV.]

What is the most likely nature of the partons producing the  $t\bar{t}$  pair at the Tevatron and at the LHC? [2]

[The standard representation of the SU(3)  $\lambda$ -matrices is

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} ]$$

END OF PAPER